$\boxed{\text{Calcolo di}}$ $P = \int \frac{dP}{d\Omega} d\Omega = \frac{q^2 |\vec{a}|^2}{16\pi^2 c^3 \varepsilon_0} \iint \frac{d\cos\theta \, d\varphi}{(1-\beta\cos\theta)^2} \left[1 - \frac{\sin^2\theta \cos^2\varphi}{\gamma^2 (1-\beta\cos\theta)^2} \right]$

× acc. circolare

$(\vec{a} \perp \vec{\beta})$ (vedi 23a)

BOZZA

$$P = \frac{q^2 |\vec{a}|^2}{16\pi^2 c^3 \varepsilon_0} \int_{-1}^{1} \frac{dx}{(1-\beta x)^3} \left[\varphi - \frac{(1-x^2)\frac{1}{2}\left(\varphi + \frac{1}{2}\sin 2\varphi\right)}{\gamma^2 (1-\beta x)^2} \right]_0^{2\pi} =$$

$$= \frac{q^2 |\vec{a}|^2}{16\pi^2 \varepsilon_0 c^3} \cdot 2\pi \int_{-1}^{1} \frac{dx}{(1-\beta x)^3} \left(1 - \frac{(1-x^2)}{2\gamma^2 (1-\beta x)^2} \right) =$$

$$= \frac{q^2 |\vec{a}|^2}{8\pi \varepsilon_0 c^3} \left\{ \int_{-1}^{1} \frac{dx}{(1-\beta x)^3}^{(*)} - \int_{-1}^{1} \frac{(1-x^2)\, dx}{2\gamma^2 (1-\beta x)^5} \right\} =$$

$$= \frac{q^2 |\vec{a}|^2}{8\pi \varepsilon_0 c^3} \left\{ 2\gamma^4 - \frac{1}{2\gamma^2} \frac{4}{3} \gamma^6 \right\} = \frac{q^2 |\vec{a}|^2}{8\pi \varepsilon_0 c^3} \frac{4}{3} \gamma^4 =$$

$$= \frac{q^2 |\vec{a}|^2}{6\pi \varepsilon_0 c^3} \gamma^4$$

Nota)^2 $\int_{-1}^{1} \frac{dx}{(1-\beta x)^3} = -\frac{1}{\beta} \int_{-1}^{1} \frac{d(1-\beta x)}{(1-\beta x)^3} = \frac{1}{2\beta} \left[\frac{1}{(1-\beta x)^2} \right]_{-1}^{1} =$

$= \frac{1}{2\beta} \left(\frac{1}{(1-\beta)^2} - \frac{1}{(1+\beta)^2} \right) = \frac{(1+\beta)^2 - (1-\beta)^2}{2\beta (1-\beta^2)^2} = 2\gamma^4$ OK

OK

JC

BOZZA

$$\boxed{\text{Calcolo di } \int_{-1}^{1} \frac{1-x^2}{(1-\beta x)^5}\, dx = \frac{4}{3}\gamma^6}$$

$$Y = 1-\beta x, \quad dy = -\beta\, dx, \quad dx = -\frac{1}{\beta}\, dy, \quad x = \frac{1-y}{\beta}.$$

$$\int_{-1}^{1} \frac{(1-x^2)}{(1-\beta x)^5}\, dx = \int_{1+\beta}^{1-\beta} \frac{1-\frac{1}{\beta^2}(1-y)^2}{y^5}\left(-\frac{dy}{\beta}\right) = \frac{1}{\beta^3}\int_{1+\beta}^{1-\beta} \frac{(1-\beta^2)-2y+y^2}{y^5}\, dy =$$

$$= \frac{1}{\beta^3}\left[\frac{(1-\beta^2)}{(-4)y^4} - \frac{2}{(-3)y^3} + \frac{1}{(-2)y^2}\right]_{1+\beta}^{1-\beta} =$$

$$= \frac{1}{\beta^3}\left[\frac{-(1-\beta^2)}{4(1-\beta)^4} + \frac{(1-\beta^2)}{4(1+\beta)^4} + \frac{2}{3(1-\beta)^3} - \frac{2}{3(1+\beta)^3} - \frac{1}{2(1-\beta)^2} + \frac{1}{2(1+\beta)^2}\right] =$$

$$= \frac{1}{\beta^3}\left[\frac{-(1+\beta)^4+(1-\beta)^4}{4(1-\beta^2)^3} + \frac{2}{3}\cdot\frac{(1+\beta)^3-(1-\beta)^3}{(1-\beta^2)^3} + \frac{1}{2}\cdot\frac{(1-\beta)^2-(1+\beta)^2}{(1-\beta^2)^2}\right] =$$

$$= \frac{1}{\beta^3}\left[\frac{-8\beta-8\beta^3}{4(1-\beta^2)^3} + \frac{2}{3}\,\frac{6\beta+2\beta^3}{(1-\beta^2)^3} + \frac{1}{2}\,\frac{-4\beta}{(1-\beta^2)^2}\right] =$$

$$= \frac{1}{\beta^2}\left[\frac{-2-2\beta^2}{(1-\beta^2)^3} + \frac{2}{3}\,\frac{6+2\beta^2}{(1-\beta^2)^3} - \frac{2}{(1-\beta^2)^2}\right] =$$

$$= \frac{1}{3\beta^2}\,\frac{-6-6\beta^2+12+4\beta^2-6(1-\beta^2)}{(1-\beta^2)^3} = \frac{1}{3\beta^2}\,\frac{4\beta^2}{(1-\beta^2)^3} =$$

$$= \frac{4}{3}\gamma^6 \quad \text{ok}$$

OK