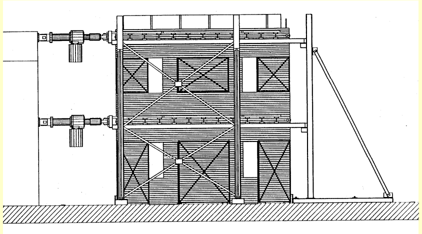
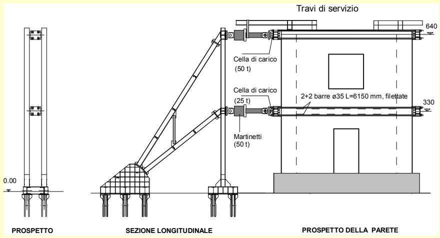
Masonry walls – Modeling in-plane response (to seismic actions)

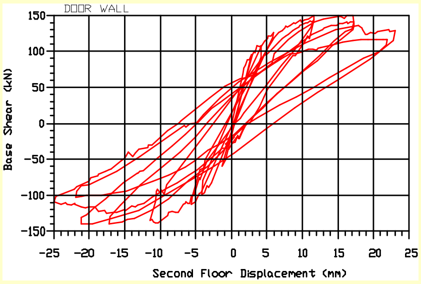
Cyclic horizontal forces, anisotropic damage, hysteretic dissipation, damage localization, inertial vertical forces …………

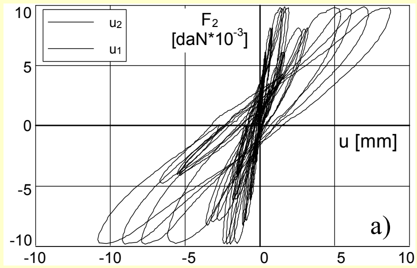


Brick masonry wall tested in Pavia, Magenes et al. (1994).

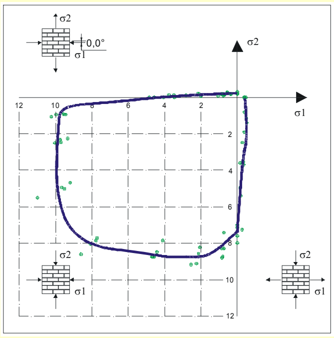


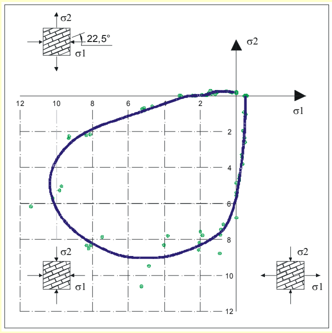
Block masonry wall in S.Sisto (Beolchini et al., 1997).

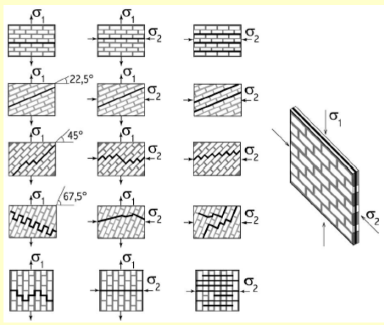


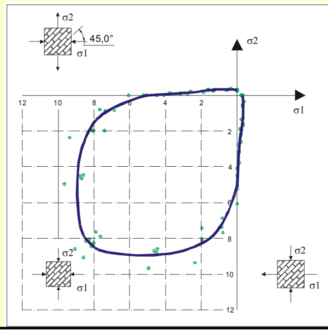


Anisotropic limit strength domains – Page, 1981

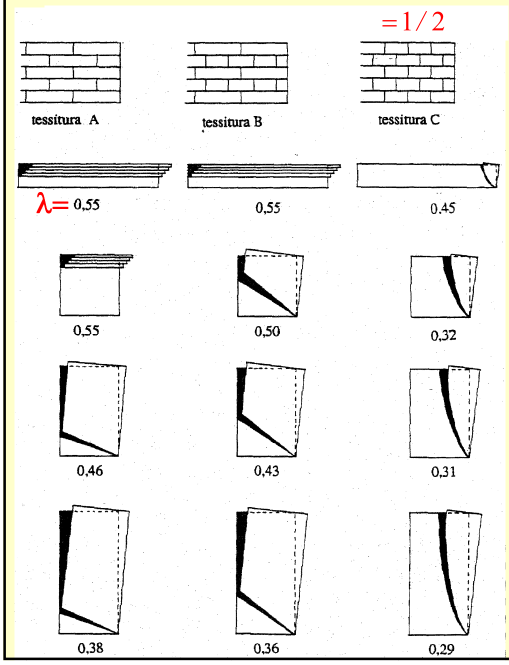








Influence of the brick/block aspect ratio and bond pattern

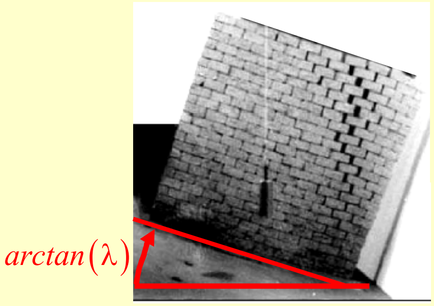


Experimental results
Dry block masonry
Giuffrè *et al.*, 1993

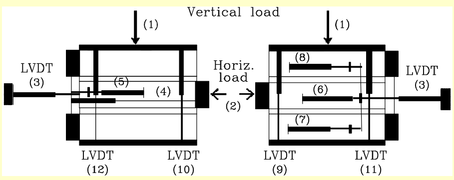
Collapse mechanisms
and limit slope angle
$arctan(\lambda)$

for varying:

- Block aspect ratio a/b
- Bond pattern
- Wall slenderness

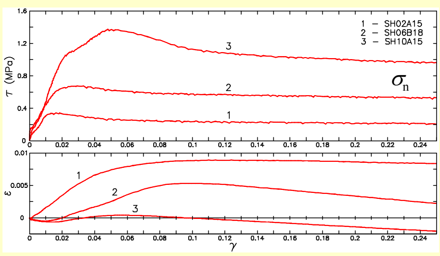


Shear testing on brick-mortar assemblages



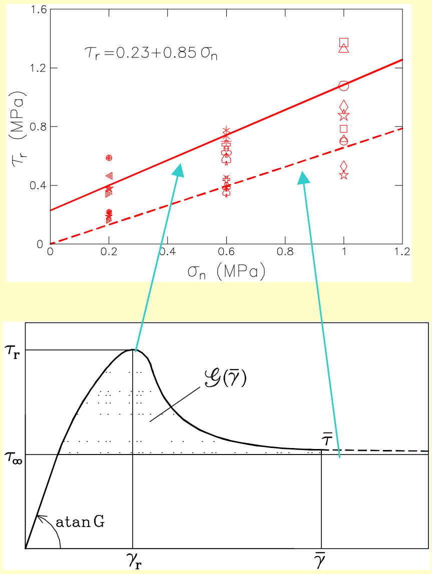
Shear test apparatus - Triplet
(Binda et al., 1995).

τ mean shear stress - γ mean shear strain



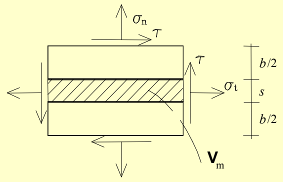
ε - mean normal extension

Experimental results



Phenomenological description

Brick-mortar interface model: coupled damage-frictional interface



Macro fields

$$\boldsymbol{\sigma}_m = \{\sigma_t\, \sigma_n\, \tau\}^t$$

$$\boldsymbol{\varepsilon}^*_m = \{0\; \varepsilon^*_m\, \gamma^*_m\}^t \quad \text{Inelastic strain}$$

$$\boldsymbol{\varepsilon}_m = \{0 \quad \varepsilon_m \quad \gamma_m\}^t \quad \text{Total strain}$$

$$\boldsymbol{\varepsilon}_m = \mathbf{K}_m \boldsymbol{\sigma}_m + \boldsymbol{\varepsilon}^*_m$$

$$\begin{cases} \varepsilon^*_m = h(\alpha_m)\; H(\sigma_n)\, \sigma_n \\ \gamma^*_m = k(\alpha_m)\,(\tau - f) \end{cases}$$

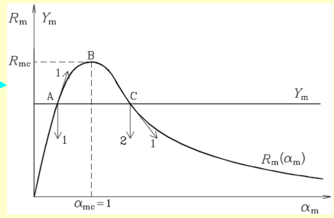
α_m damage variable
f friction

Conjugate variables

$$Y_m = \tfrac{1}{2} h'(\alpha_m)\; H(\sigma_n)\, \sigma_n^2 + \tfrac{1}{2} k'(\alpha_m)\,(\tau - f)^2 \;,\; \gamma^*_m$$

Damage evolution

$$\begin{cases} \phi_{dm} = Y_m - R_m \le 0 \\ \phi_{dm} = 0,\; \dot{\phi}_{dm} \le 0,\; \dot{\alpha}_m \ge 0,\; \dot{\phi}_{dm}\dot{\alpha}_m = 0 \end{cases}$$

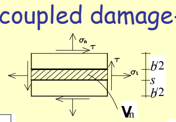


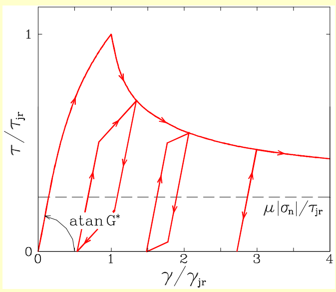
Sliding

$$\begin{cases} \phi_s = |f| + \mu\sigma_n \le 0 \\ \dot{\gamma}^*_m = \mathbf{v}\dot{\lambda} \;,\; \dot{\lambda} \ge 0 \quad \mathbf{v} = \dfrac{f}{|f|} \end{cases}$$

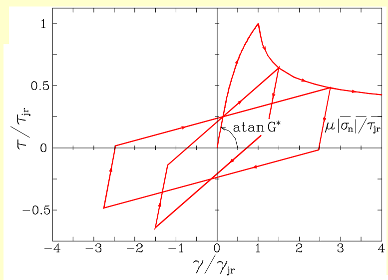
Gambarotta e Lagomarsino, 1997

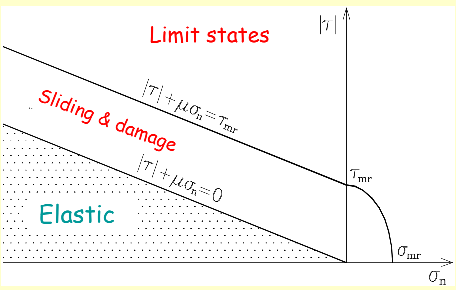
Brick-mortar interface model: coupled damage-frictional interface



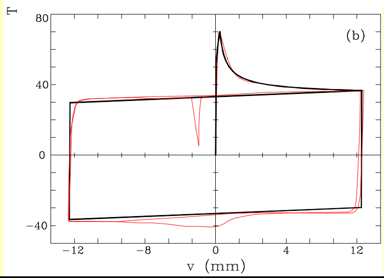


Hysteretic damage

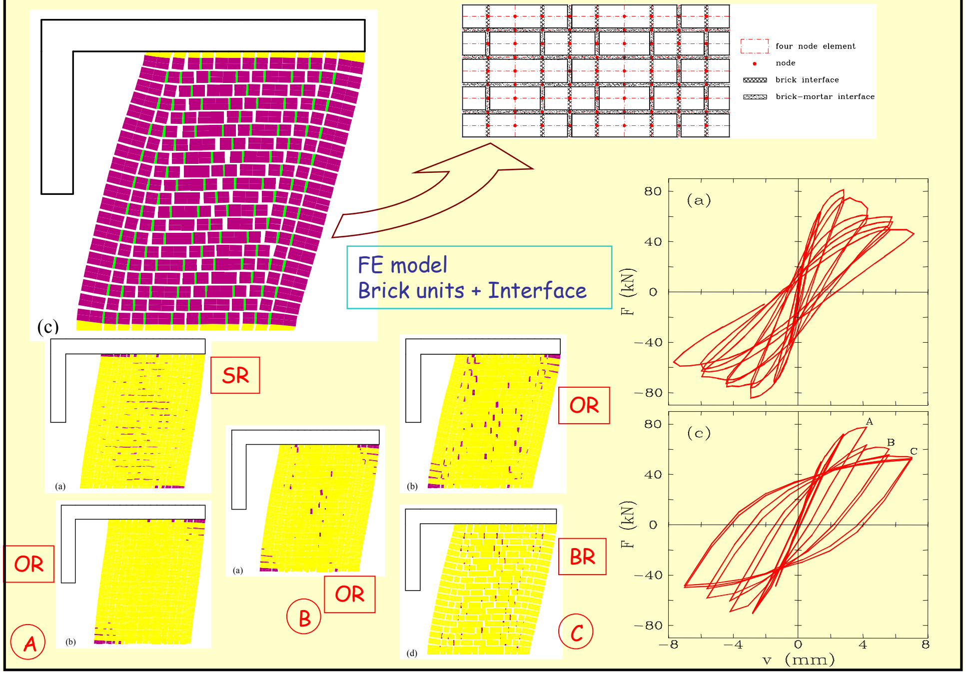




Simulation of experimental results (Atkinson et al)

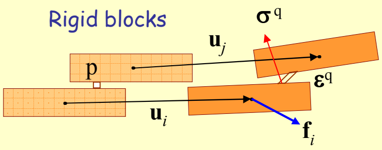


Masonry walls – Discrete models



Masonry walls – Discrete models

Formica et al., 2002



Lagrangian model: rigid block connected by damage frictional interfaces

$$\mathbf{u}_i = \{u_1 \quad u_2 \quad \varphi\}_i^T$$
$$\mathbf{f}_i = \{f_1 \quad f_2 \quad c\}_i^T$$

$$\boldsymbol{\varepsilon}^q = \{\varepsilon_n^q \quad \gamma^q\}^T$$ strain - stress at contact point q $$\boldsymbol{\sigma}^q = \{\sigma_n^q \quad \tau^q\}^T$$

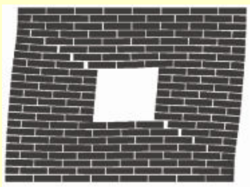
Compatibility

$$\boldsymbol{\varepsilon}^q - \mathbf{D}^q \mathbf{u} = \mathbf{0}$$

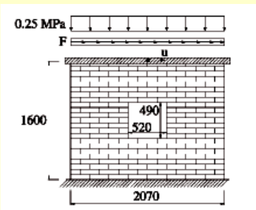
Equilibrium

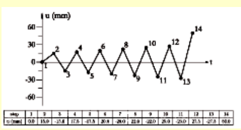
$$\sum \mathbf{D}^q \boldsymbol{\sigma}^q - \mathbf{f} = \mathbf{0}$$

\+ Coupled damage-frictional interface (Gambarotta e Lagomarsino, 1997)

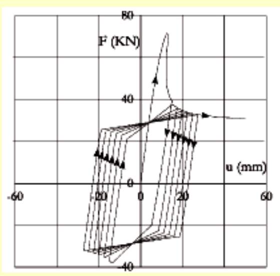


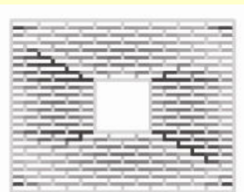
Final configuration





Mixed FE formulation
Arch-length iterative analysis

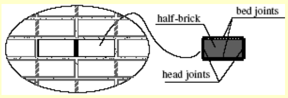




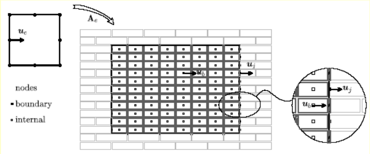
Final damage map

Discrete models – Multigrid/Multilevel approach

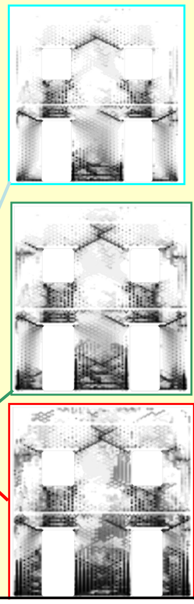
Casciaro *et al*, CMAME, 2007



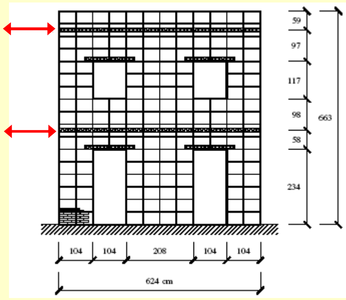
Rigid blocks + Interfaces

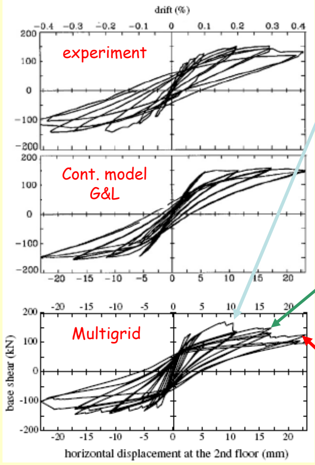


Damage

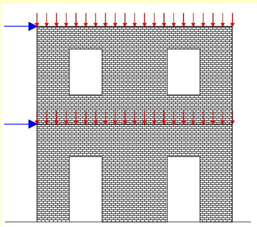


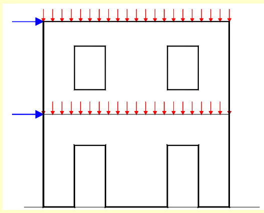
Simulation of Pavia experiment





Large masonry shear walls – 2D Cauchy equivalent continuum





Meso fields $\boldsymbol{\sigma}, \mathbf{u}, \boldsymbol{\varepsilon}, \boldsymbol{\zeta}$

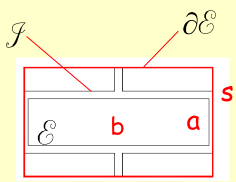
$\mathbf{u}(\mathbf{x}) = \mathbf{E}\mathbf{x} + \mathbf{u}_{per}$

$\mathbf{u}_{per}$ periodic on $\partial\mathscr{E}$

$div\boldsymbol{\sigma} = \mathbf{0}$ in $\mathscr{E}$

$\boldsymbol{\sigma}\mathbf{n}$ antiperiodic on $\partial\mathscr{E}$

$\|\boldsymbol{\sigma}\|\mathbf{n} = \mathbf{0}$ su $\mathscr{I}$



Periodic RVE

Macro fields $\boldsymbol{\Sigma}, \mathbf{E}, \mathbf{Z}$

$$\boldsymbol{\Sigma} = \frac{1}{A}\int_{\partial\mathscr{E}} \mathbf{x} \otimes \mathbf{t}\, ds$$

$$\mathbf{E} = \frac{1}{A}\int_{\partial\mathscr{E}} sym(\mathbf{u} \otimes \mathbf{n})\, ds$$

Meso – costitutive equations

Brick units $\boldsymbol{\sigma}_b \leftrightarrow \boldsymbol{\varepsilon}_b, \boldsymbol{\zeta}_b$

Mortar $\boldsymbol{\sigma}_m \leftrightarrow \boldsymbol{\varepsilon}_m, \boldsymbol{\zeta}_m$

Interface $\boldsymbol{\sigma}_i \leftrightarrow \boldsymbol{\varepsilon}_i, \boldsymbol{\zeta}_i$

ζ internal variables

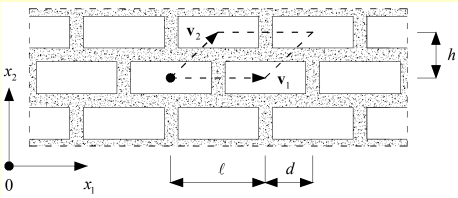
Macro – costitutive equations

$\boldsymbol{\Sigma} \leftrightarrow \mathbf{E}, \mathbf{Z}$

$\mathbf{Z}$ internal variables

Homogenization of periodic masonry   Anthoine, 1995, 2007

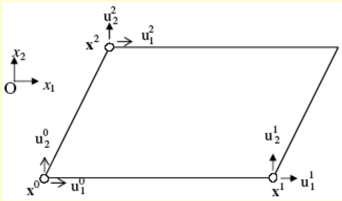
periodicity vectors $\mathbf{v}_1$ and $\mathbf{v}_2$



strain-periodic cinematically admissible displacement field

$$\mathbf{u}(\mathbf{x}) = \tilde{\mathbf{E}}\mathbf{x} + \tilde{\mathbf{W}}\mathbf{x} + \mathbf{u}^{p}(\mathbf{x})$$

$$\tilde{\mathbf{W}} = \mathbf{0}, \quad \mathbf{u}^{0} = \mathbf{0}$$



$$u_1(\mathbf{x}+\mathbf{v}_1) - u_1(\mathbf{x}) = u_1^1,$$
$$u_2(\mathbf{x}+\mathbf{v}_1) - u_2(\mathbf{x}) = u_2^1,$$
$$u_1(\mathbf{x}+\mathbf{v}_2) - u_1(\mathbf{x}) = \frac{d}{\ell}u_1^1 + \frac{h}{\ell}u_2^1 = u_1^2,$$
$$u_2(\mathbf{x}+\mathbf{v}_2) - u_2(\mathbf{x}) = u_2^2,$$

$$\tilde{\mathbf{T}} = \frac{1}{A}\int \mathbf{t}\otimes\mathbf{x}\, ds = \frac{1}{A}\sum_{i=0}^{2}\mathbf{f}_i\otimes\mathbf{x}_i = \mathbb{C}_{\text{hom}}\tilde{\mathbf{E}}$$

2D micropolar orthotropic continuum models – Periodic rigid block masonry

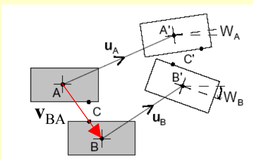
Besdo, Sulem & Műhlhaus, Di Carlo, Rizzi, Trovalusci & Masiani, Sulem & Vardoulakis .....

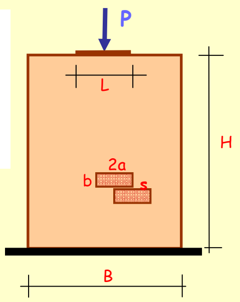
->Trovalusci & Masiani, 2003,05 Running bond 2D

- Lagrangian model: dofs block A $\mathbf{u}_\mathrm{A}$, $\mathbf{W}_\mathrm{A}$
- Homogeneity of the generalized displacements gradients $\mathbf{H}$ - $\mathbf{K}$ in the RVE

  $\mathbf{H} = grad\mathbf{u}$, $\mathbf{K} = grad\mathbf{W}$, $\quad \boldsymbol{\Gamma} = \mathbf{H} - \mathbf{W}$
- Relative generalized displacement at interface C

$$\begin{cases} \mathbf{u}_\mathrm{C} = \boldsymbol{\Gamma}\mathbf{v}_{BA} + (\mathbf{K}\mathbf{v}_{BX})\mathbf{v}_{CB} - (\mathbf{K}\mathbf{v}_{AX})\mathbf{v}_{CA} \\ \mathbf{W}_\mathrm{C} = \mathbf{K}\mathbf{v}_{BA} \end{cases}$$





- $\mathbf{t}_\mathrm{C}$, $\mathbf{T}_\mathrm{C}$ elastic generalized contact forces at interface C $\mathbf{u}_\mathrm{A}$, $\mathbf{W}_\mathrm{A}$
- Equivalence of the work expended per unit area in the Lagrangian and in the Polar continuum

$$\pi_{LAG}(\boldsymbol{\Gamma},\mathbf{K}) = \frac{1}{A}\sum_{C}\left(\mathbf{t}_C\cdot\mathbf{u}_C + \frac{1}{2}\mathbf{T}_C\cdot\mathbf{W}_C\right) = \pi_{HOM}(\boldsymbol{\Gamma},\mathbf{K}) = \boldsymbol{\Sigma}\cdot\boldsymbol{\Gamma} + \frac{1}{2}\mathbf{M}\cdot\mathbf{K}$$

- Average stress $\boldsymbol{\Sigma}$ and couple stress $\mathbf{M}$ tensors
- Constitutive equation in the case of centro-symmetric unit cell =>

$$\begin{cases} \boldsymbol{\Sigma} = \mathbb{Y}\boldsymbol{\Gamma} \\ \mathbf{M} = \mathbb{C}\mathbf{K} \end{cases}$$

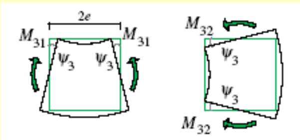
$$Y_{1111} = \delta^{-1}\left(k_t^{=} + 2k_n^{\parallel}\right), \qquad \delta = b/a$$
$$Y_{2222} = \delta k_n^{=}$$
$$Y_{1212} = \delta k_t^{=}$$
$$Y_{2121} = \delta^{-1}\left(k_n^{=} + 2k_t^{\parallel}\right)$$
$$C_{31} = a^2\left(4k_n^{=} + 3k_t^{=}\delta^2 + 2k_n^{\parallel}\delta^2 + 12k_t^{\parallel}\right)/12\delta$$
$$C_{32} = a^2\delta^2\left(4k_n^{=} + 3k_t^{=}\delta^2 + 12k_t^{\parallel}\right)/12$$

2D micropolar continuum models – Periodic masonry with elastic units and mortar

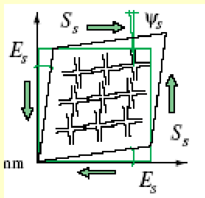
->Casolo, 2004,06 – Running bond

$$\Gamma_{11}=E_{11} \qquad E_{ii}=U_{i,i}$$
$$\Gamma_{22}=E_{22} \qquad E_{12}=\tfrac{1}{2}\left(U_{1,2}+U_{2,1}\right)$$
$$\Gamma_{12}=E_{12}+\theta \qquad \theta=\phi-\Omega$$
$$\Gamma_{21}=E_{12}-\theta \qquad \Omega=\tfrac{1}{2}\left(U_{2,1}-U_{1,2}\right)$$

$$\begin{Bmatrix}\Sigma_{11}\\ \Sigma_{22}\\ \Sigma_{12}\\ \Sigma_{21}\\ M_1\\ M_2\end{Bmatrix}=\begin{bmatrix}Y_{1111} & Y_{1122} & 0 & 0 & 0 & 0\\ Y_{2211} & Y_{2222} & 0 & 0 & 0 & 0\\ 0 & 0 & Y_{1212} & Y_{1221} & 0 & 0\\ 0 & 0 & Y_{2112} & Y_{2121} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{3131} & 0\\ 0 & 0 & 0 & 0 & 0 & C_{3232}\end{bmatrix}\begin{Bmatrix}E_{11}\\ E_{22}\\ E_{12}+\theta\\ E_{21}-\theta\\ K_1\\ K_2\end{Bmatrix}$$



(a)

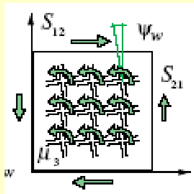


Prescribed displacements at the boundary of the RVE $\mathbf{u}=E_{12}sym(\mathbf{e}_1\otimes\mathbf{e}_2)\mathbf{x}+\mathbf{u}^p$

Evaluate:
Mean (symmetric) stress Σ^s_{12}

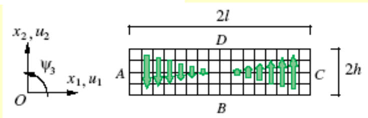
Block rotation (specifically defined) $\phi_s=\psi_s$

(b)



Prescribed rotation $\phi_w=\psi_w$ of each block with periodic boundary condition $\mathbf{u}=\mathbf{u}^p$

Evaluate:
Mean couple density
$$\mu_3=\Sigma_{21}-\Sigma_{12}$$
$$(E_{12}=\Omega=0)$$

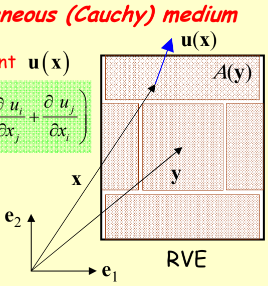


2D micropolar homogenization – Forest & Sab, 1998

Heterogeneous (Cauchy) medium

displacement $\mathbf{u}(\mathbf{x})$

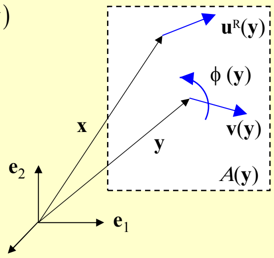
$$\varepsilon_{ij}(\mathbf{x})=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$$



Generalized (Cosserat) continuum

displacement $\mathbf{v}(\mathbf{y})$, $\phi(\mathbf{y})$

$$E_{ij}(\mathbf{y})=\frac{1}{2}\left(\frac{\partial v_i}{\partial y_j}+\frac{\partial v_j}{\partial y_i}\right)$$
$$\Omega(\mathbf{y})=\frac{1}{2}\left(\frac{\partial v_2}{\partial y_1}-\frac{\partial v_1}{\partial y_2}\right)$$
$$\theta(\mathbf{y})=\phi(\mathbf{y})-\Omega(\mathbf{y})$$
$$K_i(\mathbf{y})=\frac{\partial\phi}{\partial y_i}$$



$$\mathbf{u}^R(\mathbf{x})=\mathbf{v}(\mathbf{y})+\phi(\mathbf{y})\mathbf{e}_3\times(\mathbf{x}-\mathbf{y}) \quad \text{local rigid motion}$$

Characterization of the rigid motion that best fits the actual displacement field

$$min\int_A\left|\mathbf{u}(\mathbf{x})-\mathbf{v}(\mathbf{y})+\phi(\mathbf{y})\mathbf{e}_3\times(\mathbf{x}-\mathbf{y})\right|^2 da \Rightarrow$$

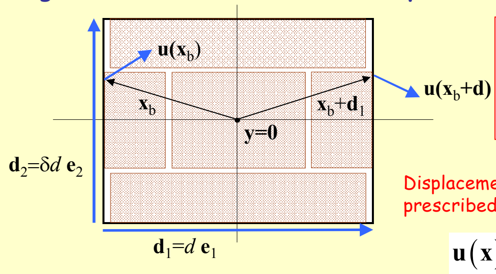
$$\mathbf{v}(\mathbf{y})=\frac{1}{A}\int_A\mathbf{u}da=\langle\mathbf{u}\rangle_A \qquad J_p=\int_A|\mathbf{x}-\mathbf{y}|^2da$$
$$\phi(\mathbf{y})=\frac{1}{J_p}\int_A\left[u_2(x_1-y_1)-u_1(x_2-y_2)\right]da$$

Generalized displacement components

Macro-displ gradient
$$\frac{\partial v_i}{\partial y_j}=\frac{1}{A}\int_A u_i n_j\,da=\left\langle\frac{\partial u_i}{\partial x_j}\right\rangle_A$$

Curvature
$$K_i=\frac{\partial\phi}{\partial y_i}=\frac{A}{J_p}\left[\frac{1}{A}\int_A\left(\frac{\partial u_2}{\partial x_j}x_1-\frac{\partial u_1}{\partial x_j}x_2\right)da-\frac{\partial v_2}{\partial y_j}y_1+\frac{\partial v_1}{\partial y_j}y_2\right]$$

Homogenization criterion based on equivalence of strain energy density in RVE



Rectangular RVE

$$min\int_A \Delta\mathbf{u}\cdot\mathbf{F}^T\mathbf{F}\Delta\mathbf{u}\,da$$

$$\Delta\mathbf{u}=\mathbf{u}^R(\mathbf{x})-\mathbf{u}(\mathbf{x})\quad \mathbf{F}=\mathbf{e}_1\otimes\mathbf{e}_1+\delta\,\mathbf{e}_2\otimes\mathbf{e}_2$$

Displacement field in the RVE and prescribed displacement at the boundary

$$\mathbf{u}(\mathbf{x})=\mathbf{u}^*(\mathbf{x})+\mathbf{u}^p(\mathbf{x})$$

- $\mathbf{u}^p(\mathbf{x})$ displacement field fulfilling the periodicity conditions at the boundary C

$$\mathbf{u}^p(\mathbf{x}_b)=\mathbf{u}^p(\mathbf{x}_b+\mathbf{d}_i)\quad i=1,2\qquad \mathbf{x}_b\in C$$

- $\mathbf{u}^*(\mathbf{x})$ displacement polynomial of grade 3 satisfying the following conditions:

$\mathbf{E}^*(\mathbf{y})=\boldsymbol{\varepsilon}^*(\mathbf{y})$ of grade 2 - **scale invariant**

$\phi^*(\mathbf{y})$ affine function

$$\mathbf{u}^*(\mathbf{x})=\begin{cases} u_1^*=B_{11}\tilde{x}_1+B_{12}\tilde{x}_2-C_{23}\tilde{x}_2^2+2C_{13}\tilde{x}_1\tilde{x}_2+D_{12}\left(\tilde{x}_2^3-3\tilde{x}_1^2\tilde{x}_2\right)\\ u_2^*=B_{21}\tilde{x}_1+B_{22}\tilde{x}_2-C_{13}\tilde{x}_1^2+2C_{23}\tilde{x}_1\tilde{x}_2-D_{12}\left(\tilde{x}_1^3-3\tilde{x}_1\tilde{x}_2^2\right)\end{cases}\qquad \tilde{x}_i=x_i/d$$

Boundary conditions on the RVE $\quad \mathbf{u}(\mathbf{x}_b+\mathbf{d}_i)-\mathbf{u}(\mathbf{x}_b)=\mathbf{u}^*(\mathbf{x}_b+\mathbf{d}_i)-\mathbf{u}^*(\mathbf{x}_b)\quad i=1,2\qquad \mathbf{x}_b\in C$

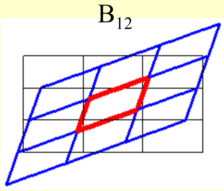
Cauchy continuum homogenization $\quad \mathbf{u}^*(\mathbf{x})=\mathbf{E}^*\mathbf{x}\quad$ $\mathbf{E}^*$ mean strain tensor

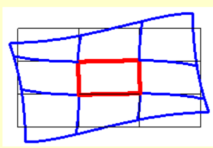
Components of generalized strain at the centre $\mathbf{y}=\mathbf{0}$ of the RVE

- 3 components of the symm part of the strain tensor $\quad E_{ij}(\mathbf{y}=\mathbf{0})=E_{ij}^*(\mathbf{y}=\mathbf{0})=\bar{E}_{ij}$
- 1 comp. of the skew part of the strain tensor $\quad \theta(\mathbf{y}=\mathbf{0})=\phi(\mathbf{y}=\mathbf{0})-\Omega(\mathbf{y}=\mathbf{0})=\bar{\theta}$
- 2 components of the torsion curvature tensor $\quad \mathbf{K}(\mathbf{y}=\mathbf{0})=\{\bar{K}_1\quad \bar{K}_2\}^T=\bar{\mathbf{K}}$

Assuming $\Omega(\mathbf{y}=\mathbf{0})=0$
=> $B_{12}=B_{21}$

B_{12}





D_{12}

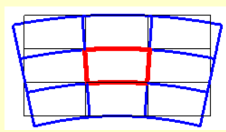
$$B_{11}/d=\bar{E}_{11},\quad B_{22}/d=\bar{E}_{22},\quad B_{12}/d=\bar{E}_{12}$$

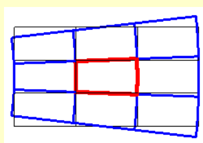
$$\frac{\delta^2}{10d}D_{12}+\frac{6}{\delta^3 d^4}\int_A\left(\delta^2 u_2^p x_1-u_1^p x_2\right)da=\bar{\theta}$$

$$-\frac{2}{d^2}C_{13}+\frac{6}{\delta d^4}\int_A u_{2,1}^p x_1\,da=\bar{K}_1$$

$$\frac{2}{d^2}C_{23}-\frac{6}{\delta^3 d^4}\int_A u_{1,2}^p x_2\,da=\bar{K}_2$$

C_{13}



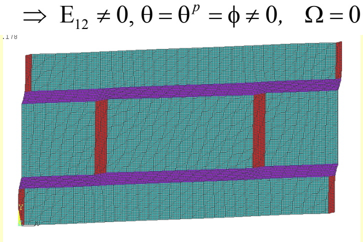


C_{23}

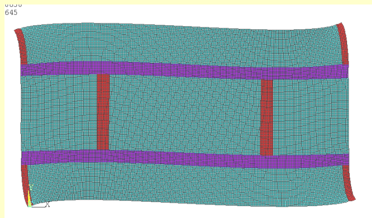
Homogenization of a regular pattern - English Bond

$B_{12} = 1$

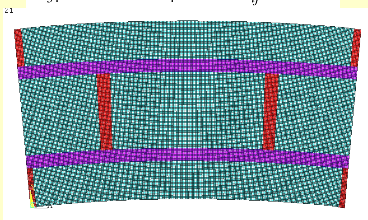
$\Rightarrow E_{12} \neq 0, \theta = \theta^p = \phi \neq 0, \quad \Omega = 0$



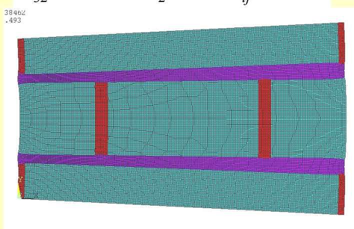
$D_{12} = 1 \quad \Rightarrow E_{12} = \Omega = 0, \quad \theta \neq 0$



$C_{31} = 1 \quad \Rightarrow K_1 \neq 0, \quad E_{ij} = \theta = 0$



$C_{32} = 1 \quad \Rightarrow K_2 \neq 0, \quad E_{ij} = \theta = 0$

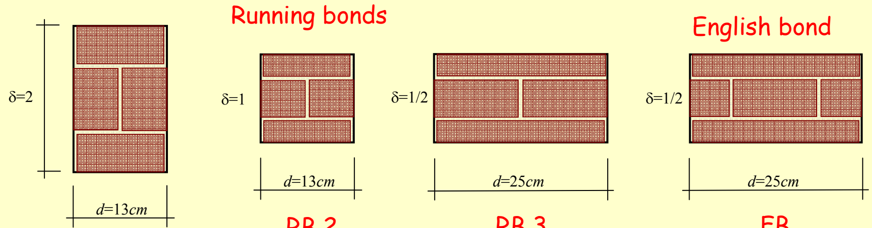


$E_b = 5000 MPa, \; E_m = 500 MPa, \; \nu_b = \nu_m = 0.1, \; s = 10mm$

Elastic generalized moduli for varying masonry patterns

Running bonds

English bond



RB 1 RB 2 RB 3 FB

$E_b = 5000 MPa, \; E_m = 500 MPa, \; \nu_b = \nu_m = 0.1, \; s = 10mm$

	Y_{1111} (Mpa)	Y_{2222} (Mpa)	Y_{1122} (Mpa)	Y_{1212} (Mpa)	Y_{2121} (Mpa)	Y_{1221} (Mpa)	C_{31} (N)	C_{32} (N)	ℓ_1 (mm)	ℓ_2 (mm)
RB1	2,865E+03	2,814E+03	2,205E+02	1,407E+04	1,021E+04	-9,899E+03	1,529E+07	4,169E+06	58	36
RB2	2,776E+03	2,098E+03	1,845E+02	7,130E+03	5,141E+03	-4,351E+03	4,553E+06	3,080E+06	36	35
RB3	3,466E+03	2,154E+03	1,990E+02	6,741E+03	4,770E+03	-3,838E+03	5,937E+06	1,203E+07	40	68
FB	3,101E+03	2,126E+03	1,911E+02	7,143E+03	4,461E+03	-3,876E+03	5,816E+06	1,180E+07	39	71

Y_{ijhk} scale invariant

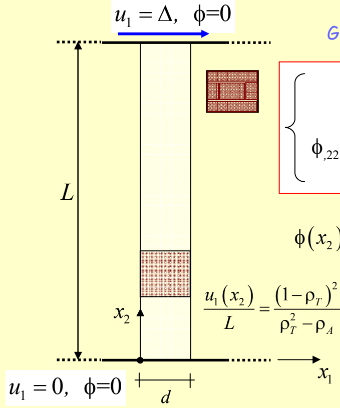
$$\frac{C_{3i}^d}{C_{3i}^D} = \left(\frac{d}{D}\right)^2$$

internal lengths

$$\ell_1 = \sqrt{\frac{Y_{2121} C_{31}}{Y_{1212} Y_{2121} - Y_{1221}^2}} \qquad \ell_2 = \sqrt{\frac{Y_{1212} C_{32}}{Y_{1212} Y_{2121} - Y_{1221}^2}}$$

Bacigalupo, Gambarotta, 2008

Shearing of a masonry walls – Boundary shear layer model



Generalized displacements $u_1(x_2), \ \phi(x_2) \quad u_2 = 0$

$$\begin{cases} \Sigma_{12} = Y_{1212}\left(u_{1,2} + \phi\right) = const \\ \phi_{,22} - \dfrac{Y_{1212}Y_{2121} - Y_{1221}^2}{Y_{1212}C_{23}}\phi = \dfrac{Y_{1212} - Y_{1221}}{Y_{1212}C_{23}}\Sigma_{12} \end{cases}$$

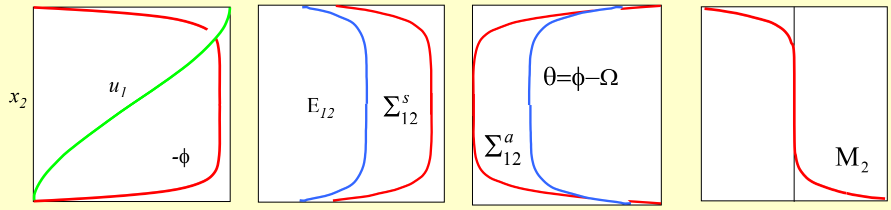
internal length $\mathbf{e_2}$

$$\ell_2 = \sqrt{\frac{Y_{1212}C_{32}}{Y_{1212}Y_{2121} - Y_{1221}^2}}$$

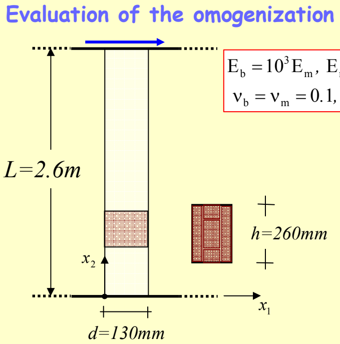
$$\phi(x_2) = -\frac{1-\rho_T}{\rho_T^2 - \rho_A}\left[cosh\left(\frac{x_2}{\ell_2}\right) + \frac{1 - cosh(L/\ell_2)}{sinh(L/\ell_2)} sinh\left(\frac{x_2}{\ell_2}\right) - 1\right]\frac{\Sigma_{12}}{Y_{1212}}$$

$$\frac{u_1(x_2)}{L} = \frac{(1-\rho_T)^2}{\rho_T^2 - \rho_A}\frac{\ell_2}{L}\left[sinh\left(\frac{x_2}{\ell_2}\right) + \frac{1 - cosh(L/\ell_2)}{sinh(L/\ell_2)}\left(cosh\left(\frac{x_2}{\ell_2}\right) - 1\right)\right]\frac{\Sigma_{12}}{Y_{1212}} + \left(1 - \frac{(1-\rho_T)^2}{\rho_T^2 - \rho_A}\right)\frac{x_2}{L}\frac{\Sigma_{12}}{Y_{1212}}$$

$$\rho_A = \frac{Y_{2121}}{Y_{1212}} \qquad \rho_T = \frac{Y_{1221}}{Y_{1212}}$$

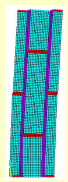


Evaluation of the omogenization of running bond masonry – vertical cell

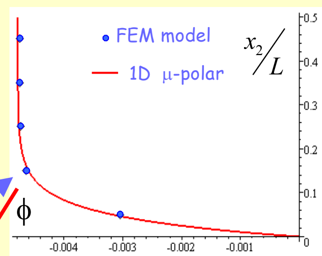


$E_b = 10^3 E_m, \ E_m = 500MPa,$
$\nu_b = \nu_m = 0.1, \ \ s = 10mm$

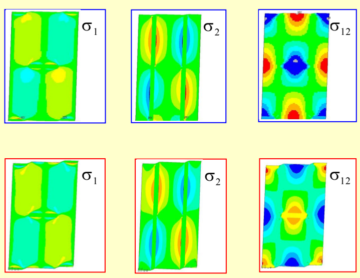
Y_{1212} (Mpa)	Y_{2121} (Mpa)	Y_{1221} (Mpa)	C_{32} (N)	ℓ_2 (mm)
1,079E+05	1,806E+05	-1,366E+05	1,091E+08	120

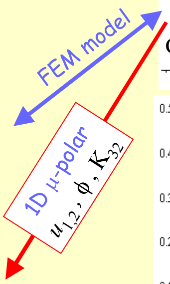


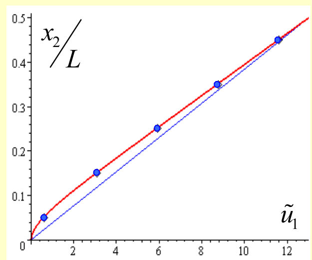
rotation



Stress components in 2nd cell

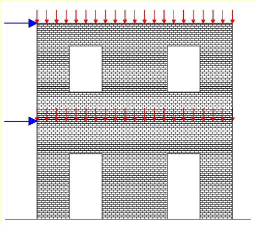


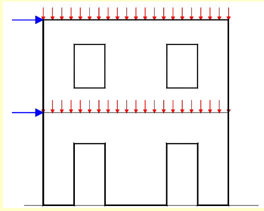




displacement

Damage in masonry shear walls





Meso fields $\boldsymbol{\sigma}, \mathbf{u}, \boldsymbol{\varepsilon}, \boldsymbol{\zeta}$

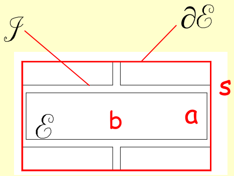
$\mathbf{u}(\mathbf{x}) = \mathbf{E}\mathbf{x} + \mathbf{u}_{per}$

$\mathbf{u}_{per}$ periodic on $\partial\mathcal{E}$

$div\boldsymbol{\sigma} = \mathbf{0}$ in $\mathcal{E}$

$\boldsymbol{\sigma}\mathbf{n}$ antiperiodic on $\partial\mathcal{E}$

$\|\boldsymbol{\sigma}\|\mathbf{n} = \mathbf{0}$ su $\mathcal{I}$



Periodic RVE

Macro fields $\boldsymbol{\Sigma}, \mathbf{E}, \mathbf{Z}$

$$\boldsymbol{\Sigma} = \frac{1}{A}\int_{\partial\mathcal{E}} \mathbf{x}\otimes\mathbf{t}ds$$

$$\mathbf{E} = \frac{1}{A}\int_{\partial\mathcal{E}} sym(\mathbf{u}\otimes\mathbf{n})ds$$

Meso – costitutive equations

Brick units $\boldsymbol{\sigma}_b \leftrightarrow \boldsymbol{\varepsilon}_b, \boldsymbol{\zeta}_b$

Mortar $\boldsymbol{\sigma}_m \leftrightarrow \boldsymbol{\varepsilon}_m, \boldsymbol{\zeta}_m$

Interface $\boldsymbol{\sigma}_i \leftrightarrow \boldsymbol{\varepsilon}_i, \boldsymbol{\zeta}_i$

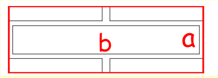
ζ internal variables

Macro – costitutive equations

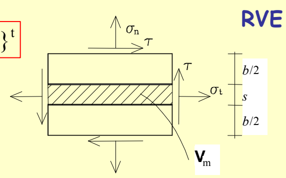
$\boldsymbol{\Sigma} \leftrightarrow \mathbf{E}, \mathbf{Z}$

$\mathbf{Z}$ internal variables

Continuum damage-frictional model - Layered model - $b \gg a$



(Gambarotta & Lagomarsino, 1997, Uva & Salerno, 2006)



$$\boldsymbol{\varepsilon} = \mathbf{K}_M\,\boldsymbol{\sigma} + \eta_m\boldsymbol{\varepsilon}^*_m + \eta_b\boldsymbol{\varepsilon}^*_b$$

$\boldsymbol{\varepsilon}^*_b = \{0\ \varepsilon_b\ \gamma_b\}^t$ Brick unit

$\boldsymbol{\varepsilon} = \{\varepsilon_1\ \varepsilon_2\ \gamma\}^t$ Mean strain

$\boldsymbol{\sigma} = \{\sigma_1\ \sigma_2\ \tau\}^t$ Mean stress

$\boldsymbol{\varepsilon}^*_m = \{0\ \varepsilon_m\ \gamma_m\}^t$ interface

Interface – damage & friction

$$\begin{cases} \varepsilon_m = c_{mn}\,\alpha_m\ H(\sigma_2)\,\sigma_2 \\ \gamma_m = c_{mt}\,\alpha_m\,(\tau - f) \end{cases}$$

Internal variables: α_m damage & f interface friction

Conjugate variables $\Longrightarrow$ $Y_m = \frac{1}{2}c_{mn}H(\sigma_2)\,\sigma_2^2 + \frac{1}{2}c_{mt}\ (\tau - f)^2$, γ_m

Brick unit – damage in compression

$$\begin{cases} \varepsilon_b = c_{bn}\,\alpha_b\,H(-\sigma_2)\,\sigma_2 \\ \gamma_b = c_{bt}\,\alpha_b\tau \end{cases}$$

Internal variable: α_b brick damage

Conjugate variable $\Longrightarrow$ $Y_b = \frac{1}{2}c_{bn}\ H(-\sigma_2)\,\sigma_2^2 + \frac{1}{2}c_{bt}\ \tau^2$

Limit conditions:

- **Damage**

$\phi_{dm} = Y_m - R_m(\alpha_m) \le 0$

$\phi_{db} = Y_b - R_b(\alpha_b) \le 0$

- **Friction**

$\phi_s = |f| + \mu\sigma_2 \le 0$

sliding

$\dot{\gamma}_m = v\dot{\lambda}$, $\dot{\lambda} \ge 0$

$v = f/|f|$

4. Continuum damage-friction model

Layered micro-model

(Gambarotta e Lagomarsino, 1997)

Evolution of the internal variables

$\sigma_2 \geq 0$

Opened interface

$$\phi_{dm} = \tfrac{1}{2} c_{mn} \sigma_2^2 + \tfrac{1}{2} c_{mt} \tau^2 - R_m(\alpha_m) \leq 0$$

$$\phi_{db} = \tfrac{1}{2} c_{bt} \tau^2 - R_b(\alpha_b) \leq 0$$

$$\begin{Bmatrix} \dot\phi_{dm} \\ \dot\phi_{db} \end{Bmatrix} = -\begin{bmatrix} R'_m & 0 \\ 0 & R'_b \end{bmatrix} \begin{Bmatrix} \dot\alpha_m \\ \dot\alpha_b \end{Bmatrix} + \begin{Bmatrix} c_{mn}\sigma_2\dot\sigma_2 + c_{mt}\tau\dot\tau \\ c_{bt}\tau\dot\tau \end{Bmatrix} \leq \mathbf{0}$$

$$\{\dot\phi_{dm} \quad \dot\phi_{db}\}\{\dot\alpha_m \quad \dot\alpha_b\}^t = \mathbf{0} \qquad \{\dot\alpha_m \quad \dot\alpha_b\}^t \geq \mathbf{0}$$

$\sigma_2 < 0$

Closed interface

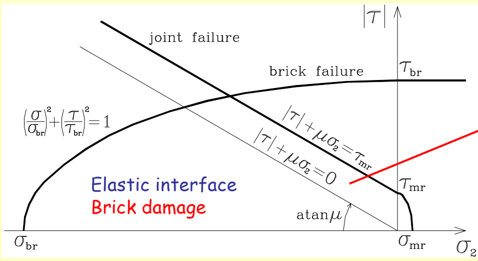
$$\phi_{dm} = \frac{1}{2}\frac{\gamma_m^2}{c_{mt}\alpha_m^2} - R_m(\alpha_m) \leq 0$$

$$\phi_s = \left|\tau - \frac{\gamma_m}{c_{mt}\alpha_m}\right| + \mu\sigma_2 \leq 0$$

$$\phi_{db} = \tfrac{1}{2} c_{bn} \sigma_2^2 + \tfrac{1}{2} c_{bt} \tau^2 - R_b(\alpha_b) \leq 0$$

$$\begin{Bmatrix} \dot\phi_{dm} \\ \dot\phi_s \\ \dot\phi_{db} \end{Bmatrix} = \begin{bmatrix} -\frac{\gamma_m^2}{c_{mt}\alpha_m^3} - R'_m & \frac{\nu\gamma_m}{c_{mt}\alpha_m^2} & 0 \\ \frac{\nu\gamma_m}{c_{mt}\alpha_m^2} & \frac{-1}{c_{mt}\alpha_m} & 0 \\ 0 & 0 & R'_b \end{bmatrix} \begin{Bmatrix} \dot\alpha_m \\ \dot\lambda \\ \dot\alpha_b \end{Bmatrix} + \begin{Bmatrix} 0 \\ \nu\dot\tau + \mu\dot\sigma_2 \\ c_{bn}\sigma_2\dot\sigma_2 + c_{bt}\tau\dot\tau \end{Bmatrix} \leq \mathbf{0}$$

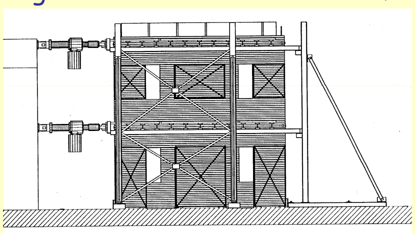
$$\{\dot\phi_{dm} \quad \dot\phi_s \quad \dot\phi_{db}\}\{\dot\alpha_m \quad \dot\lambda \quad \dot\alpha_b\}^t = \mathbf{0} \qquad \{\dot\alpha_m \quad \dot\lambda \quad \dot\alpha_b\}^t \geq \mathbf{0}$$



Limit states

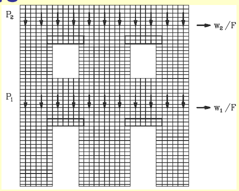
Damage in the interface and brick units

Large shear walls – simulation of experimental results

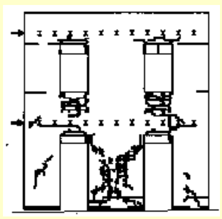


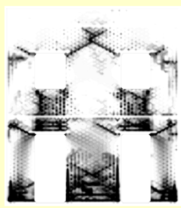
Brick masonry wall tested in Pavia, Magenes et al. (1994).

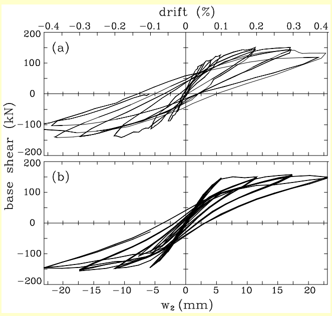
FEM model



Damage (exp)





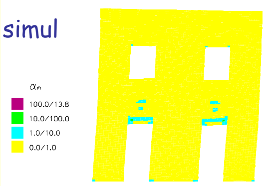


Cyclic response of the *door wall*: a) experimental; b) numerical simulation.

exp

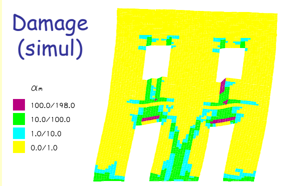
DRIFT 0.1%

simul

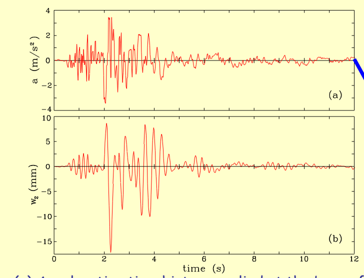


DRIFT 0.3%

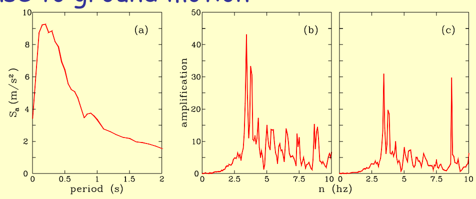
Damage (simul)



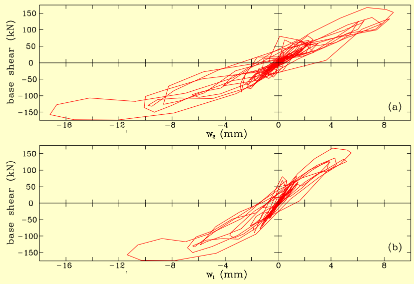
Large shear walls – dynamic response to ground motion



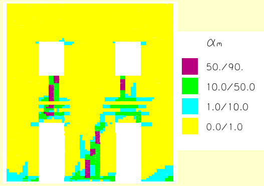
(a) Acceleration time history applied at the base of the wall.
(b) Displacement time history on the second floor.



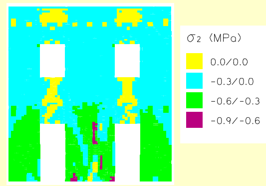
(a)Acceleration response spectrum of the input base motion. Amplification function with respect to the base of the wall: (b) first floor displacement, (c) second floor displacement.



Cyclic response of the large scale wall: (a) second floor; (b) first floor.



Damage

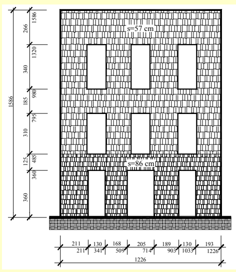


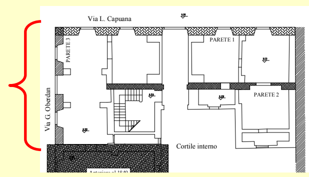
Stress

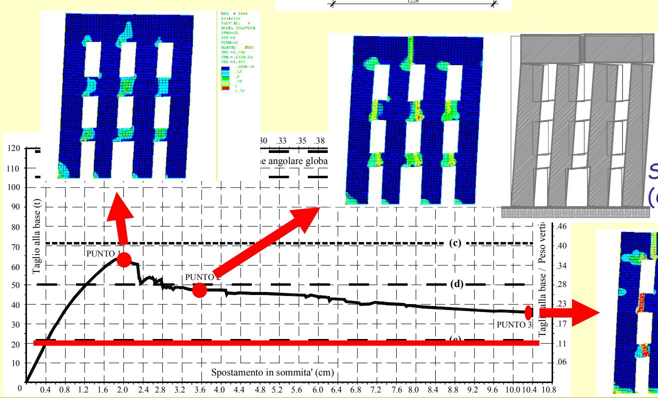
Large shear walls – response to horizontal forces

Brencich et al, 2001
Masonry building in Catania
GNDT

Horizontal forces superimposed on vertical dead loads

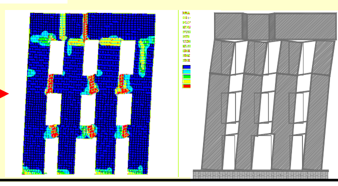




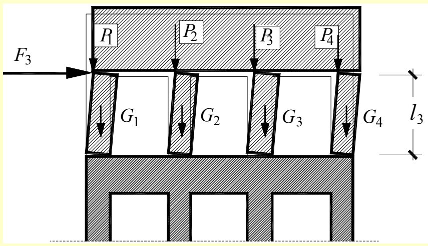


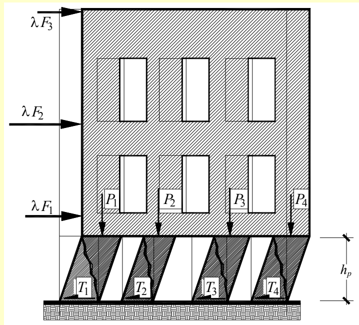
Anthoine et al., 2003

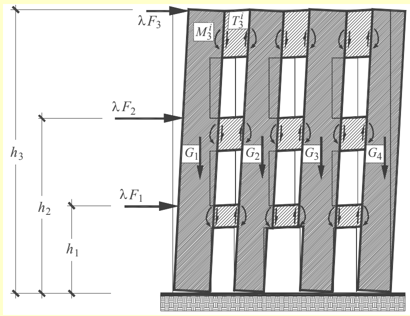
Simplified collapse mechanism (Como e Grimaldi)

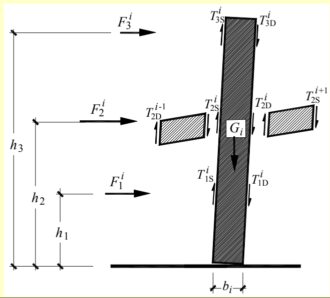


4. Large shear walls – simplified approaches

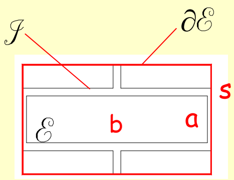








Modelling inelastic response of shear walls – Cauchy continuum models



Representative cell

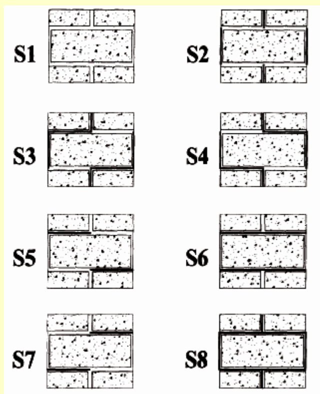
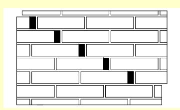


Fig. 2. Possible damaged states of old masonry material.

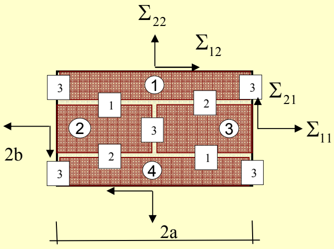
Limit Analysis
Alpa & Monetto, 1994
De Felice & de Buhan, 1995
Milani et al, 2006

Damage models
Luciano e Sacco, 1997
Massart et al, 2004-



Strain localization

Limit strength domains by homogenization of the periodic unit cell

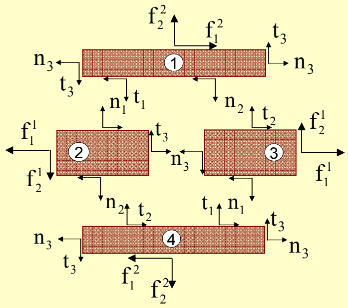


Macro-stress components

$$\Sigma_{11} = \frac{1}{2} f_1^1 + \frac{1}{2} n_3$$

$$\Sigma_{22} = f_2^2$$

$$\Sigma_{12} = f_1^1 = \Sigma_{22} = \frac{1}{2} f_2^1 + \frac{1}{2} t_3$$



Equilibrium of block1 1 & 2

$$t_1 + t_2 - 2\Sigma_{12} = 0$$

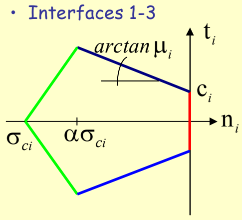
$$n_1 + n_2 - 2\Sigma_{22} = 0$$

$$t_1 - t_2 + 2\frac{b}{a} n_3 - 2\frac{b}{a}\Sigma_{11} = 0$$

$$n_1 - n_2 + 2\frac{b}{a} t_3 - 2\frac{b}{a}\Sigma_{12} = 0$$

Limit domains

- Interfaces 1-3



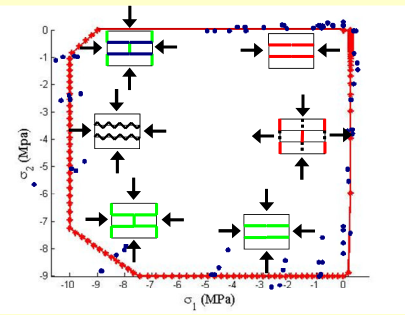
- Brick tensile strength

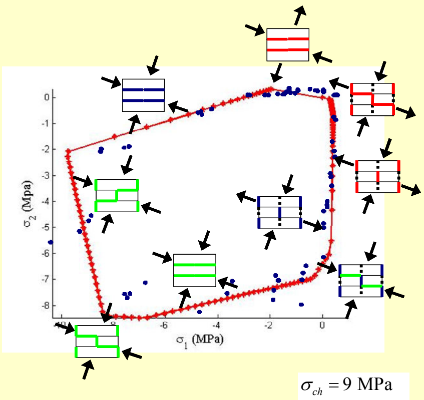
$$\frac{f_1^1}{f_{bt}} + \frac{|f_2^1|}{f_{bs}} - 1 \le 0$$

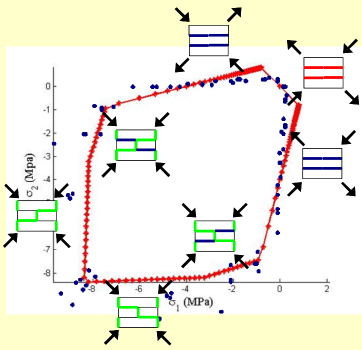
- Masonry compressive strength parallel to the bed joints

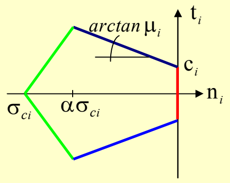
$$\Sigma_{11} + \sigma_{c1} \ge 0$$

Limit strength domains: simulation of experimental results by Page





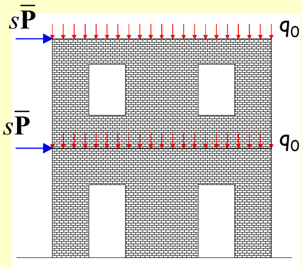




$\sigma_{ch} = 9$ MPa
$\mu_h = 0.6$
$c_h = 0.8$ MPa
$\sigma_{cv} = 7.5$ MPa
$\mu_v = 0.5$
$c_v = 0.8$ MPa
$f_{bt} = 0.5$ MPa
$f_{bta} = 1.1$ MPa
$f_{c1m} = 10$ MPa

Bacigalupo, Gambarotta, 2008

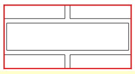
Shear walls – Multi-scale limit analysis (cohesive-frictional models)



Admissible macro-stress fields (Suquet, 1983)

Macro Σ, E

micro σ, ε



$$S^{hom} = \left\{ \Sigma \;\middle|\; \begin{array}{l} \Sigma = \frac{1}{A}\int_{\partial \mathcal{E}} \mathbf{x} \otimes \mathbf{t}\, ds \\ div\,\sigma = 0 \quad \forall \mathbf{x} \in \mathcal{E} \\ \|\sigma\|\mathbf{n} = \mathbf{0} \quad \forall \mathbf{x} \in \mathcal{J} \\ \sigma\mathbf{n} \ \text{anti-periodic on } \partial\mathcal{E} \\ \sigma(\mathbf{x}) \in S^{\alpha} \quad \forall \mathbf{x} \in \mathcal{E}^{\alpha},\ \alpha = \text{b, m} \\ \sigma(\mathbf{x}) \in S^{i} \quad \forall \mathbf{x} \in \mathcal{J} \end{array} \right\}$$

- Alpa Monetto, 1994, De Buhan, De Felice, 1997
  S^b, S^m unbounded, S^i Coulomb

Dual kinematic definition of S^{hom}

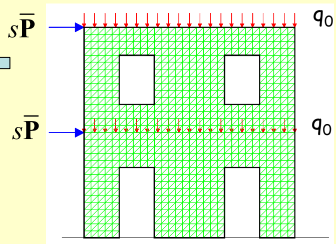
- Milani et al, 2006
  S^b, S^m Mohr-Coulomb – cut-off
  S^i not active?????

Lower bound

$$\begin{cases} s_L = \max(s_s), \\ \mathbf{C}\Sigma_V = \mathbf{c}, \\ \mathbf{Q}\Sigma_V - s_s\bar{\mathbf{q}} = \mathbf{q}_0, \\ \mathbf{Y}^T\Sigma_V \le \mathbf{y}. \end{cases}$$

Upper bound

$$\begin{cases} s_U = \min(s_k) = \min\left(-\mathbf{q}_{0l}^T\mathbf{a} + \mathbf{z}^T\dot{\boldsymbol{\lambda}}\right), \\ \mathbf{Ba} - \mathbf{Z}^T\dot{\boldsymbol{\lambda}} = \mathbf{0} \\ \mathbf{Aa} = \mathbf{0}, \\ \bar{\mathbf{q}}_l^T\mathbf{a} = 1, \\ \dot{\boldsymbol{\lambda}} \ge \mathbf{0}. \end{cases}$$

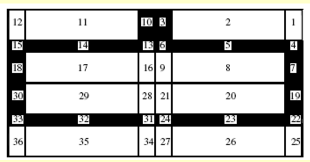


FE discretization

- Equilibrium model
- Compatible model

Anderheggen e Knöpfel, Sloan & coworkers, Pastor et al., Maier & coworkers.... ......

Limit strength domains: simulation of experimental results by Page



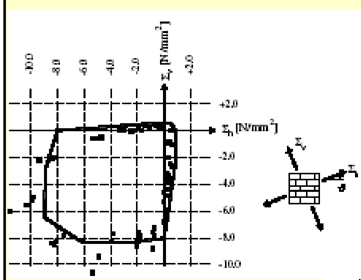
Representative cell and sub-domains

Polynomial distributions assumed for the stress components at each sub-domain

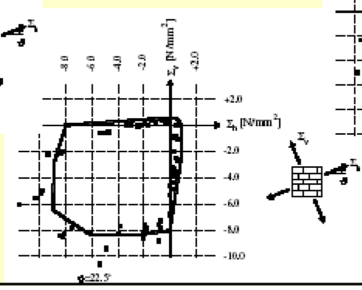
$$\sigma_{ij}^{(k)} = \mathbf{z}(\mathbf{x})^T \mathbf{s}_{ij} \qquad \mathbf{x} \in Y^k$$

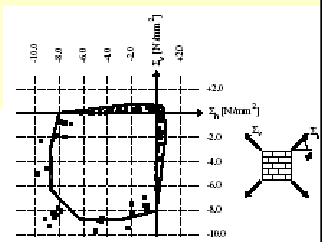
Equilibrium and anti-periodicity of $\sigma\mathbf{n}$ on $\partial\mathcal{E}$

Statical admissibility of the stress field

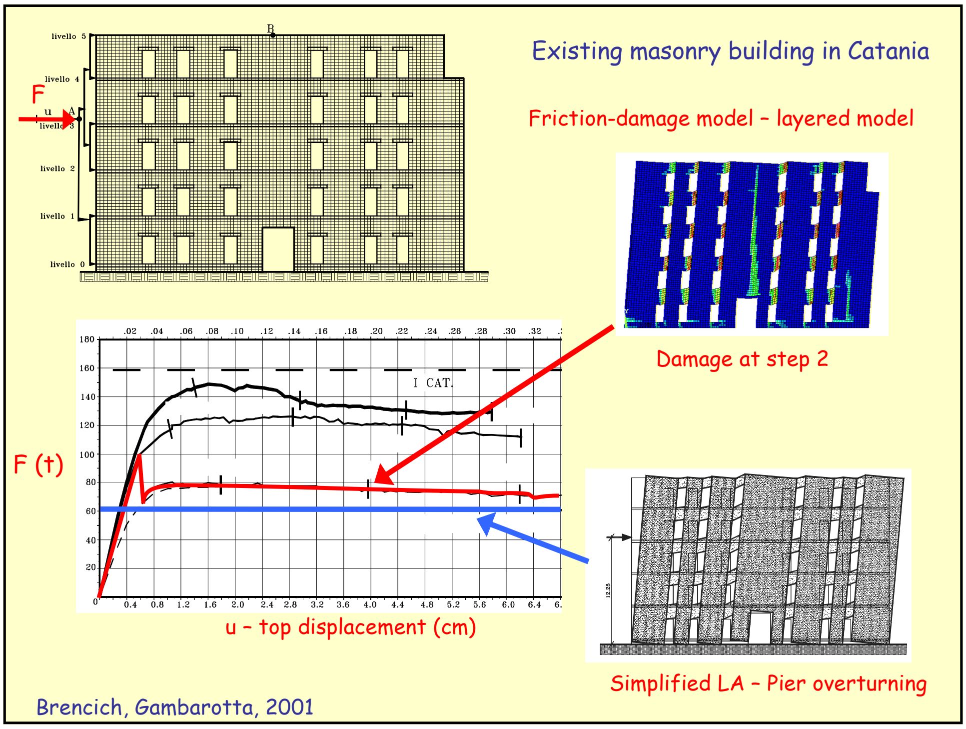


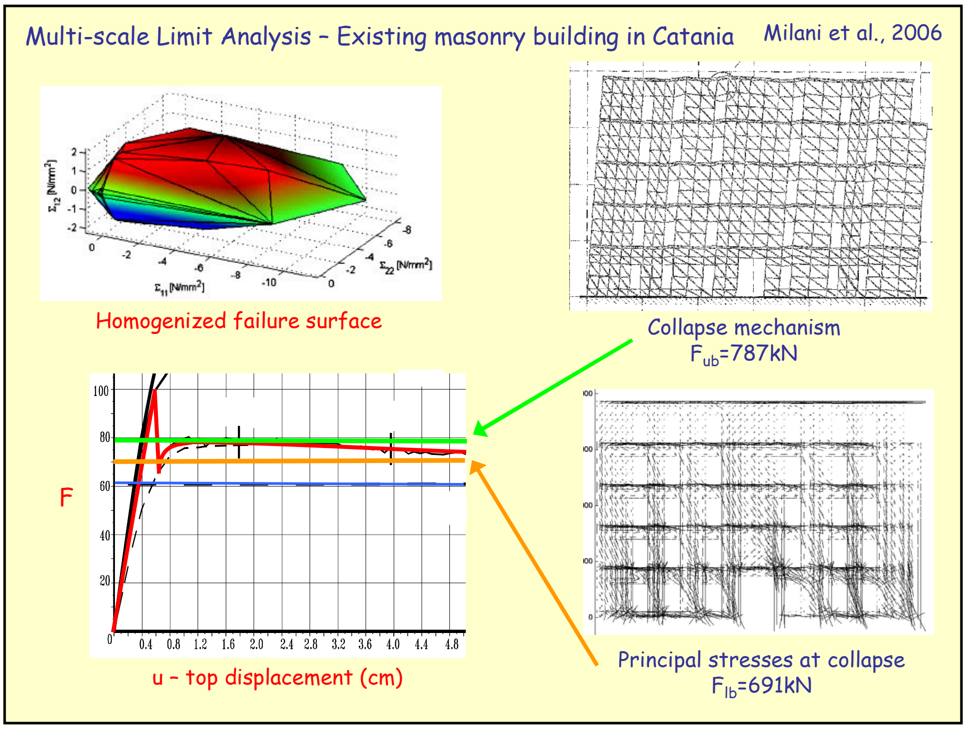
Biaxial limit domains





Milani, Tralli, 2007

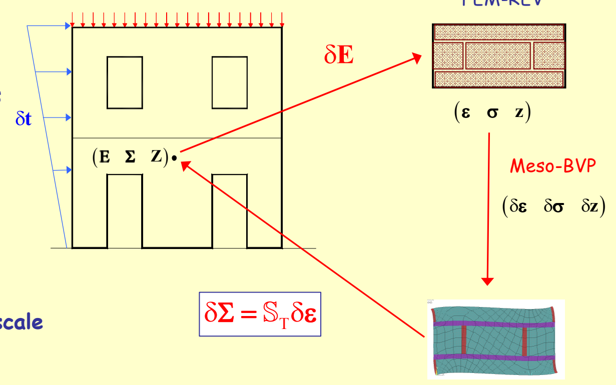

Existing masonry building in Catania
Friction-damage model – layered model
F
u
livello 5
livello 4
livello 3
livello 2
livello 1
livello 0
Damage at step 2
F (t)
I CAT.
u – top displacement (cm)
Simplified LA – Pier overturning
Brencich, Gambarotta, 2001


Multi-scale Limit Analysis – Existing masonry building in Catania
Milani et al., 2006
Homogenized failure surface
Collapse mechanism
F_{ub}=787kN
F
u – top displacement (cm)
Principal stresses at collapse
F_{lb}=691kN

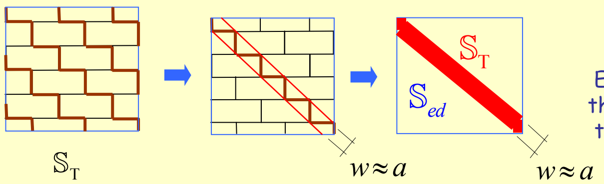
Multi-scale analysis of damaging shear walls

Massart et al., 2007

Anisotropy evolution effects combined with a mesoscopically motivated regularization technique for damage localization



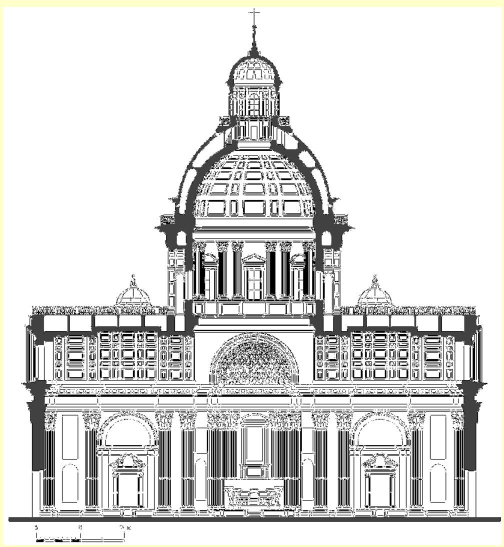
Localization at the M-scale

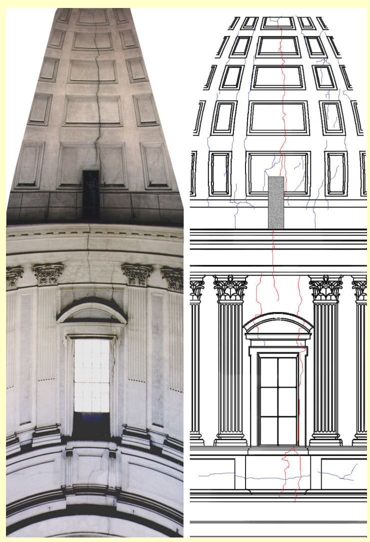


Embedded localization band: the width is determined from the periodicity of the meso-structure

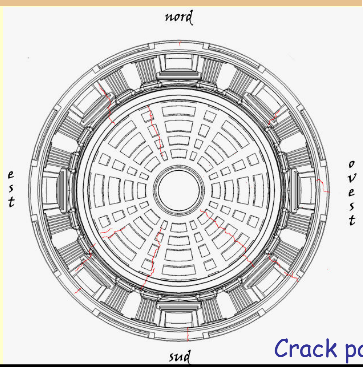
Masonry domes

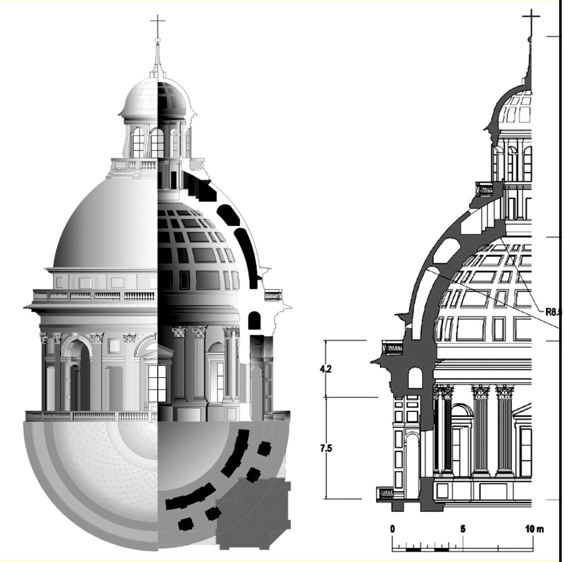
Basilica di S. Maria di Carignano - Genova





Dome-drum interaction: Basilica di Carignano in Genova (G. Alessi, 1540-1600)





Crack pattern in the inner dome (from below)

Basilica di Carignano: Safe theorem

Statically admiddible states

Hypotheses

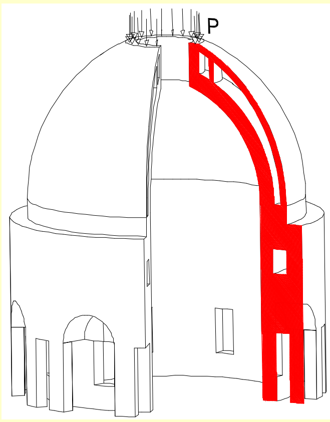
- NTR material
- Infinite compressive strength
- No sliding failures admitted

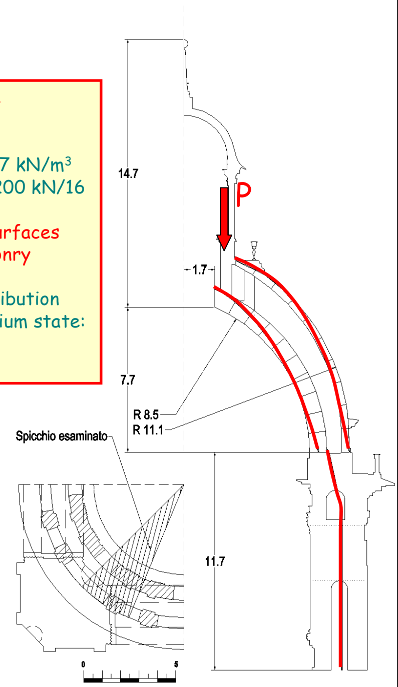
Equilibrium of a slice

Loads:
- masonry weight γ=17 kN/m^3
- lantern weight P=1200 kN/16

Search for thrust surfaces lying within the masonry

Lantern weight distribution for the safe equilibrium state:
85% inner shell
15% outer shell





Gambarotta et al., 2002

Upper Bound Theorem

If $\exists\, \dot{\mathbf{u}} \in KinAdm$ such that:

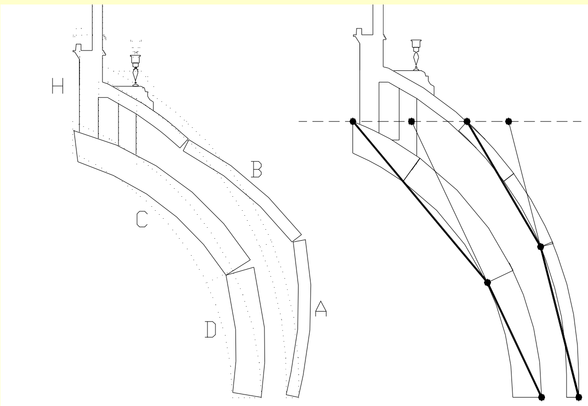
$$\dot{W} = \int_{\mathscr{B}^-} \mathbf{b} \bullet \dot{\mathbf{u}}^- dv + \int_{\mathscr{B}^+} \mathbf{b} \bullet \dot{\mathbf{u}}^+ dv = \dot{W}_a + \dot{W}_{res} \geq 0$$

⇒ The structure will collapse

$\mathbf{b}$ - *unit volume weigth*

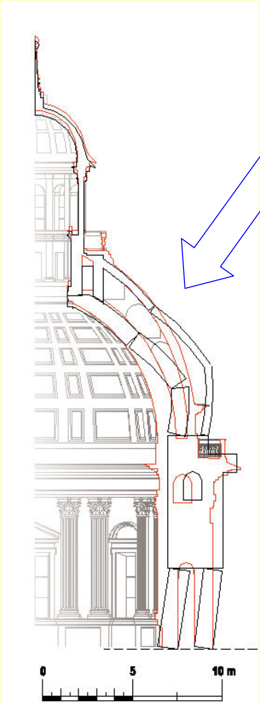
$\mathbf{u}^+$ - *upward velocity*

$\mathbf{u}^-$ - *downward velocity*



1. Local mechanism
Inner and outer domes

$$\eta_1 = \frac{|\dot{W}_{res}|}{\dot{W}_a} \approx 2 > 1 \Rightarrow \dot{W} < 0$$



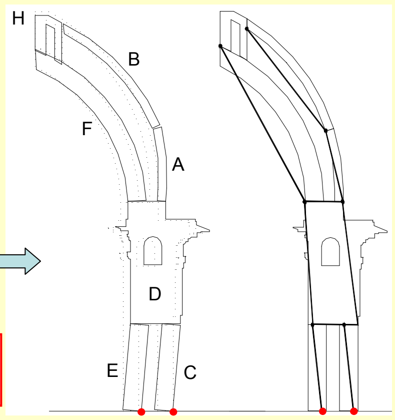
Overall Mechanism domes-drum int.

Global mechanism 1.

$$\eta_2 = \frac{|\dot{W}_{res}|}{\dot{W}_a} \approx 8.5$$

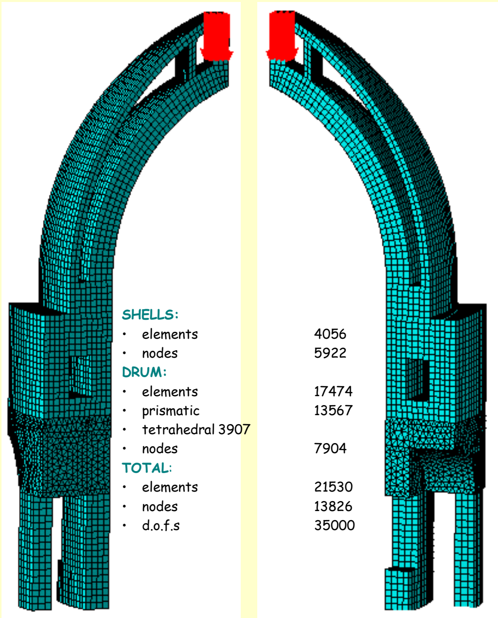
Global mechanism 2.

$$\eta_3 = \frac{|\dot{W}_{res}|}{\dot{W}_a} \approx 1.8 \div 7$$



Influence of the compressive strength on the location of the centre of rotation of the drum slice

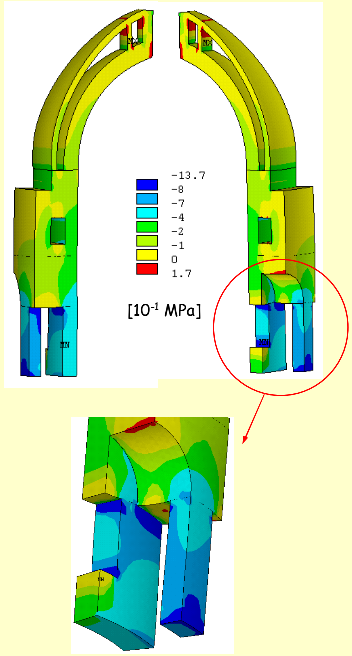
FE Model –1/8 slice



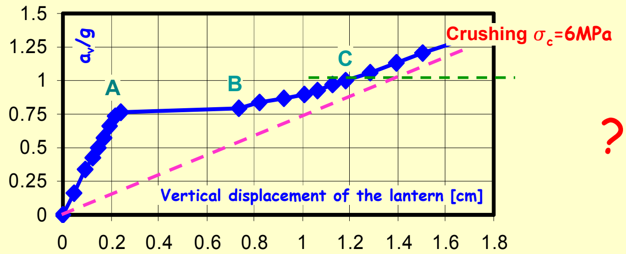
SHELLS:
- elements 4056
- nodes 5922

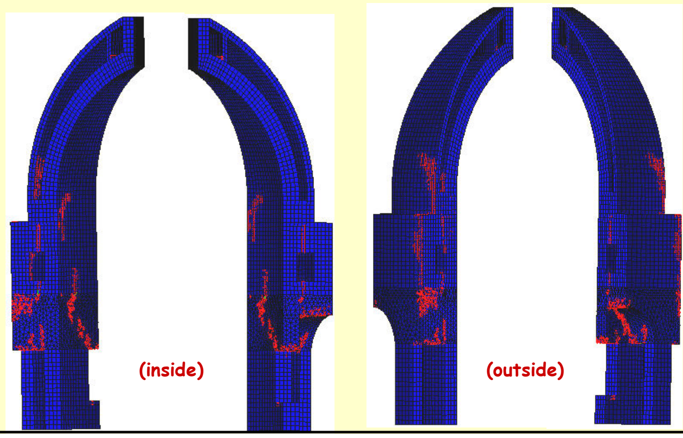
DRUM:
- elements 17474
- prismatic 13567
- tetrahedral 3907
- nodes 7904

TOTAL:
- elements 21530
- nodes 13826
- d.o.f.s 35000



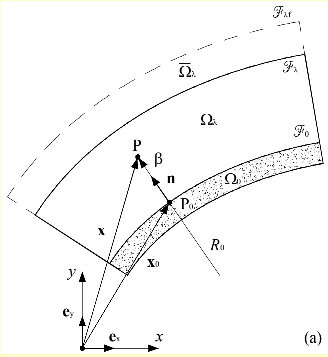
Incremental analysis



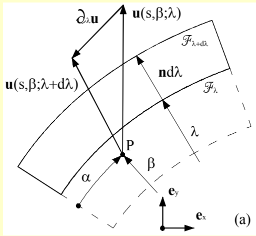


Crack pattern
State A

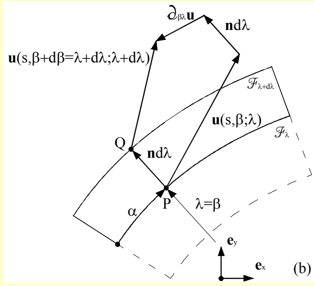
Influence of the construction sequence – structural growth



Reference domain



displacement increment due to growth



displacement increment accretion line

Stess field

$$\mathbf{T}(s,\beta;\lambda_f)=\int_\beta^{\lambda_f}\mathbb{C}\, sym\nabla\mathbf{g}(s,\beta,\lambda)\,\mathrm{d}\lambda$$

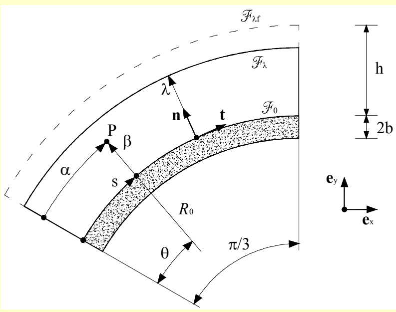
Strain field

$$\mathbf{E}(s,\beta;\lambda)=sym\nabla\overline{\mathbf{u}}_0(s)+\int_0^\beta sym\nabla\overline{\mathbf{g}}(s,\beta=\lambda;\lambda)\,\mathrm{d}\lambda+\left[\overline{\mathbf{g}}(s,\beta;\lambda=\beta)-\mathbf{g}(s,\beta;\lambda=\beta)\right]\odot\mathbf{n}+$$

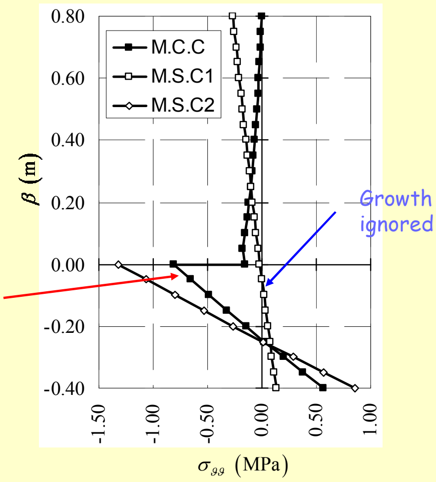
$$+\int_\beta^\lambda sym\nabla\mathbf{g}(s,\beta;\lambda')\,\mathrm{d}\lambda'$$

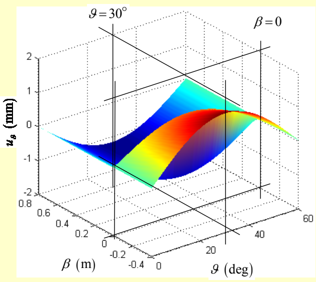
Influence of the construction sequence – structural growth

Example: Triumphal arch



Normal stresses at crown section





Displacement field
Tangential component

Bacigalupo, Gambarotta, 2008

Acknowledgement

Andrea, Bacigalupo, Antonio Brencich, Andrea Cavicchi, Christian Corradi,

Sergio Lagomarsino, Renata Morbiducci, Enrico Sterpi

Thank You for Listening