

Friction and adhesion :
models, variational formulations, solvers
and applications

Michel RAOUS, Directeur de Recherche CNRS

Laboratoire de Mécanique et d'Acoustique

31, chemin Joseph Aiguier
13402 Marseille Cedex 20 - France

Seminars

Introduction

The present course is based on a CISM course taught in Udine a few years ago ([163]) and a Master course presently taught in Marseille at the Université de la Méditerranée ([170]). We outline some of the classical results obtained in contact mechanics and present in addition some contributions resulting from the research carried out in our group "Mécanique et Modélisation du Contact" in the Laboratoire de Mécanique et d'Acoustique in Marseille.

The main topics of the researches carried out by the group concern :

- the development of models coupling adhesion, unilateral contact and friction ; they have been used for example for modelling the interaction between cracks and fiber/matrix interfaces in composite materials ([32] [164] [136] [138] [166]) and the metal/concrete contact for reinforced concrete in civil engineering ([103][171]),
- the study of the dynamic instabilities associated with friction ; Several works conducted in collaboration with J.A.C. Martins at the IST in Lisbon have established various sufficient conditions for flutter or divergence instability and the dynamical unstable (non smooth) solutions have been computed; this has been used to characterize squeal phenomena for rubber/glass contact for car industry [19] [190] [167] [130] [191]; recent works of A. Léger are concerning the notion itself of stability for the friction and the related theories ([16]),
- the study of the mathematical aspects of the previous problems (existence, uniqueness, regularity, convergence,...); both quasi-static and dynamics problems, both linear elasticity or visco-elasticity and finite deformations are considered in the works of P. Ballard, M. Cocu, A. Léger, E. Pratt and M. Raous ([13], [14], [15], [44] to [53]).
- the study of adhesive and wear phenomena in biomechanics; wear phenomena for bones and interface between bone and ligament have been studied under the direction of P. Chabrand ([23] [180]);
- the modelling of frictional unilateral contact in finite plastic deformations ; that work conducted by P. Chabrand has concerned metal forming applications ([151] [67] [40] [128]),
- the development of accelerating numerical processes for contact solvers ; in particular, we developed multigrid or subdomain decomposition methods (Fast Adaptive Composite grids) and Arbitrary Lagrangian Eulerian formulations for the frictional contacts ([88] [128]),

The numerical methods presented in this course are implemented in our own finite element codes Gyptis and Euxene written in the Modulef standard. Specific moduli and new developments are also implemented in the code LMGC from the University of Montpellier. In finite deformations (either in plasticity for metal forming or in hyper-elasticity for rubber), the implementation was done in SIMEM3 a finite element code developed during a collaboration with the Renault company.

Although it will not be attempted here to give an exhaustive bibliography, we would like to mention a few significant general contributions to contact mechanics. Among the numerous books on the various topics related to contact mechanics, the following are worth noting :

- on the modelling and mechanical aspects [Jonhson, 1987], [Rabinowicz, 1995], [Kalker, 1990], [Frémond, 2002],
- on tribology [Dowson, 1979], [Georges, 2000],
- on the mathematical aspects (functional and convex and non convex analysis) [Duvaut-Lions, 1972], [Moreau-Panagiotopoulos (Eds), 1988], [Panagiotopoulos, 1985], [Panagiotopoulos, 1993],
- on both mathematical and numerical aspects [Kikuchi-Oden, 1988], [Glowinski-Lions-Tremolieres, 1976], [Antes-Panagiotopoulos, 1992], [Wriggers, 2004].
- etc.

Numerous references can be found in [Zhong-Mackerle, 1992].

Many papers can also be found in the proceedings of a series of congresses, dealing with contact and unilateral problems, such as the series :

. Unilateral Problems in Structural Mechanics organized in Italy by G. Del Piero and F. Maceri in 1985, 1987, 1991 and 1997,

. Contact Mechanics International Symposium organized by A. Curnier (Lausanne, 1992), by M. Raous, M. Jean and J.J. Moreau (Carry-le-Rouet, 1995), by J.A.C. Martins and M. Monteiro Marques (Peniche, 2001) and P. Wriggers (Hannover, 2005).

On the tribological aspects, the proceedings of the annual "Leeds-Lyon Symposium on Tribology" and those of the scientific meetings of the "Société Tribologique de France" are worth mentioning.

Generally speaking, sessions focusing on contact problems take place in the most important international congresses related to structure mechanics.

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