Energetically orthogonal fracture mode partitioning of the J-integral for cohesive interfaces

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The cohesive zone model (CZM) was developed in the '60s of the last century, independently, by Dugdale [1] to investigate plastic fracture and Barenblatt [2] to account for the finite strength of brittle materials. More recently, numerical implementations of the CZM have gained increasing popularity, in particular for the analysis of delamination in composite laminates [3].

According to the CZM, the damage phenomena occurring in the fracture process zone (FPZ) ahead of the crack tip are described by cohesive laws, which relate the stresses acting on the fracture surface with the corresponding relative displacements. In a plane problem, the cohesive laws express the normal and tangential interfacial stresses, σ_n and σ_t, as functions of the normal and tangential relative displacements, δ_n and δ_t. Cohesive laws can be classified in different ways. A first, fundamental distinction is between cohesive laws that can be derived from a cohesive potential function, $\Phi(\delta_t, \delta_n)$, and those that cannot. Another distinction is between uncoupled cohesive laws, for which σ_t is a function of only δ_t and σ_n is a function of only δ_n, and coupled cohesive laws, for which the interfacial stresses depend on both the relative displacements [4].

A powerful tool for the theoretical and experimental investigation of cohesive laws [5] is offered by the path-independent J-integral introduced by Rice [6]. For a general cohesive interface,

$$J = \int_{(0,0)}^{(\Delta_t,\Delta_n)} \sigma_t(\delta_t, \delta_n)\, \mathrm{d}\delta_t + \sigma_n(\delta_t, \delta_n)\, \mathrm{d}\delta_n, \tag{1}$$

where Δ_t and Δ_n are the tangential and normal relative displacement at the crack tip, respectively.

If the cohesive laws are potential-based, then the interfacial stresses can be obtained as follows:

$$\sigma_t = \frac{\partial \Phi}{\partial \delta_t} \qquad \text{and} \qquad \sigma_n = \frac{\partial \Phi}{\partial \delta_n}. \tag{2}$$

By substituting Eqs. (2) into (1),

$$J = \int_{(0,0)}^{(\Delta_t,\Delta_n)} \frac{\partial \Phi}{\partial \delta_t}\, \mathrm{d}\delta_t + \frac{\partial \Phi}{\partial \delta_n}\, \mathrm{d}\delta_n = \int_{(0,0)}^{(\Delta_t,\Delta_n)} \mathrm{d}\Phi = \Phi(\Delta_t, \Delta_n), \tag{3}$$

where the final result is independent of the integration path in the plane of δ_t and δ_n because $\mathrm{d}\Phi$ is an exact differential.

If the cohesive laws are uncoupled, then the cohesive potential function can be decomposed as

$$\Phi(\delta_t, \delta_n) = \Phi_{\mathrm{I}}(\delta_n) + \Phi_{\mathrm{II}}(\delta_t), \tag{4}$$

where $\Phi_{\mathrm{I}}(\delta_n)$ and $\Phi_{\mathrm{II}}(\delta_t)$ are the mode I and mode II cohesive potential functions, respectively. As a consequence, the J-integral, Eq. (1), can be split into the sum of a mode I contribution,

$$J_{\mathrm{I}} = \int_{(0,0)}^{(\Delta_t,\Delta_n)} \sigma_n(\delta_n)\, \mathrm{d}\delta_n = \int_0^{\Delta_n} \sigma_n(\delta_n)\, \mathrm{d}\delta_n = \Phi_{\mathrm{I}}(\Delta_n), \tag{5}$$

and a mode II contribution,

$$J_{\mathrm{II}} = \int_{(0,0)}^{(\Delta_t,\Delta_n)} \sigma_t\left(\delta_t\right)\,\mathrm{d}\delta_t = \int_0^{\Delta_t} \sigma_t\left(\delta_t\right)\,\mathrm{d}\delta_t = \Phi_{\mathrm{II}}\left(\Delta_t\right). \tag{6}$$

However, if the cohesive laws are coupled, then the fracture mode partitioning is not trivial as $\sigma_t\left(\delta_t,\delta_n\right)\,\mathrm{d}\delta_t$ and $\sigma_n\left(\delta_t,\delta_n\right)\,\mathrm{d}\delta_n$ are not exact differentials. As a consequence, the line integrals in Eqs. (5) and (6) depend on the integration paths [7]. Moreover, it can be shown by examples that even physically inconsistent, negative values of J_{I} and J_{II} can be obtained from the above equations.

This work explains how the J-integral can be split into the sum of two physically consistent, positive definite, mode I and mode II contributions. To this aim, the concept of energetic orthogonality between fracture modes is exploited. More in detail, it is assumed that mode I is related to a null tangential relative displacement at the crack tip, $\Delta_t = 0$, and that the forces related to mode II are energetically orthogonal to those related to mode I. The same concept has already been applied to partition fracture modes in finite element models by the virtual crack closure technique (VCCT) [8, 9] and in beam-theory models of laminated beams [10]. Here, the method will be first illustrated with respect to linear, coupled cohesive laws and then extended to nonlinear, coupled cohesive laws.

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