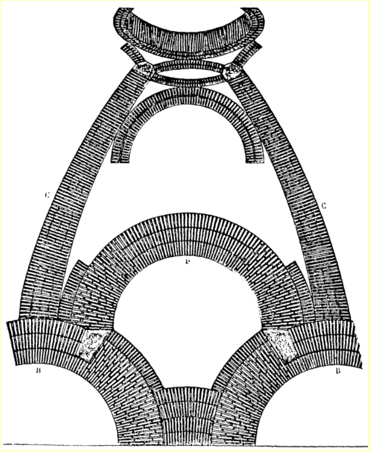
ad Alfredo Corsanego



Piranesi: Pantheon (Choisy)

Archi, pareti, volte: quali modelli per le costruzioni in muratura?

Luigi Gambarotta

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Università di Genova
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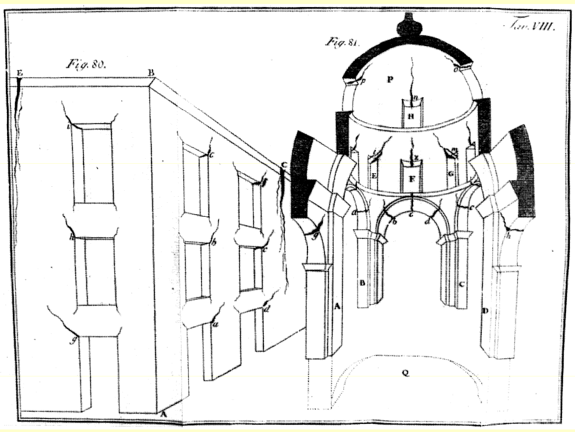
Historic masonry constructions: from damage to safety



V. Lamberti, Statica degli edifici, Napoli, 1781

1. Knowledge about historic constructions:
- Historical investigation
- Construction techniques and materials
- Survey-damage

2. Mechanical modeling:
- Simulation
- interpretation of damage – diagnosis
- Safety evaluation
- Evaluation of strengthening techniques

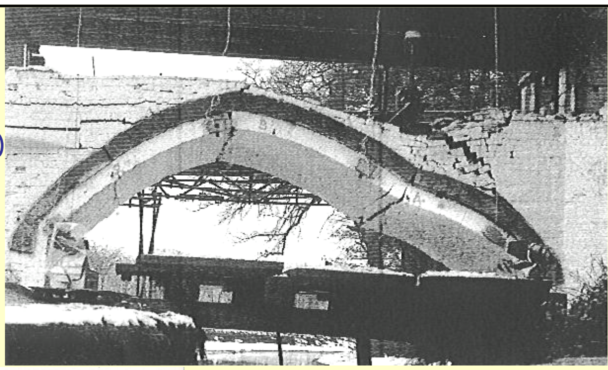
3. Design
- Retrofitting (if required)
- Monitoring

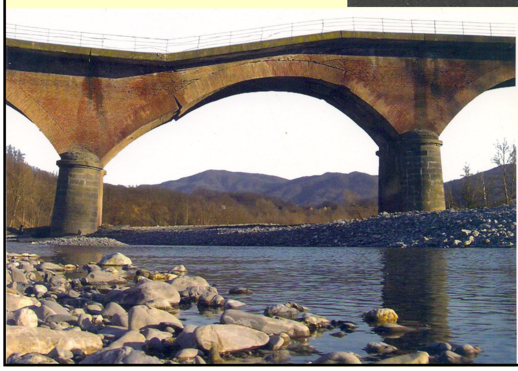
Arches

Umbria-Marche Earthquake, 1997

Masonry bridges

Prestwood
Bridge (Page, 1993)





Road bridge
Arquata S., Alessandria

Masonry walls

Out-of-plane collapse

Umbria-Marche Earthquake, Colfiorito, 1997

Masonry walls

In-plane collapse

Umbria-Marche Earthquake, Colfiorito, 1997

Vaults

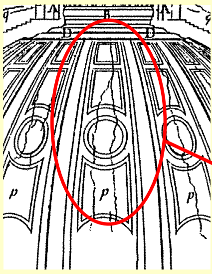
South Piedmont
Earthquake, 2003

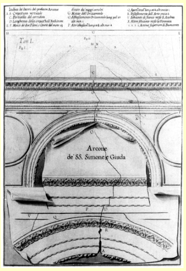
Umbria-Marche Earthquake, 1997

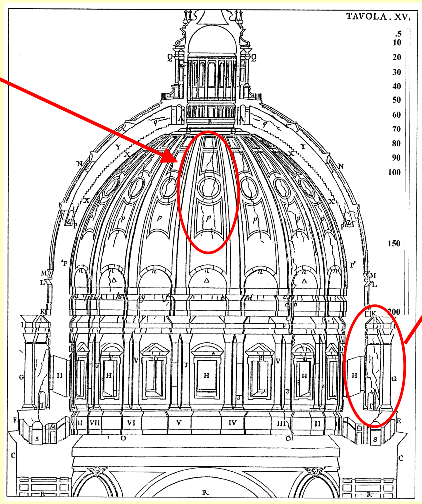
S. Pietro Dome in Roma

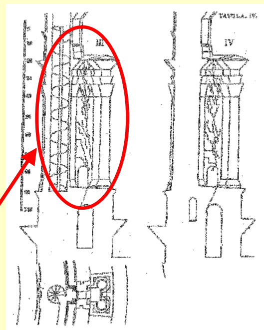
Michelangelo - Della Porta e Fontana, 1590

- Boscovich, Le Seur, Jacquier, 1743
- Poleni, 1748 – Vanvitelli
- Burri, Beltrami, Di Stefano, Como

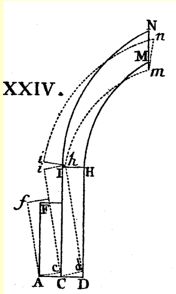


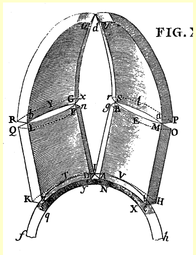


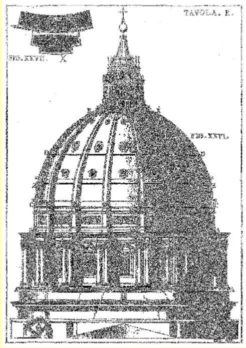


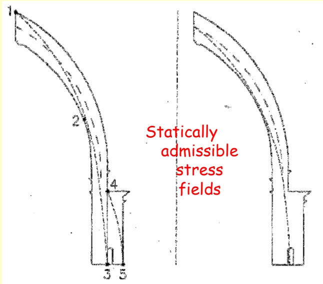


Collapse mechanism
"Tre Matematici"
Boscovich, Le Seur, Jacquier,
1743





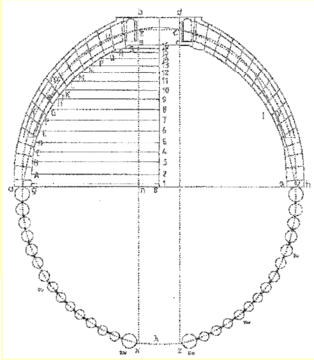


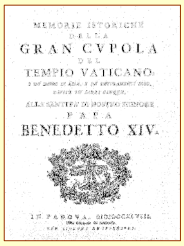


Least abutment thrust, Como, 1998

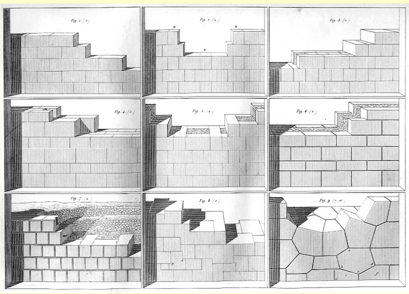
Elastic NTR solution Como, 1998

Equilibrium analysis, Poleni
1748

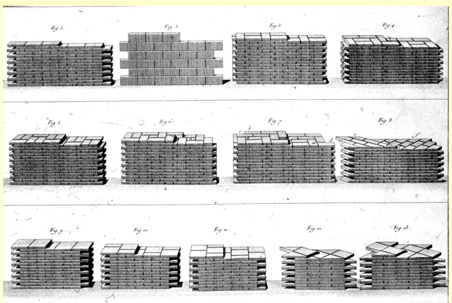


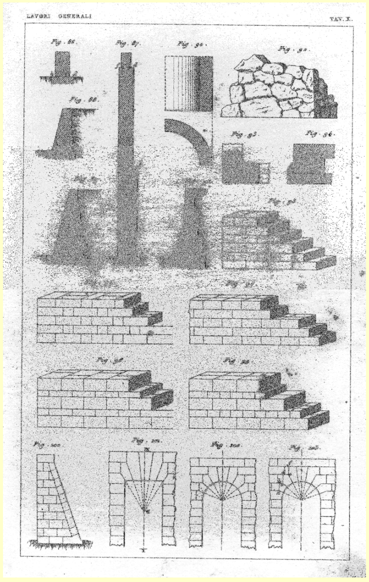


Ancient building construction techniques
and rules of practice



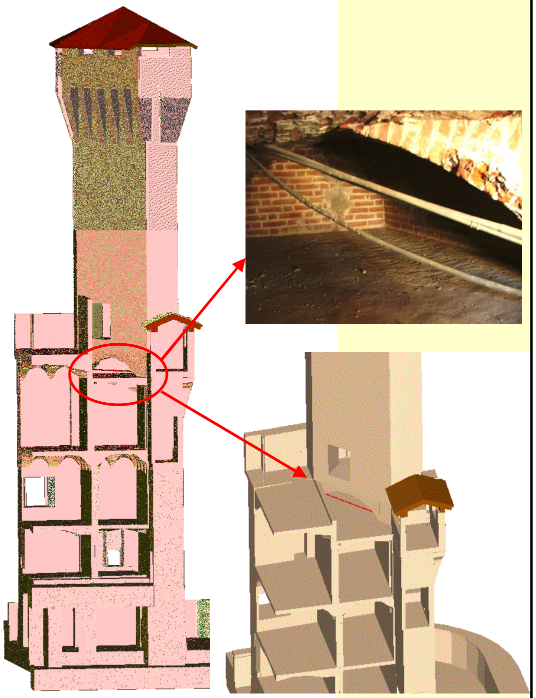
Rondelet, 1817





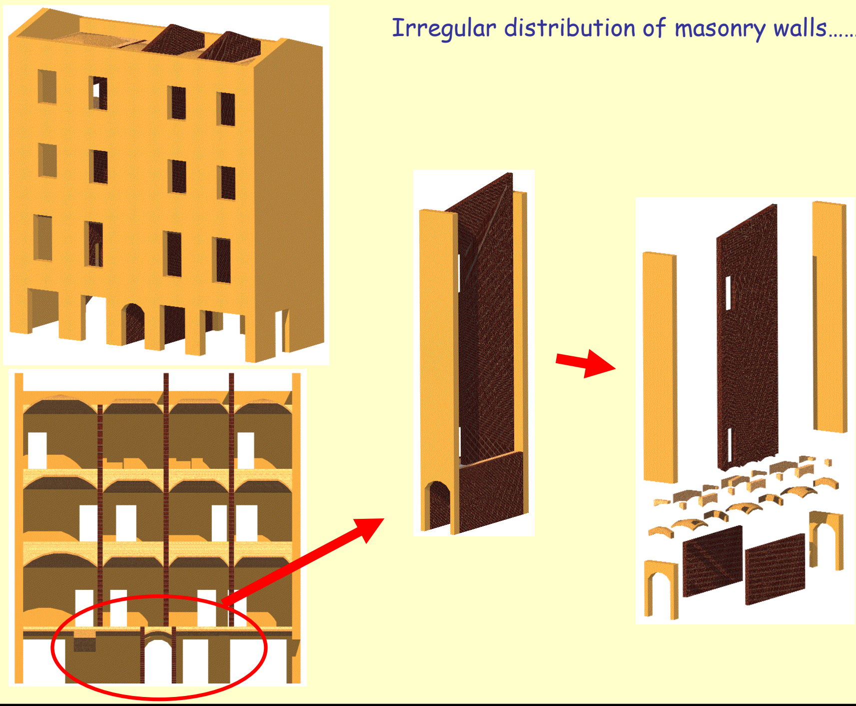
Curioni, 1885

In the absence of rules……



S. Cristoforo Castle, Piedmont

Irregular distribution of masonry walls…….



Modeling: general aspects

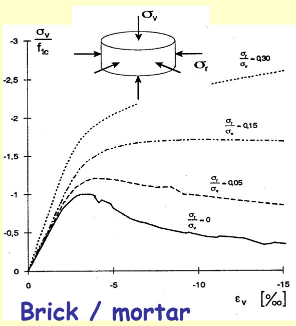
The masonry material

- heterogeneous material (periodic – random bond pattern)
- components: brick unit, stone block, mortar layer
- quasi-brittle behavior
- different types of bond pattern – thick masonry walls
- large variability of mechanical parameters
- to be calibrated by *in situ* & laboratory tests
- constitutive modeling based on the geometry and pattern of the components and their constitutive models

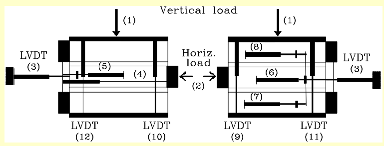
The masonry construction

- Construction vs. Structure
- Interaction among vaults, walls, columns, arches, etc.
- Building – foundation interactions
- Modification and extension of the construction during its life
- Building to building interaction (Historic centers and urban aggregations)

Experimental response of the constituents

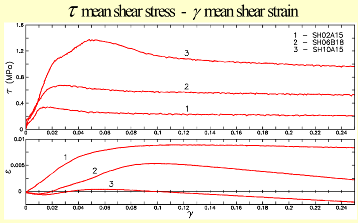


Brick-mortar interface

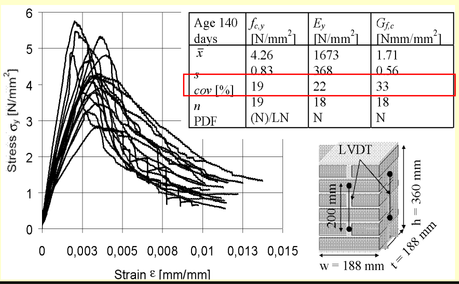


Shear test apparatus - Triplet (Binda et al., 1995).

τ mean shear stress - γ mean shear strain



ε - mean normal extension

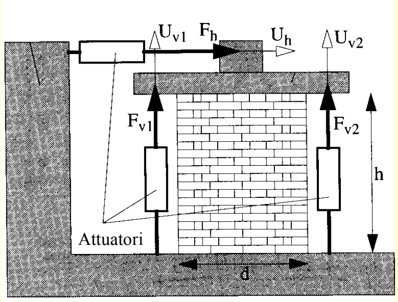


Age 140 days	$f_{c,y}$ [N/mm²]	E_y [N/mm²]	$G_{f,c}$ [Nmm/mm²]
$\bar{x}$	4.26	1673	1.71
s	0.83	368	0.56
cov [%]	19	22	33
n	19	18	18
PDF	(N)/LN	N	N

Masonry pillars: Stress-strain results and statistical summary (Schueremans & van Gemert, 2006)

Hysteretic behavior of shear walls

Imposed horizontal displacement on compressed walls



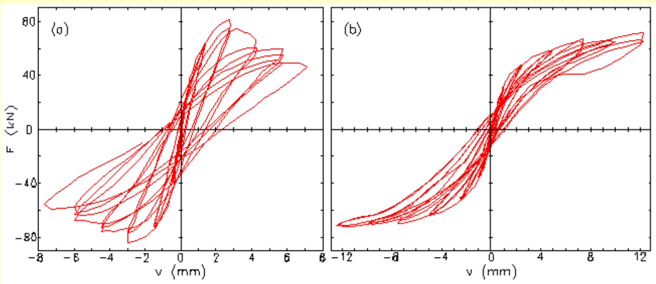
Cyclic shear test set up (Anthoine et al., 1994)

Hysteresis & damage

Dominant NL elastic response NTR

Squat wall

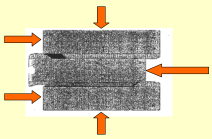
b=100cm
h=135cm

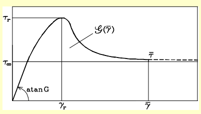


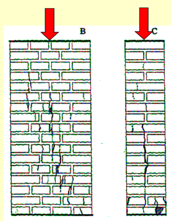
Slender wall

b=100cm
h=200cm

Modeling: introductory aspects







The constitutive ingredients

Elasticity

Unilateral contact

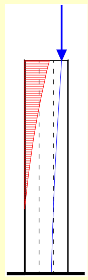
Plasticity

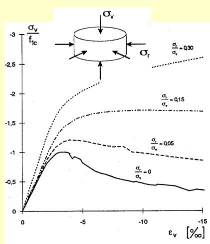
Friction

Damage

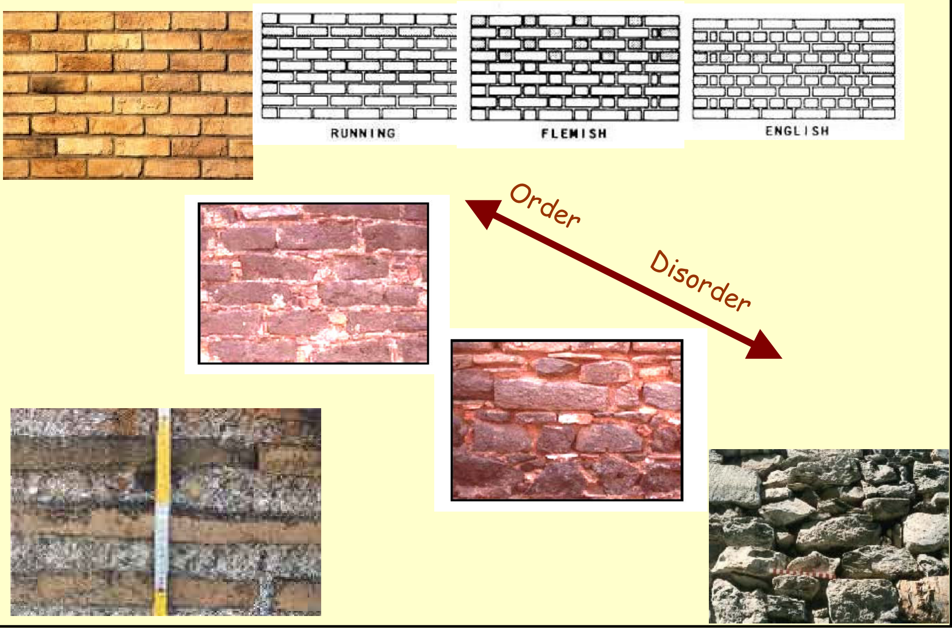
Fracture

Viscoelasticity



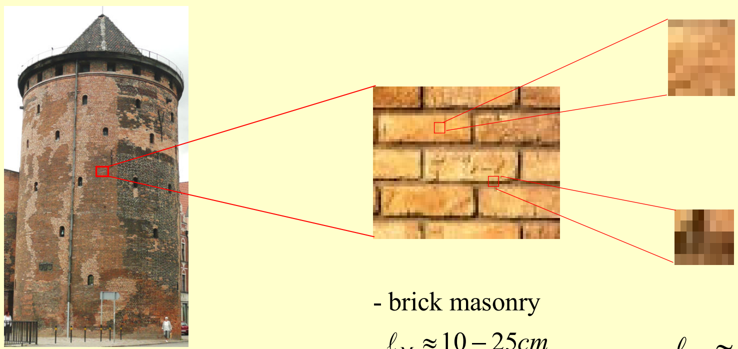


Geometry of the components and bond patterns



Masonry heterogeneity – length scales

Structural scale | Mesoscopic scale | Microstructural scale



brick

mortar

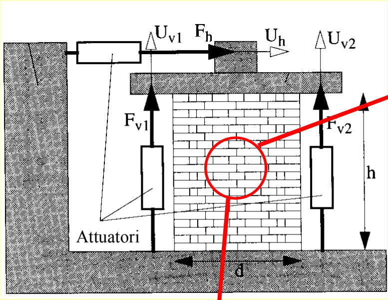
$\ell_S \approx 1-40m$

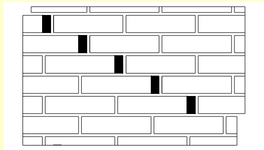
- brick masonry
  $\ell_M \approx 10-25cm$
- stone masonry
  $\ell_M \approx 15-75cm$

$\ell_m \approx 1mm$

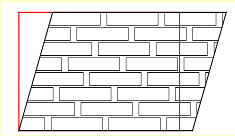
Homogenization

Periodic bond pattern masonry

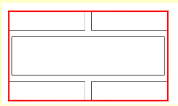




Strain localization



Homogeneous macro-strain



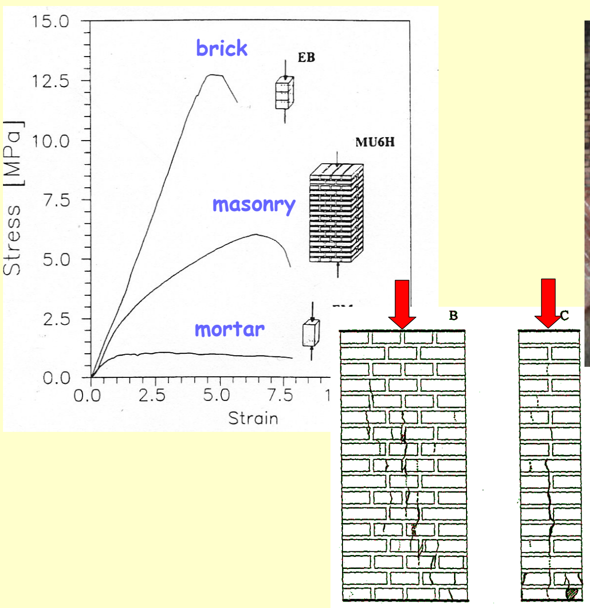
Macro Σ, E

meso σ, ε

RVE

Alpa & Monetto, JMPS, 1994, Anthoine, JSS, 1995 ……………

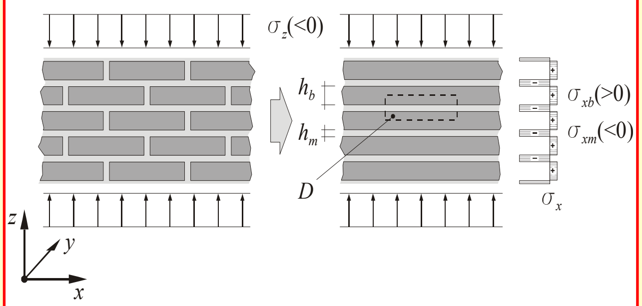
Compressive strength (solid brick masonry)



Generalized hinge at a masonry arch

Modeling compressed solid brick masonry

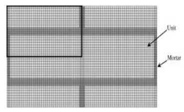
Equivalent layered medium
Hilsdorf, 1969, Francis, 1971…

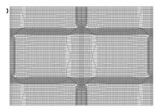


h_b - brick unit thickness
h_m - mortar layer thickness

$\alpha = h_b/h_m$

$$f_M = \frac{\alpha\, f_b^t + f_m^t}{\alpha \dfrac{f_b^t}{f_b^c} + \dfrac{f_m^t}{f_m^c}}$$





RVE FE Models

Massart *et al.*, 2004 -
Lourenco *et al*, 2006…

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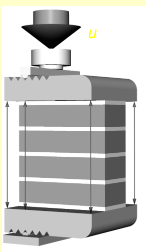
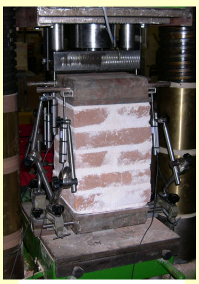
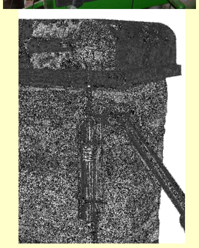
3. Columns and arches

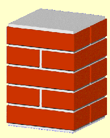
Eccentrically loaded columns & arches



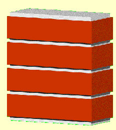




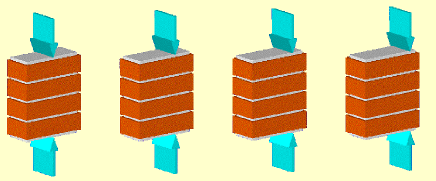
Experimental set up



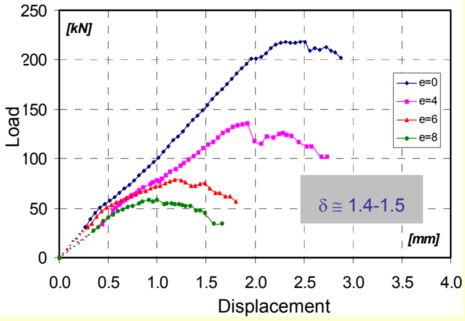
1 unit stack

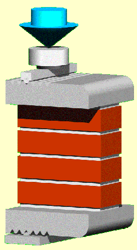


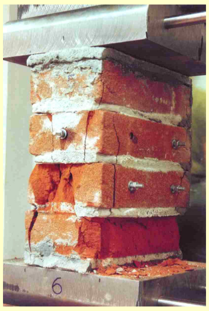
2 unit stack



Eccentrically loaded columns & arches

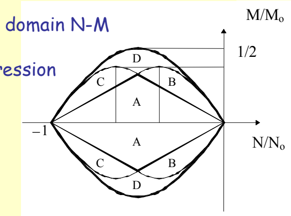




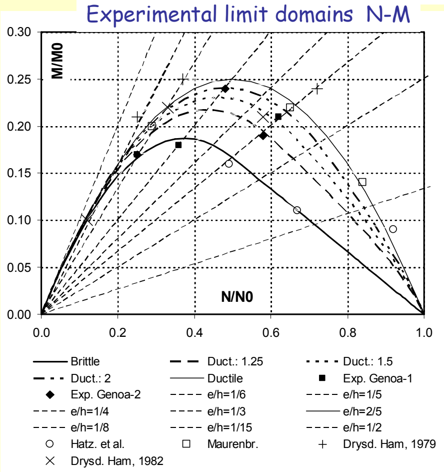


Theoretical limit domain N-M

NTR + EPP compression



Experimental limit domains N-M

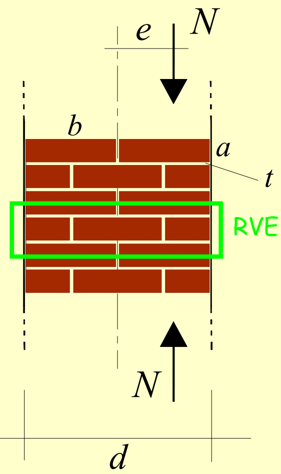


Brencich e Gambarotta, 2005, 2008

Failure of eccentrically compressed masonry pillars (solid brick masonry)

Main issues

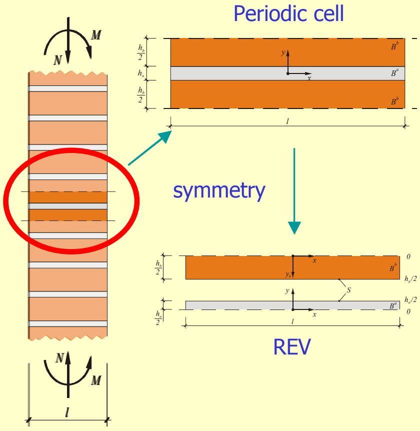
- Influence of the brick bond / geometry of the units
- Influence of the materials: constitutive models of brick-mortar joint - interfaces
- Free edge effects (comparison with layered models)
- Stress concentration/singularity due to mismatch of material parameters
- Validity limits of 2D models



$$\frac{N_l}{f_c^b d} = F\left(\frac{b}{d}, \frac{a}{b}, \frac{t}{b}, \frac{f_t^b}{f_c^b}, \frac{f_c^m}{f_c^b}, \frac{f_t^m}{f_c^b}, \ldots\ldots\ldots, \frac{e}{b}\right)$$

Eccentrically loaded columns & arches

LB strength of stacked bond prisms



Assumed tension field

$$\Phi^a(x,y) = a_0^p\left[f_0^p(x) + r f_0^d(x)\right] + \sum_{n=1}^{N}\sum_{m=1}^{M_a} a_{nm} f_n(x) g_m^a(y)$$

$$\Phi^b(x,y) = a_0^p\left[f_0^p(x) + r f_0^d(x)\right] + \sum_{n=1}^{N}\sum_{m=1}^{M_b} b_{nm} f_n(x) g_m^b(y)$$

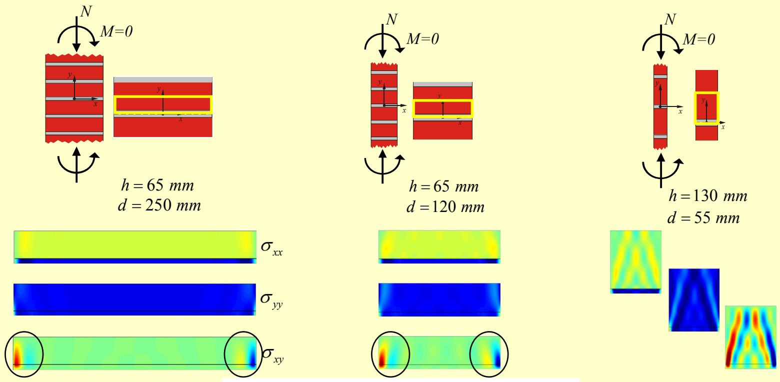
+ BC's on f() and g()
+ plastic admissibility - Mohr-Coulomb criterion
+ unilateral – frictional brick-layer interface

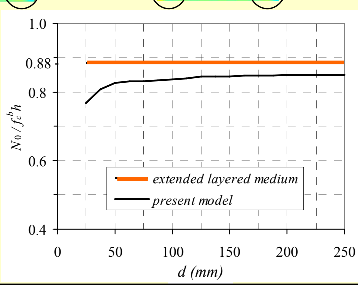
⇩

PPLIN
$$\begin{cases} \max N = \mathbf{c}^T\mathbf{a} \\ \mathbf{S}\,\mathbf{a} \le \tilde{\mathbf{d}} \\ \mathbf{A}_{eq}\ \mathbf{a} = \mathbf{0} \\ \mathbf{A}_{att}\ \mathbf{a} \le \mathbf{0} \end{cases}$$

Concentric axial force

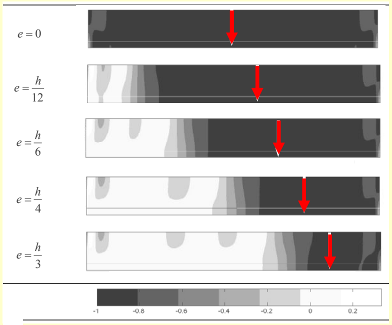
Influence of the RVE height/width ratio





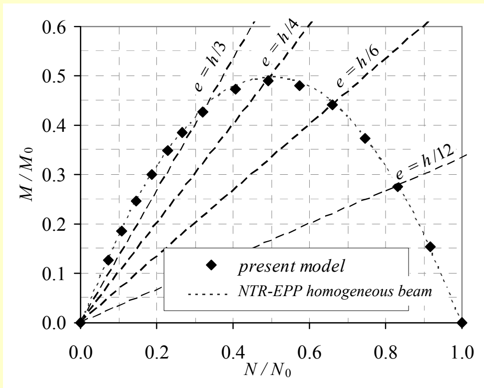
Eccentric axial force

Vertical compressive stress at the limit state



- No influence of the friction coefficient in the range $\mu \in [0, 0.6]$

N-M limit domain



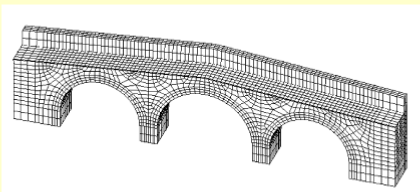
Limit domain of NTR-EPP
(NoTensileResistant - ElasticPerfectlyPlastic)
homogeneous beam

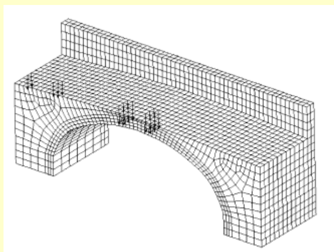
$$f(N,M) = \frac{1}{2}\frac{|M|}{M_0} + \frac{N}{N_0} + \left(\frac{N}{N_0}\right)^2 = 0$$

$$N_0 = f_M^c \, d$$

$$M_0 = N_0 \, d/4 = f_M^c \, d^2/4$$

3. Masonry arch bridges

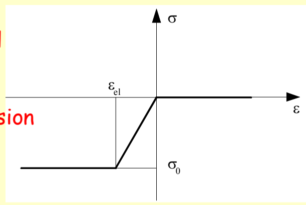


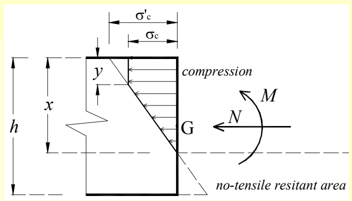


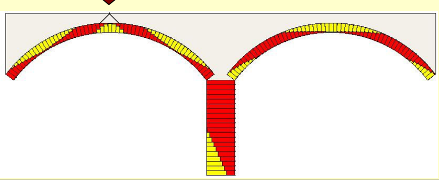
Incremental analysis - Castigliano
2D - Homogeneous beam model

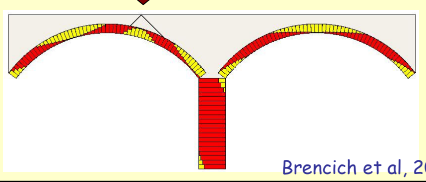
uni-axial model

NRT
+ EPP compression









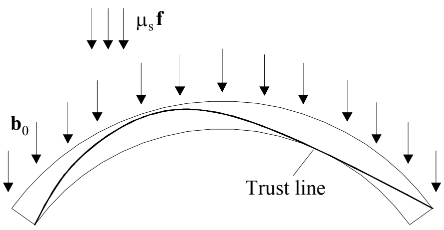
Brencich et al, 2003

Limit analysis - NTR model

Kooharian, Heyman, ………..

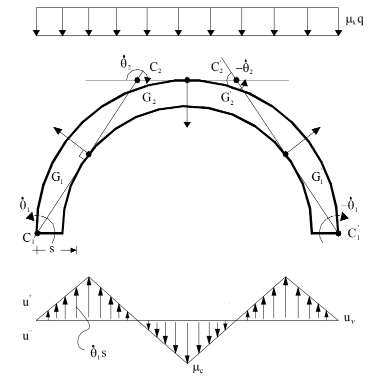
Hypotheses:

1. No tensile resistance NTR
2. Unbounded compressive strength
3. No sliding failure admitted
4. Small displacement and rotations



Statically admissible stress fields

Safe theorem $\mu_c = max\ \mu_s$.



$$\mu_k = -\int_{\mathscr{S}} \gamma\ hb\ \dot{u}_v(s)ds \Big/ \int_{\mathscr{S}} q\ \dot{u}_v(s)\ ds$$

Kinematically admissible mechanisms

Kinematic theorem $\mu_c = min\ \mu_k$

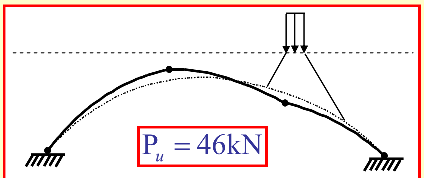
Masonry bridges: Vault – fill interaction

Tests on full scale masonry bridges: Prestwood Bridge

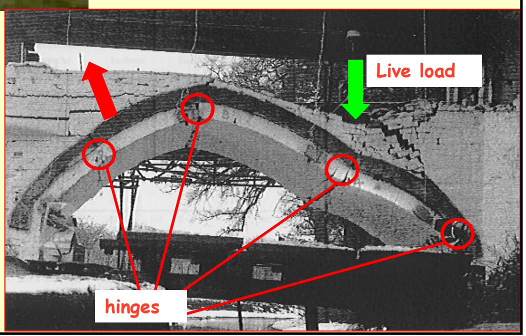
Page, 1993

$P_{exp} = 228kN$

Heavy not resisting fill

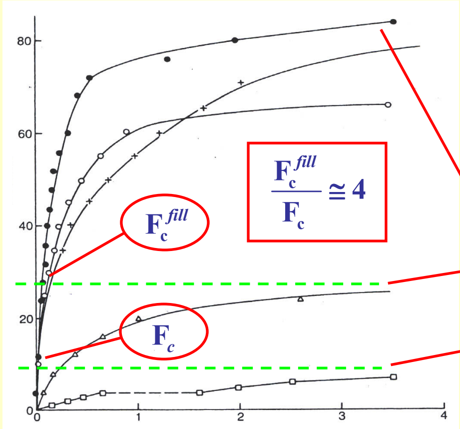


$P_u = 46kN$



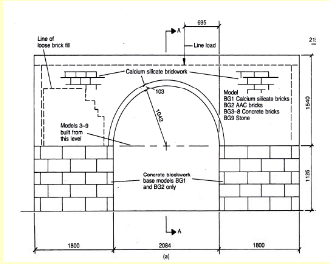
Tests on model scale bridges

(Royles & Hendry, 1991)



Total load applied F (kN)

v (mm)



Complete bridge

Vault and fill

Vault

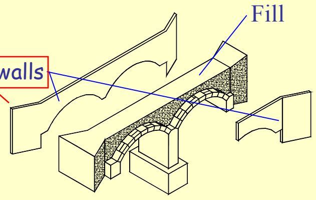
Crisfield (1985)
Choo et al. (1991)
Owen et al. (1998)
Bicanic et al. (2003)

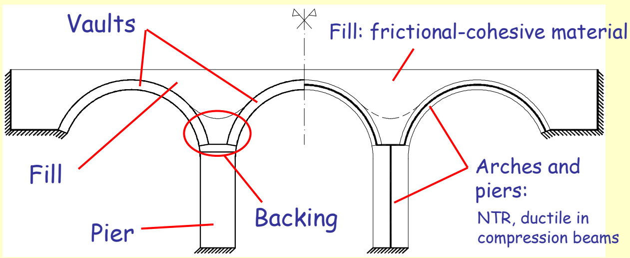
Sperimentazione su modelli – Effetto del riempimento - A. Brencich

Sperimentazione su modelli – Effetto del riempimento - A. Brencich

Two dimensional model of the bridge

The two-dimensional model is obtained by neglecting the **in plane** resistance of the spandrels

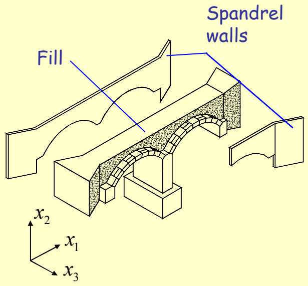


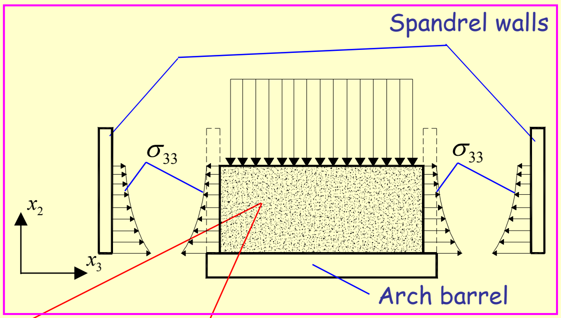


Why a two-dimensional model ?
- Focus on longitudinal collapse mechanisms and arch-fill interaction
- Difficulties in describing the 3D behaviour of materials and components

Limits of validity (within the objectives of the research)
- Interaction between longitudinal and transverse collapse mechanisms

Effects of spandrel walls on fill resistance





Assumptions about the fill:

$-\tilde{\sigma}_c \leq \sigma_{33} \leq 0$

$\tau_{13} = \tau_{23} = 0$

$\sigma_{ij,3} = 0$

The resistance of the fill is affected by the containing capacity of the spandrels, which has to be taken into account also in a plane model of the fill

The effects of spandrels and tie-rods on the fill resistance (due to the containing effect) are approximately taken into account by limiting the out-of plane stress σ_{33}

The plane state conditions necessary to obtain a two-dimensional description of the fill imply an **underestimation of its resistance (safe assumptions for the fill resistance)**

Method of analysis: Limit Analysis approach

Limit Analysis approach

- **Lower Bound** → Equilibrium model (C & G, 2007)
- **Upper Bound** → Compatible model (C & G, 2005, 2006)

A range of values is obtained within which the true collapse load of the mechanical model is contained

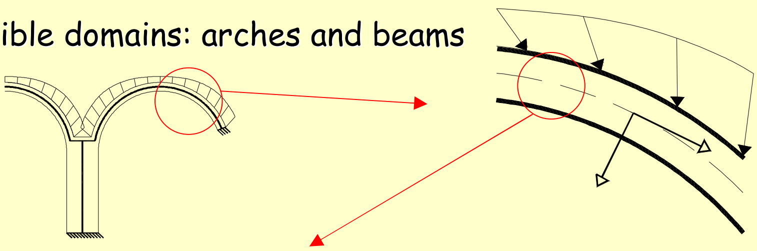
Assumptions (and limits)

- Ductile behaviour
- Associated flow rule
- Small displacements

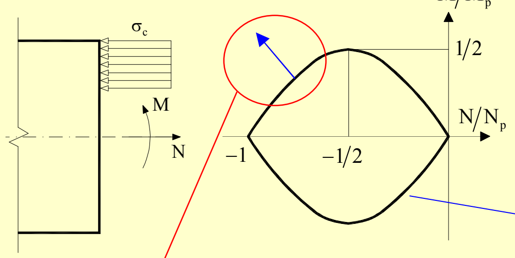
Why Limit Analysis ?

- Reduction of the number of constitutive parameters with respect to an incremental analysis (reduction of uncertainties)
- Direct evaluation and description of the collapse configuration
- Evaluation of a range of values within which the true collaps load of the mechanical model is contained

Admissible domains: arches and beams



NTR, ductile in compression beam



$$f_b(\boldsymbol{\sigma}_b) = \frac{|M|}{M_p} + \frac{2N}{N_p}\left(1 + \frac{N}{N_p}\right) \le 0$$

$$N_p = bh\sigma_c$$

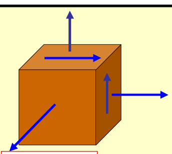
$$M_p = \frac{1}{4}bh^2\sigma_c$$

Admissible domain

Associated flow rule

$$\dot{\varepsilon} = \frac{1}{2}\left(1 + \frac{2N}{N_p}\right)\dot{\lambda}; \quad \dot{\chi} = \frac{\mathrm{Sign}(M)}{h}\dot{\lambda}; \quad \dot{\lambda} \ge 0$$

Admissible domains: fill (1)



$$\sigma_3 \ge -\tilde{\sigma}_c$$

(1) (Mohr-Coulomb criterion in the plane of the model)

$$f^1_{mc}(\boldsymbol{\sigma}) = \sqrt{(\sigma_{11} - \sigma_{22})^2 + 4\tau_{12}^2} + (\sigma_{11} + \sigma_{22})\sin\varphi - 2c\cos\varphi \le 0$$

Constitutive parameters:

- φ - angle of internal friction
- c - cohesion
- $\tilde{\sigma}_c$ - Allowed containing stress (from spandrel)

Mohr-Coulomb criterion, out-of-plane components:

$$f^2_{mc}(\boldsymbol{\sigma}) = \sqrt{(\sigma_{11} - \sigma_{22})^2 + 4\tau_{12}^2} + (\sigma_{11} + \sigma_{22})\sin\varphi - \frac{4c\cos\varphi}{1+\sin\varphi} - \frac{2(1-\sin\varphi)}{1+\sin\varphi}\sigma_3 \le 0$$

$$f^3_{mc}(\boldsymbol{\sigma}) = \sqrt{(\sigma_{11} - \sigma_{22})^2 + 4\tau_{12}^2} - (\sigma_{11} + \sigma_{22})\sin\varphi - \frac{4c\cos\varphi}{1-\sin\varphi} - \frac{2(1+\sin\varphi)}{1-\sin\varphi}\sigma_3 \le 0$$

$$\sigma_3^{max} = \min(\sigma_1, \sigma_2)\frac{1-\sin\varphi}{1+\sin\varphi} + \frac{2c\cos\varphi}{1+\sin\varphi} = \left[\frac{1}{2}(\sigma_{11} + \sigma_{22}) - \frac{1}{2}\sqrt{(\sigma_{11} - \sigma_{22})^2 + 4\tau_{12}^2}\right]\frac{1-\sin\varphi}{1+\sin\varphi} + \frac{2c\cos\varphi}{1+\sin\varphi}$$

Maximum admissible out-of-plane stress σ_3 (Minimum thrust on the spandrels)

Maximum allowed containing stress from spandrels

$$\sigma_3 = \sigma_3^{max} \ge -\tilde{\sigma}_c$$

(2) (Containing stress delimitation)

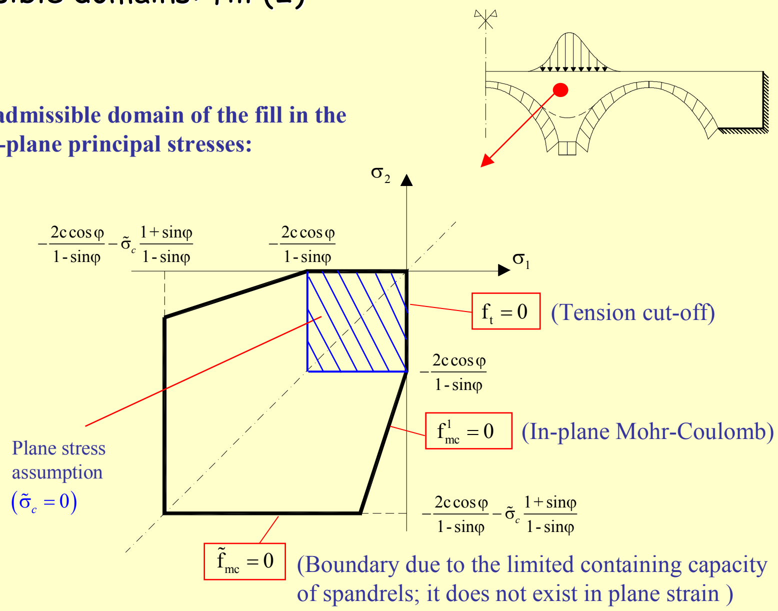
$$\tilde{f}_{mc}(\boldsymbol{\sigma}) = -(\sigma_{11} + \sigma_{22}) + \sqrt{(\sigma_{11} - \sigma_{22})^2 + 4\tau_{12}^2} - 2\tilde{\sigma}_c\frac{1+\sin\varphi}{1-\sin\varphi} - \frac{4c\cos\varphi}{1-\sin\varphi} \le 0$$

(3) (Tension cut-off)

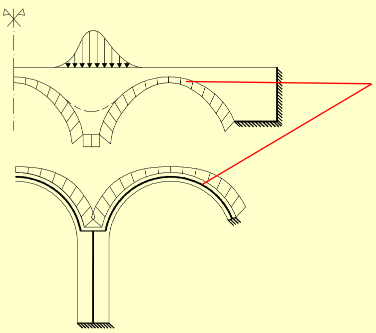
$$f_t(\boldsymbol{\sigma}) = \sqrt{(\sigma_{11} - \sigma_{22})^2 + 4\tau_{12}^2} + (\sigma_{11} + \sigma_{22}) \le 0$$

Admissible domains: fill (2)

Resulting admissible domain of the fill in the space of in-plane principal stresses:

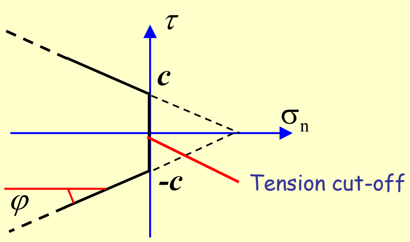


Admissible domains: arch-fill interface



Slidings between arch and fill are allowed and ruled by Coulomb law with tension cut-off:

$$\begin{cases} f^c(\boldsymbol{\sigma}) = \sigma_n \tan\overline{\varphi} + |\tau| - \overline{c} \le 0 \\ f^t(\boldsymbol{\sigma}) = \sigma_n \le 0 \quad \text{(Tension cut-off)} \end{cases}$$

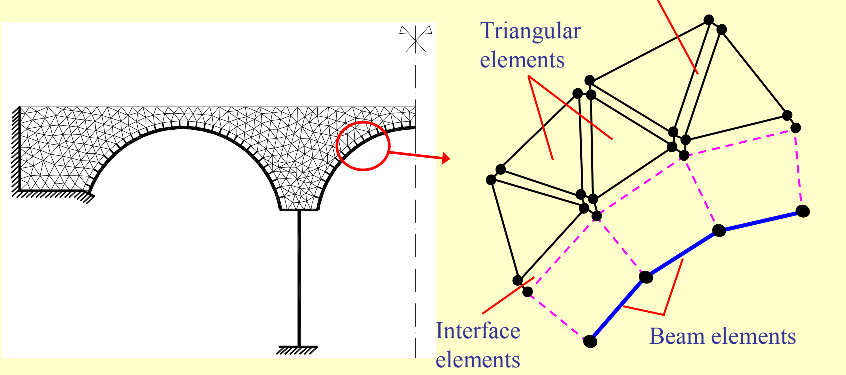


Constitutive parameters:

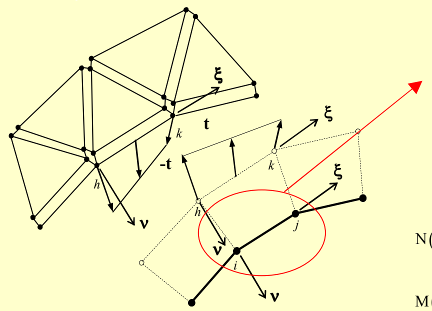
$\overline{\varphi}$ - angle of friction

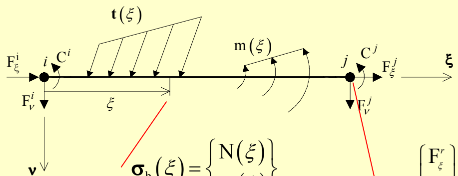
$\overline{c}$ - cohesion (assumed zero)

FE discretization of the bridge



Equilibrium model: beam element





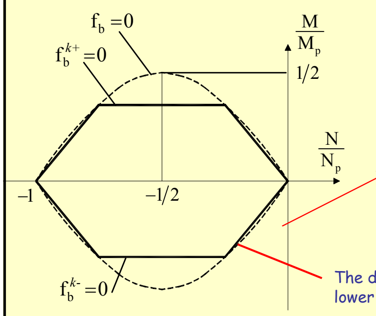
$$\boldsymbol{\sigma}_b(\xi) = \begin{Bmatrix} N(\xi) \\ M(\xi) \end{Bmatrix} \qquad \mathbf{s}_b^r = \begin{Bmatrix} F_\xi^r \\ F_\nu^r \\ C^r \end{Bmatrix}$$

$$N(\xi) = -F_\xi^i + \ell\left(\frac{\xi}{\ell}\right)\left(1-\frac{1}{2}\left(\frac{\xi}{\ell}\right)\right)\tau(0) + \frac{1}{2}\ell\left(\frac{\xi}{\ell}\right)^2\tau(\ell) - \xi\,\boldsymbol{\xi}^T\mathbf{b}_b$$

$$M(\xi) = -C^i - F_\nu^i\xi - \frac{1}{2}\ell^2\left(\frac{h}{\ell}\right)\left(\frac{\xi}{\ell}\right)\left(1-\frac{1}{2}\left(\frac{\xi}{\ell}\right)\right)\tau(0) + \frac{1}{2}\ell^2\left(\frac{\xi}{\ell}\right)^2\left(1-\frac{1}{3}\left(\frac{\xi}{\ell}\right)\right)\sigma_\nu(0) +$$
$$-\frac{1}{4}\ell^2\frac{h}{\ell}\left(\frac{\xi}{\ell}\right)^2\tau(\ell) + \frac{1}{6}\ell^2\left(\frac{\xi}{\ell}\right)^3\sigma(\ell) - \frac{1}{2}\xi^2\boldsymbol{\nu}^T\mathbf{b}_b,$$

Internal force vector from equilibrium

$$\boldsymbol{\sigma}_b(\xi) = \mathbf{D}_b^i\mathbf{s}_b^i + \mathbf{D}_b^h(\xi)\boldsymbol{\sigma}^h + \mathbf{D}_b^k(\xi)\boldsymbol{\sigma}^k + \mathbf{d}_b(\xi)$$



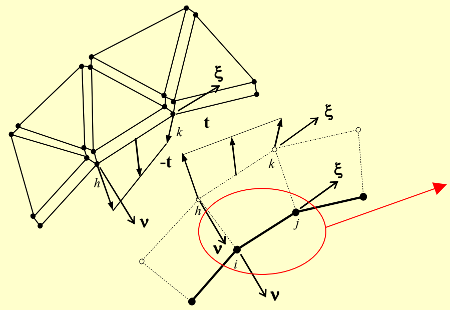
$$\mathbf{f}_b(\xi) = \mathbf{N}_b^T\boldsymbol{\sigma}_b(\xi) - \mathbf{r}_b \le \mathbf{0}$$

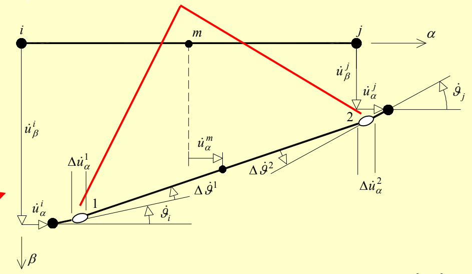
$$\xi_t = \frac{t-1}{p_s-1}\ell \longrightarrow \mathbf{f}_b^t = \mathbf{N}_b^T\boldsymbol{\sigma}_b(\xi_t) - \mathbf{r}_b \le \mathbf{0}, \quad t = 1 \ldots p_s$$

The discrete admissible domain is inside the original one in order to maintain the lower bound property of the load multiplier

Compatible model: beam element

Two hinges at ends



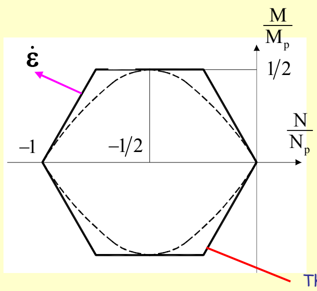


Compatibility

$$\dot{\boldsymbol{\varepsilon}}^e = \mathbf{B}^e \dot{\mathbf{u}}^e$$

$$\dot{\boldsymbol{\varepsilon}}^e = \begin{Bmatrix} \Delta \dot{u}_\alpha^1 \\ \Delta \dot{\vartheta}^1 \\ \Delta \dot{u}_\alpha^2 \\ \Delta \dot{\vartheta}^2 \end{Bmatrix} \quad \dot{\mathbf{u}}^e = \begin{Bmatrix} \dot{u}_\alpha^i \\ \dot{u}_\beta^i \\ \dot{\vartheta}^i \\ \dot{u}_\alpha^j \\ \dot{u}_\beta^j \\ \dot{\vartheta}^j \\ \dot{u}_\alpha^m \end{Bmatrix}$$

$$\mathbf{B}^e \dot{\mathbf{u}}^e - \mathbf{N}_b \dot{\boldsymbol{\lambda}}^e = \mathbf{0}$$



$$\mathbf{f}^e = \mathbf{N}^{eT} \boldsymbol{\sigma}^e - \mathbf{r}^e \le \mathbf{0}$$

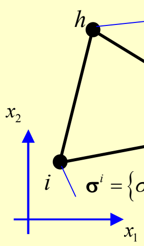
$$\boldsymbol{\sigma}^e = \begin{Bmatrix} \boldsymbol{\sigma}_b(0) \\ \boldsymbol{\sigma}_b(l) \end{Bmatrix}$$

$$\dot{\boldsymbol{\varepsilon}}^e = \mathbf{N}^e \dot{\boldsymbol{\lambda}}^e$$ Associated flow rule

$$D_p^e = \mathbf{r}_b^T \dot{\boldsymbol{\lambda}}^e$$ Dissipated power

The discrete admissible domain is outside the original one in order to maintain the upper bound property of the load multiplier

Equilibrium model: triangular element



$$\boldsymbol{\sigma}^h = \{\sigma_{11}^h \quad \sigma_{22}^h \quad \tau_{12}^h\}^T$$

$$\boldsymbol{\sigma}^j = \{\sigma_{11}^j \quad \sigma_{22}^j \quad \tau_{12}^j\}^T$$

$$\boldsymbol{\sigma}^i = \{\sigma_{11}^i \quad \sigma_{22}^i \quad \tau_{12}^i\}^T$$

$$N^i(\mathbf{x}) = a^i x_1 + b^i x_2 + c^i$$

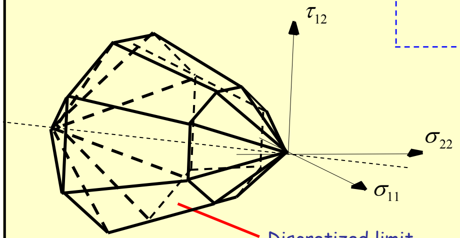
$$N^p(\mathbf{x}^q) = \delta_{pq}$$

$$\boldsymbol{\sigma}(\mathbf{x}) = \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{Bmatrix} = N^i(\mathbf{x})\boldsymbol{\sigma}^i + N^j(\mathbf{x})\boldsymbol{\sigma}^j + N^h(\mathbf{x})\boldsymbol{\sigma}^h$$

$$\begin{cases} \sigma_{11,1} + \tau_{12,2} + b_1 = 0 \\ \tau_{12,1} + \sigma_{22,2} + b_2 = 0 \end{cases}$$

Equilibrium

$$\mathbf{D}_t^i \boldsymbol{\sigma}^i + \mathbf{D}_t^j \boldsymbol{\sigma}^j + \mathbf{D}_t^h \boldsymbol{\sigma}^h + \mathbf{d}_t = \mathbf{0}$$



Discretized limit domain

$$\mathbf{f}(\boldsymbol{\sigma}(\mathbf{x})) = \mathbf{N}^T \boldsymbol{\sigma}(\mathbf{x}) - \mathbf{r} \le \mathbf{0}$$

$\boldsymbol{\sigma}(\mathbf{x})$ linearly interpolated

$$\mathbf{f}(\boldsymbol{\sigma}^k) = \mathbf{N}^T \boldsymbol{\sigma}^k - \mathbf{r} \le \mathbf{0}, \quad k = i, j, h$$

Plastic admissibility

Upper and lower bound collapse load

$$\mathbf{q} = \mathbf{q}_0 + s\bar{\mathbf{q}}$$

Dead load

Live load

Live load multiplier

Maximum statically admissible load multiplier

$$\begin{cases} s_{lb} = \max(s_s), & \\ \mathbf{C}\boldsymbol{\sigma} = \mathbf{c}, & \text{Equilibrium} \\ \mathbf{Q}\boldsymbol{\sigma} - s_s\bar{\mathbf{q}} = \mathbf{q}_0, & \text{Boundary conditions} \\ \mathbf{N}^T\boldsymbol{\sigma} \le \mathbf{r}. & \text{Plastic admissibility} \end{cases}$$

Dual

$$\begin{cases} s_{lb} = \min(\mathbf{c}^T\boldsymbol{\alpha}_1 + \mathbf{q}_0^T\boldsymbol{\alpha}_2 + \mathbf{y}^T\boldsymbol{\alpha}_3), \\ \mathbf{C}^T\boldsymbol{\alpha}_1 + \mathbf{Q}^T\boldsymbol{\alpha}_2 + \mathbf{Y}\boldsymbol{\alpha}_3 = \mathbf{0}, \\ \bar{\mathbf{q}}^T\boldsymbol{\alpha}_2 = 1, \\ \boldsymbol{\alpha}_3 \ge \mathbf{0}. \end{cases}$$

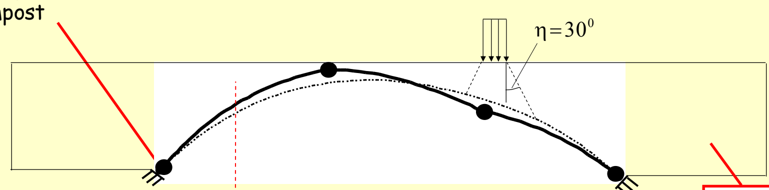
Minimum kinematically admissible load multiplier

$$\begin{cases} s_{ub} = \min(\mathbf{q}^{0T}\mathbf{v} + \mathbf{r}^T\dot{\boldsymbol{\lambda}}) & \text{Compatibility and associated flow rule} \\ \mathbf{B}\mathbf{v} - \mathbf{N}\dot{\boldsymbol{\lambda}} = \mathbf{0} & \text{Boundary conditions and internal constraints} \\ \mathbf{A}\mathbf{v} = \mathbf{0} & \\ \bar{\mathbf{q}}^T\mathbf{v} = 1 & \text{Positivity of the power of live load} \end{cases}$$

Linear Programming problems solved by internal point method (Mehrotra)

PRESTWOOD BRIDGE
Non resistant fill model: collapse mechanism

Hinge at impost

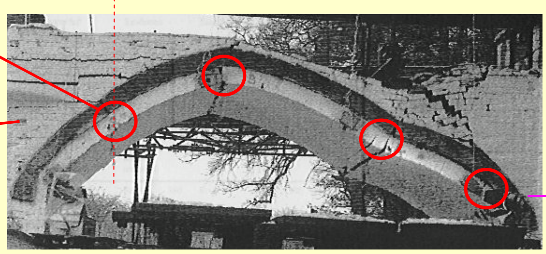


$P_u = 46\text{kN}$

Numerical collapse load (fill as dead load): 20% of the experimental result

Hinge at haunch

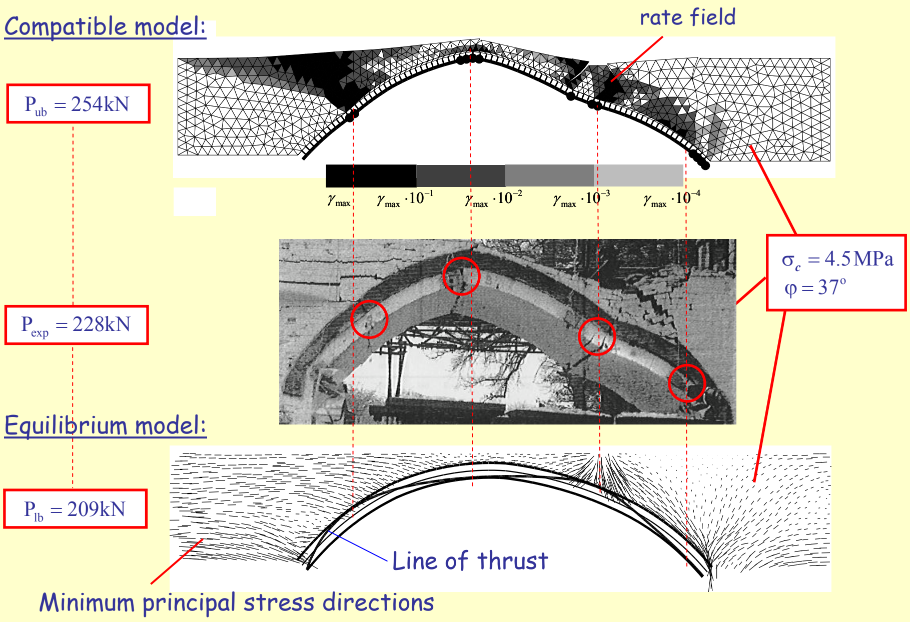
$\sigma_c = 4.5\text{MPa}$
$\varphi = 37°$



$P_{exp} = 228\text{kN}$

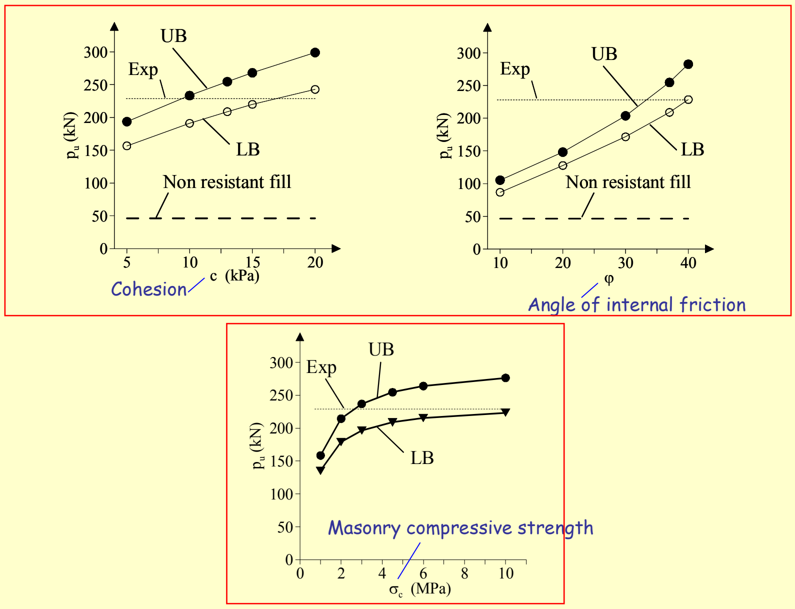
(Page, 1987)

PRESTWOOD BRIDGE: Plane strain fill



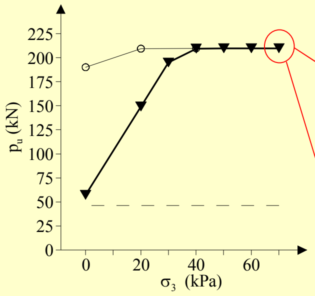
PRESTWOOD BRIDGE: Plane strain fill

Sensitivity of the collapse load P_u to the constitutive parameters

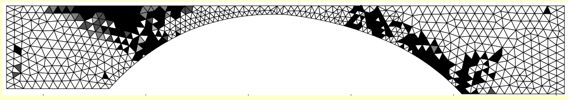


PRESTWOOD BRIDGE

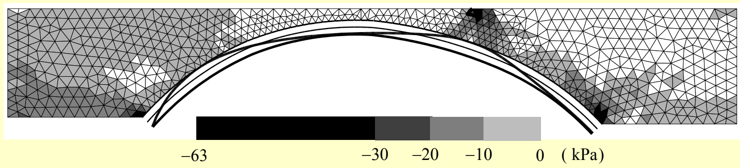
Effects of the containing stress of the spandrels



In-plane Mohr-Coulomb limit domain activation:

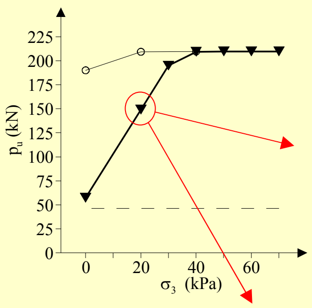


Containing stress distribution:

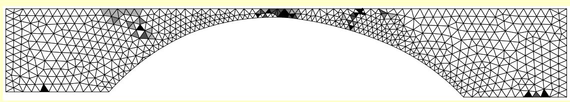


PRESTWOOD BRIDGE

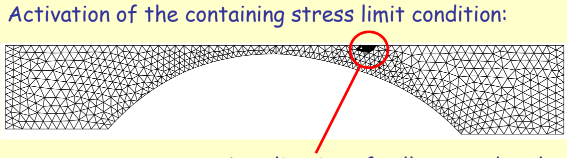
Effects of the containing stress



In-plane Mohr-Coulomb limit domain activation:

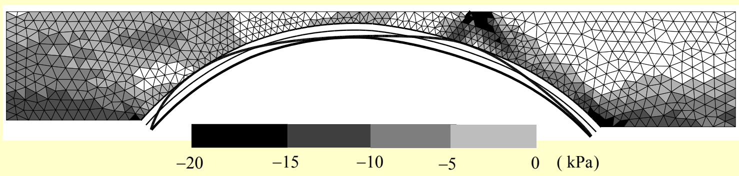


Activation of the containing stress limit condition:



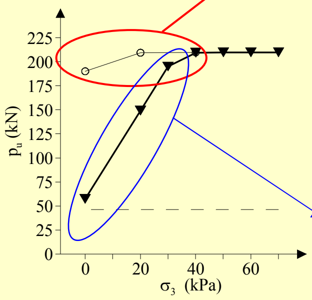
Localisation of collapse under the live load position

Containing stress distribution:



PRESTWOOD BRIDGE

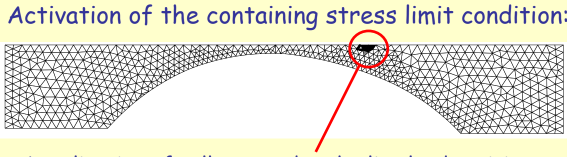
Effects of the containing stress



Results obtained by avoiding plastic strains in the fill below the position of the applied live load: in this case the reduction of the collapse load is very small

This result shows that a diffused deep reduction of the fill resistance due to a limited containing stress from spandrels does not affect the result, once localised effects under the load are avoided

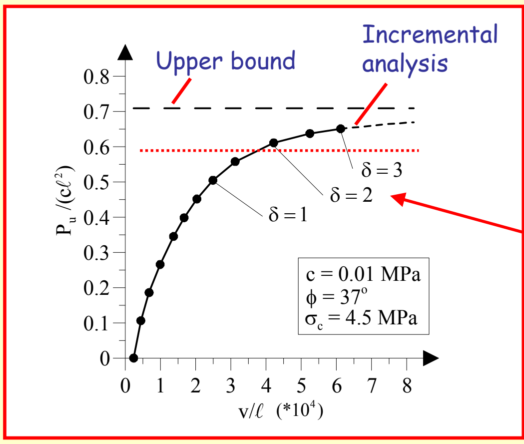
Activation of the containing stress limit condition:



Localisation of collapse under the live load position

Prestwood Bridge

Load\deflection curve and ductility demand



Masonry ductility:

$$\delta = \frac{\varepsilon}{\varepsilon_c}$$

Vertical displacement v

Multi span bridge: Fill – arches – piers interaction

Fill model properties:

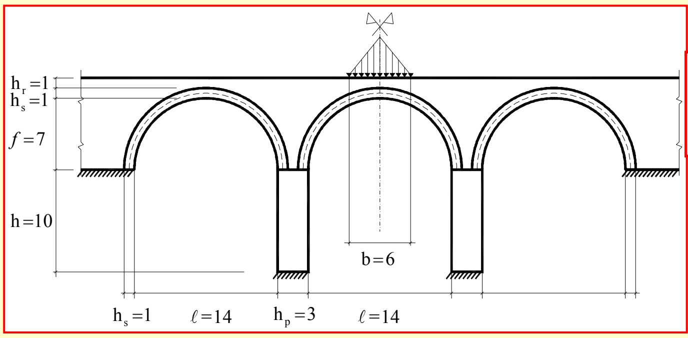
Fill density $\rho = 18\ \mathrm{kN/m^3}$

Discrete domain planes $p = 36$

Arch model properties:

Masonry density $\rho = 18\ \mathrm{kN/m^3}$

Discrete domain planes $p = 48$

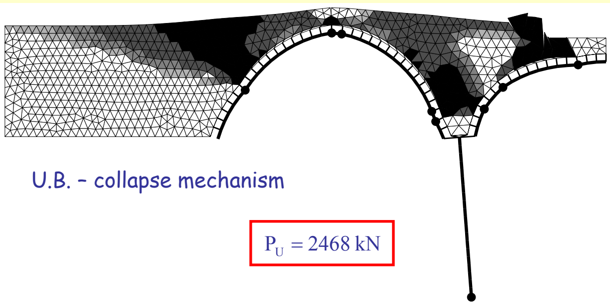


$\sigma_c = 12\,\mathrm{MPa}$

$\varphi = 30^{\circ}$

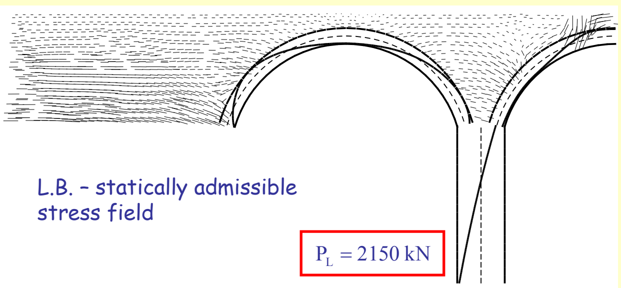
$c = 20\,\mathrm{kPa}$

Multi span bridge



U.B. – collapse mechanism

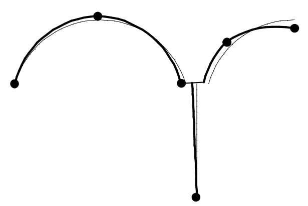
$P_U = 2468\ \mathrm{kN}$



L.B. – statically admissible stress field

$P_L = 2150\ \mathrm{kN}$

Non resistant fill



$P_u = 625\ \mathrm{kN}$

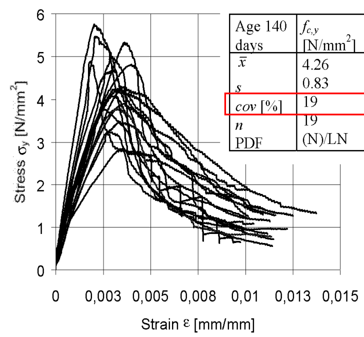
Probabilistic models of masonry arches (including non-linear material response)

- Monte Carlo simulation + Limit Analysis (Ng and Fairfield, 2002)
- Fuzzy non linear analysis (Biondini *et al.*, 2002)
- Probabilistic Limit Analysis -Hystorical Buildings (Augusti *et al.*, 2001, 2002)

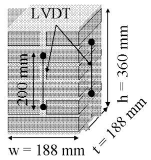
Uniaxial compression

Masonry pillars:
Stress-strain results
and statistical summary

(Schueremans & van Gemert, 2006)



Age 140 days	$f_{c,y}$ [N/mm²]	E_y [N/mm²]	$G_{f,c}$ [Nmm/mm²]
$\bar{x}$	4.26	1673	1.71
s	0.83	368	0.56
cov [%]	19	22	33
n	19	18	18
PDF	(N)/LN	N	N



Probabilistic limit domain of the generalized hinge

Hypotheses:

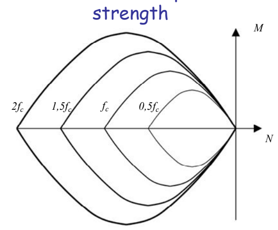
- the compressive strength $\tilde{f}_c$ is a random variable
- which is correlated in the section
- the tensile strength is deterministically vanishing

Probabilistic limit domain of the GH

$$\phi\left(N, M, \tilde{f}_c\right) = \frac{1}{2}\left(\frac{N}{bh\tilde{f}_c}\right)\left[1+\left(\frac{N}{bh\tilde{f}_c}\right)\right] + \frac{|M|}{bh^2\tilde{f}_c} = 0$$

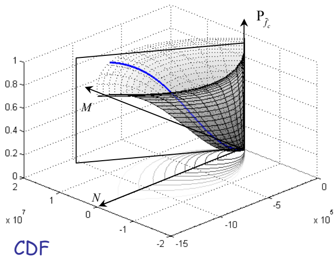
depending on the random variable $\tilde{f}_c$

Effect of the compressive strength

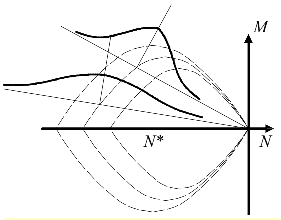


Probability of failure of the masonry section

$$P_F = P\left(\tilde{f}_c \le -\frac{N}{bh\left(\frac{2|M|}{N}+1\right)}\right)$$



CDF



PDF

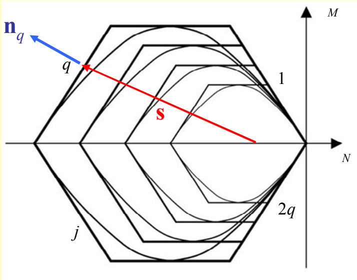
Piecewise linearization of the limit domain - $2q$ limit lines

Probability of failure of i-th sectional mechanism

$$P_{F_i} = P\left(\frac{(i-1)^2}{(q-1)^2} \tilde{f}_c \le \mathbf{n}_i^T \mathbf{s} \right) = P(\tilde{r}_i \le r_i)$$

$r_i = \mathbf{n}_i^T \mathbf{s}$ — Projection of the internal force on i-th mechanism

$\tilde{r}_i = \frac{(i-1)^2}{(q-1)^2} \tilde{f}_c$ — Strength of i-th mechanism **random variable**

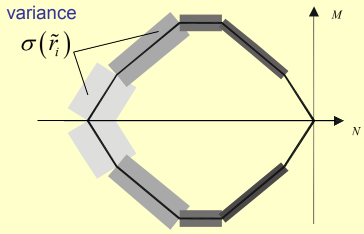


$$D_{\text{int},i} = \mathbf{s}^T \boldsymbol{\varepsilon}_i = r_i \lambda_i$$

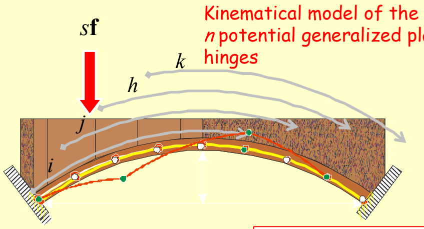
for $i=1$ the limit domain is deterministic

$$\phi_1^+(\mathbf{s}) = \mathbf{n}_1^T \mathbf{s} \le 0$$

$$\begin{cases} \tilde{D}_{\text{int},i} = \tilde{r}_i \lambda_i = \lambda_i \left(\frac{i-1}{q-1} \right)^2 \tilde{f}_c \\ \bar{D}_{\text{int},i} = \bar{r}_i \lambda_i = \lambda_i \left(\frac{i-1}{q-1} \right)^2 \bar{f}_c \\ \sigma(\tilde{D}_{\text{int},i}) = \lambda_i \sigma(\tilde{r}_i) = \lambda_i \left(\frac{i-1}{q-1} \right)^2 \sigma(\tilde{f}_c) \end{cases}$$



Arch discretization + Selection of the set of all possible mechanisms



Kinematical model of the arch
n potential generalized plastic hinges

$$N_m = C_n^4 R_q^4 = \frac{n!}{4!(n-4)!}(2q)^4$$

Mechanism enumeration

- Rate of dissipation (random variable) $$\tilde{D}_j = \tilde{\mathbf{r}}^T \boldsymbol{\lambda}_j = \sum_{p=1}^{4} \tilde{r}_p \lambda_p = \sum_{p=1}^{4} \psi_p \tilde{f}_{c\,p} \lambda_p$$
- External Power (deterministic) $$W_j = W_{0,j} + s_j W_{a,j} \ge 0$$
- **Kinematically admissible multiplier (random variable)** $$\tilde{s}_j = \frac{\tilde{D}_j - W_{0,j}}{W_{a,j}}$$

N_{KA}
No. admissible mechanisms

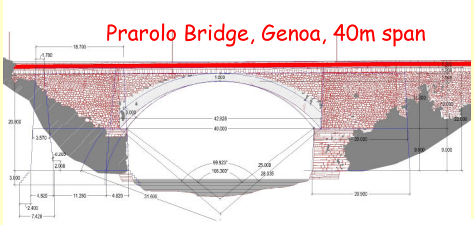
Hypotheses on the compressive strengths at the n hinges:

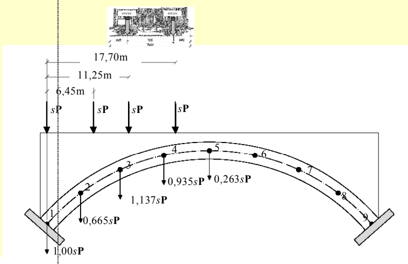
- stochastically independent
- Gaussian-distributed (stable) with the same expected strength and variance

j-th mechanism

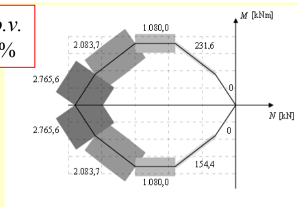
$$P_C\left[E_j\right] = P_C\left[\tilde{s}_j < s\right]$$

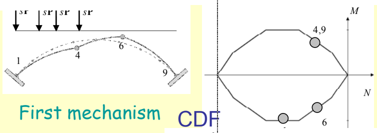
Masonry bridges: probabilistic analysis

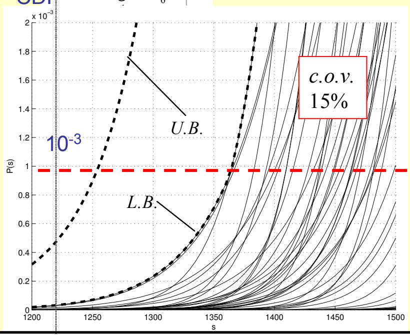




c.o.v. 15%







- Collapse of the arch (realization of at least one event) $[E]=\bigcup_j [E_j]$
- CORNELL bounds

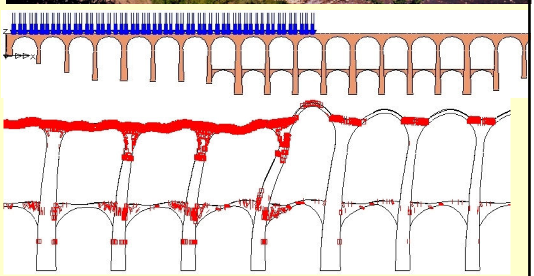
$$\max P_C(E_j) \le P_C[E] \le \sum_j P_C(E_j) \le 1$$

good approximations for small probabilities <<10^{-3}

Masonry railway bridges

Open problems

- 2D vs. 3D modeling
- Interactions among structural elements (spandrel walls……….)
- Uncertainty of material properties
- Non linear analysis including damage and cracking
- Dynamic response to time-varying loadings
- Foundation settlements
- Material degradation



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