Problemi di Ottimizzazione di sistemi alari

Emanuele Rizzo

Dipartimento di Ingegneria Aerospaziale *"L. Lazzarino"*
Università di Pisa

Scuola di Dottorato in Ingegneria *"Leonardo da Vinci"*
Pisa, Italy

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1. Outline.
2. Introduction
   - New scenarios for the aviation of the future.
3. Results of the theory of optimization.
   - The general NLP problem.
   - Algorithms.
   - An algorithm for global optimization.
4. Test Cases.
   - Math benchmarking problems.
   - Aerodynamic Optimization of wings.
5. Application to a ULM PrandtlPlane airplane.
   - General Layout of PP-ULM.
   - Optimization Model.
   - Some solutions.
6. Conclusions, results and way forward.
   - Suggested Links

EU VISION 2020: main targets to be achieved.

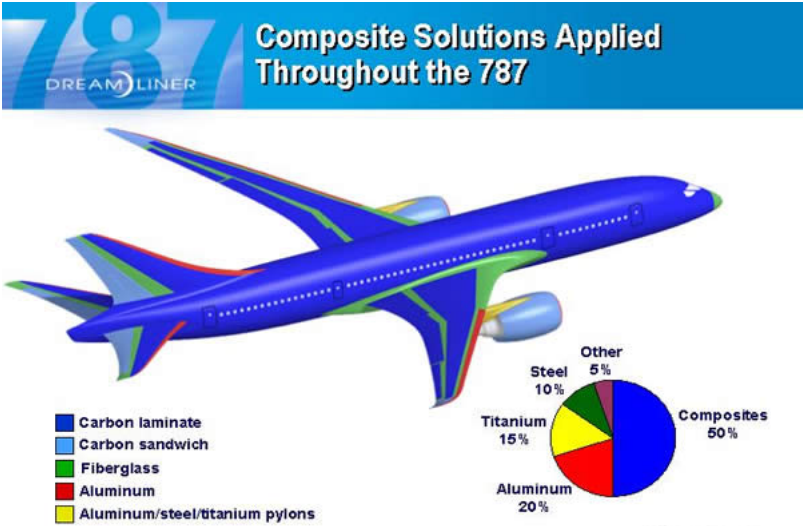
The European Union published the VISION 2020 report setting a new set of ambitious targets for the next generation air transport system:

1. 80% reduction of NOx emissions
2. Halving perceived aircraft noise
3. Five-fold reduction in accidents
4. 50% cut in CO2 emissions per passenger-km
5. 99% of all flights within 15 minutes of timetable

Keywords: more efficiency, more safety, more integration among the airport/transport systems.

State of the art.

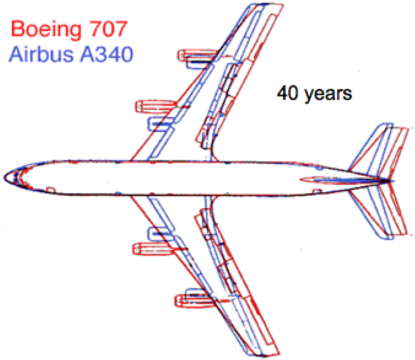
Boeing 787: conventional architecture, composite materials.



State of the art.

Airbus A380: the upper limit for conventional aircraft.

State of the art.
B707 vs A340



from *Aircraft Design: Synthesis and Analysis* I. Kroo, http://adg.stanford.edu/aa241/AircraftDesign.html

How does the aircraft of the next generation look like?

The Blended Wing Body (BWB).

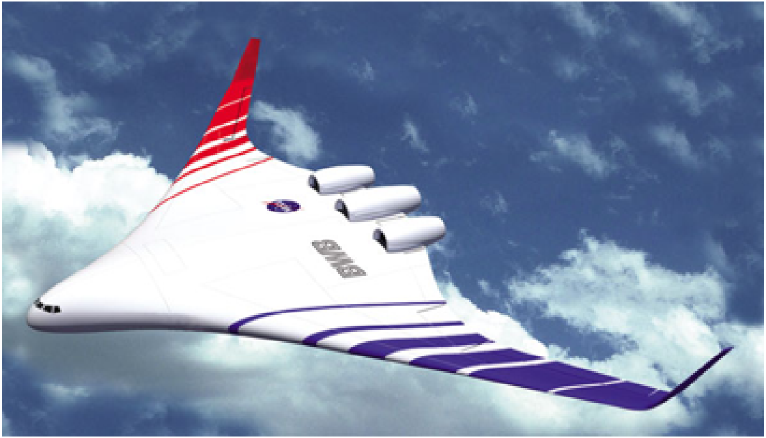


Figure: BWB tries to minimize wetted area.

How does the aircraft of the next generation look like?
The C-Wing.

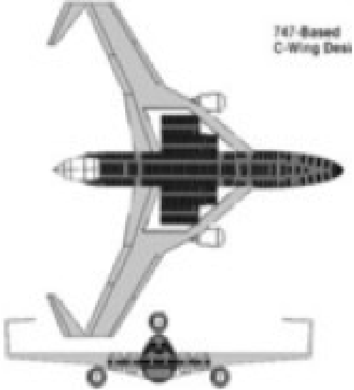



Figure: C-Wing: an example of non-planar wing system.

How does the aircraft of the next generation look like?

The PrandtlPlane.

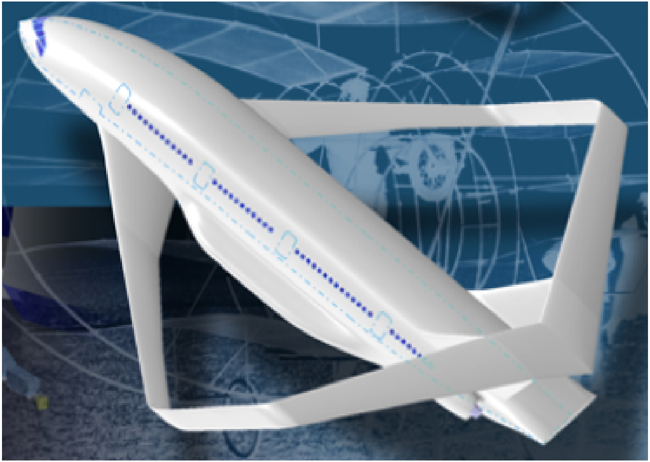
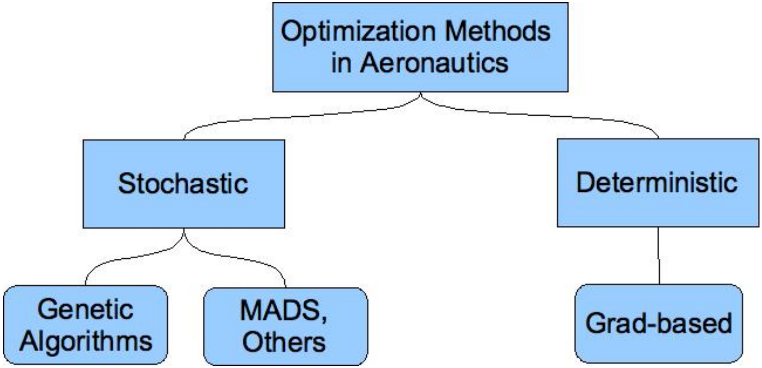


Figure: Based on the Best Wing System by Prandtl: the induced drag is minimum.

Optimization in Aeronautics.



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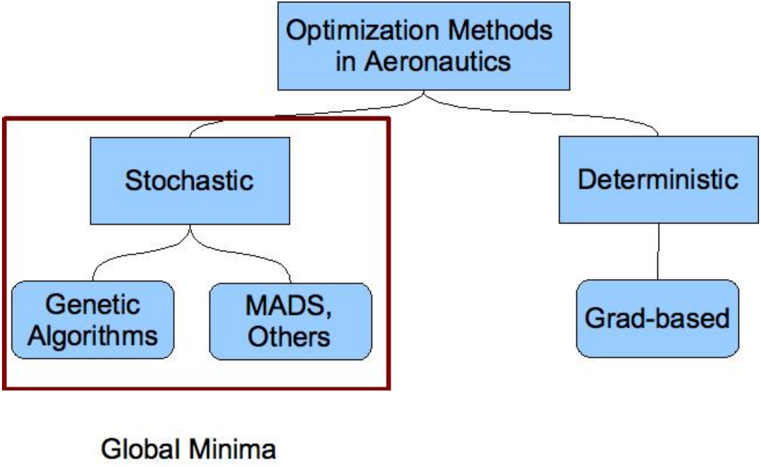

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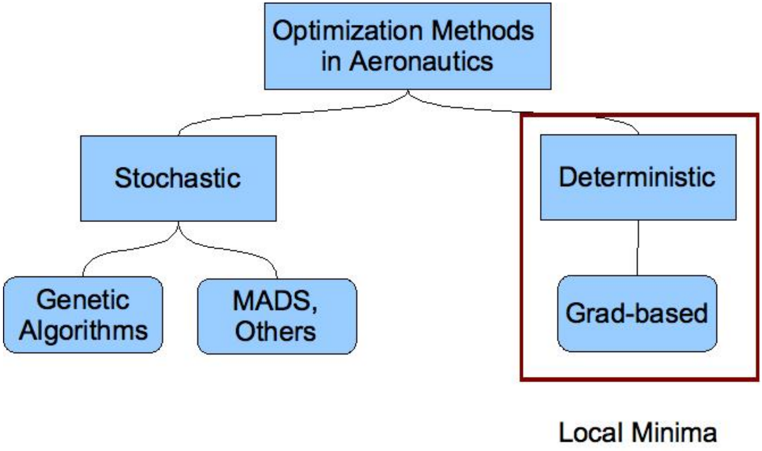

Main characteristics

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1. Good exploration properties of the design space (Stochastic Alg.)
2. Seek a global minimum (Stochastic Alg.)
3. Fast & Robust (Grad-based Alg.)
4. Applicable to a wide range of problems

NLP problem.

$$\begin{cases} \min f(x) \\ g(x) \leq 0 \\ h(x) = 0 \\ x \in \mathbb{R}^n \end{cases} \tag{1}$$

with $f : \mathbb{R}^n \to \mathbb{R}$, $f \in C^1$, $g : \mathbb{R}^n \to \mathbb{R}^p$, $g \in C^1$, and $h : \mathbb{R}^n \to \mathbb{R}^m$, $h \in C^1$.
Problem 1 is called a Non-Linear Programming (NLP) problem when at least one among the objective functions, inequality constraints and equality constraints is nonlinear with respect to x.

Optimality Conditions.

For the *unconstrained* problem:

$$\begin{cases} \min f(x) \\ x \in \mathbb{R}^n \end{cases}$$

- x^* is a stationary point if: $\nabla f(x^*) = 0$ (Necessary Optimality Condition).

Optimality Conditions.

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- x^* is a stationary point if: $\nabla f(x^*) = 0$ (Necessary Optimality Condition).
- x^* is a *minimum* if the Hessian matrix $\nabla^2 f(x^*)$ is positive definite (Second Order Sufficient Condition).

Optimality Conditions.

For the *constrained* problem 1 necessary conditions are provided by the following

Theorem (Karush-Khun-Tucker (KKT))

Let the Lagrange function be $L(x, \lambda, \mu) = f(x) + \sum_{i=1}^{p} \lambda_i g_i(x) + \sum_{j=1}^{m} \mu_j h_j(x)$*, let the constraint qualification be verified in* x^**, then the point* x^* *is a solution of problem 1 if exist* μ^* *and* $\lambda^* \geq 0$ *satisfying the following system:*

$$\begin{cases} \nabla_x L(x^*, \lambda^*, \mu^*) = 0 & \\ \sum_{i=1}^{p} \lambda_i g_i(x^*) = 0 & \\ h_j(x^*) = 0 & j = 1, \ldots, m \end{cases}$$

Unconstrained Methods (grad-based)

In general, an Optimization Algorithm generates a sequence

$$x^{k+1} = x^k + t_k d^k = x^k - t_k H^k \nabla f(x^k) \text{ such that } f(x^{k+1}) \leq f(x^k).$$

where H^k is a positive defined matrix.

If $H^k = I \Rightarrow$ steepest descent.

If $H^k = [\nabla^2 f(x^k)]^{-1} \Rightarrow$ Newton's method (quadratic rate of convergence)

If $H^k = [\widehat{\nabla^2 f(x^k)}]^{-1} \Rightarrow$ quasi-Newton's method (superlinear rate of convergence).

$t_k = \arg\min_t f(x^k + td^k)$: exact search of stepsize.

t_k such that $f(x^k + td^k) \leq f(x^k) + \gamma t \nabla f(x^k) d^k$: Armijio's inexact search (LSA).

t_k such that LSA + $\nabla f(x^k + t_k d^k) d^k \geq \beta \nabla f(x^k) d^k$: Wolfe's condition.

Unconstrained Derivative-free Methods (Global Pattern Search)

A direct search method.

Theory can be found in (1997):

SIAM J. OPTIM.
Vol. 7, No. 1, pp. 1–25, February 1997

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ON THE CONVERGENCE OF PATTERN SEARCH ALGORITHMS*

VIRGINIA TORCZON†

- Pattern: a set of independent directions spanning all $\mathbb{R}^n$ (the search directions d^k).

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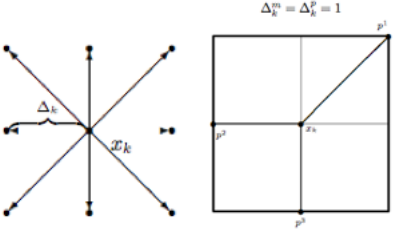
- Pattern: a set of independent directions spanning all $\mathbb{R}^n$ (the search directions d^k).
- A Frame Parameter Δ_k is defined (the stepsize t_k) such that:

$$\lim_{k \to \infty} \Delta_k = 0.$$

Unconstrained Derivative-free Methods (Global Pattern Search)

A direct search method.

The directions may be kept constant during the iterative process (classical GPS) or they may vary randomly as in MADS algorithm[1].



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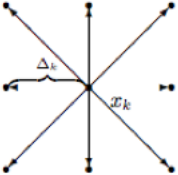

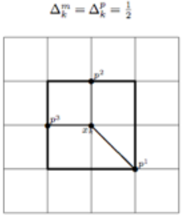

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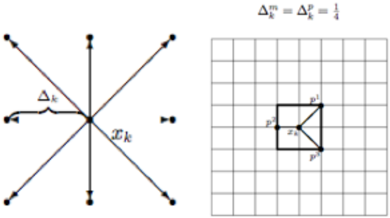

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Unconstrained (Constrained) Derivative-free Methods

A direct search method.

Derivative-free but some properties must hold to demonstrate convergence:

1. If f is strictly differentiable at the solution x^* then x^* is a minimum (KKT) point;
2. If f is Lipschitz near x^* then x^* is a stationary point of f on Ω.

GPS performances.

There is a deterioration of the rate of convergence as the dimensions of space increase.

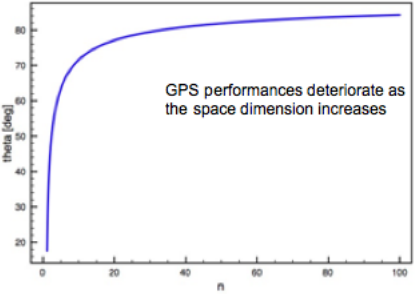

Figure: Angle between the GPS search and $-\nabla f$ directions.

Constrained problem.

In the present work, the constrained problem has been faced by SQP algorithm already implemented in the function *fmincon* in the software Matlab and by *Penalty Methods*. In particular, the *Augmented Lagrangian* penalty function has been extensively used.

- *Courant*:

$$\min\{f(x) + \frac{1}{\epsilon_k}\left[||\max\left[g(x), 0\right]||^2 + ||h(x)||^2\right]\}$$

- *Augmented Lagrangian*:

$$\min\{f(x)+\lambda\max\{g(x), -\frac{\epsilon_k}{2}\lambda\}+\mu h(x)+\frac{1}{\epsilon_k}||\max\{g(x), -\frac{\epsilon_k}{2}\lambda\}||^2+\frac{1}{\epsilon_k}||h(x)||^2\}$$

$$\{\epsilon_{k+1}\} < \{\epsilon_k\}$$

Why global optimization?

- Engineering models are often "black boxes", which means that mathematical properties of the objective function and constraints cannot be analysed.

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- Engineering models are often “black boxes”, which means that mathematical properties of the objective function and constraints cannot be analysed.
- Often, algorithms converge to a point that does not fulfill the NOC conditions; in the most part of practical cases, the algorithm terminates when the maximum number of iteration, linked to a maximum CPU elapsed time, is reached.
- The solution results in an “improved point” instead of an “optimum point”; often this is enough for engineering purposes if the initial design (the starting point x_0) is sufficiently close to the solution. When completely new projects are faced, the starting point may be far from the final solution: algorithms with specific characteristics of exploring the design space, like global optimization algorithms, are necessary.

An Algorithm.

The local smoothing technique.

This algorithm[4] is based on the following main ideas:

- the objective function has an underlying structure (*funnel structure*) so that it can be viewed as a superimposition of "noise" to the underlying function

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- the step function is smoothed, minimized and its minimum gives directions on the basin of attraction.

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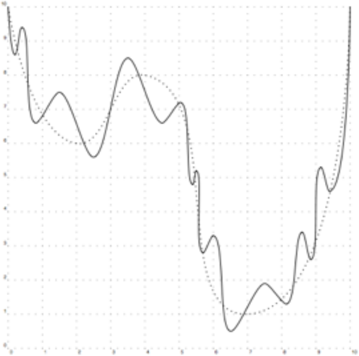
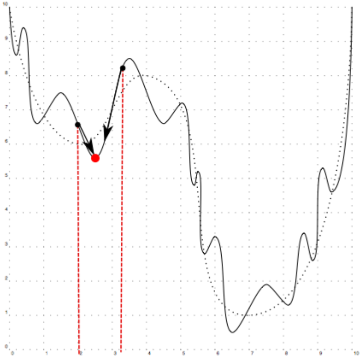
The local smoothing technique.
Funnel structure.



The local smoothing technique.
Local minimization.



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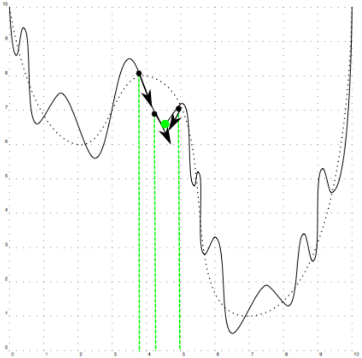
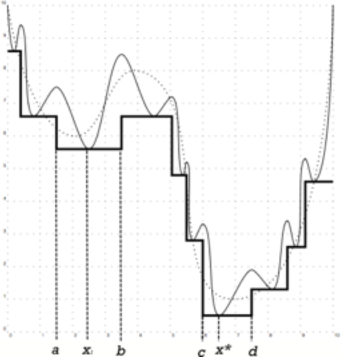
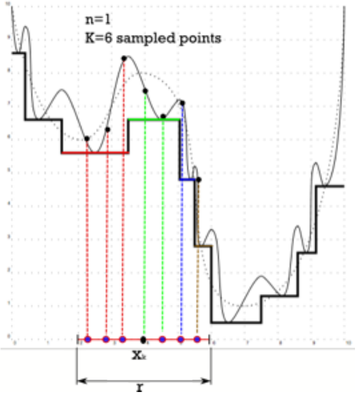


The local smoothing technique.
Step function.

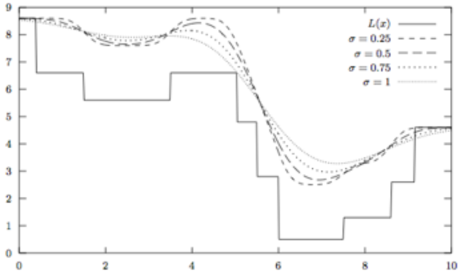


The local smoothing technique.

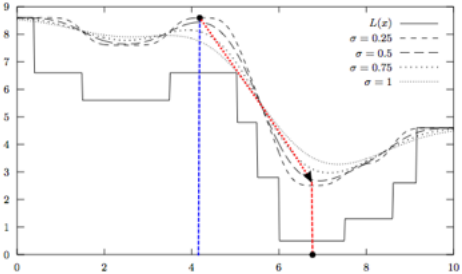
Sampling points to construct the step function.



The local smoothing technique.
Gaussian smoothing.



The local smoothing technique.
Minimizing the smoothed function.



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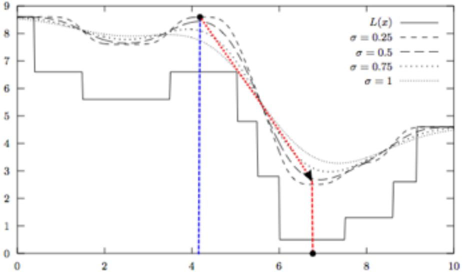


$$\widehat{L_g^B}(x) = \frac{\sum_{i=1}^{K} L(y_i) g(\|y_i - x\|)}{\sum_{i=1}^{K} g(\|y_i - x\|)} \; ; \; g(z) = e^{-z^2/(2\sigma)^2}$$

$$\sigma = \frac{r}{\sqrt[n]{K}} \; ; \; \sigma = \frac{\prod_{i=1}^{n} r_i}{\sqrt[n]{K}} \; ; \; r = \frac{|ub - lb|}{\delta}, \delta \in \mathbb{R}, \delta \geq 1$$

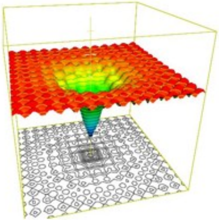
Math Test 1:
Ackley's function.

$$f(x) = -20e^{-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}} - e^{\frac{1}{n}\sum_{i=1}^{n}\cos 2\pi x_i} + 20 + e$$

Domain: $-32.768 \leq x \leq 32.768$.

Point of global minimum: $x^* = \begin{pmatrix} 0 & 0 & \ldots & 0 \end{pmatrix}$.

Global Minimum: $f(x^*) = 0$.



n	LOCSMOOTH with: SQP local solver		MADS local solver	
	$f(x^*)$	NFval	$f(x^*)$	NFval
2	10.1203	824	$3.03\ 10^{-6}$	10837
5	5.9783	7422	$9.1\ 10^{-6}$	22013
10	15.0208	15392	$0.1395\ 10^{-4}$	95319
15	13.8393	26243	$0.1577\ 10^{-4}$	190992
20	19.8439	10395	$0.1559\ 10^{-4}$	554557
25	19.9737	16066	$0.1479\ 10^{-4}$	390157
30	20.2286	8974	$0.1536\ 10^{-4}$	1000328

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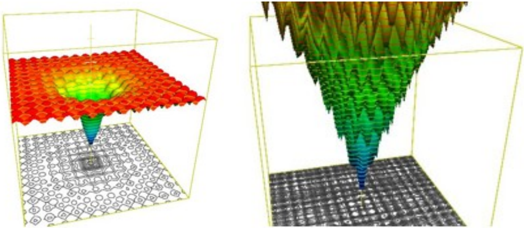
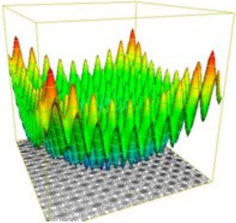


Math Test 2:
Rastrigin's function.

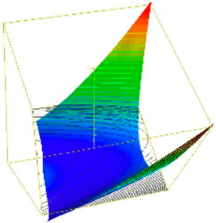
$f(x) = 10n + \sum_{i=1}^{n} x_i^2 - 10\cos 2\pi x_i$
Domain: $-5.12 \leq x \leq 5.12$.
Point of global minimum: $x^* = \begin{pmatrix} 0 & 0 & \dots & 0 \end{pmatrix}$.
Global Minimum: $f(x^*) = 0$.



n	LOCSMOOTH with SQP local solver $f(x^*)$	NFval	LOCSMOOTH with MADS local solver $f(x^*)$	NFval
2	0.995	437	$2.11\ 10^{-10}$	3137
5	4.9798	1690	$6.35\ 10^{-9}$	24986
10	9.9496	7038	$0.244\ 10^{-7}$	37806
15	21.889	6553	$0.889\ 10^{-7}$	89374
20	20.8940	10363	$0.685\ 10^{-7}$	257906
25	0.995	14181	$0.833\ 10^{-7}$	432970
30	18.9042	18614	$0.114\ 10^{-6}$	820628

Math Test 3:
Rosenbrock's function.

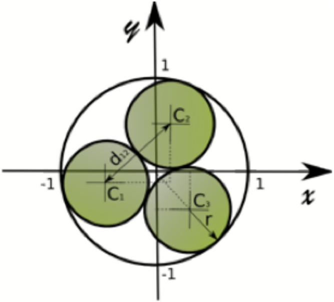
$f(x) = \sum_{i=1}^{n-1} 100(x_{i+1} - x_i)^2 + (1 - x_i)^2$
Domain: $-2.48 \leq x \leq 2.48$.
Point of global minimum: $x^* = \begin{pmatrix} 1 & 1 & \dots & 1 \end{pmatrix}$.
Global Minimum: $f(x^*) = 0$.



n	*LOCSMOOTH with*			
	SQP local solver		*MADS local solver*	
	$f(x^*)$	NFval	$f(x^*)$	NFval
2	$6.14\ 10^{-11}$	3968	$8.97\ 10^{-10}$	58916
5	$1.73\ 10^{-10}$	9416	$0.0866\ 10^{-3}$	1243050
10	$0.996\ 10^{-9}$	22419	$0.1059\ 10^{-3}$	3345535
15	$0.6965\ 10^{-9}$	39776	$0.0933\ 10^{-3}$	980867
20	$0.159\ 10^{-9}$	45309	$0.1644\ 10^{-3}$	6758729
25	$0.6145\ 10^{-8}$	34251	$0.1955\ 10^{-3}$	11527938
30	$0.2937\ 10^{-8}$	119417	$0.0202\ 10^{-3}$	13462135

Math Test 4:
Circle Packaging problem.

Given a circle of radius 1 and an interger number K, find the maximum radius r of K non-overlapping circles inside the unit-radius circle.



Math Test 4:
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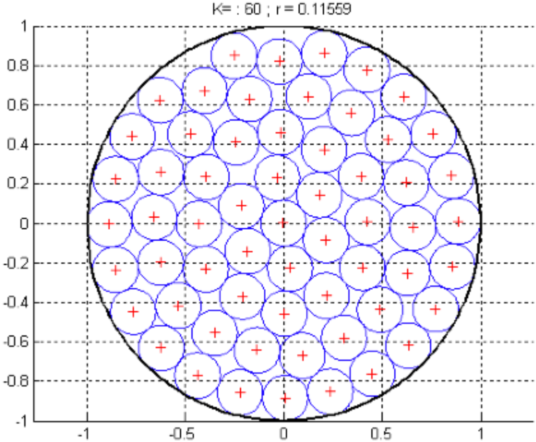
Given a circle of radius 1 and an interger number K, find the maximum radius r of K non-overlapping circles inside the unit-radius circle.

$$\begin{cases} \max r & \\ \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \geq 2r & 1 \leq i \leq k, 1 \leq j \leq k, i \neq j \\ \sqrt{x_i^2 + y_i^2} + r \leq 1 & i = 1, \dots, k \\ 0 \leq x_i \leq 1 & i = 1, \dots, k \\ 0 \leq y_i \leq 1 & i = 1, \dots, k \\ 0 \leq r \leq 1 & \end{cases}$$

The number of variables is $2K + 1$, the number of non-linear, non-convex constraints for the non-overlapping condition is $K(K - 1)/2$, and the number of non-linear, non-convex constraints for the container condition is K.

Math Test 4:
Circle Packaging problem.



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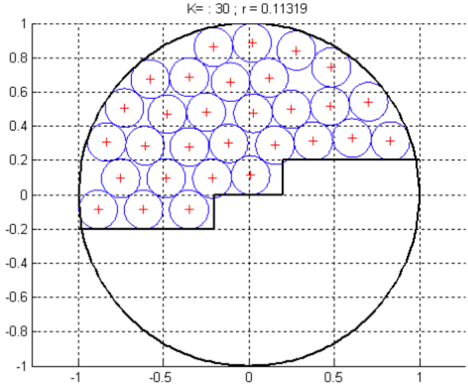
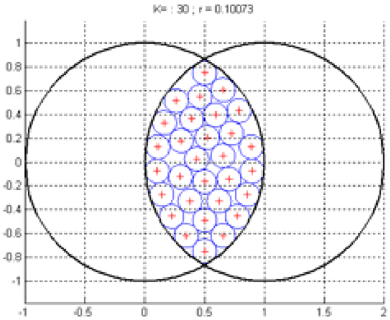




Aerodynamic Optimization of wings.

Results of Calculus of Variations.

The problem of the minimum induced drag in wings and lifting system was faced for the first time by Munk[5,6] by means the calculus of variations. In particular, by considering a lifting line of length b, the following isoperimetric problem can be formulated:

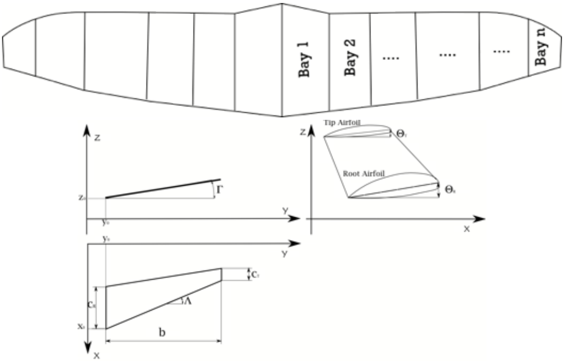
$$\begin{cases} \min \frac{\varrho}{4\pi} \int_{-b/2}^{b/2} \int_{-b/2}^{b/2} \Gamma(y_1) \frac{d\Gamma}{dy} \frac{dy dy_1}{y-y_1} \\ \varrho V_0 \int_{-b/2}^{b/2} \Gamma(y) dy = W \end{cases}$$

The unknown is the circulation function $\Gamma(y)$.
As a result, Munk stated his two theorems (the cosinus theorem and the staggered wings theorem), and he make more general the Prandtl's result for the optimum wing, that is:

$$w_i(y) = const$$

[5] Isoperimetrische Aufgaben aus der Theorie des Fluges – 1919
[6] The minimum induced drag in airfoils, NACA 121

Aerodynamic Optimization of wings.
Numerical Approach.

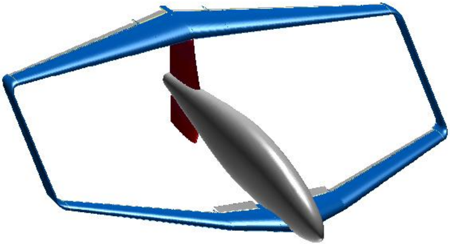


One trapezoidal wing bay (wing trunk) is defined by 12 parameters.

Aerodynamic Optimization of wings.

Numerical Approach.

In order to solve the problem, an aerodynamic solver based on the Vortex Lattice Method (VLM) has been used. This code offers the advantages of fast simulations and good prediction of the aerodynamic derivatives for subsonic flows. The viscous part of drag has been simulated by means the flat plate analogy, with a model that considers all the flow laminar or turbulent depending on Reynolds number.



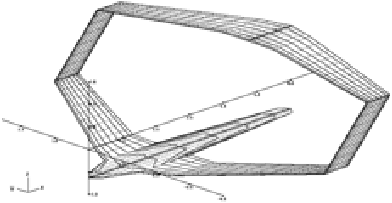
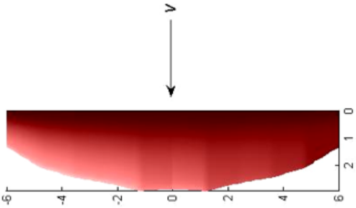
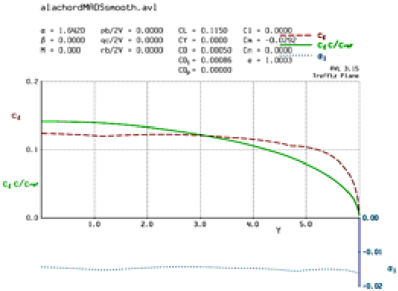


Figure: VLM representation

Aerodynamic Optimization of wings.

Minimum induced drag of a wing.

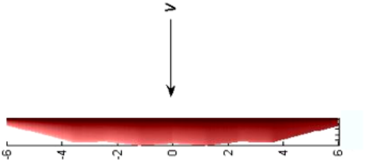


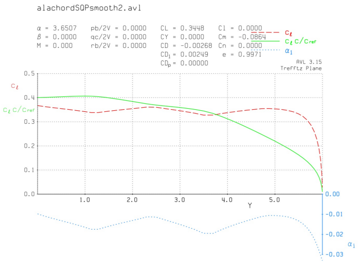


MADS+LocSmooth result: $e = 1$.

Aerodynamic Optimization of wings.

Minimum induced drag of a wing.

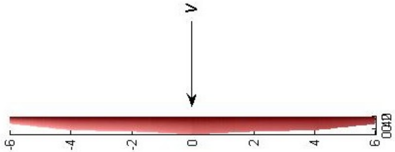


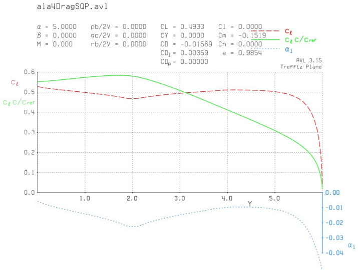


SQP+LocSmooth result: $e = 0.997$.

Aerodynamic Optimization of wings.

Minimum total (induced + viscous) drag of a wing.

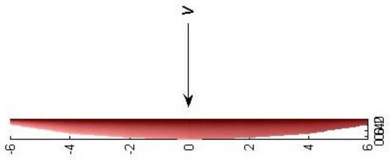


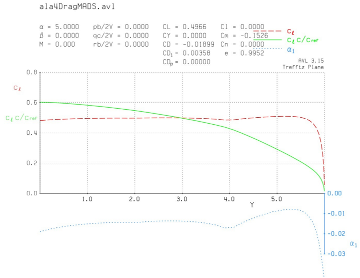


SQP+LocSmooth result: $D/D_0 = 0.654$.

Aerodynamic Optimization of wings.

Minimum total (induced + viscous) drag of a wing.

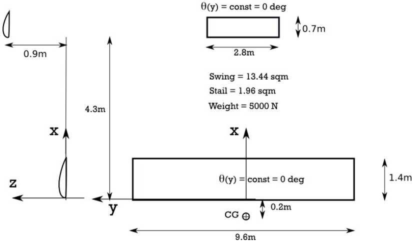


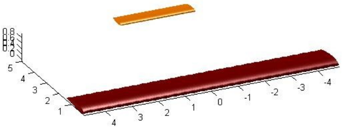


MADS+LocSmooth result: $D/D_0 = 0.647$.

Aerodynamic Optimization of wings.

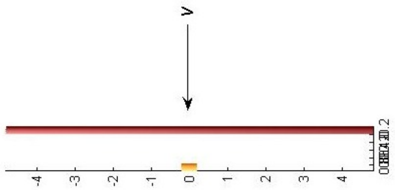
Minimum total (induced + viscous) drag of a wing-tail combination.

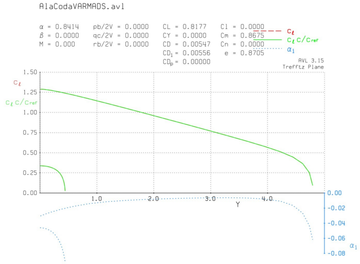




Aerodynamic Optimization of wings.

Minimum total (induced + viscous) drag of a wing-tail combination.

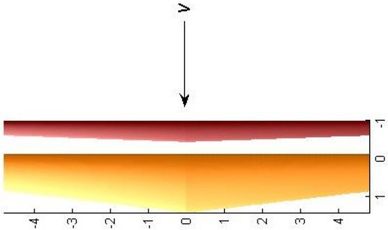


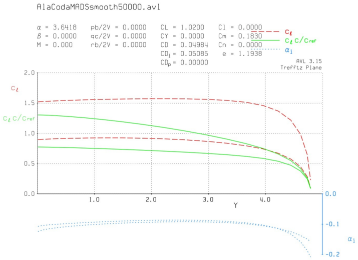


SQP + LocSmooth solver $D/D_0 = 0.933$

Aerodynamic Optimization of wings.

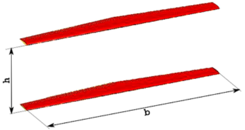
Minimum total (induced + viscous) drag of a wing-tail combination: very large weight.

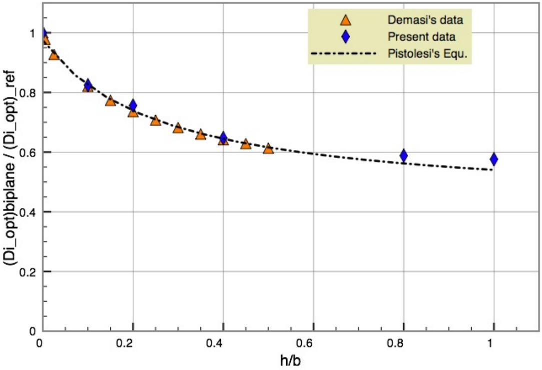




Aerodynamic Optimization of wings.

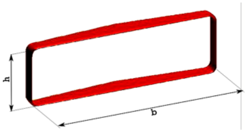
Minimum Induced Drag of a biplane.

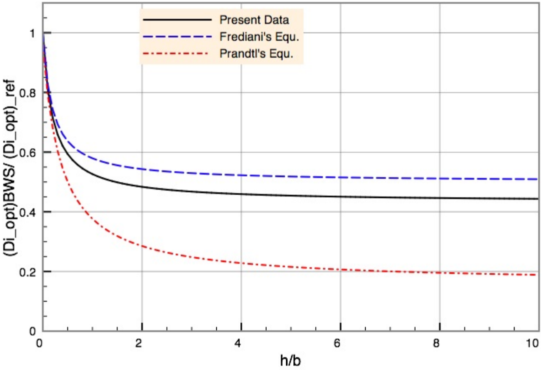




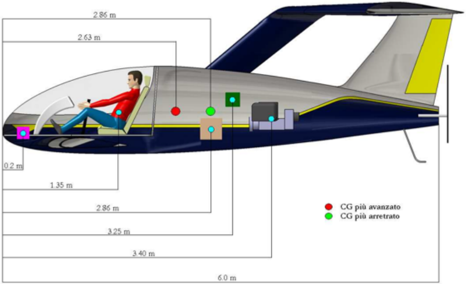
Aerodynamic Optimization of wings.

Minimum Induced Drag of a BWS.



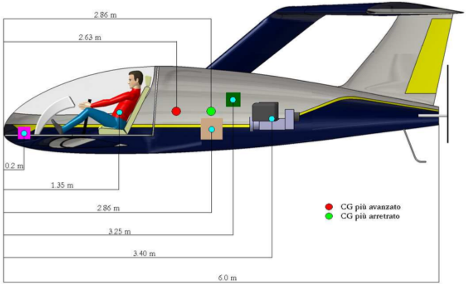


The PrandtlPlane: ULM case.



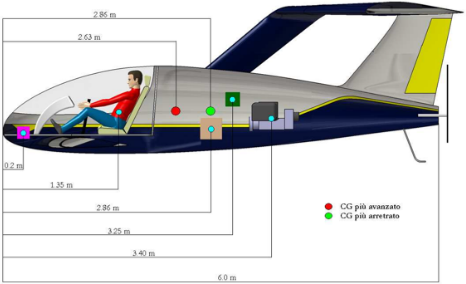
- Safety

The PrandtlPlane: ULM case.



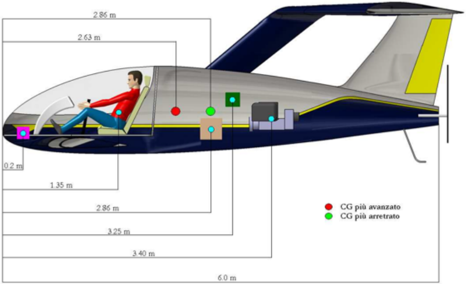
- Safety
  - Visibility

The PrandtlPlane: ULM case.



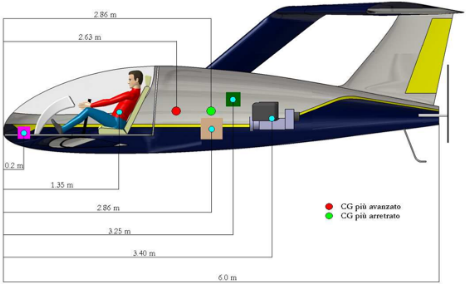
- Safety
  - Visibility
  - Engine position

The PrandtlPlane: ULM case.



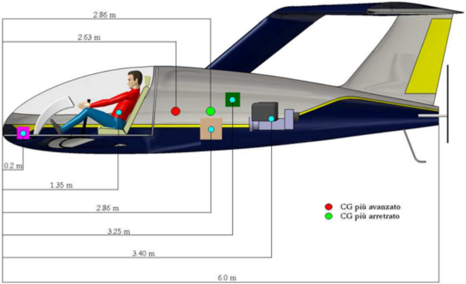
- Safety
  - Visibility
  - Engine position
  - Anticrash System

The PrandtlPlane: ULM case.



- Safety
  - Visibility
  - Engine position
  - Anticrash System
- Comfort

The PrandtlPlane: ULM case.

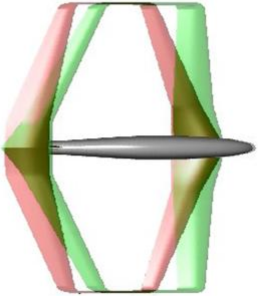


- Safety
  - Visibility
  - Engine position
  - Anticrash System
- Comfort
- Design

The aerodynamic model has been tested by a flying model (see video later)

Geometry Variations.

Every half wing is divided into 2 bays (3 sections). The position of the wing-fuselage connection is fixed. An example of modification of the geometry.



Trim & Stability Constraints.

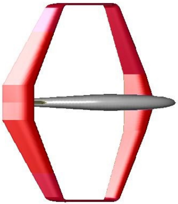
In order to have trim and stability, the following system must be satisfied:

$$\begin{cases} \sum_i L_i = W \\ \sum_i M_i|_{CG} = 0 \\ \frac{\partial M}{\partial \alpha} < 0 \end{cases} \Rightarrow \begin{cases} \sum_i L_i = W \\ X_{CG} = X_{PC} \\ MoS_{min} \leq MoS \leq MoS_{max} \end{cases}$$

The starting geometry.

The starting geometry.

The starting geometry



Item	Value
Wing Surface	$15.34 m^2$
Lift	$5010 N$
Induced Drag	$36.23 N$
Viscous Drag	$348.91 N$
Total Drag (no fuselage)	$385.14 N$
MoS	-9%
Trim Condition	YES

PrandtlPlane ULM case.

In terms of optimization ...

$$\begin{cases} \min D(x) = D_{ind}(x) + D_{visc}(x) \\ L_{cr,Lan} = (n_z W)_{cr,Lan} \\ M_{cr,Lan}|_{CG} = 0 \\ MoS_{min} \leq MoS \leq MoS_{max} \\ C_L|_{Lan} \leq C_L|_{Stall} \\ lb \leq x \leq ub \end{cases} \Longrightarrow \begin{cases} \min D(x) = D_{ind}(x) + D_{visc}(x) \\ W^*_{min}|_{cr,Lan} \leq L_{cr,Lan} \leq W^*_{max}|_{cr,Lan} \\ |X_{CG} - X_{PC}|_{cr,Lan} \leq \delta \\ MoS_{min} \leq MoS \leq MoS_{max} \\ C_L|_{Lan} \leq C_L|_{Stall} \\ lb \leq x \leq ub \end{cases}$$

$$x = \begin{pmatrix} c_i & \theta_i & \Lambda_j & \alpha_{cr} & \alpha_{Lan} & \delta_e & \delta_f \end{pmatrix}$$
$i = 1, \ldots, 6$
$j = 1, \ldots, 5$

Mathematical Model.

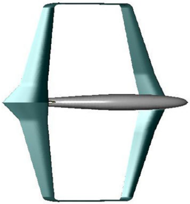
PrandtlPlane ULM case.

Boundaries.

$$
\begin{array}{rcll}
0.5m & \leq c_i \leq & 1.5m & i = 1, \ldots, 6 \\
-10deg & \leq \theta_i \leq & 10deg & i = 1, \ldots, 6 \\
0deg & \leq \Lambda_i \leq & 35deg & i = 1, 2, \text{ forward wing} \\
-35deg & \leq \Lambda_i \leq & 0deg & i = 3, 4, \text{ rearward wing} \\
-3deg & \leq \alpha_{AV} \leq & 3deg & \\
0deg & \leq \alpha_{BV} \leq & 16deg & \\
-20deg & \leq \delta_e \leq & 20deg & \\
0deg & \leq \delta_f \leq & 30deg & \\
4950N & \leq W \leq & 5050N & \\
5\% & \leq MoS_{cr,Lan} \leq & 20\% &
\end{array}
$$

The optimum point x^*

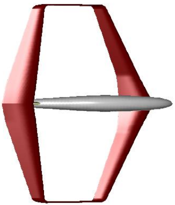
The optimum geometry



Item	Value	Variation
Wing Surface	$13.47m^2$	$\searrow$
Lift	$5004N$	$\searrow$
Induced Drag	$36.53N$	$\nearrow$
Viscous Drag	$282.81N$	$\searrow$
Total Drag (no fuselage)	$319.34N$	$\searrow$
MoS	7.4%	$\nearrow$
Trim Condition	YES	$\leftrightarrow$

Changing constraints:

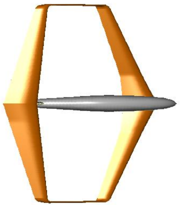
The optimum geometry



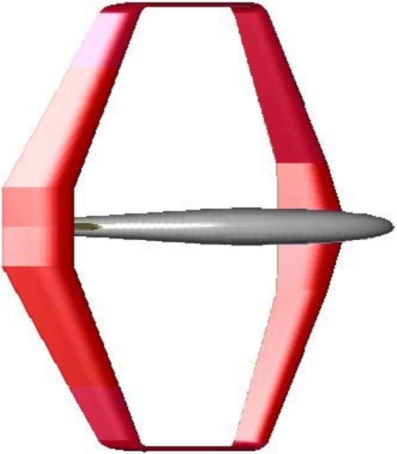
Item	Value	Variation
Wing Surface	$14.26 m^2$	$\nearrow$
Lift	$4990 N$	$\searrow$
Induced Drag	$36.59 N$	$\nearrow$
Viscous Drag	$311.87 N$	$\nearrow$
Total Drag (no fuselage)	$348.46 N$	$\nearrow$
MoS	5%	$\searrow$
Trim Condition	YES	$\leftrightarrow$

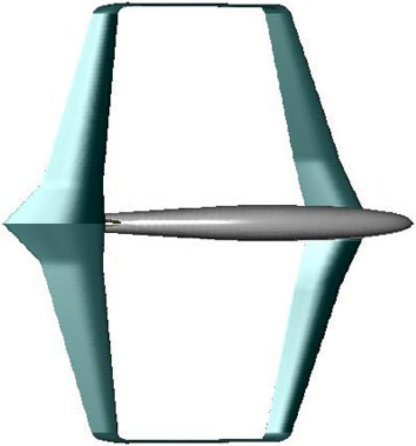
Final tuning:

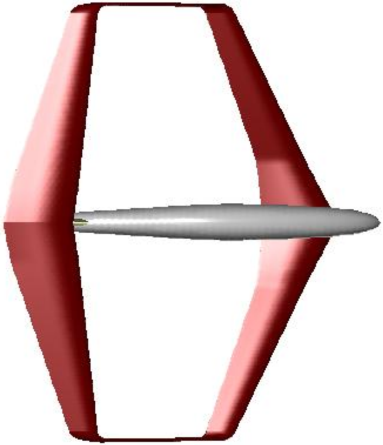
The optimum geometry

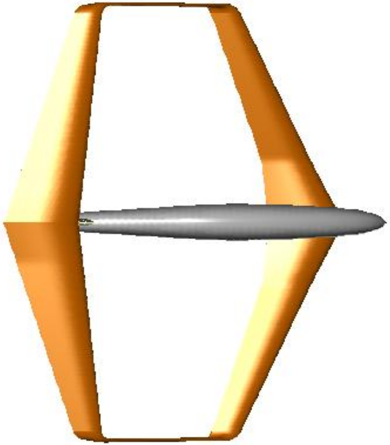


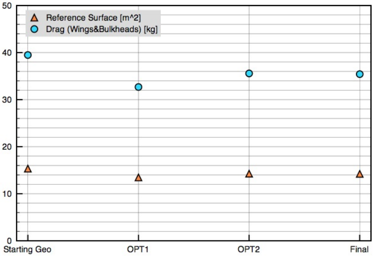
Item	Value	Variation
Wing Surface	$14.22m^2$	$\searrow$
Lift	$4976N$	$\searrow$
Induced Drag	$36.49N$	$\searrow$
Viscous Drag	$311N$	$\searrow$
Total Drag (no fuselage)	$347.49N$	$\searrow$
MoS	5.74%	$\nearrow$
Trim Condition	YES	$\leftrightarrow$

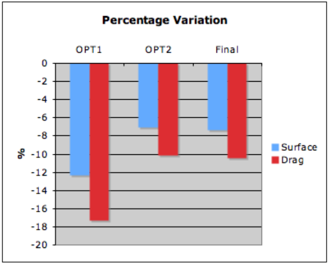


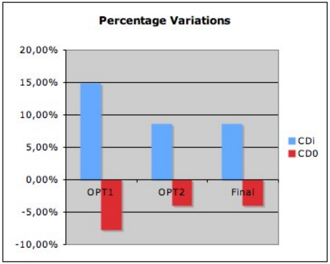


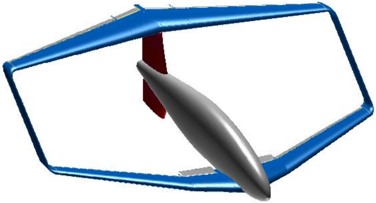


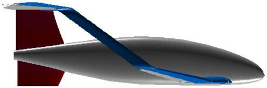


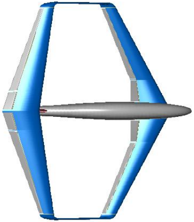

50
40
30
20
10
0
Reference Surface [m^2]
Drag (Wings&Bulkheads) [kg]
Starting Geo
OPT1
OPT2
Final

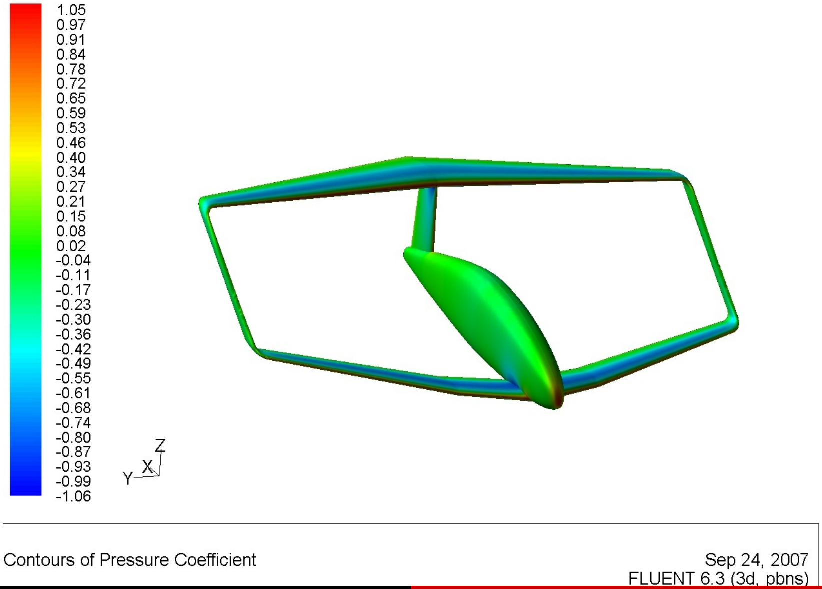

Percentage Variation
OPT1
OPT2
Final
%
Surface
Drag
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20

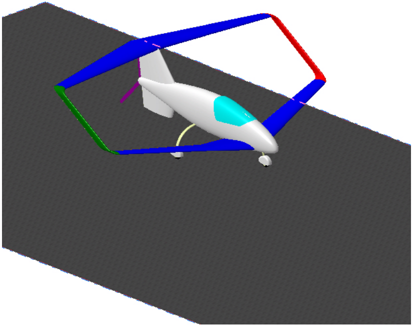

Percentage Variations
20,00%
15,00%
10,00%
5,00%
0,00%
-5,00%
-10,00%
OPT1
OPT2
Final
CDi
CD0

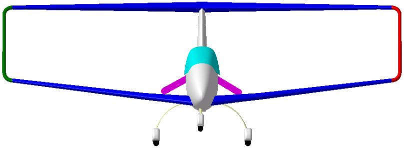


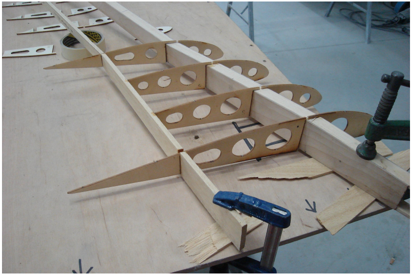





1.05
0.97
0.91
0.84
0.78
0.72
0.65
0.59
0.53
0.46
0.40
0.34
0.27
0.21
0.15
0.08
0.02
-0.04
-0.11
-0.17
-0.23
-0.30
-0.36
-0.42
-0.49
-0.55
-0.61
-0.68
-0.74
-0.80
-0.87
-0.93
-0.99
-1.06
Z
X
Y
Contours of Pressure Coefficient
Sep 24, 2007
FLUENT 6.3 (3d, pbns)





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Conclusions, results and way forward.

- A general overview of the current state of the art in aeronautics has been presented
- A short and quick overview of some optimization algorithms applied during the research have been presented and they have been tested on some benchmarking problems
- The same algorithms have been applied to design the preliminary wing planform of an innovative aircraft configuration
- The construction of a prototype of the aircraft has been funded and it is under construction

- `http://www.optimization-online.org/`
- `http://ntrs.nasa.gov/search.jsp`
- `http://www.prandtlplane.it/`
- *Variational Analysis and Aerospace Engineering*, Springer N. Y.