The ellipse of crack-tip flexibility for the partitioning of fracture modes

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A crack in a solid body will generally propagate according to a combination of the three basic fracture modes (I or opening, II or sliding, and III or tearing). Thus, the energy release rate, G, will be the sum of three modal contributions, G_{I}, G_{II}, and G_{III} [1]. In the finite element context, the virtual crack closure technique (VCCT) is widely used to calculate the energy release rate and its modal contributions [2]. Accordingly, G is related to the work done by the forces, $\mathbf{r}$ and $-\mathbf{r}$, applied at the crack-tip nodes to close up the crack, once propagated by a finite length, Δa (Fig. 1a).

In I/II mixed-mode fracture problems, the crack-tip relative displacement, $\Delta\mathbf{s} = [\Delta u, \Delta w]^T = \mathbf{F}\mathbf{r}$, where $\mathbf{F}$ is the crack-tip flexibility matrix. The conic section associated to $\mathbf{F}$ turns out to be an ellipse, Γ, named the *ellipse of crack-tip flexibility* (Fig. 1b), similar to Culmann's ellipse of elasticity [3]. The ellipse of crack-tip flexibility helps visualise the relationship between the crack-tip force, $\mathbf{r}$, and relative displacement, $\Delta\mathbf{s}$, whose directions correspond to conjugate diameters [4]. Furthermore, the ellipse can be used to decompose the crack-tip force vector, $\mathbf{r}$, into energetically orthogonal components, which enable a physically consistent partitioning of fracture modes [5].

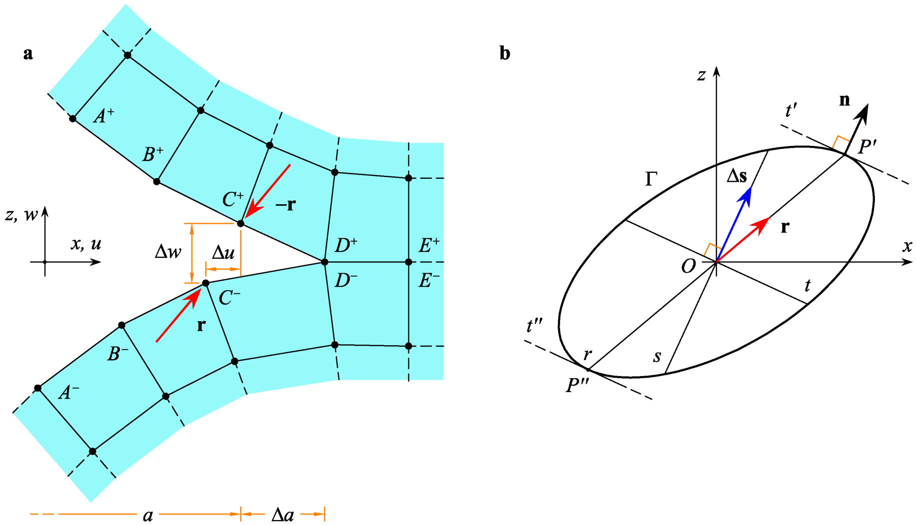

Figure 1: (a) virtual crack closure; (b) ellipse of crack-tip flexibility.

References

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