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On the Measurement of Residual Strains and Stresses in Composite Laminates

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ENSMA

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Recherche en Matériaux, Mécanique et Énergétique pour les Transports, l'Énergie et l'Environnement

Head: Prof. Y. GERVAIS
Prof. J.C. GRANDIDIER

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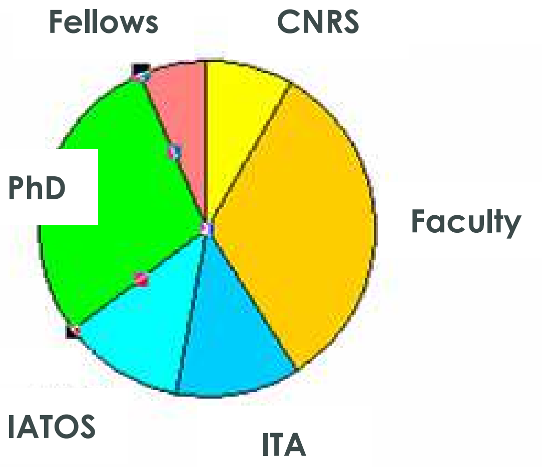
PPRIME UPR CNRS 3346: around 540 people

CNRS Researchers	DR	18	**45**
	CR	27	
Faculty	Prof.	64	**176**
	MCF	112	
Technical staff	ITA	52	**109**
	IATOS	57	
Non permanent staff	PhD students (45 / an)	160	**190**
	Fellows	33	



HDR **120** (50%)

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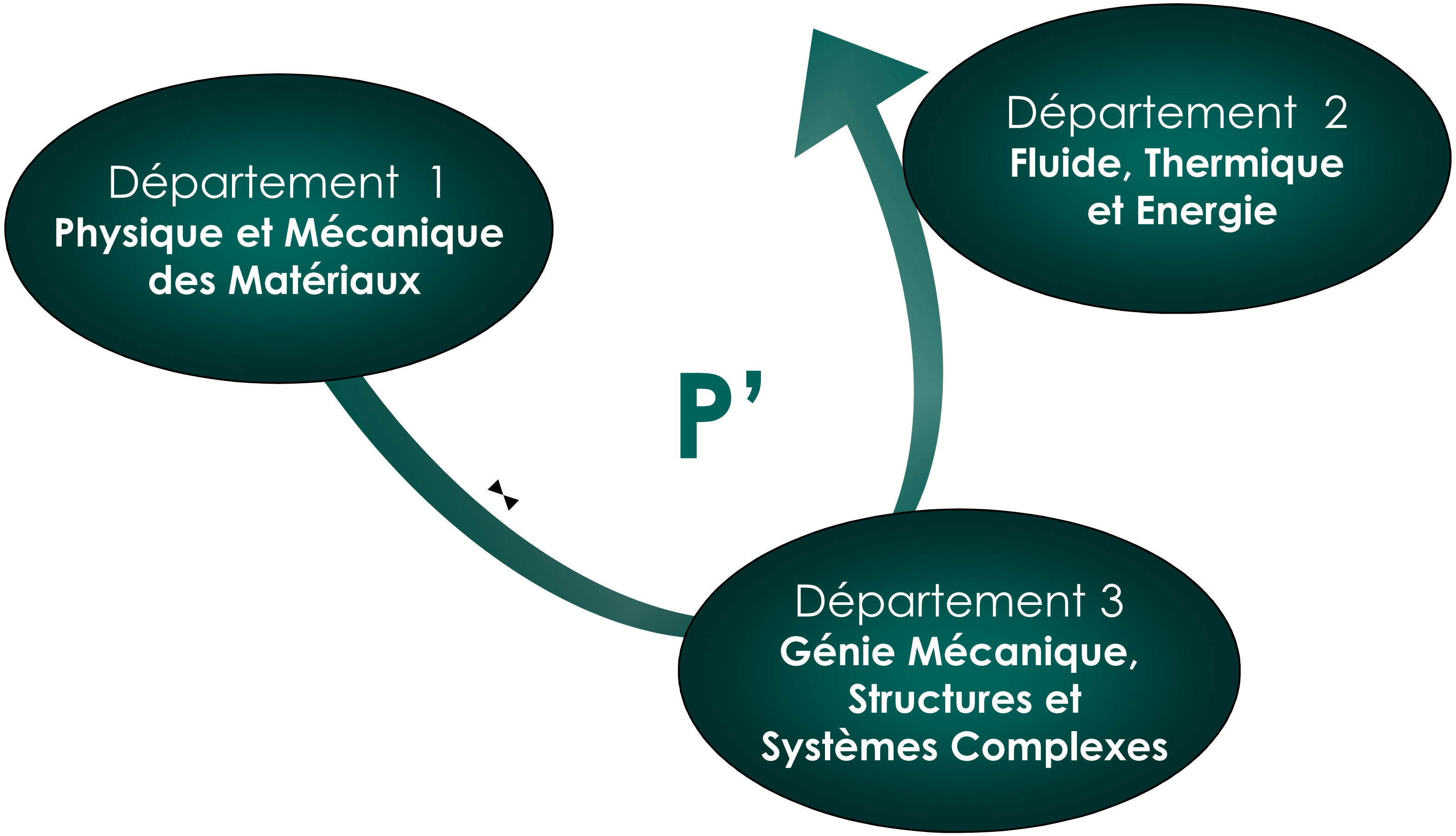

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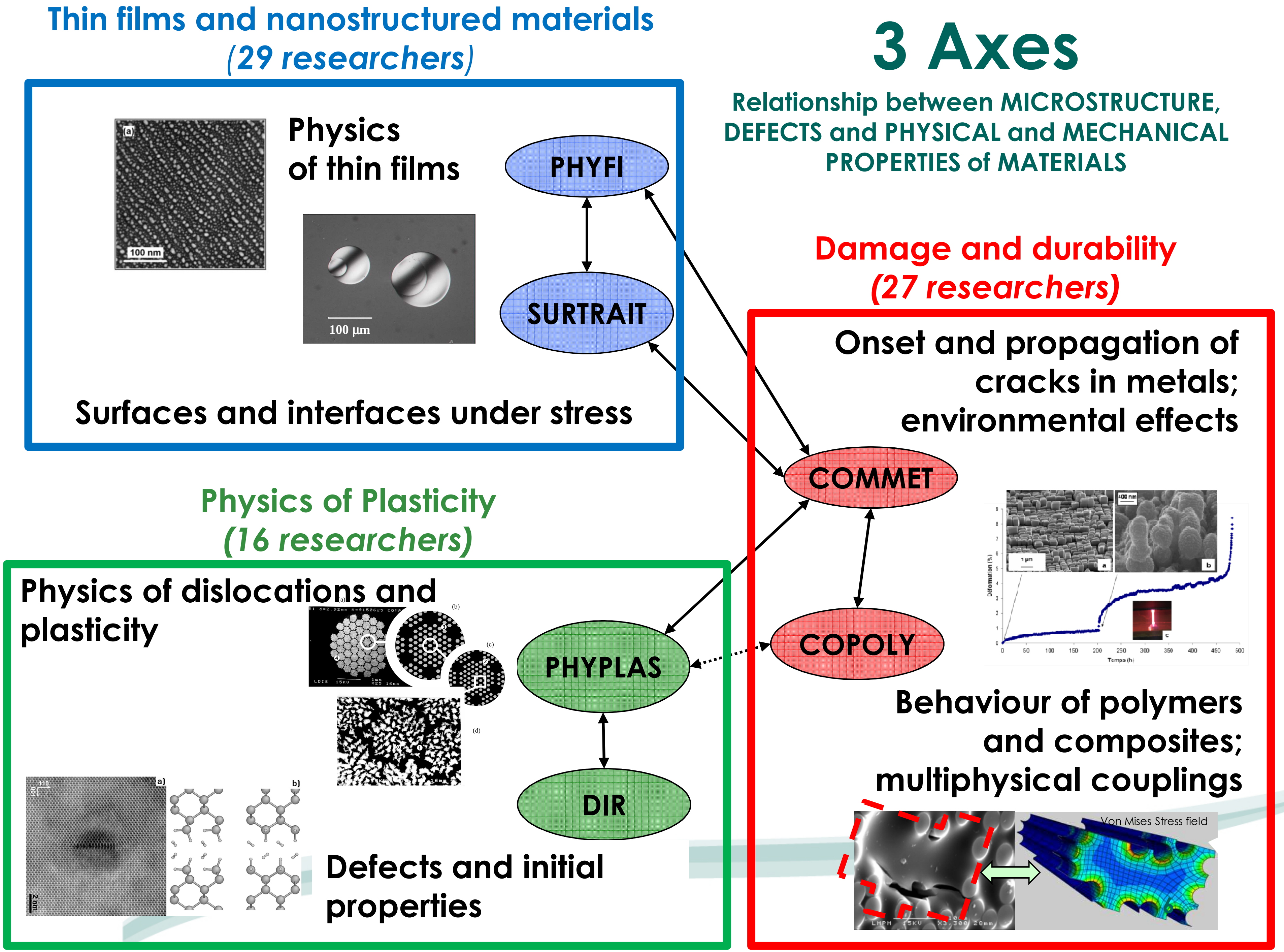
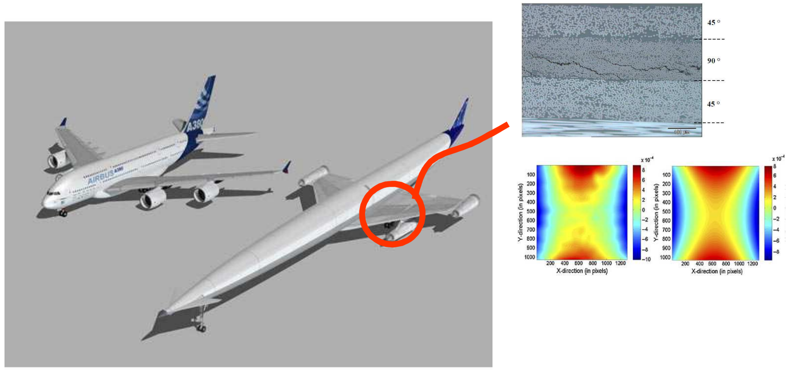
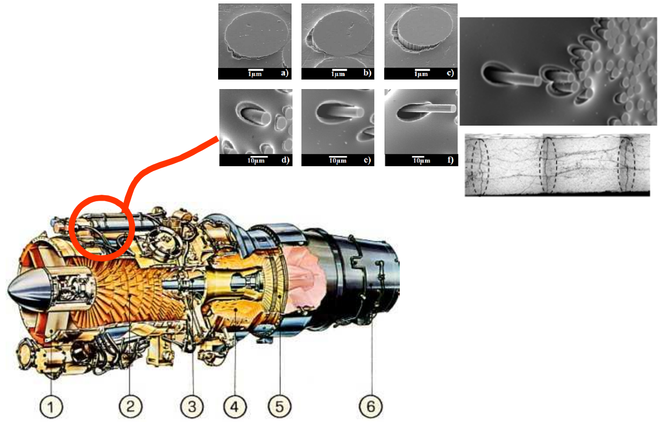

3 Axes
Relationship between MICROSTRUCTURE, DEFECTS and PHYSICAL and MECHANICAL PROPERTIES of MATERIALS
Thin films and nanostructured materials
(29 researchers)
Physics of thin films
PHYFI
SURTRAIT
100 nm
100 µm
Surfaces and interfaces under stress
Damage and durability
(27 researchers)
Onset and propagation of cracks in metals; environmental effects
COMMET
COPOLY
Behaviour of polymers and composites; multiphysical couplings
Von Mises Stress field
Physics of Plasticity
(16 researchers)
Physics of dislocations and plasticity
PHYPLAS
DIR
Defects and initial properties

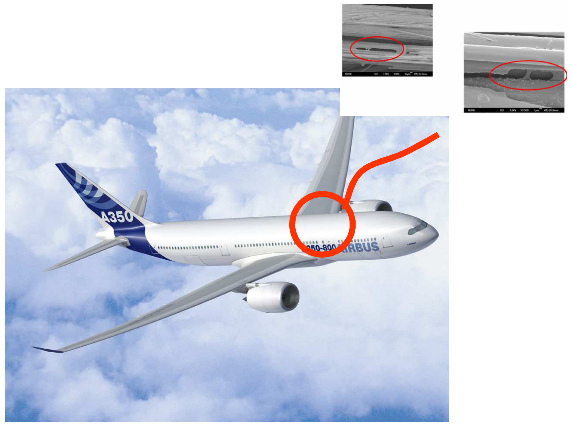
Multi-Physical and Multi-Scale Approach for the Study of Ageing and Durability of Composite Materials



Supersonic Aircraft



"Warm" structures of turbo-engines



Fuselage structures

- ✓ **design and optimisation of dedicated tests**, at different scales
- ✓ development of **models for understanding test results**

Scientific Partnership

- ✓ LMT ENS Cachan (Prof. P. Ladeveze)
- ✓ PIMM ENSAM Paris (Prof. J. Verdu, Prof. X. Colin)
- ✓ MMSMat EC Paris (Dr. J. Bai)
- ✓ Université de Versailles (Prof. P. Vannucci)
- ✓ LGMT UPS Toulouse (Prof. P. Olivier)
- ✓ GeM Nantes (Prof. F. Jacquemin)
- ✓ Politecnico di Milano, Politecnico di Torino

Industrial Partnership

- ✓ EADS IW Suresnes (Dr. J. Cinquin, Dr. C. Paris)
- ✓ SAFRAN (AIRCELLE, SNECMA)
- ✓ AIRBUS SAS Toulouse
- ✓ PSA

On the Measurement of Residual Strains and Stresses in Composite Laminates

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Residual strains and stresses in composite laminates

Two interesting questions

Examples

Residual Strains and Stresses in Composite Laminates

Residual strains and stresses in composite laminates

… residual stress due to curing

… hygrothermoelastic stress due to moisture absorption and temperature changes

… "chemical" stress related to species diffusion/reaction

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Residual strains and stresses in composite laminates

$$\mathbf{E} = \mathbf{E}^{M} + \mathbf{E}^{NM}$$

$$\mathbf{E}^{M} = \mathbb{S}\mathbf{T}$$

$$\mathbf{E}^{NM} = \mathbf{E}^{T} + \mathbf{E}^{H} + \mathbf{E}^{C} = \alpha\Delta T + \beta\Delta m + \gamma\Delta c$$

$$\mathbf{T} = \mathbb{C}(\mathbf{E} - \mathbf{E}^{NM})$$

$$\mathbf{T} = \mathbb{Q}(\mathbf{E} - \mathbf{E}^{NM})$$

Residual stress ?

Internal stress ?

Initial stress ?

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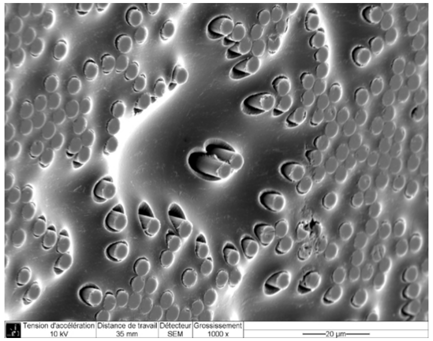
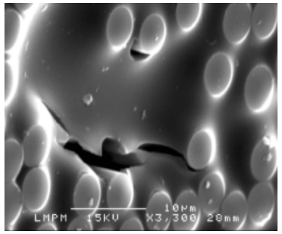
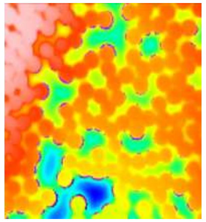
Residual strains and stresses in composite laminates

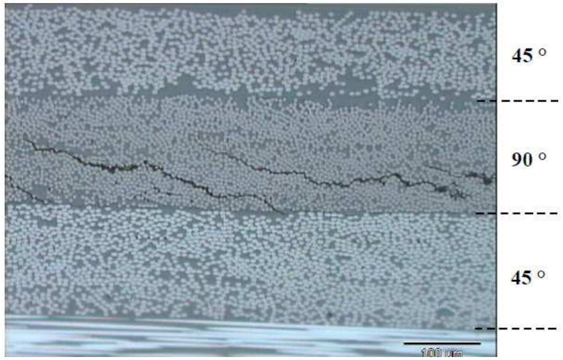
Is there any evidence of such stress ?







microscopic scale

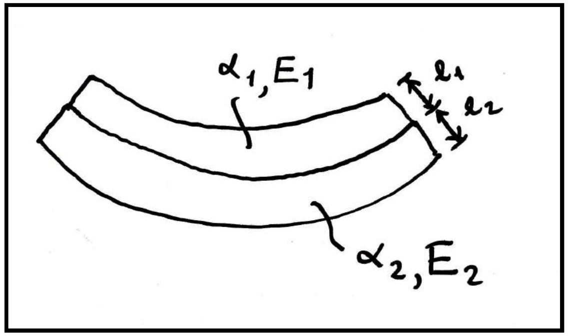


mesoscopic scale

meso/macroscopic scale

…

Timoshenko's bimetallic strip

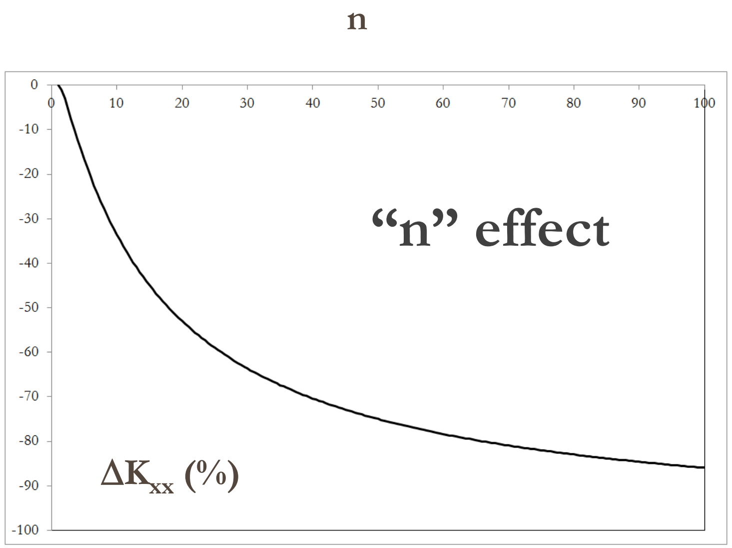


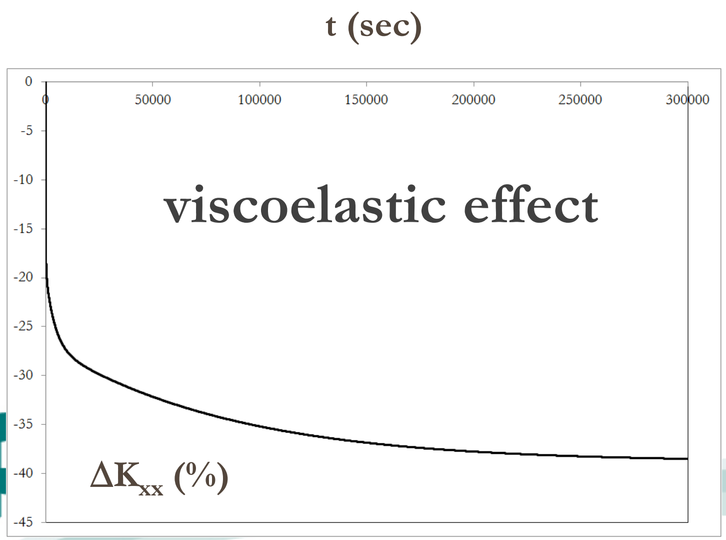
$$K_{xx} = \frac{6(\alpha_2 - \alpha_1)\Delta T(1+m)^2}{e\left(3(1+m)^2 + (1+mn)\left(m^2 + \frac{1}{mn}\right)\right)}$$

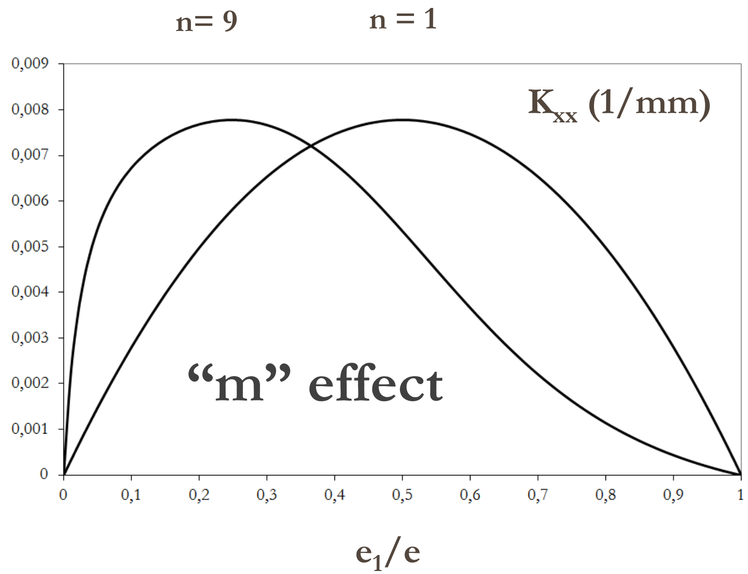
$$m = \frac{e_1}{e_2} \qquad n = \frac{E_1}{E_2}$$

Timoshenko, Journal of the Optical Society of America, 11: 233-255, 1925

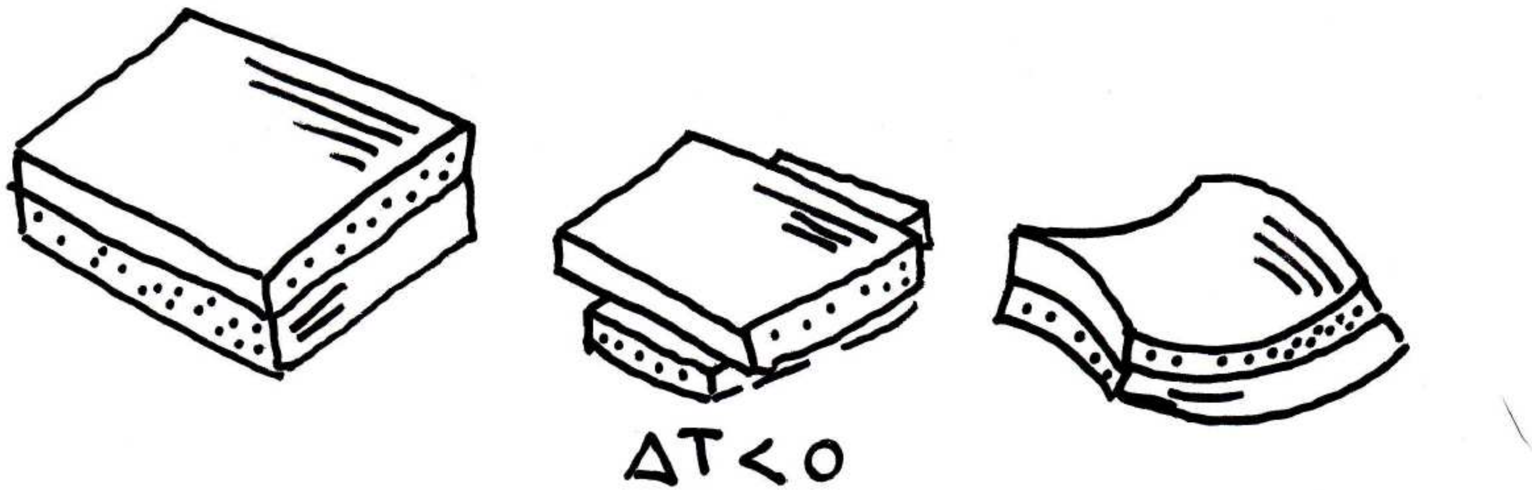
Timoshenko's bimetallic strip



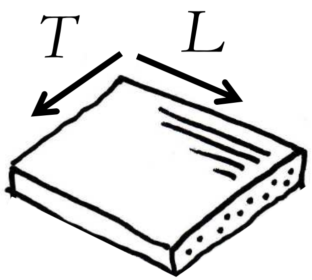




0/90 unsymmetric composite plates



Calculation of residual stresses in composite laminates (CLT)



$$\mathbf{T}_k = \mathbb{Q}_k(\mathbf{E} - \mathbf{E}_k^{NM}) = \mathbb{Q}_k(\mathbf{E} - \alpha_k \Delta T)$$

$$\mathbb{Q}_k \equiv \begin{pmatrix} Q_{LL} & Q_{LT} & 0 \\ Q_{LT} & Q_{TT} & 0 \\ 0 & 0 & Q_{SS} \end{pmatrix}$$

$$\alpha_k \equiv \begin{pmatrix} \alpha_L \\ \alpha_T \\ 0 \end{pmatrix}$$

$$T_0 = \frac{Q_{LL} - 2Q_{LT} + 4Q_{SS} + Q_{TT}}{8} \qquad R_0 = \frac{Q_{LL} - 2Q_{LT} - 4Q_{SS} + Q_{TT}}{8}$$

$$T_1 = \frac{Q_{LL} + 2Q_{LT} + Q_{TT}}{8} \qquad R_1 = \frac{Q_{LL} - Q_{TT}}{8}$$

Vannucci et al., International Journal of Solids and Structures, 38: 9281-9295, 2001

Calculation of residual stresses in composite laminates (CLT)

$$\mathbf{T}_k = \mathbb{Q}_k(\mathbf{E} - \alpha_k \Delta T)$$

$$\mathbf{E} = \mathbf{E}^0 + z\mathbf{K}$$

$$\mathbf{N} = \int_e \mathbf{T}_k dz \qquad \mathbf{M} = \int_e \mathbf{T}_k z dz$$

$\bar{T}_0, \quad \ldots$ $\qquad$ $\bar{T}, \quad \ldots$

$$\mathbf{N} = \mathbb{A}\mathbf{E}^0 + \mathbb{B}\mathbf{K} - \mathbf{U}\Delta T$$

$$\mathbf{M} = \mathbb{B}\mathbf{E}^0 + \mathbb{D}\mathbf{K} - \mathbf{V}\Delta T$$

$\hat{T}, \quad \ldots$

$\hat{T}_0, \quad \ldots \qquad \tilde{T}_0, \quad \ldots$

Vannucci et al., International Journal of Solids and Structures, 38: 9281-9295, 2001

Calculation of residual stress in composite laminates (CLT)

$$\mathbf{E}^0 = \mathbb{A}^*\mathbf{N} + \mathbb{B}^*\mathbf{M} + \mathbf{U}^*\Delta T$$

$$\mathbf{K} = \mathbb{B}^{**}\mathbf{N} + \mathbb{D}^*\mathbf{M} + \mathbf{V}^*\Delta T$$

$$\mathbb{A}^* = \left(\mathbb{A} - \mathbb{B}\mathbb{D}^{-1}\mathbb{B}\right)^{-1}$$

$$\mathbb{D}^* = \left(\mathbb{D} - \mathbb{B}\mathbb{A}^{-1}\mathbb{B}\right)^{-1}$$

$$\mathbb{B}^* = -\left(\mathbb{A} - \mathbb{B}\mathbb{D}^{-1}\mathbb{B}\right)^{-1}\mathbb{B}\mathbb{D}^{-1}$$

$$\mathbb{B}^{**} = -\left(\mathbb{D} - \mathbb{B}\mathbb{A}^{-1}\mathbb{B}\right)^{-1}\mathbb{B}\mathbb{A}^{-1}$$

$$\mathbf{U}^* = \left(\mathbb{A} - \mathbb{B}\mathbb{D}^{-1}\mathbb{B}\right)^{-1}\left(\mathbf{U} - \mathbb{B}\mathbb{D}^{-1}\mathbf{V}\right)^{-1}$$

$$\mathbf{V}^* = \left(\mathbb{D} - \mathbb{B}\mathbb{A}^{-1}\mathbb{B}\right)^{-1}\left(\mathbf{V} - \mathbb{B}\mathbb{A}^{-1}\mathbf{U}\right)^{-1}$$

$$\mathbf{T}_k = \mathbb{Q}_k(\mathbf{E} - \alpha_k \Delta T)$$

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Two interesting questions

Two interesting issues:

- conditions of max/min residual stress/distortion ?
- small ΔT ?

Max/min residual stress/distortion in laminates

$$\mathbf{E}^0 = \mathbb{A}^*\mathbf{N} + \mathbb{B}^*\mathbf{M} + \mathbf{U}^*\Delta T$$

$$\mathbf{K} = \mathbb{B}^{**}\mathbf{N} + \mathbb{D}^*\mathbf{M} + \mathbf{V}^*\Delta T$$

$$\mathbb{A}^* = \left(\mathbb{A} - \mathbb{B}\mathbb{D}^{-1}\mathbb{B}\right)^{-1}$$

$$\mathbb{D}^* = \left(\mathbb{D} - \mathbb{B}\mathbb{A}^{-1}\mathbb{B}\right)^{-1}$$

$$\mathbb{B}^* = -\left(\mathbb{A} - \mathbb{B}\mathbb{D}^{-1}\mathbb{B}\right)^{-1}\mathbb{B}\mathbb{D}^{-1}$$

$$\mathbb{B}^{**} = -\left(\mathbb{D} - \mathbb{B}\mathbb{A}^{-1}\mathbb{B}\right)^{-1}\mathbb{B}\mathbb{A}^{-1}$$

$$\mathbf{U}^* = \left(\mathbb{A} - \mathbb{B}\mathbb{D}^{-1}\mathbb{B}\right)^{-1}\left(\mathbf{U} - \mathbb{B}\mathbb{D}^{-1}\mathbf{V}\right)^{-1}$$

$$\mathbf{V}^* = \left(\mathbb{D} - \mathbb{B}\mathbb{A}^{-1}\mathbb{B}\right)^{-1}\left(\mathbf{V} - \mathbb{B}\mathbb{A}^{-1}\mathbf{U}\right)^{-1}$$

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Vannucci, Journal of Elasticity, DOI 10.1007/s10659-012-9415-0, 2012

Max/min residual stress/distortion in laminates

$$\mathbf{N} = \mathbb{A}\mathbf{E}^0 + \mathbb{B}\mathbf{K} - \mathbf{U}\Delta T$$
$$\mathbf{M} = \mathbb{B}\mathbf{E}^0 + \mathbb{D}\mathbf{K} - \mathbf{V}\Delta T$$

$$\mathbf{E}^0 = \mathbb{A}^*\mathbf{N} + \mathbb{B}^*\mathbf{M} + \mathbf{U}^*\Delta T$$
$$\mathbf{K} = \mathbb{B}^{**}\mathbf{N} + \mathbb{D}^*\mathbf{M} + \mathbf{V}^*\Delta T$$

$$\mathbb{B} = 0 \Rightarrow \mathbf{V} = 0$$

thermo-elastic uncoupling

$$B = \sqrt{\hat{R}_0^2 + 4\hat{R}_1^2} = 0$$
$$V = \hat{R} = 0$$

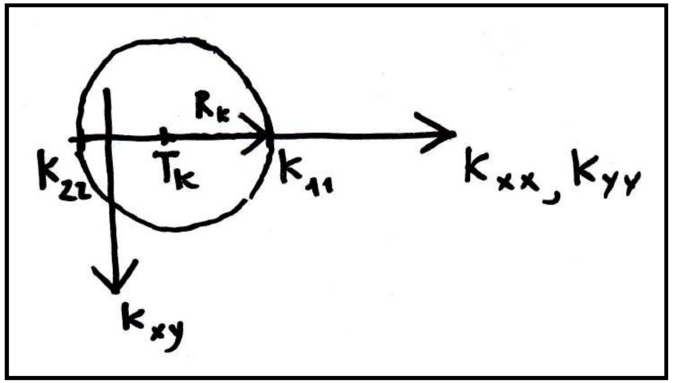
max thermo-elastic coupling

$0_m/90_n$ with $e_0 = e_{90}$ maximise the thermal solicitations induced by ΔT (max V)

Vannucci , Journal of Elasticity, 64: 13-28, 2001

Maximum curvatures of $0_m/90_n$ laminates

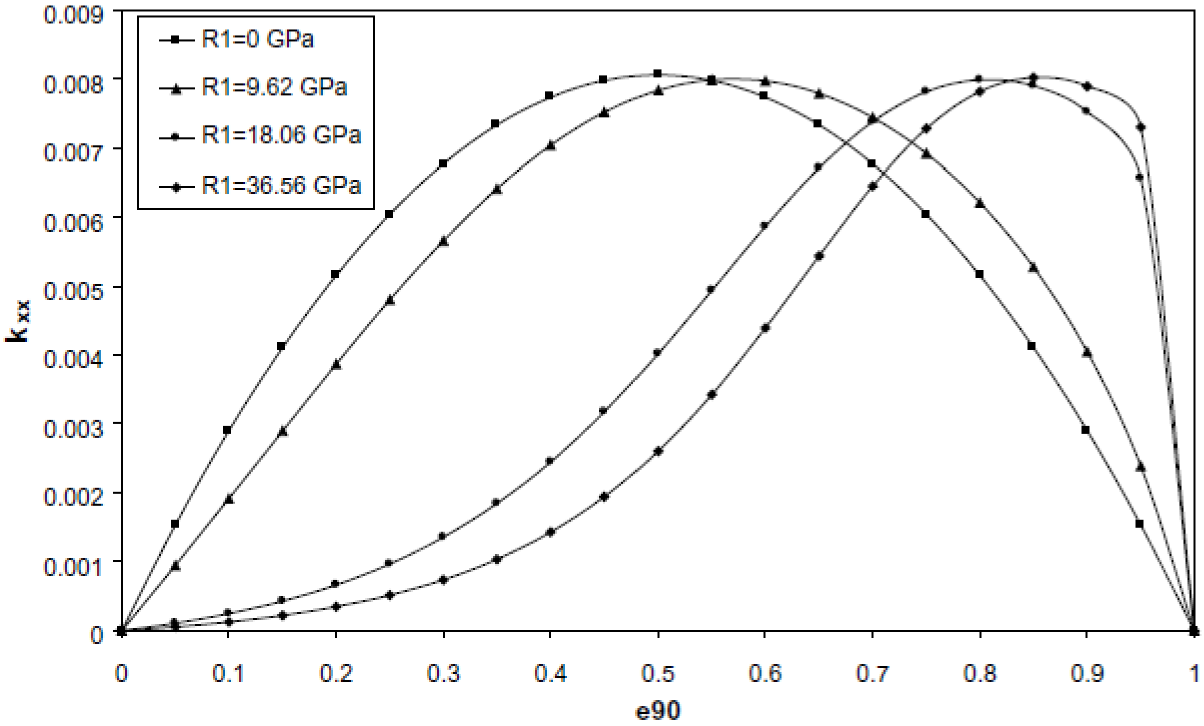
$$\begin{pmatrix} T_{NT} \\ R_{NT} \\ T_{MT} \\ R_{MT} \end{pmatrix} = \begin{pmatrix} 4\bar{T}_1 & 4\bar{R}_1 & 0 & 4\hat{R}_1 \\ 4\bar{R}_1 & 2(\bar{T}_0 + \bar{R}_0) & 4\hat{R}_1 & 0 \\ 0 & 4\hat{R}_1 & 4\tilde{T}_1 & 4\tilde{R}_1 \\ 4\hat{R}_1 & 0 & 4\tilde{R}_1 & 2(\tilde{T}_0 + \tilde{R}_0) \end{pmatrix} \begin{pmatrix} T_E \\ R_E \\ T_K \\ R_K \end{pmatrix}$$



$$8R_1\hat{e}_{90}^2 - 2(T_0 + 2T_1 + R_0 - 4R_1)\hat{e}_{90} + (T_0 + 2T_1 + R_0 - 4R_1) = 0$$

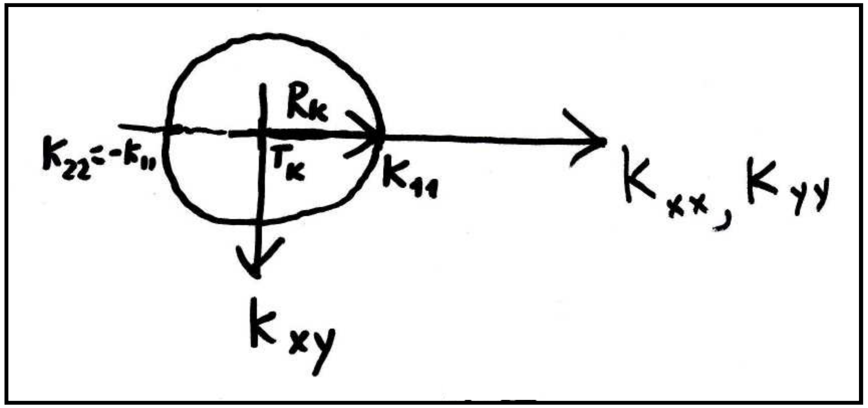
Gigliotti et al., Composite Structures[2], 68: 177-184, 2005

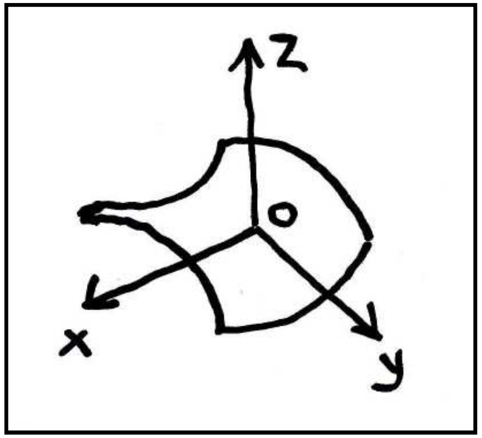
Maximum curvatures of $0_m/90_n$ laminates



Gigliotti et al., Composite Structures, 68: 177-184, 2005

$0_m/90_n$ laminates, m = n



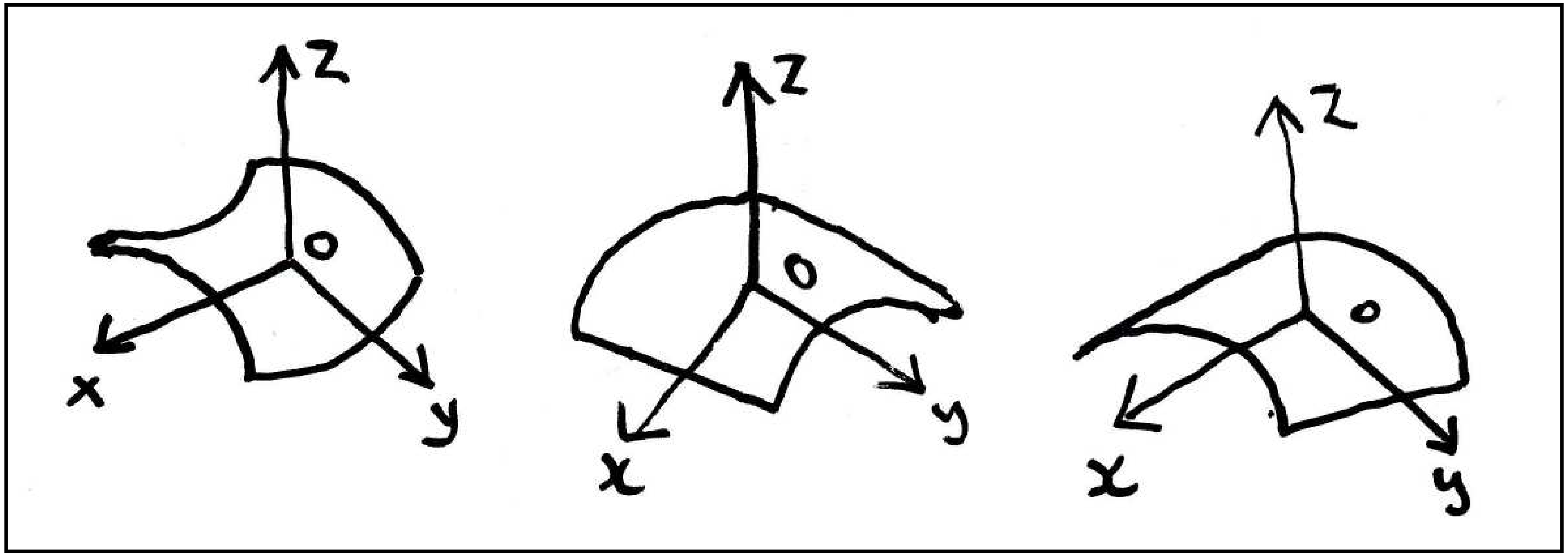


$$K_{xx} = -K_{yy} = \frac{1}{8}\frac{T_0T_1 + T_1R_0 - 2R_1^2}{2T_0T_1 + 2T_1R_0 - 3R_1^2}(\alpha_T - \alpha_L)\Delta T$$

$$w(x,y) = \frac{1}{2}K_{xx}(x^2 - y^2)$$

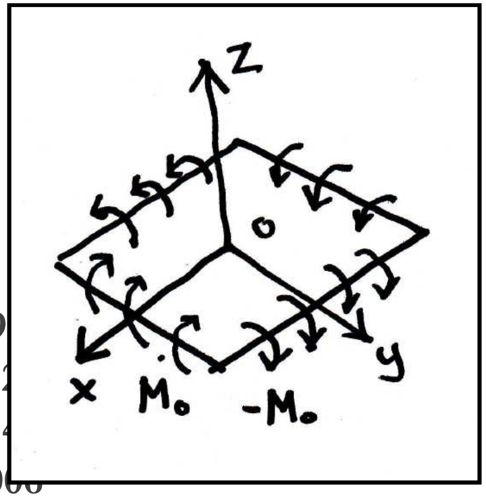
$$R_1 = 0 \qquad K_{xx} = -K_{yy} = \frac{1}{16}(\alpha_T - \alpha_L)\Delta T$$

ΔT small ?



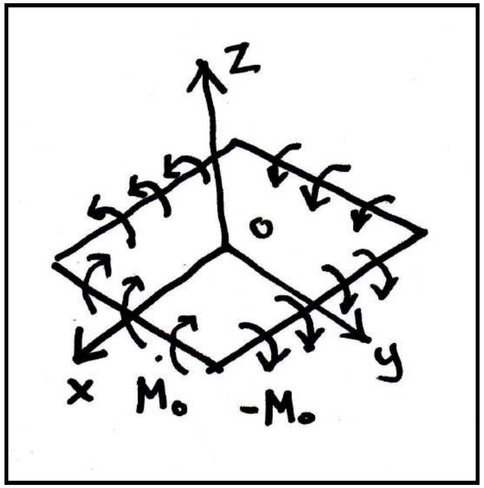
Hyer , Journal of Composite Materials, 15: 175-194, 1981

ΔT small ?
Plates with strain mismatch



Hyer, Journal of Composite Materials, 15: 296-310, 1981
Dano et al., International Journal of Solids and Structures, 40: 59
Gigliotti et al., Composite Science and Technology, 64: 109-128, 2
Galletly et al., International Journal of Solids and Structures, 41: 4
Guest et al., Proceedings of the Royal Society A, 462: 839-854, 2006
Seffen, Proceedings of the Royal Society A, 463: 67-83, 2007
Diaconu et al., Thin-Walled Structures, 46: 689-701, 2008
Mattioni et al., International Journal of Solids and Structures, 46: 151-164, 2009
Fernandes et al., International Journal of Solids and Structures, 47: 1449-1458, 2010
Giomi et al., Proceedings of the Royal Society A 468: 511-530, 2012
Di Carlo et al., Proceedings of the XX AIMETA Congress, Bologna, 2011

ΔT small ?
Analogous problem of a plate bent by double bending load



$$w(x,y) = \frac{M_o}{2D(1-\nu)}(x^2 - y^2)$$

$$w(x,y) = \frac{1}{2}\frac{1}{8}\frac{T_0T_1 + T_1R_0 - 2R_1^2}{2T_0T_1 + 2T_1R_0 - 3R_1^2}(\alpha_T - \alpha_L)\Delta T(x^2 - y^2)$$

La risposta è semplice. Si tratta di un problema di biforcazione fra due stati di equilibrio. Se chiamiamo $\pm M_0$ l'intensità delle coppie agenti sui due lati opposti, e supponiamo che M_0 sia piccolo, allora la configurazione di minima energia totale [$\mathcal{E}_1 = \mathcal{U}_1$ (energia di deformazione) $- \mathcal{L}_1$ (lavoro delle coppie)] è quella a "sella". M_0 per valori maggiori di M_0, la configurazione di minima energia è quella cilindrica. Si può determinare M_0.

Il problema da Lei posto è molto interessante.

P. V.

Minimum Total Energy

$$E = \int_V \frac{1}{2} \mathbf{E} \cdot \mathbb{Q} \mathbf{E} dV + E_{ext}$$

$$\delta E = 0$$

$$\delta^2 E > 0$$

$$E^o_{xx} = \frac{\partial u(x,y)}{\partial x} + \frac{1}{2}\left(\frac{\partial w(x,y)}{\partial x}\right)^2$$

$$E^o_{yy} = \frac{\partial v(x,y)}{\partial y} + \frac{1}{2}\left(\frac{\partial w(x,y)}{\partial y}\right)^2$$

$$K_{xx} = \frac{\partial^2 w(x,y)}{\partial x^2} \Big/ \left(1 + \left(\frac{\partial w(x,y)}{\partial x}\right)^2\right)^{3/2} \cong \frac{\partial^2 w(x,y)}{\partial x^2}$$

$$K_{yy} = \frac{\partial^2 w(x,y)}{\partial y^2} \Big/ \left(1 + \left(\frac{\partial w(x,y)}{\partial y}\right)^2\right)^{3/2} \cong \frac{\partial^2 w(x,y)}{\partial y^2}$$

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Rayleigh–Ritz approach

$$\mathbf{u}^R(x, y, z) = \sum_{i=0}^{q} \mathbf{a}_i \chi_i(x, y, z)$$

$$E^R(\mathbf{u}^R(x, y, z)) = E^R(\mathbf{a}_i)$$

$$\frac{\partial E^R}{\partial \mathbf{a}_i} = 0$$

$$\frac{\partial^2 E^R}{\partial \mathbf{a}_i \partial \mathbf{a}_i} > 0 \quad i = 1 \ldots q$$

Assumed displacement fields

4-term model

$$u^{R4}(x,y) = cx - \frac{a^2x^3}{6} - \frac{abxy^2}{4}$$

$$v^{R4}(x,y) = dy - \frac{b^2y^3}{6} - \frac{abx^2y}{4}$$

$$w^{R4}(x,y) = \frac{1}{2}(ax^2 + by^2)$$

12-term model*

$$u^{R12}(x,y) = c_7x - \frac{2}{3}c_1^2x^3 - \frac{8}{5}c_1c_3x^5 - \frac{8}{7}c_3^2x^7 - \frac{12}{7}c_1c_5x^7 - \frac{8}{3}c_3c_5x^9 - \frac{18}{11}c_5^2x^{11} + c_9xy^2 + c_{11}xy^4$$

$$v^{R12}(x,y) = c_8y - \frac{2}{3}c_2^2y^3 - \frac{8}{5}c_2c_4y^5 - \frac{8}{7}c_4^2y^7 - \frac{12}{7}c_2c_6y^7 - \frac{8}{3}c_4c_6y^9 - \frac{18}{11}c_6^2y^{11} + c_{10}x^2y + c_{12}x^4y$$

$$w^{R12}(x,y) = c_1x^2 + c_2y^2 + c_3x^4 + c_4y^4 + c_5x^6 + c_6y^6$$

*Ashwell, Journal of the Royal Aeronautical Society, 54: 708, 1950

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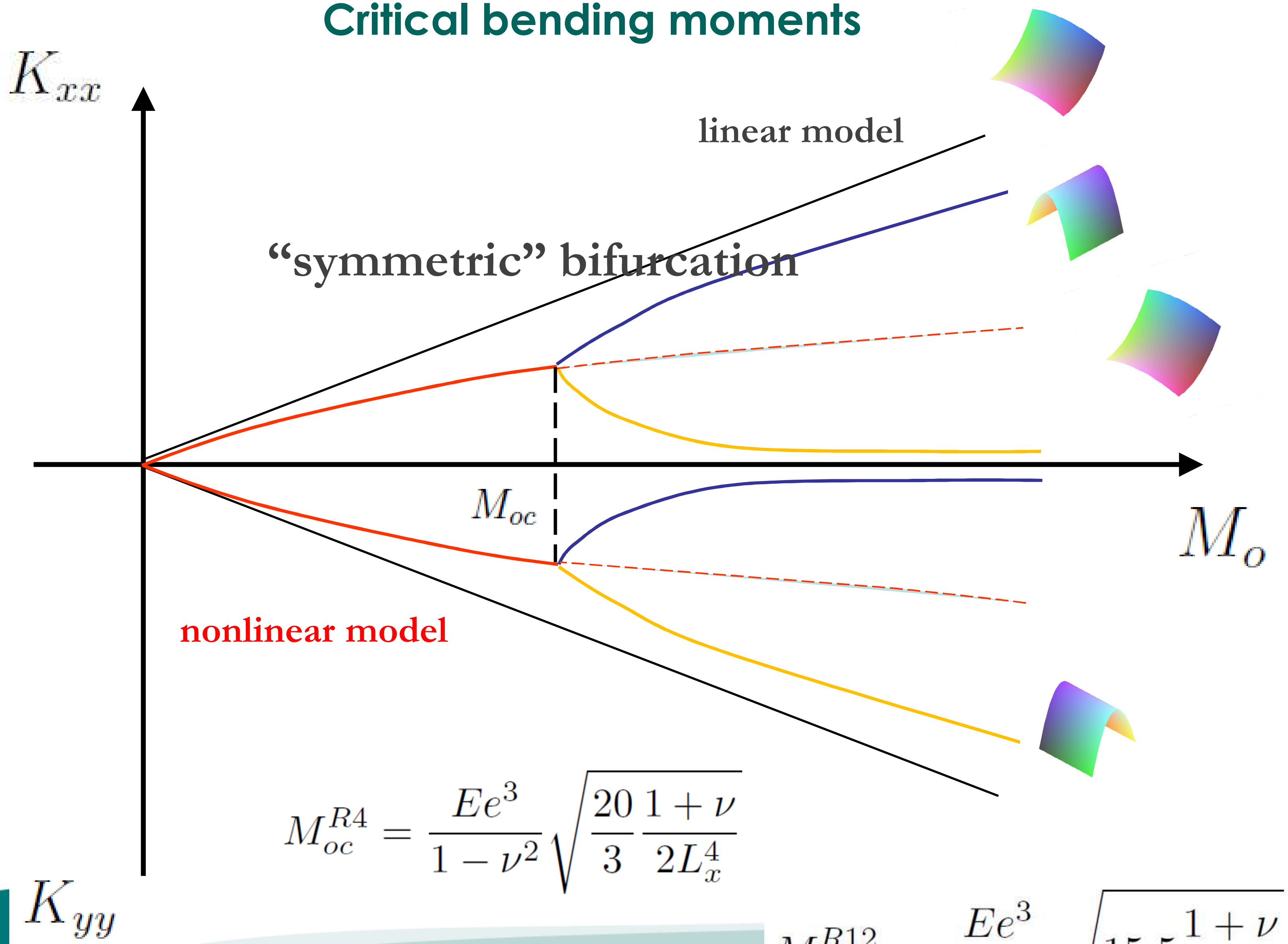
Critical bending moments



$$M_{oc}^{R4} = \frac{Ee^3}{1-\nu^2}\sqrt{\frac{20}{3}\frac{1+\nu}{2L_x^4}}$$

$$M_{oc}^{R12} = \frac{Ee^3}{1-\nu^2}\sqrt{15.5\frac{1+\nu}{2L_x^4}}$$

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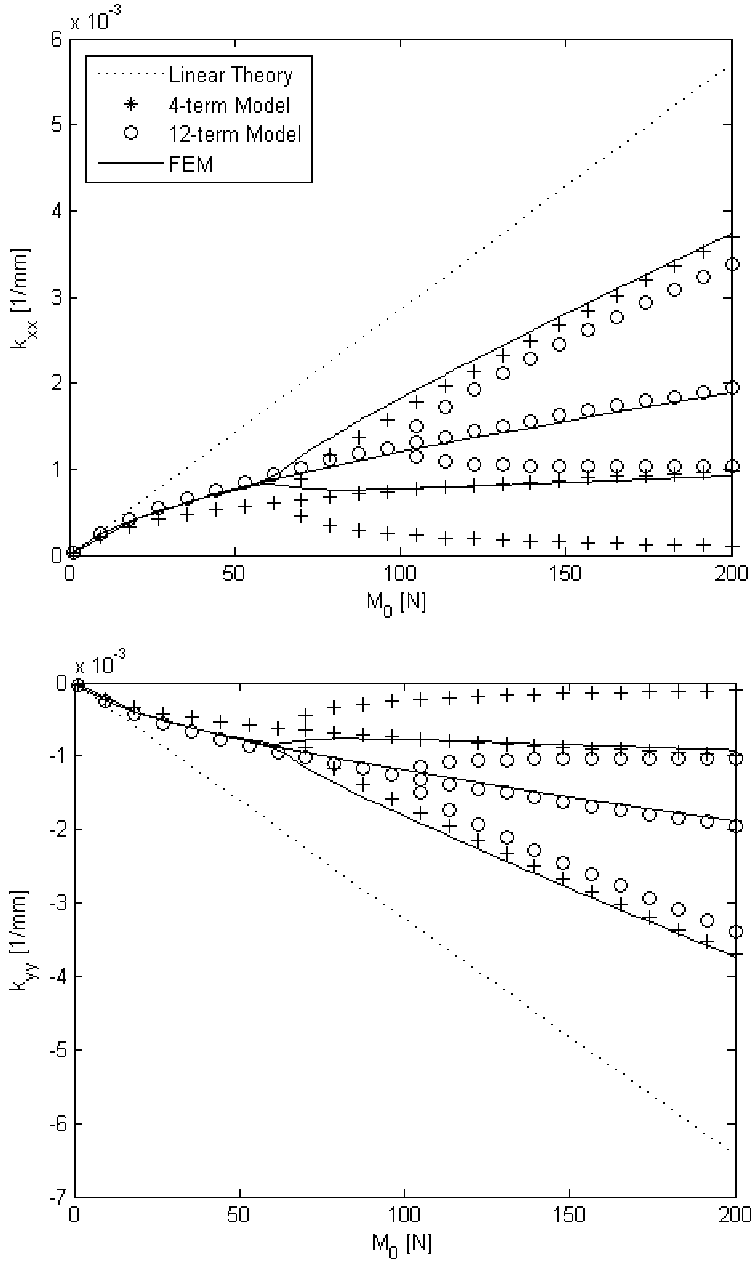
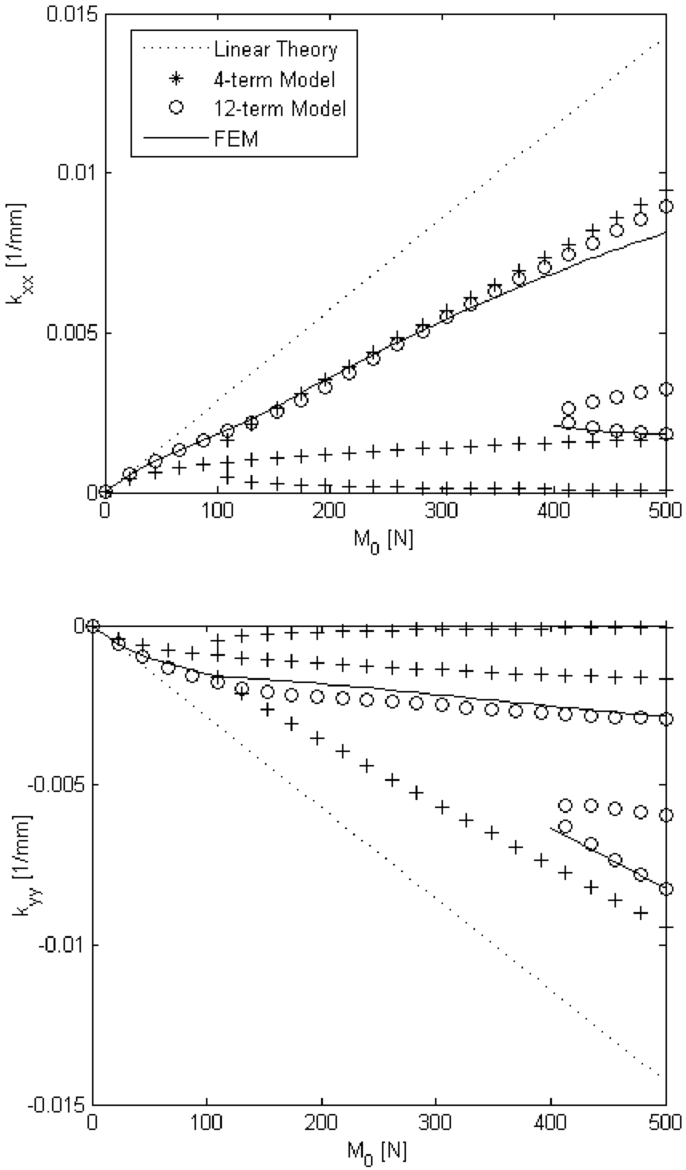
AR = 1



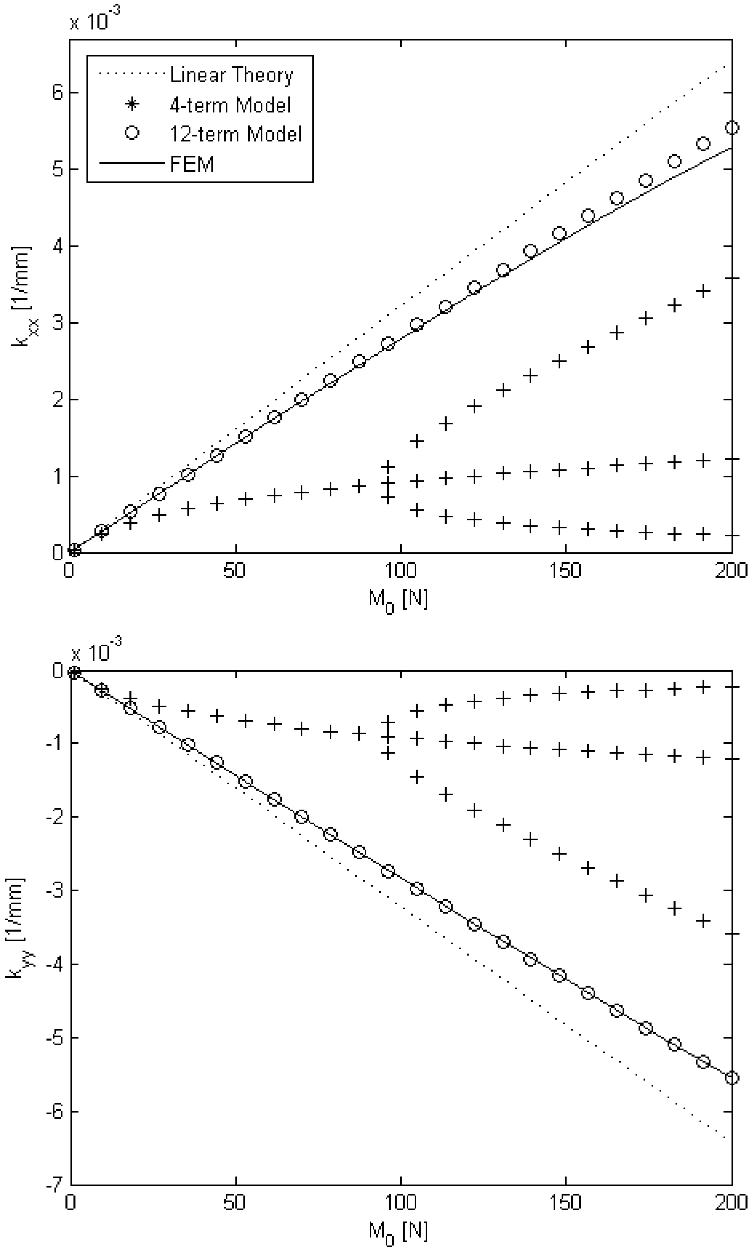
“symmetric” bifurcation

AR = 2



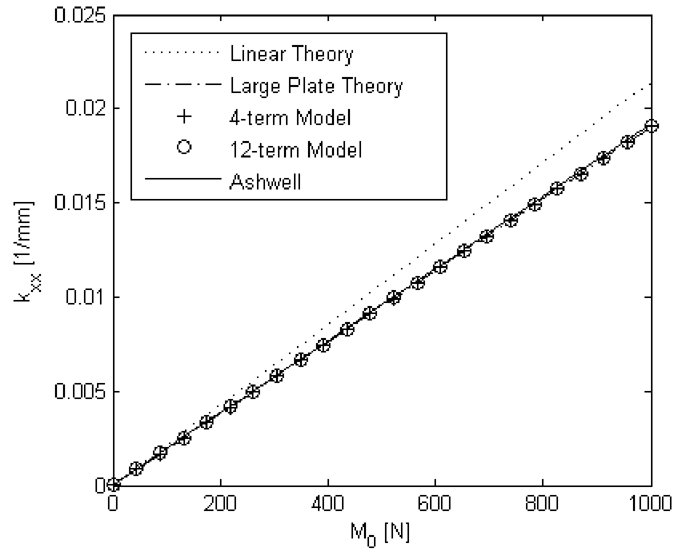
“loss” of bifurcation behaviour

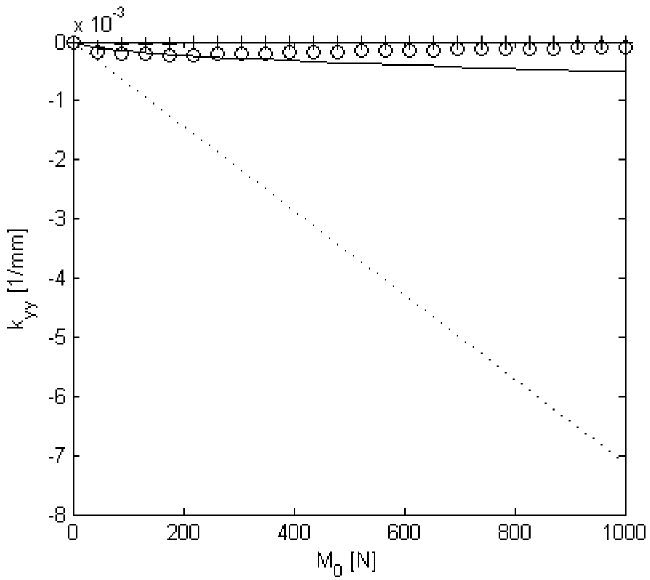
AR = 10

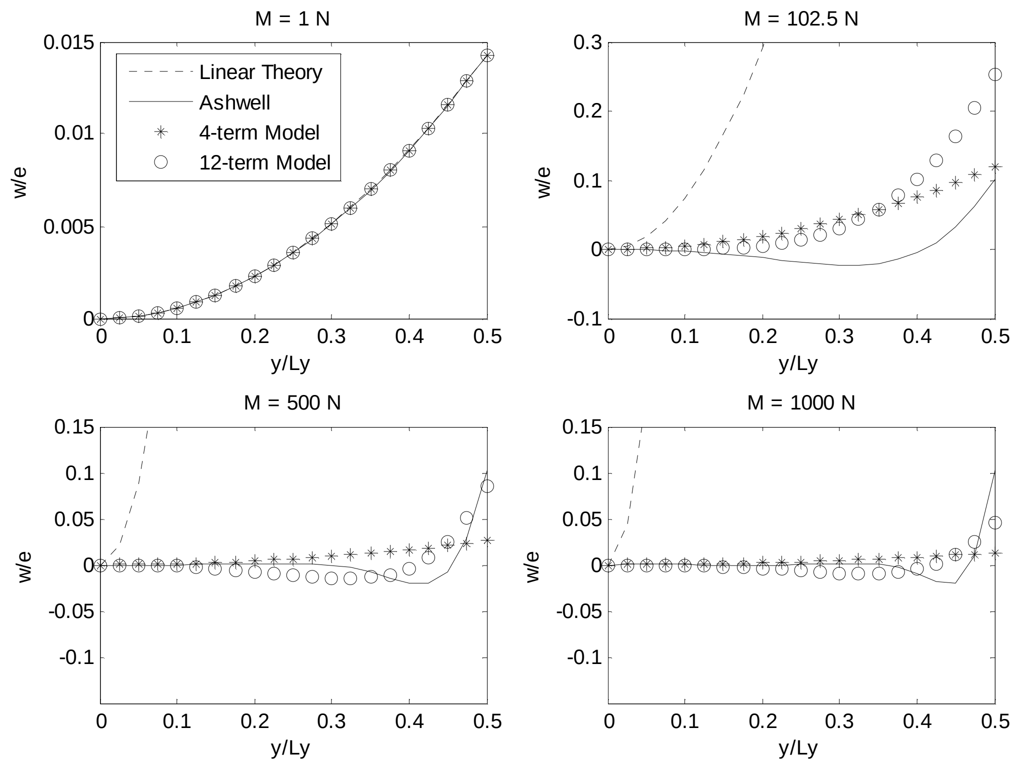


"close to linear" behaviour

Simple bending, AR = 1



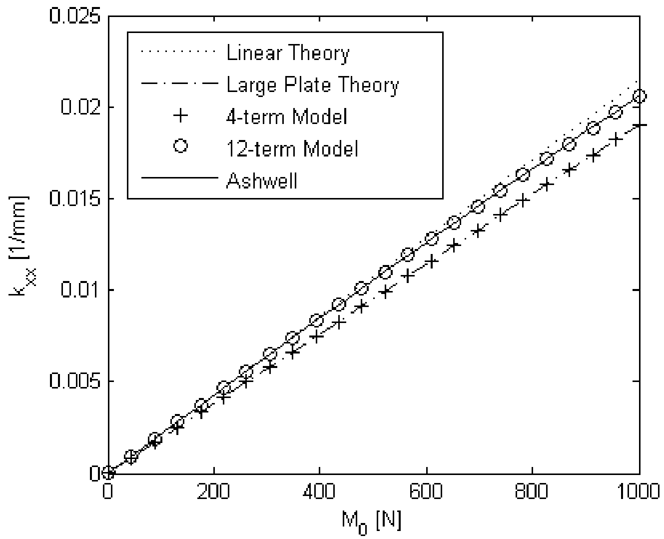


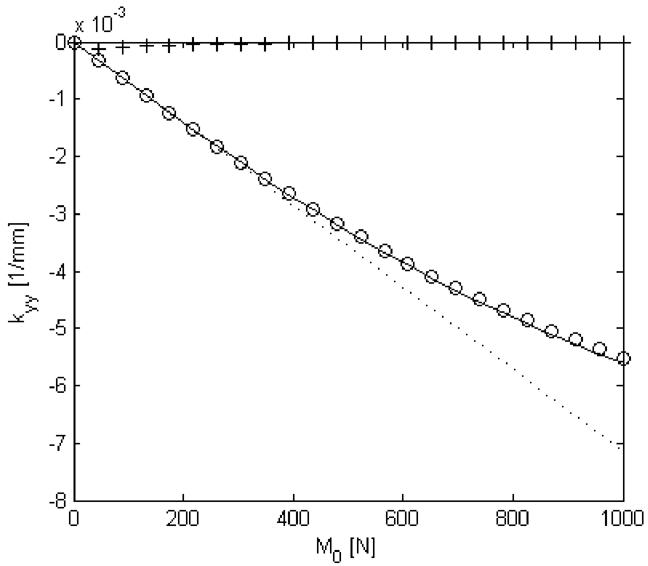


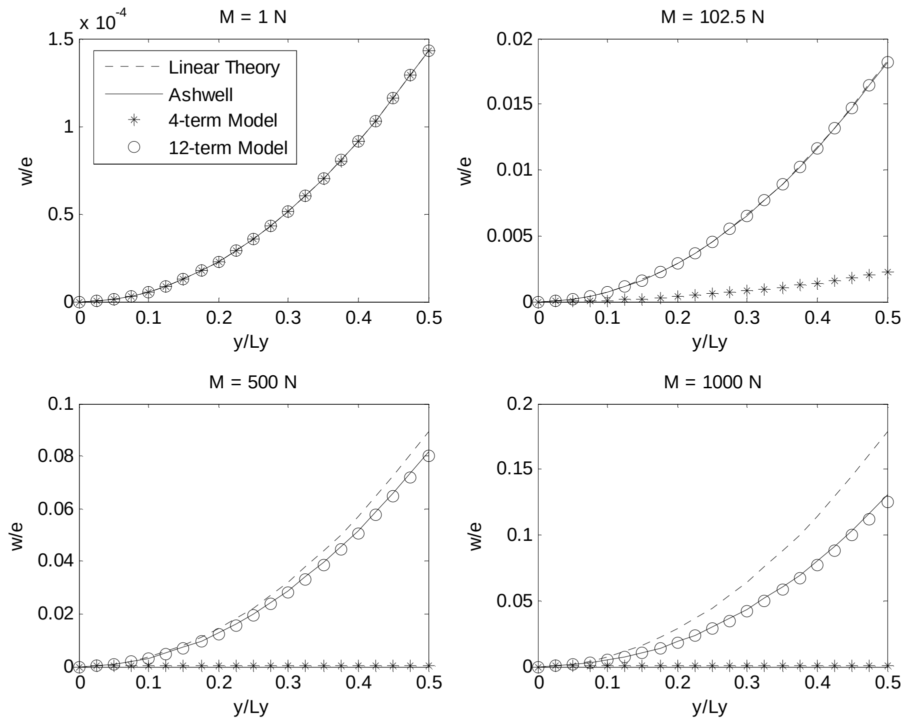
Linear Theory $$K_{xx} = \frac{M_o}{EI} \qquad K_{yy} = -\nu K_{xx}$$

Large Plate Theory $$K_{xx} = \frac{M_o}{EI}(1-\nu^2) \qquad K_{yy} = 0$$

Simple bending, AR = 10







“Efficiency” of the models

Linear model	4-term model	12-term model
0.33	1	7

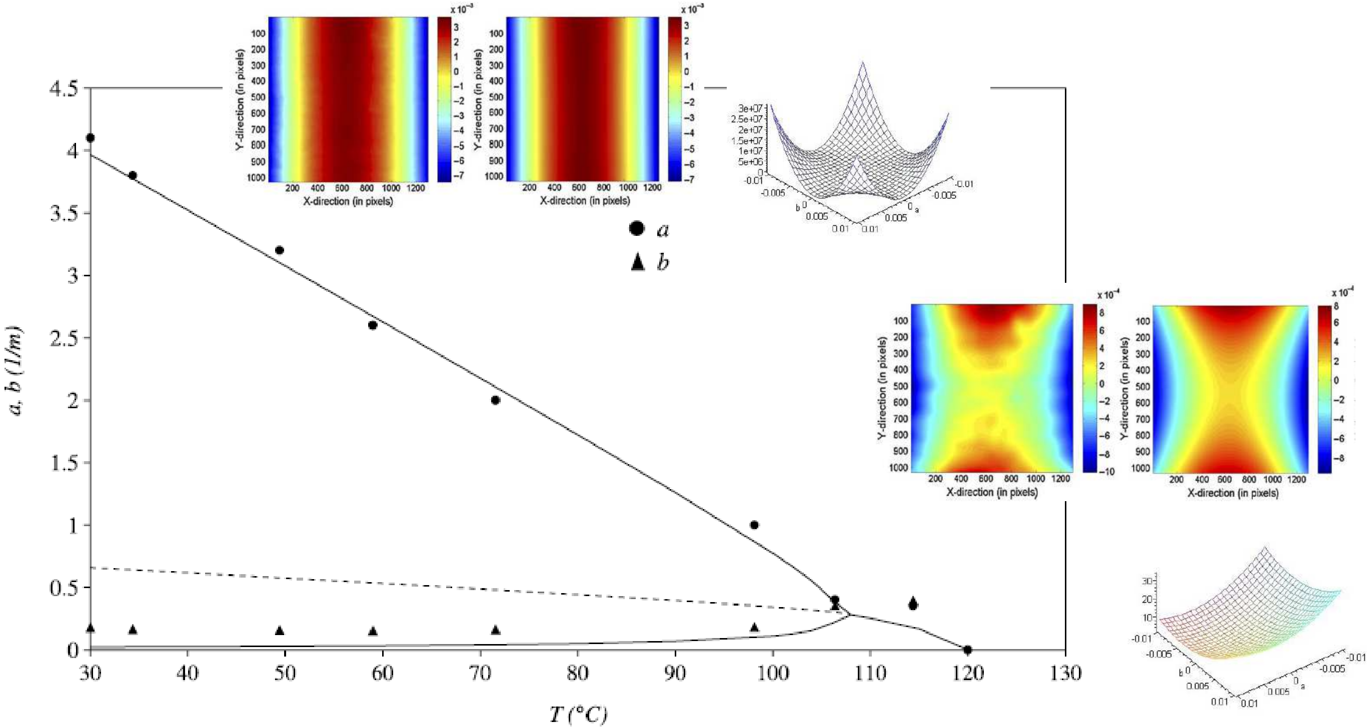
12-term model much more accurate
only slightly slower

Examples

0/90 unsymmetric laminates for monitoring thermal stress in composite materials

$$\delta E = 0$$

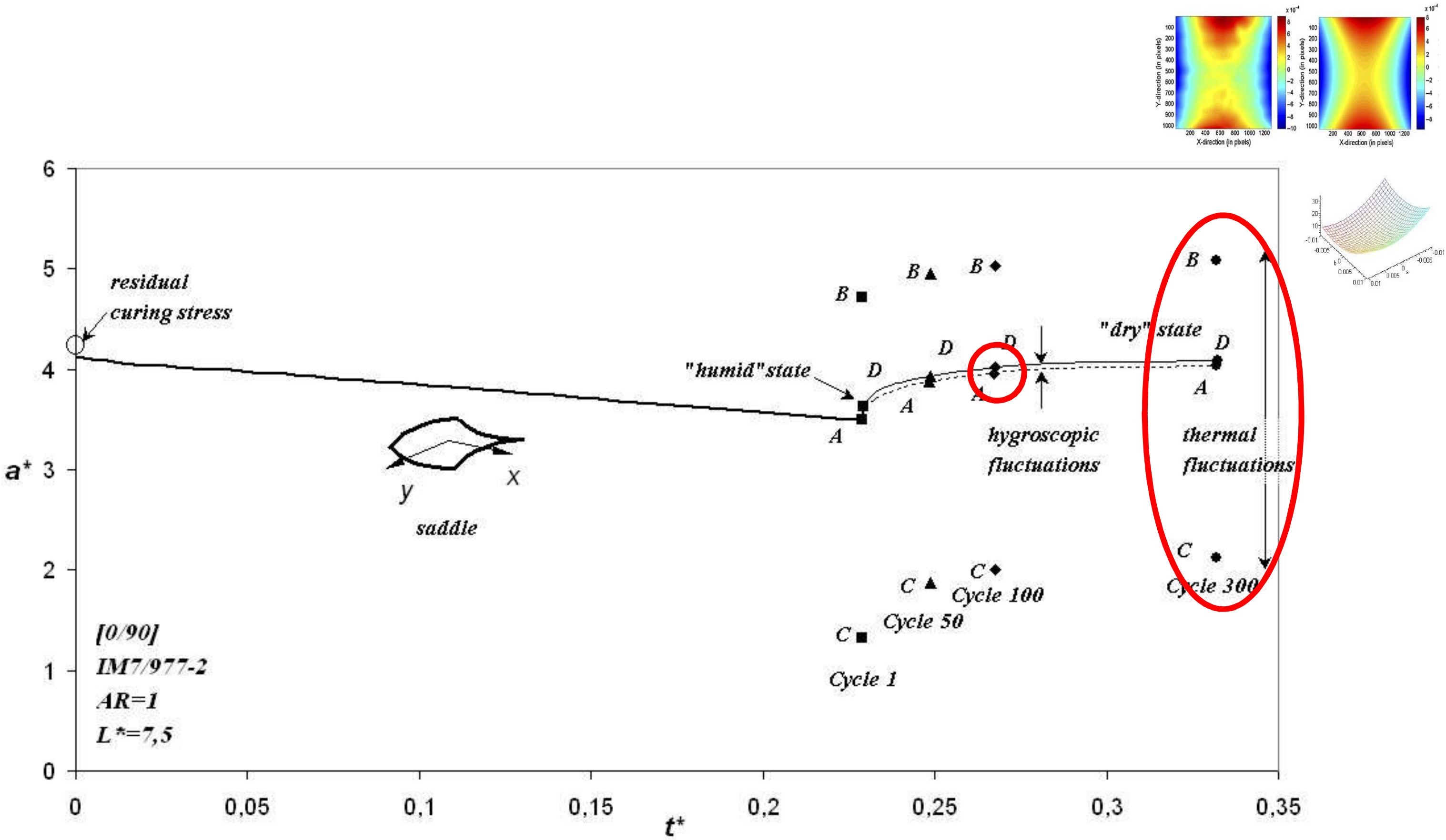
$$\delta^2 E > 0$$



Gigliotti et al., Mechanics of Materials, 39 : 729-745, 2007

Gigliotti et al., Composite Structures, 94 : 2793-2808, 2012

0/90 unsymmetric laminates for monitoring hygrothermal cyclic stress in composite materials



Gigliotti et al., Mechanics of Materials, 39 : 729-745, 2007

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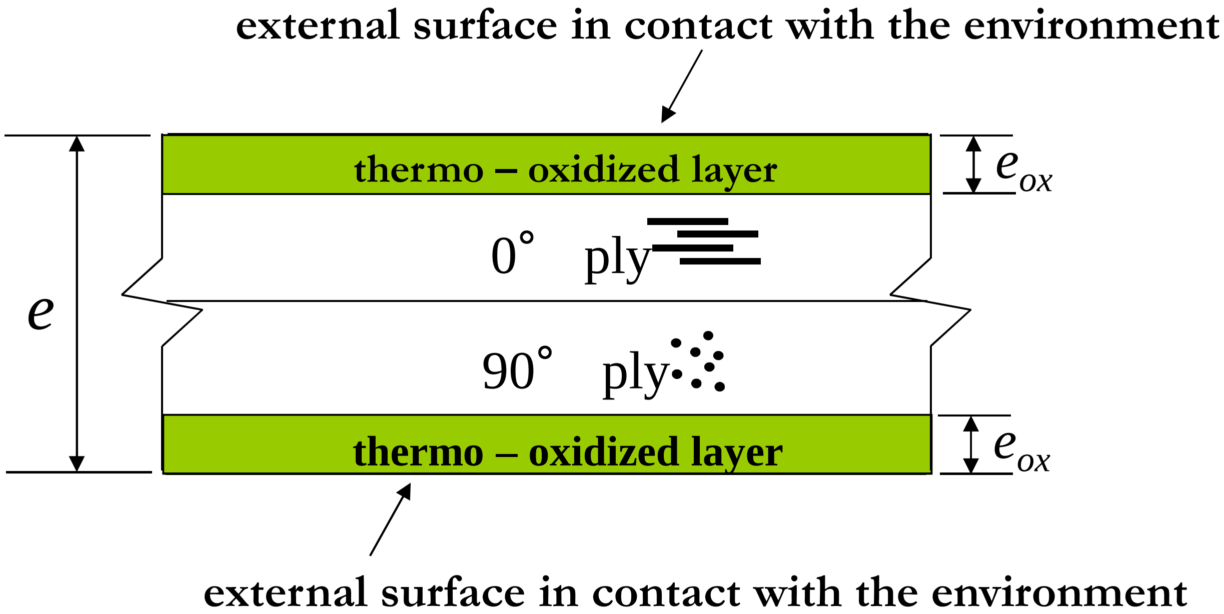
0/90 unsymmetric laminates for monitoring thermo-oxidative ageing of composite materials



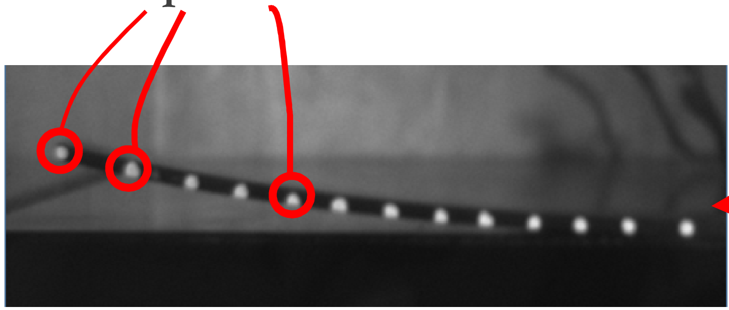
Thermo – oxidation leads to curvature changes with time

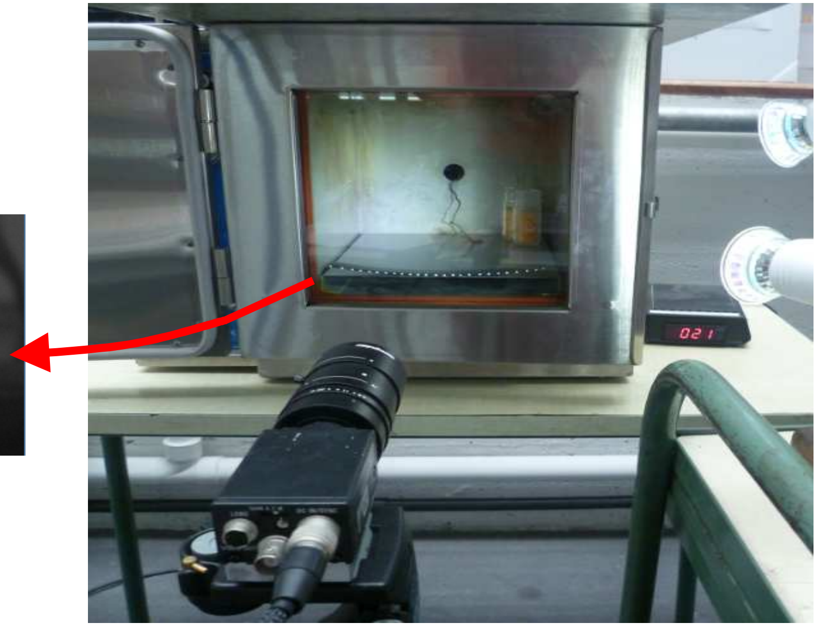
Material: IM7/977-2
Plate dimensions: 200 mm x 20 mm x 2 mm (AR = 10)

Experimental procedure:

- Drying at 80° C under vacuum
- Ageing under atmospheric air at 150° C in a oven (dry environment)
- Curvature measurement

white markers to follow shape evolution with time





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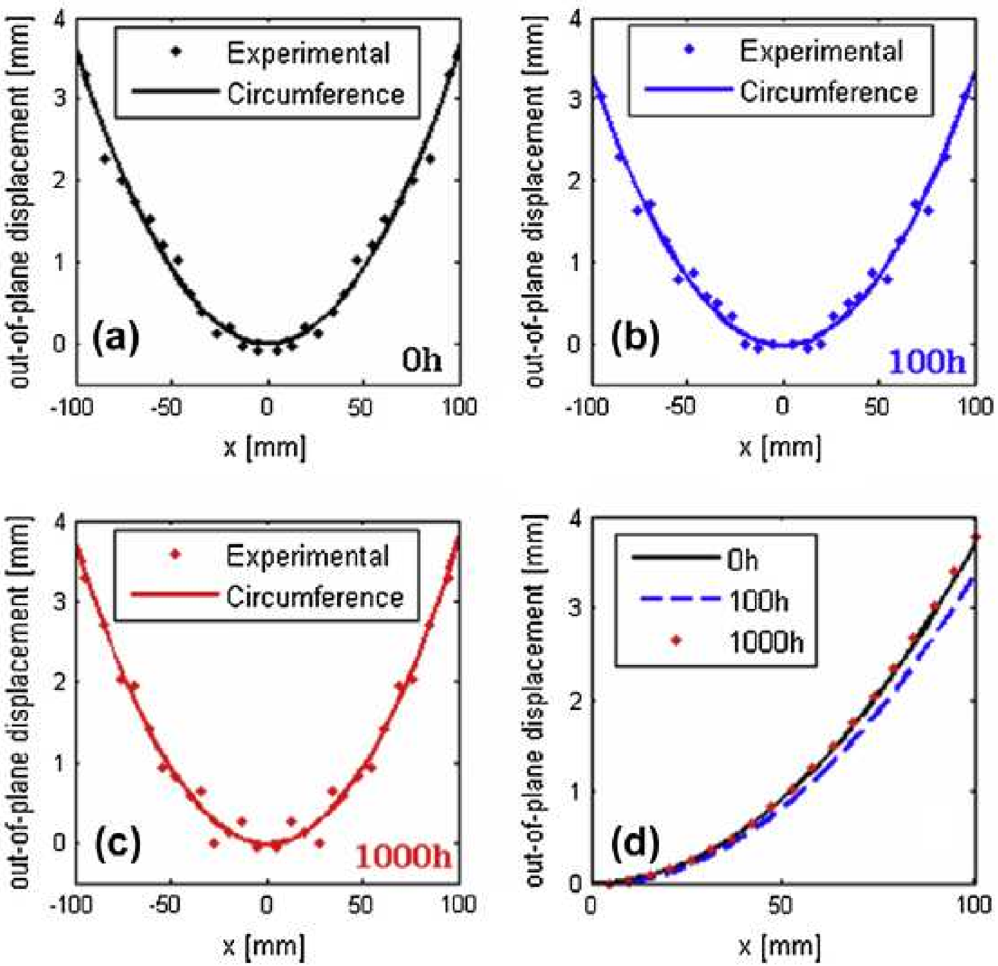
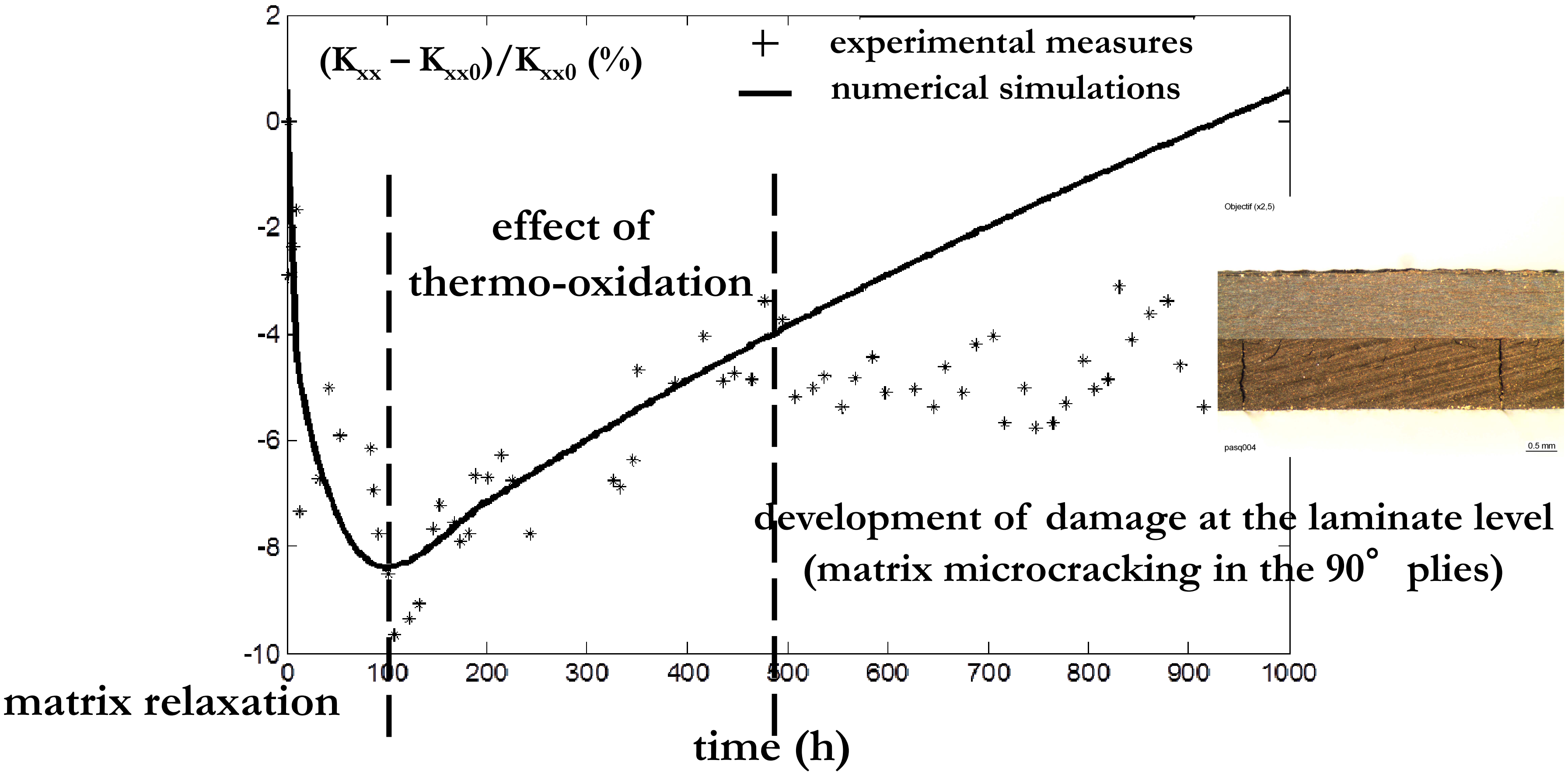
Continuous monitoring of curvature: shape evolution with time



Curvature predictions

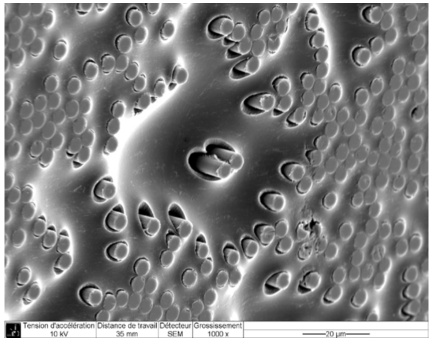


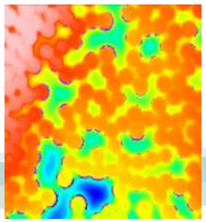
Conclusions

- Internal/residual strain/stress do exist in composite materials !
- They may evolve (relax/increase/decrease) with time following exposure to the environment
- They may lead to damage onset/propagation
- They should be taken into account – even in simplified way – in every structural analysis involving composite materials

Some perspective work …

- General problem of max/min distortion/stress in laminates !
- Detailed study of damage onset/propagation due to internal/residual stress
- Internal/residual stress at the microscopic level





Possibilities for Internships ("Stages Master") at DPMM/ENSMA

- ✓ ERASMUS exchanges (Politecnico di Milano, Politecnico di Torino, Università di Padova, Università della Calabria, UNICAL)
- ✓ PEGASUS network (PoliMI, PoliTO, Università di Pisa)
- ✓ EASN Network …

Possibilities for PhD studies at DPMM/ENSMA

- ✓ Grants form the Ministry of University and Research
- ✓ Grants from the Ministry of Industry and Defence
- ✓ Grants from Industrial partnership (CIFRE Grants)
- ✓ Grants from National or International research projects (ANR, FUI, FRAE, EU, IRT, …)
- ✓ … ("Co-tutelle")

Possibilities for Post-Doc Grants at DPMM/ENSMA

- ✓ Grants from National or International research projects (ANR, FUI, FRAE, EU, IRT, …)
- ✓ …

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Other Temporary/Permanent Staff positions at DPMM/ENSMA

- ✓ Research Engineer (Ingénieur de Recherche, CDD/CDI)
- ✓ Research Assistant (Ingénieur d'Etudes, CDD/CDI)
- ✓ Research Fellow (A.T.E.R., CDD)

National Habilitation for 2nd Rank Professorship (Maitre de Conférences)

- ✓ Must hold a PhD
- ✓ Several temporary research positions (A.T.E.R., Post-Doc, Research Fellow)
- ✓ 5 publications in International Journals
- ✓ Having some experience in writing grants
- ✓ …

National Habilitation for 1st Rank Professorship (Professeur des Universités)

- ✓ Must hold a HDR (Habilitation à Diriger des Recherches)
- ✓ 10 publications in International Journals, Chapters in Books, Reviews, Prizes
- ✓ Having good experience in writing and directing research projects, PhD students, Master students, …
- ✓ …

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