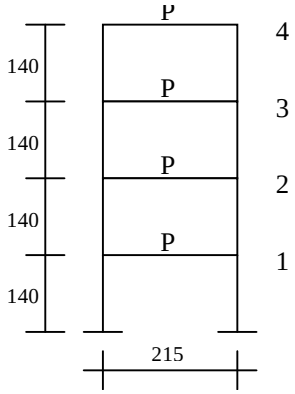
ESEMPIO 1

ANALISI DINAMICA DI TELAIO PIANO



P = 728 g

Piedritti in alluminio 15x2 mm

Traversi in acciaio 20x20 mm

E_{all} = 685000 kg/cmq

E_{acc} = 2100000 kg/cmq

Si vuole studiare il comportamento dinamico del telaio piano, ovvero se ne ricercano le frequenze proprie ed i modi di vibrare.

Ipotesi:

1. traversi infinitamente rigidi; ovvero, la rigidezza dei traversi è molto grande rispetto alla rigidezza laterale del pilastri
2. deformabilità assiale dei traversi e dei pilastri trascurabile
3. massa concentrata a livello dei traversi, piedritti privi di massa

Le incognite del problema sono le traslazioni orizzontali dei traversi; il sistema ha quattro gradi di libertà.

Per risolvere il problema si deve scrivere, per ciascun traverso, l'equazione di equilibrio dinamico alla traslazione.

Se si trascura lo smorzamento, per la singola massa M, le forze in gioco sono: la forza d'inerzia e le forze di richiamo elastiche esercitate dai piedritti.

$$m_1 = m_2 = m_3 = m_4 = M = \frac{728 * 10^{-3}}{981} = 7.42 * 10^{-4} \text{ kg sec}^2/\text{cm}$$

$$J_p = \frac{1.5 * 0.2^3}{12} = 1 * 10^{-3} \quad \text{cm}^4$$

$$k_1 = k_2 = k_3 = k_4 = K = \frac{24 * E_{all} * J_P}{h^3} = \frac{24 * 685000 * 10^{-3}}{14^3} = 5.991 \quad \text{kg/cm}$$

Sistema delle equazioni di equilibrio:

$$\begin{cases} m_1\ddot{x}_1 + k_1 x_1 + k_2(x_1 - x_2) = 0 \\ m_2\ddot{x}_2 + k_2(x_2 - x_1) + k_3(x_2 - x_3) = 0 \\ m_3\ddot{x}_3 + k_3(x_3 - x_2) + k_4(x_3 - x_4) = 0 \\ m_4\ddot{x}_4 + k_4(x_4 - x_3) = 0 \end{cases}$$

In forma matriciale:

$$[m]\{\ddot{x}\} + [k]\{x\} = 0$$

$$[m] = \begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{bmatrix} = \begin{bmatrix} M & 0 & 0 & 0 \\ 0 & M & 0 & 0 \\ 0 & 0 & M & 0 \\ 0 & 0 & 0 & M \end{bmatrix}$$

$$[k] = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 \\ 0 & -k_3 & k_3 + k_4 & -k_4 \\ 0 & 0 & -k_4 & k_4 \end{bmatrix} = \begin{bmatrix} 2K & -K & 0 & 0 \\ -K & 2K & -K & 0 \\ 0 & -K & 2K & -K \\ 0 & 0 & -K & K \end{bmatrix}$$

Si sostituisce:

$$x_j(t) = u_j * f(t) \qquad \text{con} \qquad f(t) = A \operatorname{sen} \omega t$$

il sistema diviene:

$$[k]\{u\} = \omega^2 [m]\{u\}$$

che ammette soluzioni diverse da quella banale solo quando il determinante è ≠ 0.
Per cui deve essere:

$$\Delta(\omega^2) = \left| k_{ij} - \omega^2 m_{ij} \right| = 0$$

Nel caso in esame, il sistema assume la forma:

$$\begin{cases} \left(2K-\omega^2 M\right)u_1 - Ku_2 = 0 \\ -Ku_1 + \left(2K-\omega^2 M\right)u_2 - Ku_3 = 0 \\ -Ku_2 + \left(2K-\omega^2 M\right)u_3 - Ku_4 = 0 \\ -Ku_3 + \left(K-\omega^2 M\right)u_4 = 0 \end{cases}$$

e deve essere:

$$\begin{vmatrix} \left(2K-\omega^2 M\right) & -K & 0 & 0 \\ -K & \left(2K-\omega^2 M\right) & -K & 0 \\ 0 & -K & \left(2K-\omega^2 M\right) & -K \\ 0 & 0 & -K & \left(K-\omega^2 M\right) \end{vmatrix} = 0$$

ovvero:

$$\left(2K-\omega^2 M\right) * \begin{vmatrix} \left(2K-\omega^2 M\right) & -K & 0 \\ -K & \left(2K-\omega^2 M\right) & -K \\ 0 & -K & \left(K-\omega^2 M\right) \end{vmatrix} +$$

$$+K * \begin{vmatrix} -K & -K & 0 \\ 0 & \left(2K-\omega^2 M\right) & -K \\ 0 & -K & \left(K-\omega^2 M\right) \end{vmatrix} = 0$$

$$\left(2K-\omega^2 M\right) * \left[\left(2K-\omega^2 M\right)^2 * \left(K-\omega^2 M\right) - K^2 * \left(2K-\omega^2 M\right) - K^2 * \left(K-\omega^2 M\right)\right] +$$
$$+K * \left[-K * \left(2K-\omega^2 M\right) * \left(K-\omega^2 M\right) + K^3\right] = 0$$

$$\left(K-\omega^2 M\right) * \left(K^3 - 9K^2\omega^2 M + 6K\omega^4 M^2 - \omega^6 M^3\right) = 0$$

le cui soluzioni sono:

$$\omega_1^2 = 974$$
$$\omega_2^2 = \frac{K}{M} = 8074$$
$$\omega_3^2 = 18953$$
$$\omega_4^2 = 28520$$

In corrispondenza si ha:

$\omega_1 = 31.20$ [rad/sec]	$f_1 = 4.96$ [Hz]	$T_1 = 0.201$ [sec]
$\omega_2 = 89.86$	$f_2 = 14.30$	$T_2 = 0.070$
$\omega_3 = 137.67$	$f_3 = 21.91$	$T_3 = 0.045$
$\omega_4 = 168.87$	$f_4 = 26.87$	$T_4 = 0.037$

I vettori modali si ottengono inserendo i valori di ω^2 e risolvendo il sistema:

$$[u] = \left[\{u^{(1)}\} \quad \{u^{(2)}\} \quad \{u^{(3)}\} \quad \{u^{(4)}\}\right] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1.879 & 1 & -0.347 & -1.532 \\ 2.532 & 0 & -0.879 & 1.347 \\ 2.880 & -1 & 0.653 & -0.532 \end{bmatrix}$$

Nel seguito sono riportati i tabulati di input (MDOF) e di output (MDOF.EIG, MDOF.SOL) relativi all'analisi dello stesso modello effettuata con il programma agli elementi finiti SAP90.

```
MDOF
SYSTEM
V=4
JOINTS
1       X=-10.75        Y=0     Z=0
2       X=10.75
9       X=-10.75                Z=56
10      X=10.75                         Q=1,2,9,10,1,2
:
RESTRAINTS
1,10,1  R=0,1,0,1,0,1
1,2,1   R=1,1,1,1,1,1
4,10,2  R=1,1,1,1,1,1
:
MASSES
3,9,2  M=7.42E-4,0,7.42E-4,0,0,0
:
FRAME
NM=2
1       A=0.3   J=0.0285      I=0.001,0.05625    E=685000
2       A=4     J=1.3333      I=1.333,1.333      E=2.1E6
1,1,3   M=1     LP=2,0  MS=0,3
2,2,4
3,3,5                   MS=3,5
4,4,6
5,5,7                   MS=5,7
6,6,8
7,7,9                   MS=7,9
8,8,10
9,3,4   M=2     LP=1,0  MS=3,3
10,5,6                  MS=5,5
11,7,8                  MS=7,7
12,9,10                 MS=9,9
:
```

MDOF

E I G E N S Y S T E M P A R A M E T E R S

NUMBER OF EQUATIONS	=	12
NUMBER OF MASSES	=	8
NUMBER OF VALUES TO BE EVALUATED	=	4
SIZE OF SUBSPACE	=	8

E I G E N V A L U E S A N D F R E Q U E N C I E S

MODE NUMBER	EIGENVALUE (RAD/SEC)**2	CIRCULAR FREQ (RAD/SEC)	FREQUENCY (CYCLES/SEC)	PERIOD (SEC)
1	.973250E+03	.311970E+02	4.965151	.201404
2	.807023E+04	.898345E+02	14.297602	.069942
3	.189491E+05	.137656E+03	21.908579	.045644
4	.285180E+05	.168873E+03	26.876942	.037207

B A S E F O R C E R E A C T I O N F A C T O R S

MODE #	PERIOD (sec)	X DIRECTION	Y DIRECTION	Z DIRECTION	X MOMENT	Y MOMENT	Z MOMENT
1	.201	.515E-01	.000E+00	.130E-04	.000E+00	.208E+01	.000E+00
2	.070	.157E-01	.000E+00	-.327E-04	.000E+00	-.220E+00	.000E+00
3	.046	.762E-02	.000E+00	.113E-04	.000E+00	.697E-01	.000E+00
4	.037	.331E-02	.000E+00	-.107E-04	.000E+00	-.247E-01	.000E+00

P A R T I C I P A T I N G M A S S - (percent)

MODE	X-DIR	Y-DIR	Z-DIR	X-SUM	Y-SUM	Z-SUM
1	89.336	00.000	.000	89.336	00.000	.000
2	8.338	00.000	.000	97.675	00.000	.000
3	1.957	00.000	.000	99.632	00.000	.000
4	.368	00.000	.000	100.000	00.000	.000

M O D E S H A P E S DISPLACEMENTS "U" AND ROTATIONS "R"

MODE SHAPE NUMBER 1 PERIOD = .201404 SECONDS

JOINT	U(X)	U(Y)	U(Z)	R(X)	R(Y)	R(Z)
1	.000000	.000000	.000000	.000000	.000000	.000000
2	.000000	.000000	.000000	.000000	.000000	.000000
3	8.367672	.000000	.002954	.000000	.000839	.000000
4	8.367672	.000000	.000000	.000000	.000000	.000000
5	15.729658	.000000	.004511	.000000	.000830	.000000
6	15.729658	.000000	.000000	.000000	.000000	.000000
7	21.195157	.000000	.005052	.000000	.000689	.000000
8	21.195157	.000000	.000000	.000000	.000000	.000000
9	24.104648	.000000	.005051	.000000	.000469	.000000
10	24.104648	.000000	.000000	.000000	.000000	.000000

MODE SHAPE NUMBER 2 PERIOD = .069942 SECONDS

JOINT	U(X)	U(Y)	U(Z)	R(X)	R(Y)	R(Z)
1	.000000	.000000	.000000	.000000	.000000	.000000
2	.000000	.000000	.000000	.000000	.000000	.000000
3	21.192664	.000000	-.003807	.000000	.000588	.000000
4	21.192664	.000000	.000000	.000000	.000000	.000000
5	21.198363	.000000	-.009943	.000000	-.001545	.000000
6	21.198363	.000000	.000000	.000000	.000000	.000000
7	.005236	.000000	-.014540	.000000	-.002719	.000000
8	.005236	.000000	.000000	.000000	.000000	.000000
9	-21.194544	.000000	-.015750	.000000	-.001951	.000000
10	-21.194544	.000000	.000000	.000000	.000000	.000000

MODE SHAPE NUMBER 3 PERIOD = .045644 SECONDS

JOINT	U(X)	U(Y)	U(Z)	R(X)	R(Y)	R(Z)
1	.000000	.000000	.000000	.000000	.000000	.000000
2	.000000	.000000	.000000	.000000	.000000	.000000
3	24.103898	.000000	-.000129	.000000	-.000345	.000000
4	24.103898	.000000	.000000	.000000	.000000	.000000
5	-8.365245	.000000	.000566	.000000	-.001783	.000000
6	-8.365245	.000000	.000000	.000000	.000000	.000000
7	-21.196892	.000000	.005806	.000000	.001374	.000000
8	-21.196892	.000000	.000000	.000000	.000000	.000000
9	15.729760	.000000	.008976	.000000	.002112	.000000
10	15.729760	.000000	.000000	.000000	.000000	.000000

MODE SHAPE NUMBER 4 PERIOD = .037207 SECONDS

JOINT	U(X)	U(Y)	U(Z)	R(X)	R(Y)	R(Z)
1	.000000	.000000	.000000	.000000	.000000	.000000
2	.000000	.000000	.000000	.000000	.000000	.000000
3	15.734166	.000000	-.002421	.000000	-.001139	.000000
4	15.734166	.000000	.000000	.000000	.000000	.000000
5	-24.102672	.000000	-.002579	.000000	.000040	.000000
6	-24.102672	.000000	.000000	.000000	.000000	.000000
7	21.193535	.000000	-.003425	.000000	.000395	.000000
8	21.193535	.000000	.000000	.000000	.000000	.000000
9	-8.368999	.000000	-.006031	.000000	-.001611	.000000
10	-8.368999	.000000	.000000	.000000	.000000	.000000