Passiamo a calcolare i campi dai potenziali di L-W.

$$\vec{E} = -\vec{\nabla}\varphi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$

$$\vec{B} = \vec{\nabla} \wedge \vec{A}$$

nota: siamo nel gauge di L.
GAUGE DI LORENTZ

Notiamo che $\vec{\nabla} = \left(\frac{\partial}{\partial r_1}, \frac{\partial}{\partial r_2}, \frac{\partial}{\partial r_3}\right)$ e nel calcolo abbiamo necessità delle seguenti operazioni o definizioni:

$$\boxed{\frac{dt}{dt'} = 1 - \hat{n}\cdot\vec{\beta}}$$ (già visto)

$$\boxed{\vec{v} = \vec{\beta}c = \frac{d\vec{r}'}{dt'}}$$ (def, già visto)

$$\boxed{\hat{n} = \frac{\vec{R}}{R}}$$

$$\boxed{\vec{a} = \frac{d\vec{v}}{dt'} = c\dot{\vec{\beta}}}$$ (def.)

$$\boxed{\frac{d\vec{R}}{dt'} = -\vec{v}}$$ perché $\frac{d\vec{R}}{dt'} = \frac{d(\vec{r}-\vec{r}')}{dt'} = -\frac{d\vec{r}'}{dt'} = -\vec{v}$

$$\boxed{\frac{dR}{dt'} = -\hat{n}\cdot\vec{v}}$$ perché $\frac{d\sqrt{r^2+r'^2-2\vec{r}'\cdot\vec{r}}}{dt'} = \frac{2\vec{r}'\cdot\vec{v} - 2\vec{v}\cdot\vec{r}}{2R} = \frac{\vec{v}\cdot(\vec{r}'-\vec{r})}{R}$

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segue →

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$$\vec{\nabla} R = \frac{\hat{n}}{1-\hat{n}\cdot\vec{\beta}} \qquad \left(\frac{\partial R}{\partial r_i} = \frac{n_i}{1-\hat{n}\cdot\vec{\beta}}\right)$$

$$\vec{\nabla} t' = \frac{-\hat{n}/c}{1-\hat{n}\cdot\vec{\beta}} \qquad \left(\frac{\partial t'}{\partial r_i} = \frac{-n_i/c}{1-\hat{n}\cdot\vec{\beta}}\right)$$

$$\vec{\nabla} t' = -\frac{\vec{\nabla} R}{c}$$

Dim: $t - \frac{R}{c} = t'$ e quindi

$$\underbrace{\vec{\nabla} t}_{=\vec{0}} - \frac{\vec{\nabla} R}{c} = \vec{\nabla} t' \quad \text{ok}$$

OK OK

Dimostrarsi cui

$$\frac{dR}{dr_i} = \frac{\partial R}{\partial r_i} + \frac{\partial R}{\partial t'}\frac{\partial t'}{\partial r_i} = \frac{\partial}{\partial r_i}\sqrt{r^2+r'^2-2\vec{r}\cdot\vec{r}'} + (-\hat{n}\cdot\vec{v})\frac{\partial t'}{\partial r_i} = \, ?$$

$$= \frac{2r_i - 2r'_i}{2R} - \hat{n}\cdot\vec{v}\,\frac{\partial t'}{\partial r_i} = \frac{R_i}{R} - \hat{n}\cdot\vec{v}\,\frac{\partial t'}{\partial r_i}$$

$$\Rightarrow \vec{\nabla} R = \hat{n} - \hat{n}\cdot\vec{v}\,\vec{\nabla} t' = \hat{n} + \hat{n}\cdot\vec{\beta}\,\vec{\nabla} R \Rightarrow$$

$$\Rightarrow \vec{\nabla} R = \frac{\hat{n}}{1-\hat{n}\cdot\vec{\beta}} \quad \text{ok} \quad ; \Rightarrow \vec{\nabla} t' = \frac{-\hat{n}/c}{1-\hat{n}\cdot\vec{\beta}} \quad \text{ok}$$

OK OK

Calcoliamo $\frac{\partial \vec{A}}{\partial t} = \frac{d}{dt}\left(\frac{q\vec{\beta}}{R(1-\hat{n}\cdot\vec{\beta})}\right) = q\frac{d}{dt'}\left(\frac{\vec{\beta}}{R(1-\hat{n}\cdot\vec{\beta})}\right)\frac{dt'}{dt} =$

$$= \frac{\partial \vec{A}}{\partial t} = q\,\frac{\dot{\vec{\beta}}\,R(1-\hat{n}\cdot\vec{\beta}) - \vec{\beta}\,\frac{d}{dt'}\left(R - \vec{R}\cdot\vec{\beta}\right)}{R^2(1-\hat{n}\cdot\vec{\beta})^2}\cdot\frac{1}{1-\hat{n}\cdot\vec{\beta}} =$$

$$= \frac{q}{R^2(1-\hat{n}\cdot\vec{\beta})^3}\left[\frac{\vec{a}}{c}R(1-\hat{n}\cdot\vec{\beta}) - \vec{\beta}\left(-\hat{n}\cdot\vec{\beta}c + \vec{\beta}c\cdot\vec{\beta} - \vec{R}\cdot\frac{\vec{a}}{c}\right)\right] \quad \text{OK}$$

$$\frac{\partial \vec{A}}{\partial t} = \frac{q}{R^2(1-\hat{n}\cdot\vec{\beta})^3}\left[\frac{\vec{a}}{c}R(1-\hat{n}\cdot\vec{\beta})+\vec{\beta}\left(\hat{n}\cdot\vec{\beta}c - c\beta^2+\frac{\vec{a}\cdot\vec{R}}{c}\right)\right]$$ 5a OK

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$$\vec{\nabla}\varphi = \vec{\nabla}\left(\frac{q}{R-\vec{R}\cdot\vec{\beta}}\right) = -\frac{q}{(R-\vec{R}\cdot\vec{\beta})^2}\vec{\nabla}(R-\vec{R}\cdot\vec{\beta}) =$$ OK OK

$$= -\frac{q}{R^2(1-\hat{n}\cdot\vec{\beta})^2}\left[\frac{\hat{n}}{1-\hat{n}\cdot\vec{\beta}} - \underbrace{\frac{\partial(\vec{R}\cdot\vec{\beta})}{\partial x_i}}_{\text{dipendenza esplicita}} - \underbrace{\frac{\partial(\vec{R}\cdot\vec{\beta})}{\partial t'}\vec{\nabla}t'}_{\text{implicita}}\right] =$$ OK

$$= -\frac{q}{R^2(1-\hat{n}\cdot\vec{\beta})^2}\left[\frac{\hat{n}}{1-\hat{n}\cdot\vec{\beta}} - \delta_{ij}\beta_j + \frac{\hat{n}/c}{1-\hat{n}\cdot\vec{\beta}}\left(-\vec{\beta}c\cdot\vec{\beta}+\vec{R}\cdot\frac{\vec{a}}{c}\right)\right]$$ OK

$$= -\frac{q}{R^2(1-\hat{n}\cdot\vec{\beta})^2}\left[\frac{\hat{n}}{1-\hat{n}\cdot\vec{\beta}} - \vec{\beta} + \frac{\hat{n}/c}{1-\hat{n}\cdot\vec{\beta}}\left(-c\beta^2+\frac{\vec{R}\cdot\vec{a}}{c}\right)\right] =$$

$$= -\frac{q}{R^2(1-\hat{n}\cdot\vec{\beta})^3}\left[\hat{n}-\vec{\beta}(1-\hat{n}\cdot\vec{\beta})+\hat{n}\left(-\beta^2+\frac{\vec{R}\cdot\vec{a}}{c^2}\right)\right] =$$ OK

$$\vec{\nabla}\varphi = -\frac{q}{R^2(1-\hat{n}\cdot\vec{\beta})^3}\left[\hat{n}\left(1-\beta^2+\frac{\vec{R}\cdot\vec{a}}{c^2}\right)-\vec{\beta}(1-\hat{n}\cdot\vec{\beta})\right]$$ OK

$$\vec{E} = -\vec{\nabla}\varphi - \frac{\partial\vec{A}}{c\partial t} = \frac{q}{R^2(1-\hat{n}\cdot\vec{\beta})^3}\left\{\hat{n}\left(1-\beta^2+\frac{\vec{R}\cdot\vec{a}}{c^2}\right)-\vec{\beta}(1-\hat{n}\cdot\vec{\beta}) - \frac{\vec{a}R}{c^2}(1-\hat{n}\cdot\vec{\beta}) - \vec{\beta}\left(\hat{n}\cdot\vec{\beta}-\beta^2+\frac{\vec{a}\cdot\vec{R}}{c^2}\right)\right\}$$

dimostriamo che vale:

$$\vec{E} = \frac{q(1-\beta^2)(\hat{n}-\vec{\beta})}{R^2(1-\vec{\beta}\cdot\hat{n})^3} + \frac{q\,\hat{n}\wedge[(\hat{n}-\vec{\beta})\wedge\vec{a}]}{c^2R(1-\vec{\beta}\cdot\hat{n})^3}$$ ... vero se $\longrightarrow$

Termini senza a

$$\frac{q\,(1-\beta^2)(\hat n-\vec\beta)}{R^2(1-\vec\beta\cdot\hat n)^3} = \frac{q}{R^2(1-\vec\beta\cdot\hat n)^3}\left\{ \begin{aligned} &\hat n(1-\beta^2)-\vec\beta(1-\hat n\cdot\vec\beta)+\\ &-\vec\beta(\hat n\cdot\vec\beta-\beta^2)\end{aligned}\right\} =$$

$$= \frac{q}{R^2(1-\vec\beta\cdot\hat n)^3}\left\{\hat n(1-\beta^2)-\vec\beta(1-\beta^2)\right\}$$ Ok

OK

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Termini con $\vec a$:

$$\frac{q\ \hat n\wedge[(\hat n-\vec\beta)\wedge\vec a]}{c^2R(1-\vec\beta\cdot\hat n)^3} = \frac{q\left[\hat n\,\frac{(\vec R\cdot\vec a)}{c^2}-\frac{\vec a\,R}{c^2}(1-\hat n\cdot\vec\beta)-\frac{(\vec a\cdot\vec R)}{c^2}\ -\right.}{R^2(1-\vec\beta\cdot\hat n)^3}$$

$$\hat n\wedge[(\hat n-\vec\beta)\wedge\vec a] = \hat n(\hat n\cdot\vec a)-\vec a(1-\hat n\cdot\vec\beta)-(\vec a\cdot\hat n)\vec\beta$$

|| sviluppiamo $\left(\vec A\wedge(\vec B\wedge\vec C) = \vec B(\vec A\cdot\vec C)-\vec C(\vec A\cdot\vec B)\right)$

$$\hat n\wedge(\hat n\wedge\vec a)-\hat n\wedge(\vec\beta\wedge\vec a) = \hat n(\hat n\cdot\vec a)-\vec a-\vec\beta(\hat n\cdot\vec a)+\vec a(\hat n\cdot\vec\beta)$$

$\vec a(\hat n\cdot\hat n)$

Ok!

OK

Quindi:

$$\boxed{\vec E = \frac{q(1-\beta^2)(\hat n-\vec\beta)}{R^2(1-\vec\beta\cdot\hat n)^3}+\frac{q\ \hat n\wedge[(\hat n-\vec\beta)\wedge\vec a]}{Rc^2(1-\vec\beta\cdot\hat n)^3}}$$

$$\boxed{\vec B = \hat n\wedge\vec E}$$ (non dim.)

Campo di radiazione

→ in MKSA ...