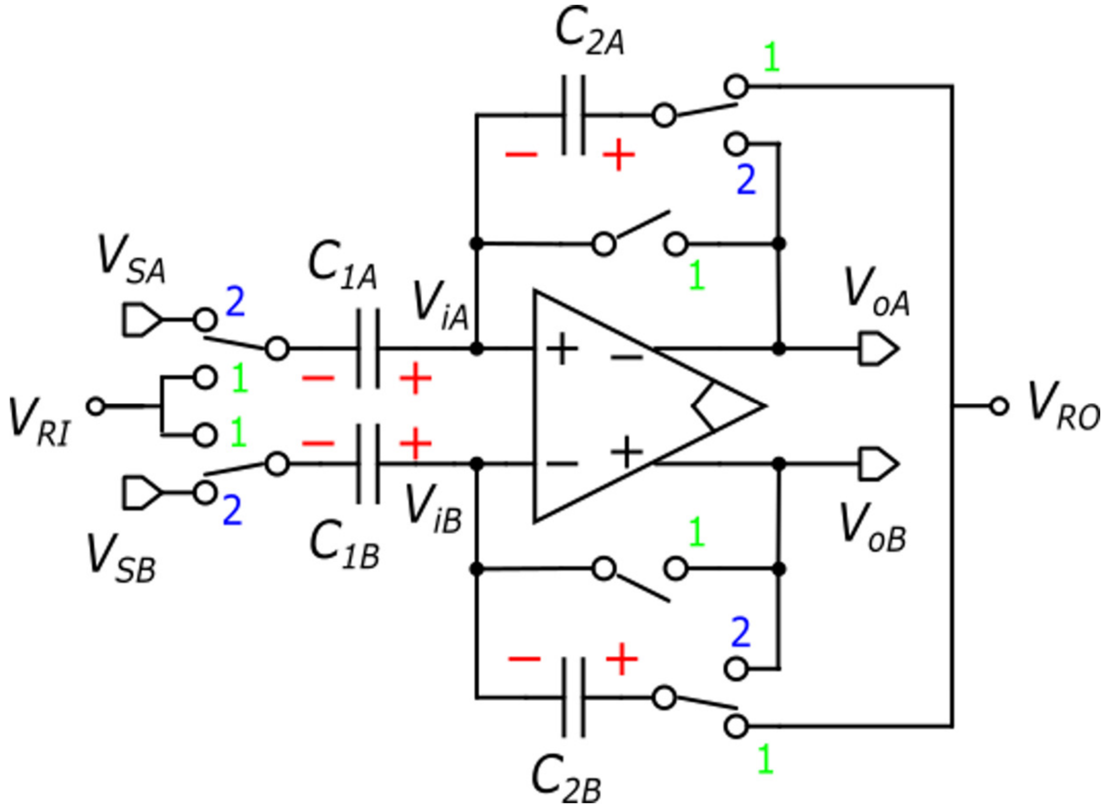
# Fully-differential switched capacitor amplifier



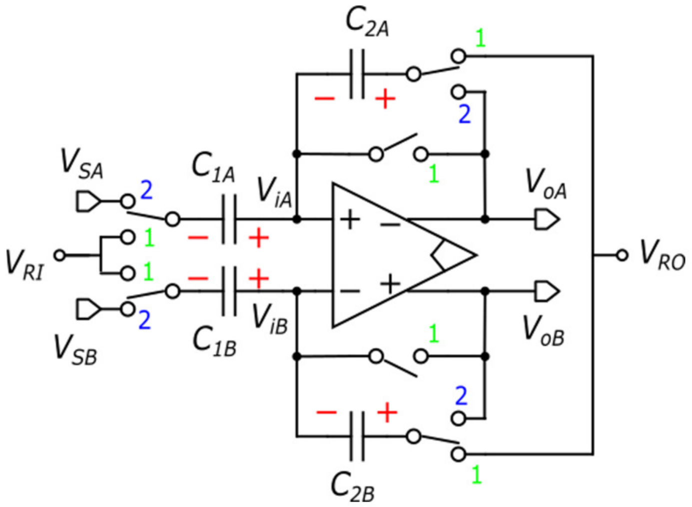

$V_{RO}$ and $V_{RI}$ are constant voltages used to periodically discharge capacitor pairs. They substitute *gnd* in a single-supply circuit

We will consider the nominal case for the capacitors:

$$\begin{cases} C_{1A} = C_{1B} \equiv C_1 \\ C_{2A} = C_{2B} \equiv C_2 \end{cases}$$

$$V_{Sd} = V_{SA} - V_{SB}$$ Input. diff. voltage

$$V_{od} = V_{oB} - V_{oA}$$ Output. diff. voltage

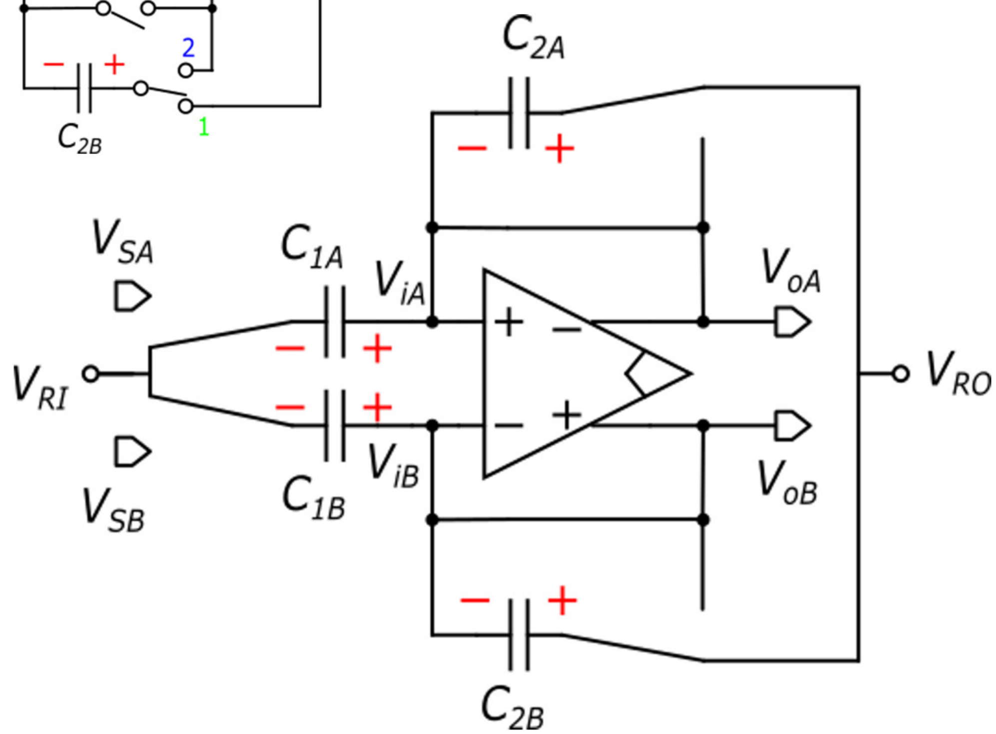



# Phase 1

$$v_{iA}^{(1)} = V_{CMO} + \frac{v_n^{(1)}}{2}; \qquad v_{iB}^{(1)} = V_{CMO} - \frac{v_n^{(1)}}{2}$$

Due to the unity-gain connection

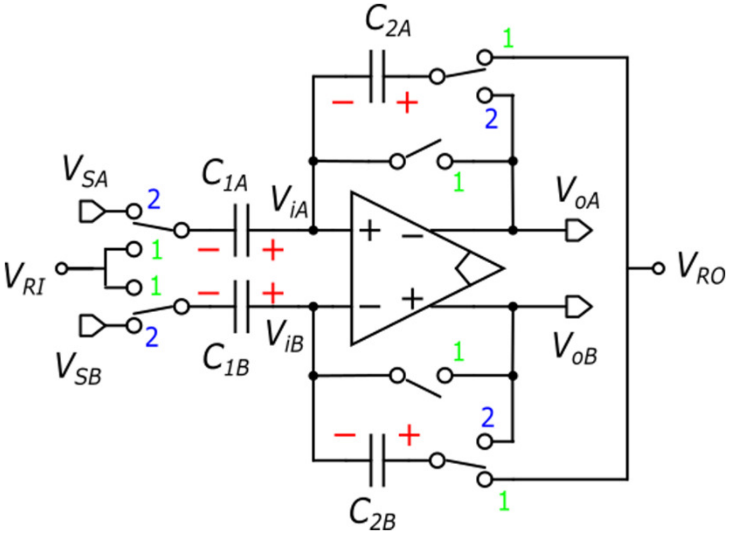
Let us write the voltages across each capacitor

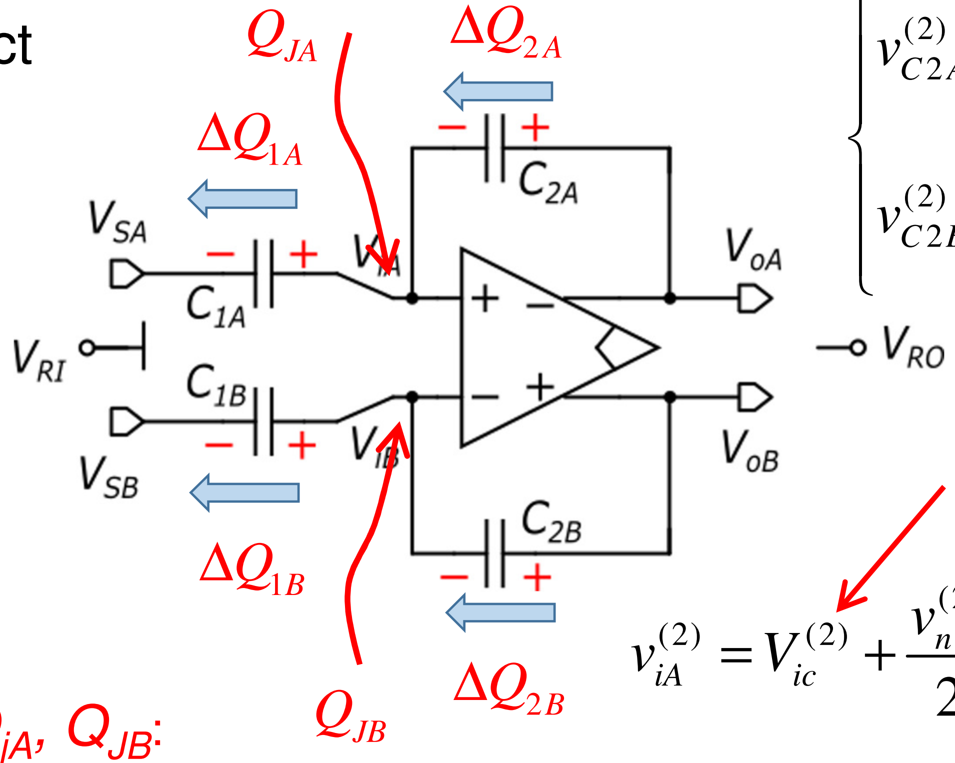



$$\begin{cases} v_{C1A}^{(1)} = V_{CMO} + \dfrac{v_n^{(1)}}{2} - V_{RI} \\ v_{C1B}^{(1)} = V_{CMO} - \dfrac{v_n^{(1)}}{2} - V_{RI} \\ v_{C2A}^{(1)} = V_{RO} - V_{CMO} - \dfrac{v_n^{(1)}}{2} \\ v_{C2B}^{(1)} = V_{RO} - V_{CMO} + \dfrac{v_n^{(1)}}{2} \end{cases}$$

# Transition to phase 2

In this analysis we will neglect the kT/C noise, for simplicity




$$\begin{cases} v_{C2A}^{(2)} = v_{C2A}^{(1)} + \dfrac{\Delta Q_{2A}}{C_2} \\ v_{C2B}^{(2)} = v_{C2B}^{(1)} + \dfrac{\Delta Q_{2B}}{C_2} \end{cases}$$

unknown

$$v_{iA}^{(2)} = V_{ic}^{(2)} + \frac{v_n^{(2)}}{2}; \qquad v_{iB}^{(2)} = V_{ic}^{(2)} - \frac{v_n^{(2)}}{2}$$

$$v_{C1A}^{(2)} = V_{ic}^{(2)} + \frac{v_n^{(2)}}{2} - V_{SA}$$

$$v_{C1B}^{(2)} = V_{ic}^{(2)} - \frac{v_n^{(2)}}{2} - V_{SB}$$

$Q_{jA}$, $Q_{JB}$: caused by charge-injection and other mechanisms

$$\begin{cases} \Delta Q_{2A} = \Delta Q_{1A} - Q_{JA} = C_1\left(v_{C1A}^{(2)} - v_{C1A}^{(1)}\right) - Q_{JA} \\ \Delta Q_{2B} = \Delta Q_{1B} - Q_{JB} = C_1\left(v_{C1B}^{(2)} - v_{C1B}^{(1)}\right) - Q_{JB} \end{cases}$$

Phase 2

$$\begin{cases} v_{C2A}^{(2)} = v_{C2A}^{(1)} + \dfrac{\Delta Q_{2A}}{C_2} \\ v_{C2B}^{(2)} = v_{C2B}^{(1)} + \dfrac{\Delta Q_{2B}}{C_2} \end{cases}$$

$$\Delta Q_{2A} = C_1\left(v_{C1A}^{(2)} - v_{C1A}^{(1)}\right) - Q_{JA}$$

$$\Delta Q_{2B} = C_1\left(v_{C1B}^{(2)} - v_{C1B}^{(1)}\right) - Q_{JB}$$

$$v_{C1A}^{(2)} = V_{ic}^{(2)} + \frac{v_n^{(2)}}{2} - V_{SA}$$

$$v_{C1B}^{(2)} = V_{ic}^{(2)} - \frac{v_n^{(2)}}{2} - V_{SB}$$

Phase 1

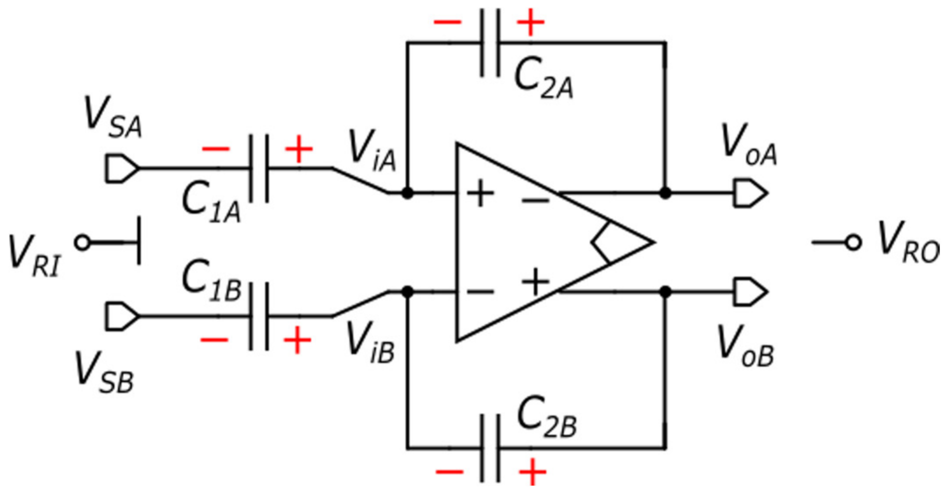
$$\begin{cases} v_{C1A}^{(1)} = V_{CMO} + \dfrac{v_n^{(1)}}{2} - V_{RI} \\ v_{C1B}^{(1)} = V_{CMO} - \dfrac{v_n^{(1)}}{2} - V_{RI} \\ v_{C2A}^{(1)} = V_{RO} - V_{CMO} - \dfrac{v_n^{(1)}}{2} \\ v_{C2B}^{(1)} = V_{RO} - V_{CMO} + \dfrac{v_n^{(1)}}{2} \end{cases}$$

$$v_{C2A}^{(2)} = V_{RO} - V_{CMO} - \frac{v_n^{(1)}}{2} + \frac{C_1}{C_2}\left(V_{ic}^{(2)} + \frac{v_n^{(2)}}{2} - V_{SA} - V_{CMO} - \frac{v_n^{(1)}}{2} + V_{RI}\right) - \frac{Q_{JA}}{C_2}$$

$$v_{C2B}^{(2)} = V_{RO} - V_{CMO} + \frac{v_n^{(1)}}{2} + \frac{C_1}{C_2}\left(V_{ic}^{(2)} - \frac{v_n^{(2)}}{2} - V_{SB} - V_{CMO} + \frac{v_n^{(1)}}{2} + V_{RI}\right) - \frac{Q_{JB}}{C_2}$$

$$v_{C2A}^{(2)} = V_{RO} - V_{CMO} - \frac{v_n^{(1)}}{2} + \frac{C_1}{C_2}\left(V_{ic}^{(2)} + \frac{v_n^{(2)}}{2} - V_{SA} - V_{CMO} - \frac{v_n^{(1)}}{2} + V_{RI}\right) - \frac{Q_{JA}}{C_2}$$

$$v_{C2B}^{(2)} = V_{RO} - V_{CMO} + \frac{v_n^{(1)}}{2} + \frac{C_1}{C_2}\left(V_{ic}^{(2)} - \frac{v_n^{(2)}}{2} - V_{SB} - V_{CMO} + \frac{v_n^{(1)}}{2} + V_{RI}\right) - \frac{Q_{JB}}{C_2}$$




$$\begin{cases} v_{oA}^{(2)} = v_{iA}^{(2)} + v_{C2A}^{(2)} \\ v_{oB}^{(2)} = v_{iB}^{(2)} + v_{C2B}^{(2)} \end{cases}$$

$$v_{iA}^{(2)} = V_{ic}^{(2)} + \frac{v_n^{(2)}}{2}; \qquad v_{iB}^{(2)} = V_{ic}^{(2)} - \frac{v_n^{(2)}}{2}$$

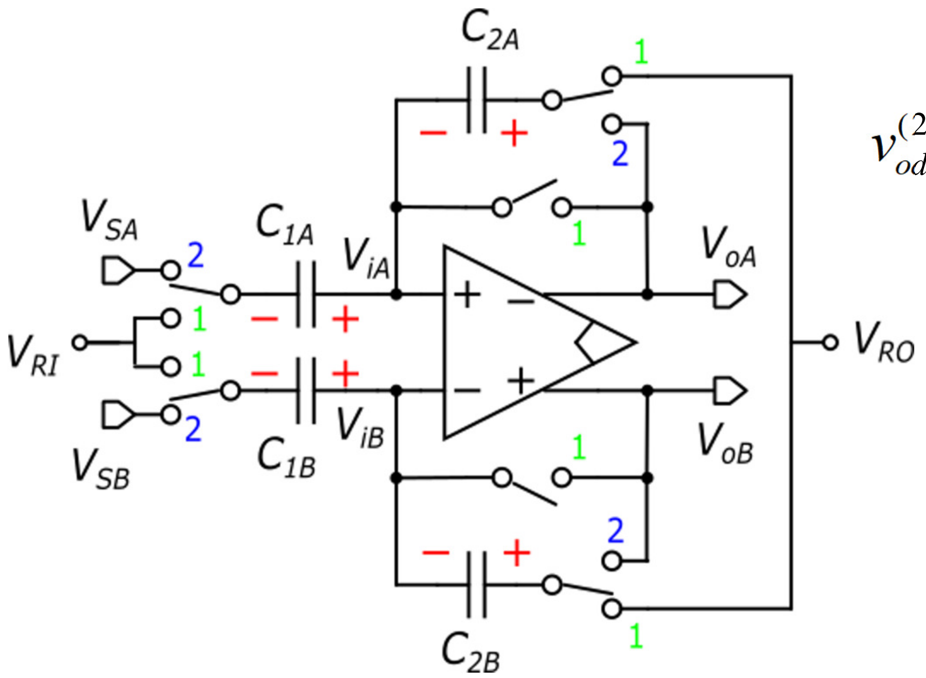
$$\begin{cases} v_{oA}^{(2)} = V_{ic}^{(2)} + \dfrac{v_n^{(2)}}{2} + V_{RO} - V_{CMO} - \dfrac{v_n^{(1)}}{2} + \dfrac{C_1}{C_2}\left(V_{ic}^{(2)} + \dfrac{v_n^{(2)}}{2} - V_{SA} - V_{CMO} - \dfrac{v_n^{(1)}}{2} + V_{RI}\right) - \dfrac{Q_{JA}}{C_2} & - \\ v_{oB}^{(2)} = V_{ic}^{(2)} - \dfrac{v_n^{(2)}}{2} + V_{RO} - V_{CMO} + \dfrac{v_n^{(1)}}{2} + \dfrac{C_1}{C_2}\left(V_{ic}^{(2)} - \dfrac{v_n^{(2)}}{2} - V_{SB} - V_{CMO} + \dfrac{v_n^{(1)}}{2} + V_{RI}\right) - \dfrac{Q_{JB}}{C_2} & + \end{cases}$$

$$v_{od}^{(2)} = v_{oB}^{(2)} - v_{oA}^{(2)}$$

Differential mode analysis

$$v_{od}^{(2)} = -v_n^{(2)} + v_n^{(1)} + \frac{C_1}{C_2}\left(-v_n^{(2)} + V_{SA} - V_{SB} + v_n^{(1)}\right) + \frac{Q_{JA} - Q_{JB}}{C_2}$$

$$v_{od}^{(2)} = \frac{C_1}{C_2}\left(V_{SA} - V_{SB}\right) - \left(v_n^{(2)} - v_n^{(1)}\right)\left(\frac{C_1}{C_2} + 1\right) + \frac{Q_{JA} - Q_{JB}}{C_2}$$




$$v_{od}^{(2)}=\underbrace{\frac{C_1}{C_2}\left(V_{SA}-V_{SB}\right)}_{V_{Sd}}\underbrace{-\left(v_n^{(2)}-v_n^{(1)}\right)\left(\frac{C_1}{C_2}+1\right)}_{\text{output noise due to the amplifier}}+\underbrace{\frac{Q_{JA}-Q_{JB}}{C_2}}_{\text{output error due to charge injection}}$$

$$A_{dd}=\frac{C_1}{C_2}\equiv A$$

Errors referred to the input $-\left(v_n^{(2)}-v_n^{(1)}\right)\left(\frac{A+1}{A}\right)$ $\qquad \frac{Q_{JA}-Q_{JB}}{C_2}$

Note that CDS is applied to the amplifier noise

In a perfectly symmetrical amplifier, charge injections on the two sides cancel each other. Only mismatch components contribute to the error.

# Common mode components

$$\begin{cases} v_{oA}^{(2)} = V_{ic}^{(2)} + \dfrac{v_n^{(2)}}{2} + V_{RO} - V_{CMO} - \dfrac{v_n^{(1)}}{2} + \dfrac{C_1}{C_2}\left(V_{ic}^{(2)} + \dfrac{v_n^{(2)}}{2} - V_{SA} - V_{CMO} - \dfrac{v_n^{(1)}}{2} + V_{RI}\right) - \dfrac{Q_{JA}}{C_2} & \times\dfrac{1}{2} \\ v_{oB}^{(2)} = V_{ic}^{(2)} - \dfrac{v_n^{(2)}}{2} + V_{RO} - V_{CMO} + \dfrac{v_n^{(1)}}{2} + \dfrac{C_1}{C_2}\left(V_{ic}^{(2)} - \dfrac{v_n^{(2)}}{2} - V_{SB} - V_{CMO} + \dfrac{v_n^{(1)}}{2} + V_{RI}\right) - \dfrac{Q_{JB}}{C_2} & \times\dfrac{1}{2} \end{cases}$$

Neglecting the charge injection (calculation of the CM components require less accuracy)

$$V_{CMO} = V_{ic}^{(2)} + V_{RO} - V_{CMO} + \frac{C_1}{C_2}\left(V_{ic}^{(2)} - \frac{V_{SA} + V_{SB}}{2} - V_{CMO} + V_{RI}\right)$$

$$V_{CMO} = V_{ic}^{(2)}\left(1 + A\right) + V_{RO} - V_{CMO}\left(1 + A\right) + A\left(-V_{SC} + V_{RI}\right)$$

# Common mode components

$$V_{CMO} = V_{ic}^{(2)}\left(1+A\right) + V_{RO} - V_{CMO}\left(1+A\right) + A\left(-V_{SC} + V_{RI}\right)$$

$$V_{CMO} - V_{RO} + V_{CMO}\left(1+A\right) - A\left(-V_{SC} + V_{RI}\right) = V_{ic}^{(2)}\left(1+A\right)$$

$$V_{ic}^{(2)} = V_{CMO} + \frac{V_{CMO} - V_{RO}}{A+1} + \frac{A}{A+1}\left(V_{SC} - V_{RI}\right)$$

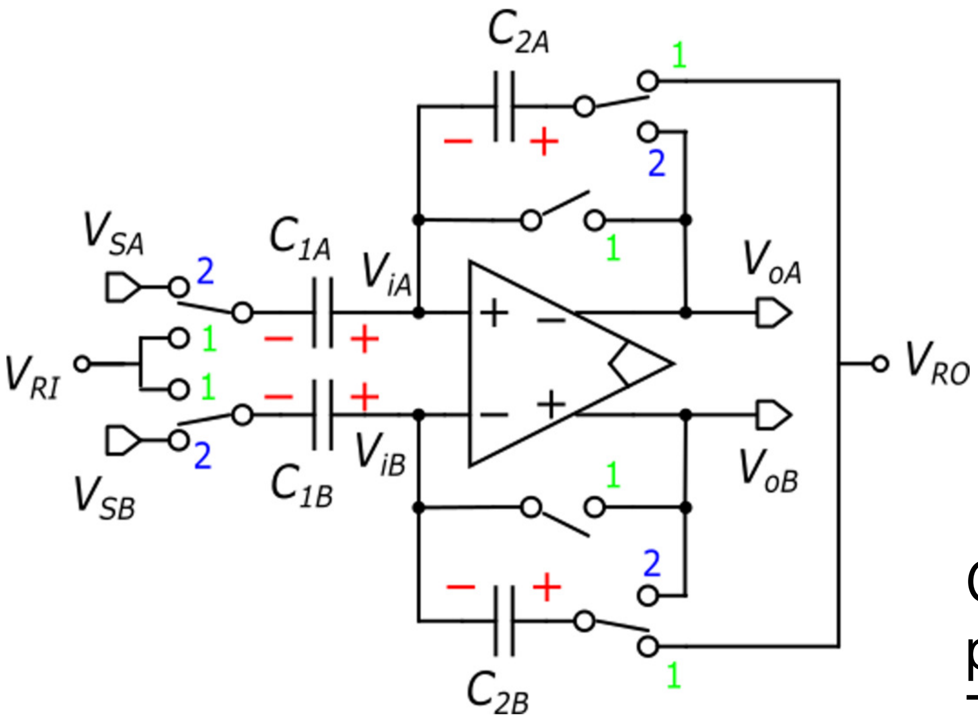
Since in phase 1 $V_{ic}$ is equal to $V_{CMO}$, it is desirable that it does not change passing into phase 2.

To obtain this, we set: $V_{RO} = V_{CMO}$

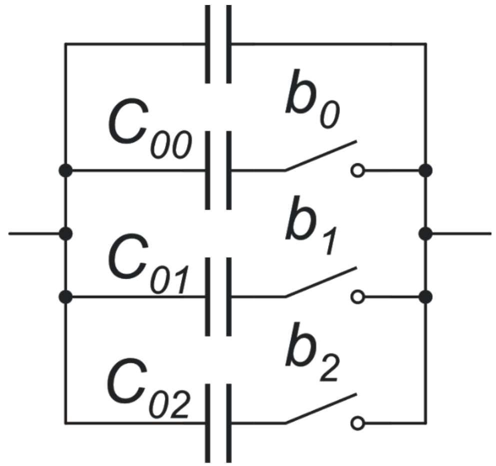
$V_{RI} = V_{SC}$

This is not always possible to guarantee, since the common mode of the source may be not predictable

# Programmable gain amplifier (PGA)




$$A = \frac{C_1}{C_2}$$




Capacitors can be made easily programmable.
The advantage with respect of resistive feedback amplifier is that the switch resistance does not affect accuracy (it affects only speed)

# Additional charge injection from non-constant $V_{ic}$

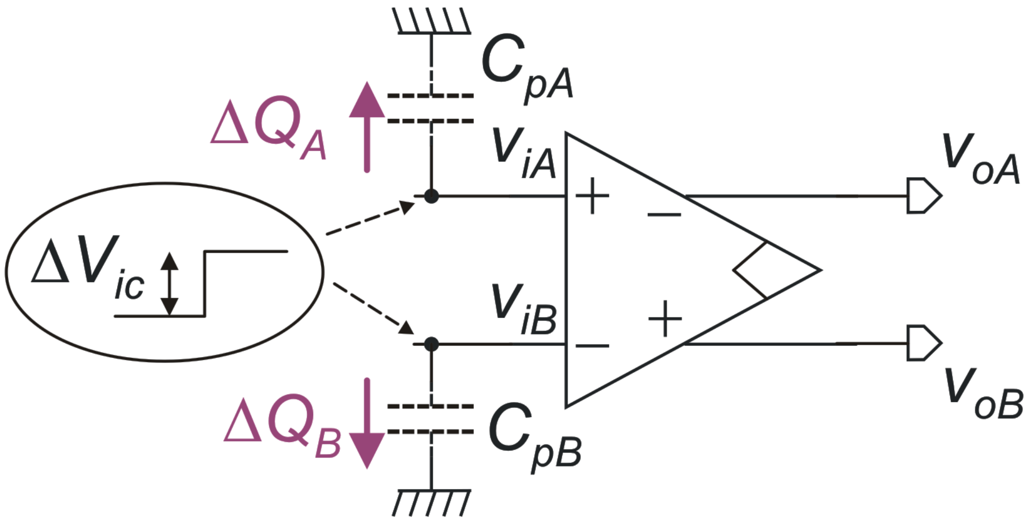
$$V_{ic}^{(1)} = V_{CMO}$$

$$V_{ic}^{(2)} = V_{CMO} + \frac{V_{CMO} - V_{RO}}{A+1} + \frac{A}{A+1}\left(V_{SC} - V_{RI}\right)$$

$$\Delta Q_A = \Delta V_{ic} C_{pA}$$

$$\Delta Q_B = \Delta V_{ic} C_{pB}$$

In this simplified picture, switches and feedback capacitors are not represented




These charges contribute to $Q_{JA}$ and $Q_{JB}$ together with charge injection from the switches

Since parasitic capacitance may have some mismatch, a differential voltage may result.