EXPERIMENTAL ANALYSIS OF TWO METHODS FOR ESTIMATING IN-SITU TENSILE FORCE IN TIE-RODS

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Abstract

The results of a series of 48 laboratory tests to investigate the reliability of two methods for determining the tensile axial force in metallic rods are presented. The first method, called “mixed method”, consists of a simple numerical identification procedure, based on the experimental measurement of the vertical displacement caused by a concentrated load, and the fundamental vibration frequency. In the second method, called “static method”, measurements of nine vertical displacements are required: the midspan section and the two quarter span sections displacements of the rod caused by a concentrated load acting in the same sections. The rod is modelled as a tie-beam with two equal elastic rotational springs at both ends; thus, the hypothesis of symmetrical scheme of constraints is considered. The variables of the tests were: the rod slenderness, the end-restraint conditions and the intensity of the tensile axial force. Experimental tests results were used to predict the axial force in the rods by numerical identification procedures. Generally, the static method was more reliable than the mixed method especially in predicting the rigidity of the elastic springs of the tie-model. Particularly, the mixed method produced high errors (more than 20 %) for the lower tensioning level and for the rods with nearly fixed ends.

Keywords: Historical buildings, laboratory tests, masonry arches and vaults, tie-rods

1. Introduction

In architectural tradition the use of metallic rods at the base of masonry arches and vaults is a commonly structural solution capable of reducing the horizontal reactions on the bearings. A correct approach to restoration designs as well as to monitoring and structural strengthening of ancient buildings require the accurate determination of the tensile forces acting in these rods. Unfortunately, the actual force in a metallic rod (denoted as N) depends on many factors, as the compatibility conditions with the structural system, the characteristics of the materials, the building and reconstruction interventions, the natural degrading effects, and the thermal excursions. Generally, the geometrical and mechanical characteristics of the rods can be easily assessed, whereas the level of constraints in the masonry is very difficult to identify. As it is well known, the two hypotheses of either hinged or fixed rod extremities to the masonry lead to very different values of N when the calculation is based on the fundamental vibration frequency (more than 100%).

Within this context, the evaluation of N based on simple analytical theories is not feasible and experimental methods are to be used. So far, different methods have already been proposed to this aim [1, 2, 3, 4, 5 and 6]. All these methods consist in analytical or numerical identification procedures based on simple structural models of the rod, in which two or more static and dynamic experimental measures are required. In most cases the goodness of a method has been associated to the simplicity of the experimental set-up rather than to the accuracy of results. Moreover, reliability analyses with direct comparison of results furnished by different methods have not yet been published.

In the present paper a study on the accuracy of two of such methods is presented. The first method, called “mixed method”, is the one firstly developed by Blasi and Sorace [4] and here considered with some modifications; the second method, called “static method”, was firstly proposed by Beconcini [5]. In these methods, the same two parameters model is used, in which the tie-rod is modelled as a beam endowed with flexural stiffness and identical end restraint conditions (hypothesis of symmetric scheme). The boundary conditions are then represented by two equal elastic rotational springs with rigidity denoted as K. Thus K, along with the tensile axial force N, represent the unknowns of the parametric identification procedures.

In the mixed method two experimental data are required: the midspan section vertical displacement when a concentrated load is applied and the fundamental vibration frequency in free oscillation regime. In the static method nine measures are required: the midspan section and the two quarter sections vertical displacements when a concentrated load is applied at the middle and at the two quarter sections of the rod. Applying such procedures, the parameters estimate is carried out using numerical solution techniques.

In the first part of the paper the two methods are briefly resumed and a series of 48 laboratory tests performed varying the rod slenderness, the end-restraint conditions and the level of the tensile axial force is presented. In the second part a critical discussion about results of the experiments is given, together with some relevant considerations on the practical application of the two methods in monumental buildings.

2. Model of the rod and axial force evaluation methods

The tie-rod is modelled as a prismatic tie-beam with reference span l, flexural rigidity EJ and cross-sectional area A, constrained at its ends by two equal elastic rotational springs, having rigidity K (Fig. 1a). From a structural viewpoint, the assumption of symmetrical restraint conditions at the two ends is acceptable in relation to the initial placement of the rod and it is essential for the application of the mixed method [4]. In reference [5] a more refined model was used in the static method, with asymmetric end restraint conditions. In this study the hypothesis of symmetrical restraint conditions of the rods was accurately verified during the laboratory tests, thus the assumption of a model with two unknowns (N and K) was justified and more appropriate for comparison purposes.

The experimental equipment necessary to carry out the tests is represented in Fig. 1b. For the mixed method it consists of an accelerometer and a fleximeter placed in the rod mid section, together with a data acquisition system to record the signals registered by the accelerometer. For the static method the instrumentation consists in two additional fleximeters placed at the two quarter rod sections, whereas the accelerometer and the acquisition system are unnecessary.

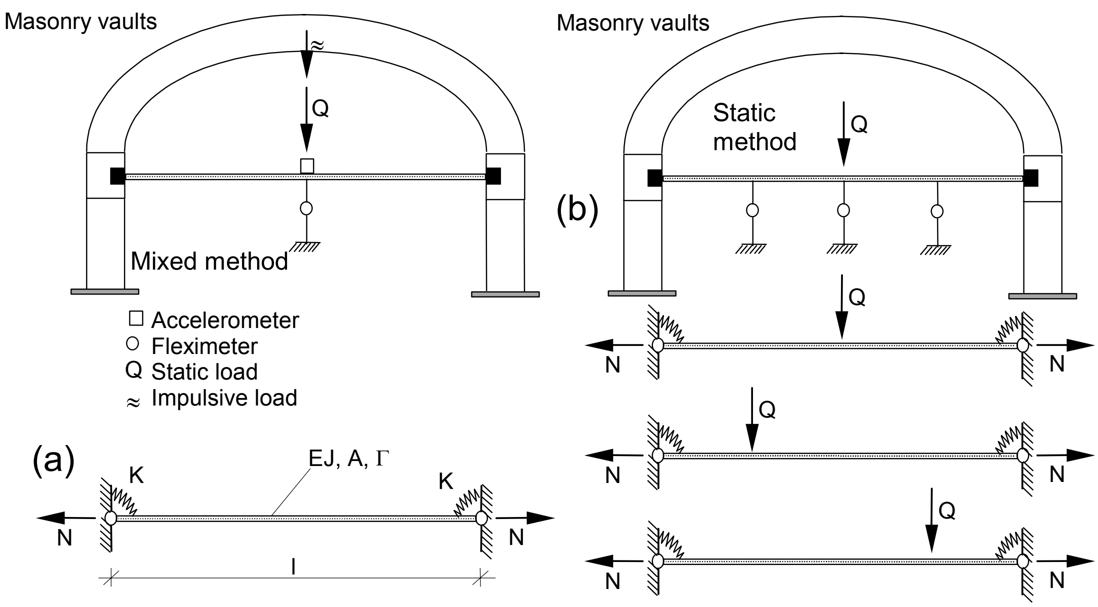

Fig. 1: (a) Model of the tie-rod; (b) Instrumentation needed for testing

2.1 Analytical relationships of the mixed method

The solving equations of the mixed method were obtained by combining some analytical expressions valid in the static and dynamic fields of tie-beams. A first relationship furnishes the midspan section vertical displacement v when a concentrated load Q is statically applied to the beam subjected to the axial force N:

$$v = \frac{Ql^3}{16EJ} \cdot \frac{1}{\gamma^3}\left(\gamma - \mathrm{tgh}\gamma - \frac{(2\cosh\gamma - 2 - \mathrm{tgh}\gamma \cdot \mathrm{senh}\gamma)}{\cosh\gamma} \cdot \frac{Kl}{Kl\,\mathrm{tgh}\gamma + 2\gamma EJ}\right); \quad \beta = \sqrt{\frac{N}{EJ}}; \quad \gamma = \frac{\beta l}{2}. \tag{1}$$

Equation (1) was derived in rigorous manner from the theory of elastic beams subjected to a tensile axial force and to a transversal concentrated load. It must be observed that in the original version of this method the following simpler and approximate equation was used [4]:

$$N = N_{crk}\left(\frac{v_{trk}}{v} - 1\right) \tag{2}$$

where N_{crk} is the first Euler critical load of the beam with the appropriate rotational springs at the ends and v_{trk} is the middle section vertical displacement due to the load Q of the same beam (without N). Further, in the paper [4], the Authors approximate N_{crk} by the well-known Newmark formula:

$$N_{crk} = C(K)\frac{\pi^2 EJ}{l^2}; \qquad C(K) = \frac{(L+0.4)^2}{(L+0.2)^2}; \qquad L = \frac{EJ}{lK}, \tag{3}$$

being C(K) the amplification coefficient depending on the rigidity K by means of the dimensionless parameter L. A second group of relationships relates the unknowns of the problem to the fundamental vibration frequency f of the rod. If f_{1k} denotes the same frequency without N, the following equations hold:

$$N = N_{crk}\left(\left(\frac{f}{f_{1k}}\right)^2 - 1\right); \qquad f_{1k} = \frac{(\alpha l)^2}{2\pi l^2}\sqrt{\frac{EJg}{\Gamma A}} \tag{4}$$

where N_{crk} has already been defined. The frequency f_{1k} is obtained by the second of Eqns. (4), where g is the gravity acceleration, Γ the specific weight of the material and (αl) is the characteristic parameter of the equation yielding the eigenvalues of the free oscillation problem of the beam elastically constrained at its ends, without axial load:

$$\left[2(\alpha l)^2 \,\mathrm{sen}(\alpha l)\,\mathrm{senh}(\alpha l)\right]L^2 + \left[2(\alpha l)\left(\mathrm{sen}(\alpha l)\cosh(\alpha l) - \cos(\alpha l)\,\mathrm{senh}(\alpha l)\right)\right]L + \left(1 - \cos(\alpha l)\cosh(\alpha l)\right) = 0\,. \tag{5}$$

In the original version of the method [4], in order to obtain a closed form solution, the Authors derived an approximate mathematical expression for the (αl) - L relationship in the range $0.015 \le L \le 1$; outside of this interval the same Authors consider the tie-beam with clamped ($\alpha l = 4.73$) or hinged extremities ($\alpha l = \pi$). To investigate the influence of the different analytical formulations the mixed method has been applied in four different versions:

M1) using the “exact” equations both for static and dynamic fields (Eqns. (1) and (5)); M2) using the “exact” equations for dynamic field and the approximate equation (2) in the static field (evaluating the critical load N_{crk} with the rigorous theory of stability); M3) the same as M2 in which the critical load N_{crk} is evaluate by the approximate Newmark formula (3); M4) using the original method [4] in which the approximate equations are used in the static and dynamic fields.

In the M1 version the two equations (1) and (5) relate the unknowns, N and K, to the measured values v and f, completing the formulation of the mathematical procedure. A standard solver for non linear systems has been used, in order to eliminate any analytical approximation. Similarly, the same solver has been used for M2 and M3. Thus, in addition to the differences due to the analytical formulation, two main kinds of errors affect the solutions given by this method: the modelling errors and the experimental inaccuracies.

2.2 Analytical relationships of the static method

The solving equations of the static method are the same used by the Author of the reference [5]. They represent the nine mathematical expressions of the midspan section and the quarter sections vertical displacements when a concentrated load Q is applied in the same sections of the tie-beam (Equation (1) is the first of these formulas). Given a tie-beam with prefixed values of N and K, we denote as v_{iex} (i=1,2,...,9) the nine experimental measures of the displacements and as v_{ith} (i=1,2,...,9) the mutual theoretical values. The identification procedure consists in determining the unknowns N and K corresponding to the minimum error in the sense of the least square approach. Thus, the following function error has been used:

$$E_q = \sum_{i=1}^{9} \left(v_{ith}(N,K) - v_{iex} \right)^2 = E_q(N,K) \tag{6}$$

As for the mixed method a standard algorithmic solver has been used to determine the unknowns of the numerical problem, thus obtaining the best estimate of the axial force N and elastic rigidity K.

3. Experimental tests

Overall, 48 tests where conducted on four rods made of circular steel bars (grade Fe 360) with 16 and 20 mm diameters. Two rods had a 16 mm diameter and were long 1800 and 2600 mm whereas the other two rods, of a 20 mm diameter, were long 2250 and 3250 mm, thus obtaining rods slenderness equal to 450 and 650. The rods were connected at their ends to rigid double bolted supports. For each end, two steel plates (100 mm long and 30 mm thick) clamped the rods end (Fig. 2a). The tensioning level was varied by adjusting the bolts at one extremity, whereas the bolts at the other end were initially tightened. In these experiments three nominal level of the normal stresses equal to 40, 80 and 120 N/mm^2 were applied to the rods, to reproduce the average range of working stresses acting in tie-rods of monumental buildings. Finally, four levels of constraint were simulated, corresponding to the two limit situations of fixed and hinged rods ends, as well as two intermediate clamping hypotheses. The hinged extremities were obtained by semi-cylindrical steel work pieces (Fig. 2b). The two intermediate levels of constraints were made by interposing a cylindrical layer of soft material between the end steel plates and the rod; a rubber layer 15 mm thick and a wood layer 8 mm thick were placed around the ends' rod.

Tensioning was measured using three pairs of strain gauges placed at the midspan section, and in proximity to the end sections. Five fleximeters were placed along the rods to measure the vertical displacements at the midspan, the two quarter and the two end sections (Fig. 2c). Static measurements were carried our for each tensioning level with a load Q = 99.11 N.

The experimental variables of the 48 tests are resumed in Table 1. It must be observed that the rods with hinged extremities had a reference span 216 mm longer than other rods. The testing program was conducted according to the following steps. The rod was tensioned at the desired level acting on the bolts, then the impulsive load was applied to record the free vibration history, finally the static load Q was applied in the three different positions to measure the vertical displacements. During each experiment, strain gauges revealed that the tensioning level was approximately constant.

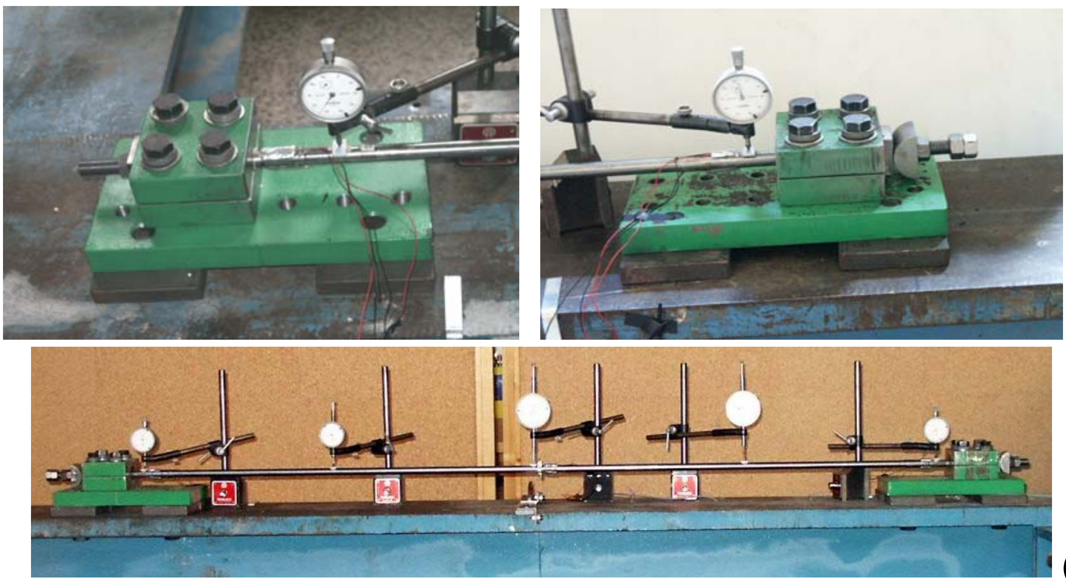
(a) (b) (c)

Fig 2: (a) Clamped extremity of a rod; (b) Hinged extremity; (c) Instrumentation for laboratory tests

Tab. 1: General outline of laboratory tests

Rod l = 1800 mm (*), ϕ = 16 mm			Rod l = 2250 mm (*), ϕ = 20 mm		
Test	Constraint	Tensioning level (N/mm^2)	Test	Constraint	Tensioning level (N/mm^2)
T1	Fixed	40	T25	Fixed	40
T2	Fixed	80	T26	Fixed	80
T3	Fixed	120	T27	Fixed	120
T4	Wood layer	40	T28	Wood layer	40
T5	Wood layer	80	T29	Wood layer	80
T6	Wood layer	120	T30	Wood layer	120
T7	Rubber layer	40	T31	Rubber layer	40
T8	Rubber layer	80	T32	Rubber layer	80
T9	Rubber layer	120	T33	Rubber layer	120
T10	Hinged	40	T34	Hinged	40
T11	Hinged	80	T35	Hinged	80
T12	Hinged	120	T36	Hinged	120
Rod l = 2600 mm (*), ϕ = 16 mm			Rod l = 3250 mm (*), ϕ = 20 mm		
Test	Constraint	Tensioning level (N/mm^2)	Test	Constraint	Tensioning level (N/mm^2)
T13	Fixed	40	T37	Fixed	40
T14	Fixed	80	T38	Fixed	80
T15	Fixed	120	T39	Fixed	120
T16	Wood layer	40	T40	Wood layer	40
T17	Wood layer	80	T41	Wood layer	80
T18	Wood layer	120	T42	Wood layer	120
T19	Rubber layer	40	T43	Rubber layer	40
T20	Rubber layer	80	T44	Rubber layer	80
T21	Rubber layer	120	T45	Rubber layer	120
T22	Hinged	40	T46	Hinged	40
T23	Hinged	80	T47	Hinged	80
T24	Hinged	120	T48	Hinged	120

(*) Reference span of the rods with hinged ends were 216 mm longer

4. Discussion of results

Main results obtained from the 48 tests are reported in Table 2. N_{EX} denotes the experimentally determined axial load, N_{ST} and N_{MI} are the numerically identified axial loads with the static and the mixed method, Δ_{ST} and Δ_{MI} represent the mutual errors in the axial force evaluation. The dimensionless parameter L representing the constraint-level is also given for the two methods (L_{ST} and L_{MI}).

Consider first the axial force N_{ST} predicted by the static method. In most of tests (thirty-three on the whole) the N_{ST} values are higher than the experimental ones, i.e. the method leads to overestimate the tensioning level of the rods. A maximum positive error of 13.9 % was obtained in the test T19, a minimum error of –9.4 % was calculated for the test T3. Considering the whole set of experiments an average absolute error of 4.3 % in predicting the axial force was registered. The errors were higher than 10% in a limited number of tests (six cases, all corresponding to rods tensioned at the lower level of stress, Fig. 3a). These general results well agree with those presented in the reference [5] when applying the same method to laboratory tests. When the method is applied to actual cases, the error committed in the estimation of N gets lower; this is presumably due to scale effects. In reference [5] a maximum error of 10 % in the evaluation of N_{ST} was determined. The slightly higher errors here obtained are well explained if we consider that in the research [5] an asymmetric tie-model was used with two different elastic stiffnesses at the rod ends. Under this hypothesis, given the redundant number of measurements employed by the method, a better numerical identification of the prototype is possible and the results improve. Nevertheless, in our experiments, nearly symmetric conditions were realised, thus making small differences in results.

A good characteristic of the static method that emerged from examination of results (which is not presented here) was the low influence in determining N_{ST} of the two end sections vertical displacements. Although these displacements are unknown when applying the method in monumental buildings they were exactly measured in the laboratory tests. Calculation of N_{ST} with and without taking into account the end displacements has showed that these effects produce negligible errors in the identification procedure of the model.

Tab. 2: Experimental results of laboratory tests

Test	N_{EX} (N)	N_{ST} (N)	N_{MI} (N)	L_{ST}	L_{MI}	Δ_{ST} (%)	Δ_{MI} (%)
Rod l = 1800 mm, ϕ = 16 mm							
T1	7717	8532	11997	0.012	0.146	10.6	55.5
T2	15606	16644	20533	0.020	0.202	6.7	31.6
T3	22826	20683	23043	0.002	0.051	–9.4	0.9
T4	8084	8876	9559	0.112	0.168	9.8	18.2
T5	14309	14440	14037	0.104	0.080	0.9	–1.9
T6	23582	23881	25422	0.103	0.256	1.3	7.8
T7	8344	9304	10461	0.299	∞	11.5	25.4
T8	16730	17364	17766	0.344	∞	3.8	6.2
T9	23734	24057	23398	0.780	∞	1.4	–1.4
T10	8798	8417	8148	∞	∞	–4.3	–7.4
T11	15520	15645	15569	∞	0.387	0.8	0.3
T12	23885	23922	25078	∞	∞	0.2	5.0
Rod l = 2600 mm, ϕ = 16 mm							
T13	9355	10467	12886	0.008	0.226	11.9	37.7
T14	17838	17724	20542	0.003	0.171	–0.6	15.2
T15	25343	25927	29012	0.014	0.390	2.3	14.5
T16	8378	9097	10170	0.060	0.212	8.6	21.4
T17	16050	17025	17531	0.127	0.224	6.1	9.2
T18	24075	24825	25348	0.100	0.164	3.1	5.3
T19	8295	9446	9995	0.305	1.242	13.9	20.5
T20	15884	16462	16968	0.232	∞	3.6	6.8
T21	23285	24905	24443	1.899	0.334	7.0	5.0
T22	8607	9747	9701	∞	∞	13.2	12.7
T23	16279	17328	17687	∞	2.477	6.4	8.7
T24	23617	24537	25200	∞	∞	3.9	6.7
Rod l = 2250 mm, ϕ = 20 mm							
T25	13735	14615	13761	0.038	0.025	6.4	0.2
T26	24152	24092	24806	0.022	0.032	–0.3	2.7
T27	36461	35172	37975	0.014	0.044	–3.5	4.2
T28	12043	12029	12011	0.056	0.060	–0.1	–0.3
T29	23688	23781	24486	0.064	0.082	0.4	3.4
T30	34570	33667	34511	0.057	0.071	–2.6	–0.2
T31	12342	13068	11984	0.272	0.171	5.9	–2.9
T32	22726	23864	22668	0.551	0.273	5.0	–0.3
T33	37290	36233	35351	0.349	0.223	–2.8	–5.2
T34	12408	12503	13215	∞	∞	0.8	6.5
T35	25048	25153	19819	∞	0.035	0.4	–20.9
T36	36361	36423	35731	∞	0.207	0.2	–1.7
Rod l = 3250 mm, ϕ = 20 mm							
T37	12763	13695	15971	0.019	0.118	7.3	25.1
T38	24323	24473	26073	0.016	0.054	0.6	7.2
T39	34714	34452	36038	0.015	0.044	–0.8	3.8
T40	12229	12205	12265	0.032	0.035	–0.2	0.3
T41	23054	22195	23365	0.030	0.069	–3.7	1.4
T42	36953	36518	36861	0.071	0.082	–1.2	–0.2
T43	12095	13374	13366	0.171	0.177	10.6	10.5
T44	24390	24759	23475	0.261	0.092	1.5	–3.8
T45	34881	32977	32760	0.126	0.104	–5.5	–6.1
T46	13030	13233	13272	∞	∞	1.6	1.9
T47	25092	24450	24632	∞	0.336	–2.6	–1.8
T48	36886	35699	36256	∞	0.283	–3.2	–1.7

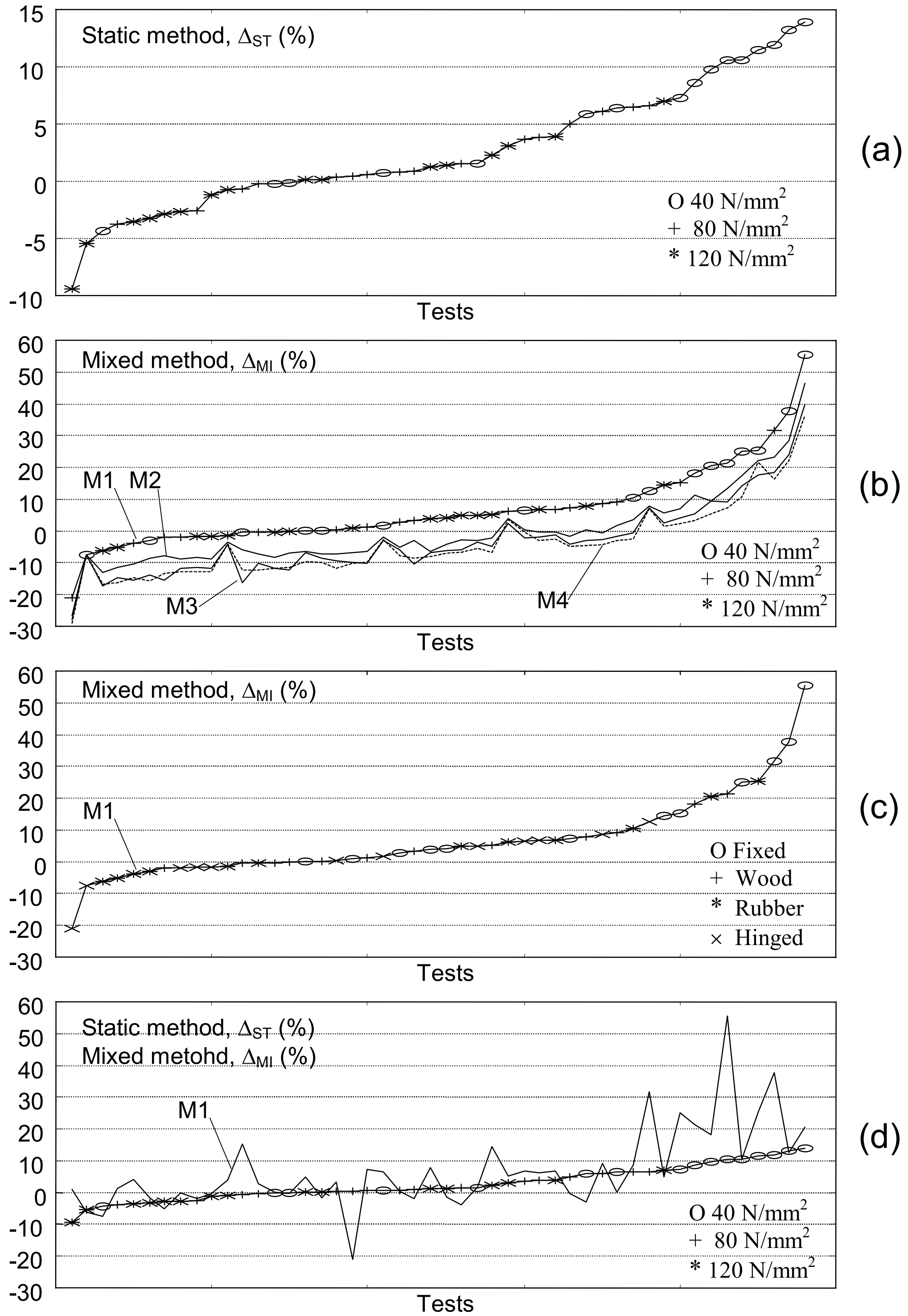

Fig. 3: (a) Error of the static method; (b) and (c) Errors of the mixed method; (d) Comparisons of errors of the two methods Δ_{ST} and Δ_{MI}

The constraint-level values of L lower than 0.015 can be technically referred to the limit situation of fixed terminal sections, whereas when L is higher than 1.0 the terminal sections can be considered as hinged. These conventional limits are reported in Fig. 4 (denoted with F and H) together with the L_{ST} values, plotted in ascending order. The figure shows that the static method was excellent in predicting the constraint-level, because the L_{ST} values are consistent with the previous limits for all tests with few exceptions. This is an important observation because it shows that the numerical identification based on this method produces results consistent with the effective end conditions of the rods.

Axial forces N_{MI} predicted by the mixed method M1 are generally less accurate than N_{ST}. In eight tests the absolute error was higher than 20 %, showing that this method is very sensitive to the modelling and experimental inaccuracies. The higher errors were obtained in the cases of fixed end extremities and when applying the lower stress level (Figs. 3b and 3c).

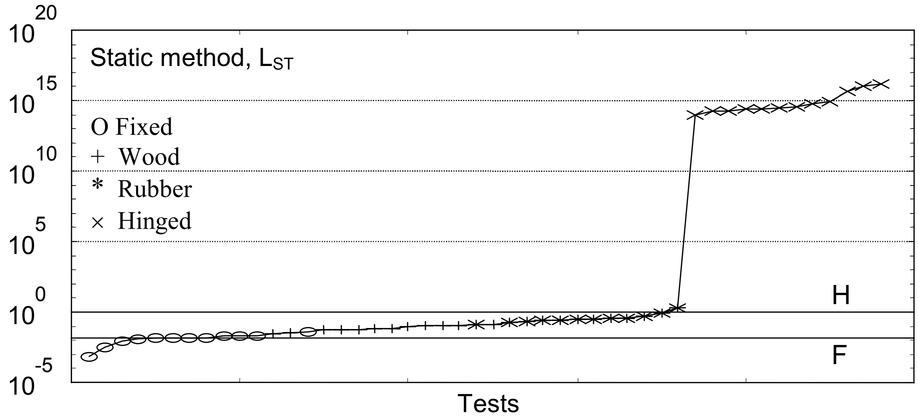

Fig. 4: Dimensionless parameter L_{ST} identified with the static method

Overall, an average error of 9.1 % was calculated. Effects of the analytical approximations in calculating the displacement v and the frequency f are shown in Fig. 3b (methods M2, M3 and M4). It is evident that these approximate procedures lead to underestimate the axial force when compared with the rigorous implementation M1. Using the M4 scheme an average difference of 10.0 % to M1 was calculated for N_{MI}. In the research [4] the Authors obtained a maximum error of 6.6 % applying the M4 scheme. This remarkable difference is probably due to the wider range of situations and to the high number of tests considered in our study. Further, this method demonstrated high numerical sensibility when calculating the unknowns depending on the eventual vertical movements of the end supports.

On the other hand, in most of intermediate restraint conditions the method predicts the axial force acceptably. Finally, values of L_{MI} (Tab. 2) clearly show that the method is unreliable in predicting the effective end restraint conditions of the rods. Determination of L_{MI} equal to infinity correspond to the situation in which no numerical solution was obtained by this method; in these cases the best approximation of the N_{EX} load was determined in the hypothesis of hinged extremities using the dynamic equation and the measured value of the frequency f.

5. Conclusions

The 48 experimental laboratory tests conducted varying the constraint-level, the tensioning level and the slenderness of the rods have showed that the static method, based on nine experimental measures, is very accurate in predicting the tensile axial force N and the elastic rigidity K. Nevertheless, the mixed method, that is simpler and it is based on two measures, furnished acceptable predictions of N in the greater part of the tested rods, corresponding to the intermediate constraint-levels. Average errors of 4.3 and 9.1 % were calculated in predicting N with the two methods, respectively. The results indicate that for tie-rods in which the end restraint conditions are in proximity of the fixed ends situation the static method is applicable with acceptable errors whereas the mixed method, in some cases, leads to substantially overestimate the load N. The same trend was observed for the rods tensioned at the lower stress. On such basis, the application of the two methods should be decided based on a first rough evaluation of the two unknowns, performed after a preliminary survey and eventually based on measuring the frequency f.

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