Stochastic Petri nets

Stochastic Petri nets

- Markov Chain grows very fast with the dimension of the system

- Petri nets: High-level specification formalism

- Markovian Stochastic Petri nets

  adding temporal and probabilistic information to the model
  the approach aimed at equivalence between SPN and MC
  idea of associating an exponentially distributed random delay
  with the PN transitions (1980)

- Non Markovian Stochastic Petri nets

  non exponentially distributed random delay

Automated tools supporting SPN for modelling and evaluation

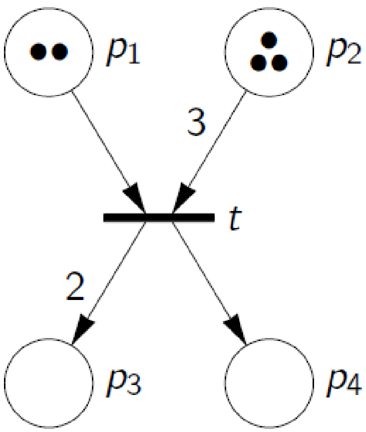
Petri nets

$$N_{P/T} = \langle P, T; F, W, M_0 \rangle$$

- Place ○
- Transition ▬
- Token •
- Arcs →

$$W : F \to \mathbb{N} - \{0\}$$ Weigth of arcs

$$M_0 : P \to \mathbb{N}$$ Initial marking



t, y transitions

$${}^\bullet t = \{p \in P \mid \langle p, t \rangle \in F\}$$ preset

$$y^\bullet = \{z \in P \mid \langle y, z \rangle \in F\}$$ postset

t enabled if: $\forall_{p \in {}^\bullet t}\ M(p) \geq W(\langle p, t \rangle)$

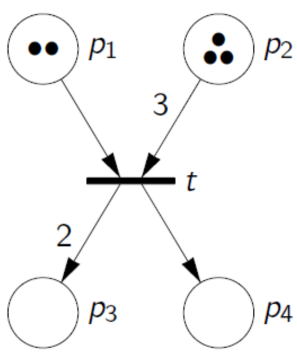
we write $M\,[t\rangle$

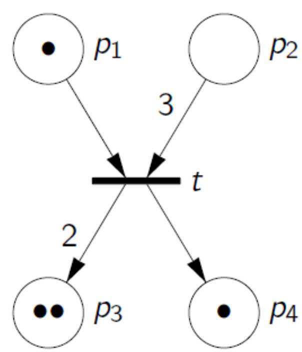
Transition firing

Firing rule

$$
\begin{array}{ll}
\forall_{p\in({}^\bullet t - t^\bullet)} & M'(p) = M(p) - W(\langle p,t\rangle) \\
\forall_{p\in(t^\bullet - {}^\bullet t)} & M'(p) = M(p) + W(\langle t,p\rangle) \\
\forall_{p\in({}^\bullet t \cap t^\bullet)} & M'(p) = M(p) - W(\langle p,t\rangle) + W(\langle t,p\rangle) \\
\forall_{p\in P-({}^\bullet t \cup t^\bullet)} & M'(p) = M(p)
\end{array}
$$

We write $M\,[t\rangle\,M'$





M0[>M1

M0=(2,3,0,0)

M1=(21,0,2,1)

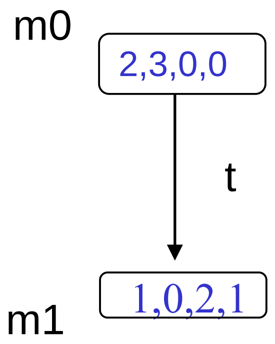
Reachable marking $R_N(M)$:

$$M \in R_N(M)$$

$$M' \in R_N(M) \wedge \exists_{t \in T} M' \left[t\right\rangle M'' \Rightarrow M'' \in R_N(M)$$

We can build the reachability graph

Reachability graph



Transition sequence

Let $M\,[t_1\rangle\, M' \wedge M'\,[t_2\rangle\, M''$ then $M\,[t_1 t_2\rangle\, M''$

If $M\,[t_1 \ldots t_n\rangle\, M^{(n)}$ then $t_1 \ldots t_n$ is a transition sequence

Analysis

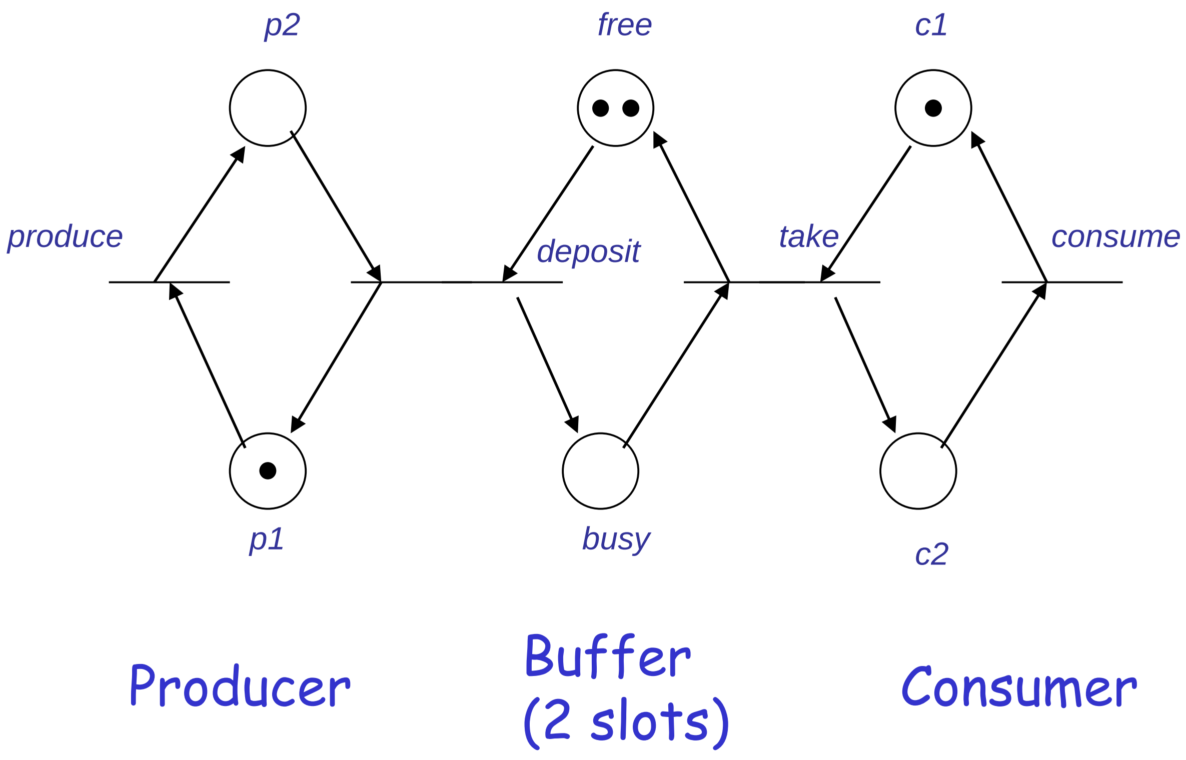
Reachable markings

Transitions never enabled

Conditions on reachable markings

………………………

Producer/Consumer example



System description with Petri nets

- Place:
  - a system component (one for every component)
  - a class of system components (CPU , Memory, ..)
  - components in a given state (CPU, FaultyCPU, ..)
  - …….
- Token:
  - number of components (number of CPUs)
  - Occurrence of an event (fault, ..)
  - …..
- Transitions:
  - Occurrence of an event (Repair, CPUFaulty, …)
  - Execution of a computation step
  - …

Timed transitions

Timed transition: an activity that needs some time to be executed

When a transition is enabled a local timer is set to a delay d;

- the timer is decresed
- when the timer elapses, the transition fires and remove tokens from input places

- if the transitions is desabled before the firing, the timer stops.

Handling of the timer (two alternatives):

Continue:
the timer maintains the value and it will be used when the transition is enabled again

Restart:
the timer is reset

Sequence of timed transitions:

$(\tau k1, Tk1) \ldots (\tau kn, Tkn)$

where

$\tau k1 <= \tau k2 <= \tau kn$

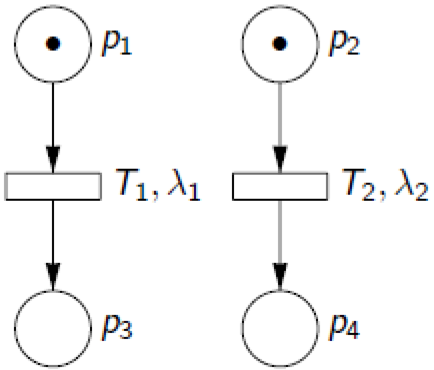
$[\tau ki, \tau ki+1)$ is the period of time between the firing of two transitions

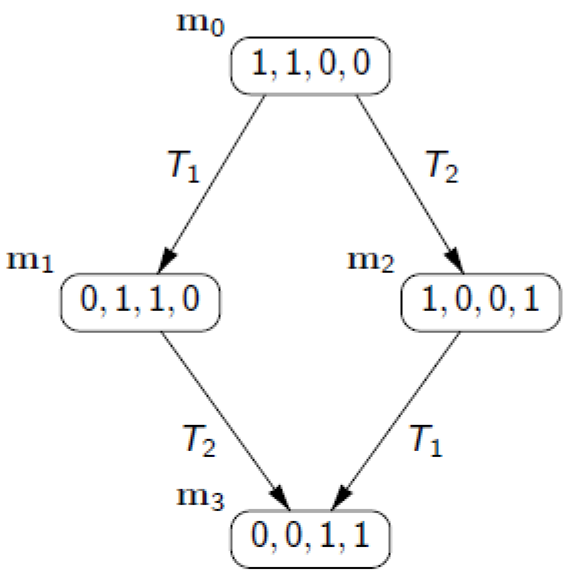
period of time the net stay in a marking

STOCHASTIC PETRI NET:

when the delay d of a timed transition is a random variable

Stochastic Petri nets (SPN) – reachability graph





A timed transition T enabled at time t, with d the random value for the transition delay, fires at time t+d if it remains enabled in the interval [t, t+d)

Markov chain

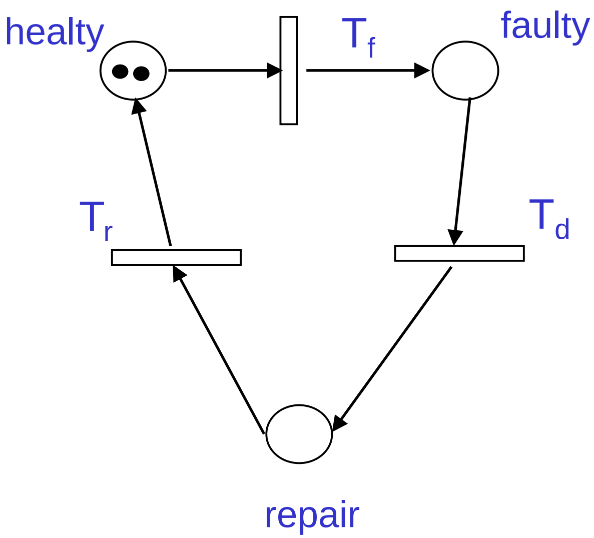
Random process $\{M(t), t>=0\}$

with $M(0) = M0$ and $M(t)$ the marking at time t

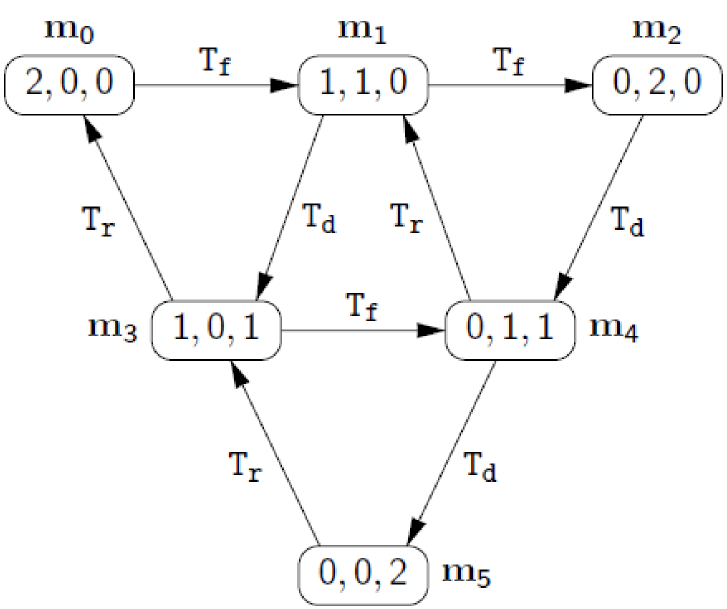
$\{M(t), t>=0\}$ is a CTMC (memoriless property of exponential distributions)

A redundant system with repair

- Two identical CPUs
- Failure of the CPU: exponentially distributed with parameter λ
- Fault detection: exponentially distributed with parameter δ
- CPU repair: exponentially distributed with parameter ⌠

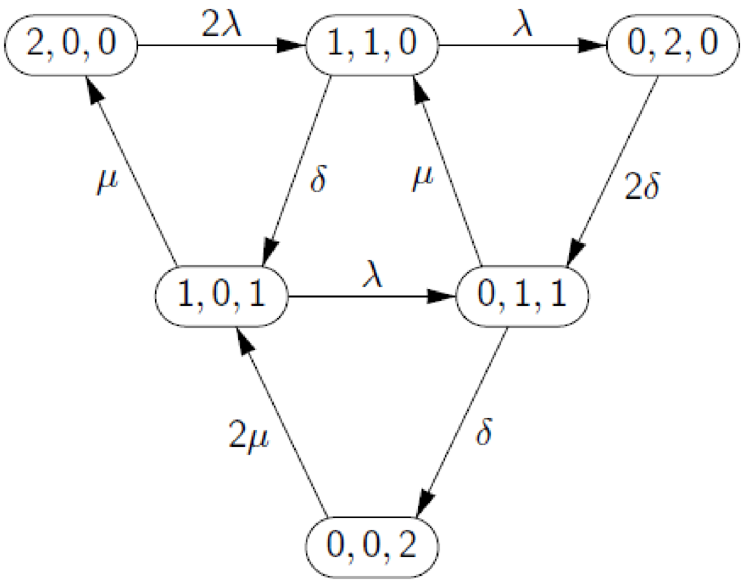


SPN



Reachability graph

Markov chain



Properties

- Steady-state probability that both processors behave correctly
- Steady-state probability of one undetected faulty processor
- Steady state probability that both processors must be repaired
- ....

M. Ajmone Marsan. Stochastic Petri nets: An elementary introduction. In G. Rozenberg, editor, *Advances in Petri Nets 1989*, volume 424 of *LNCS*, pages 1–29. Springer Verlag, 1990.