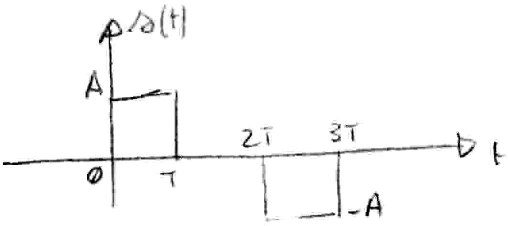
17/1/2012 ASB(270)

Esercizio 1



$A = 2\,V$

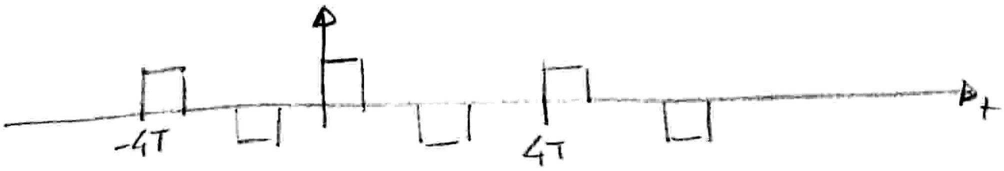
$T = 1\,s$

$$s_1(t) = A\,\mathrm{rect}\left(\frac{t - T/2}{T}\right) - A\,\mathrm{rect}\left(\frac{t - \frac{5}{2}T}{T}\right)$$

$$S_1(f) = AT\,\mathrm{sinc}(fT)\,e^{-j\pi fT} - AT\,\mathrm{sinc}(fT)\,e^{-j5\pi fT} =$$

$$= AT\,\mathrm{sinc}(fT)\,e^{-j3\pi fT}\left(e^{j2\pi fT} - e^{-j2\pi fT}\right) =$$

$$= AT\,\mathrm{sinc}(fT)\,2j\sin(2\pi fT)\,e^{-j3\pi fT}$$



$T_0 = 4T \qquad f_0 = \frac{1}{4T} = 0{,}25\ Hz$

$S_n = S_n^*$ non ci sono simmetrie

$S_0 = 0$

$|S_n| \propto \frac{1}{n}$

$$S_n = \frac{1}{T_0} S\left(\frac{n}{T_0}\right) = \frac{AT}{4T}\,\mathrm{sinc}\left(\frac{nT}{4T}\right) 2j \sin\left(\frac{2\pi T n}{4T}\right) e^{-j\frac{3\pi T n}{4T}} =$$

$$= j\frac{A}{2}\,\mathrm{sinc}\left(\frac{n}{4}\right) \sin\left(\frac{\pi n}{2}\right) e^{-j\frac{3\pi n}{4}}$$

$S_n = 0$ per n pari

$s_2(t) = |s_1(t)|$

- il periodo è dimezzato $\Rightarrow f_0 = \frac{1}{2T}$ la freq. fondamentale è raddoppiata

l'andamento dei coeff è simile. $|S_n| \propto \frac{1}{n}$

$S_n = S_n^* \qquad S_0 = 1 \qquad S_n = 0$ per n pari

$$S_n = \frac{1}{2T}\int_0^{2T} s_2(t)\,e^{-j\frac{2\pi n t}{2T}}\,dt = \frac{1}{2T}\int_0^{T} A\,e^{-j\frac{\pi n}{T}t}\,dt = \frac{A}{2T}\,\frac{T}{-j\pi n}\left(e^{-j\frac{\pi n T}{T}} - 1\right) =$$

$$= \frac{A}{2}\,\frac{1}{\pi n}\,e^{-j\frac{\pi n}{2}}\,2\sin\left(\frac{\pi n}{2}\right) = \frac{A}{2}\,\mathrm{sinc}\left(\frac{n}{2}\right)e^{-j\frac{\pi}{2}n}$$

In questo secondo caso i coefficienti sono posizionati nelle fre

$$f_K = \frac{K}{2T}$$

mentre nel primo $f_h = \frac{h}{4T}$

$n=0$ $\qquad S_0 = \frac{A}{2} = 1$

$n=1$ $\qquad S_1 = \frac{A}{2}\,\frac{1}{\frac{\pi}{2}}\,e^{-j\frac{\pi}{2}} = \frac{2}{\pi}e^{-j\frac{\pi}{2}}$

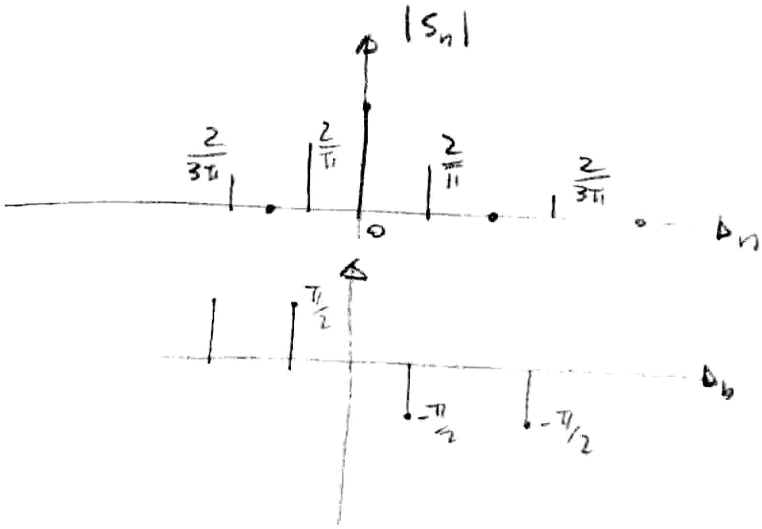
$\qquad S_{-1} = \frac{2}{\pi}e^{j\frac{\pi}{2}}$

$n=2$ $\qquad S_2 = 0$

$n=3$ $\qquad S_3 = \frac{A}{2}\left(-\frac{1}{\frac{3\pi}{2}}\right)e^{-j\frac{3\pi}{2}} = \frac{2}{3\pi}e^{-j\frac{\pi}{2}}$

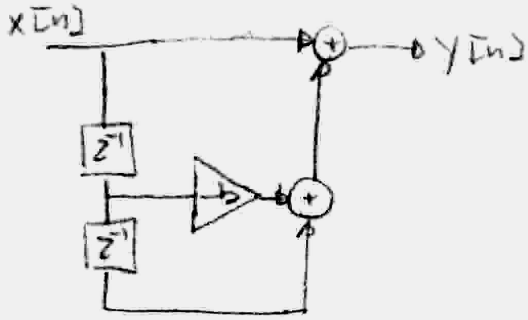
$\qquad S_{-1} = \frac{2}{3\pi}e^{+j\frac{\pi}{2}}$

$n=4$ $\qquad S_4 = 0$

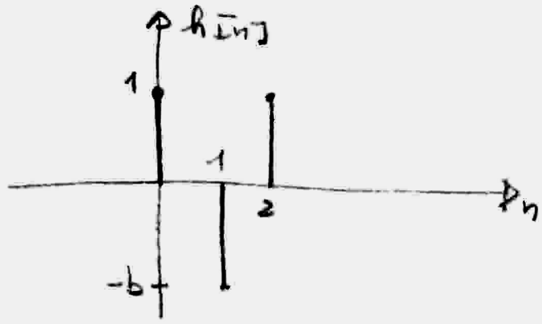


Esercizio 2

$$y[n] = x[n] - b\,x[n-1] + x[n-2]$$



$x[n] = \delta[n]$ $\qquad$ $y[n] = h[n] = \delta[n] - b\,\delta[n-1] + \delta[n-2]$

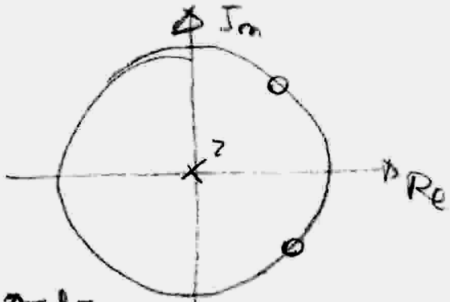


$b = \sqrt{2}$

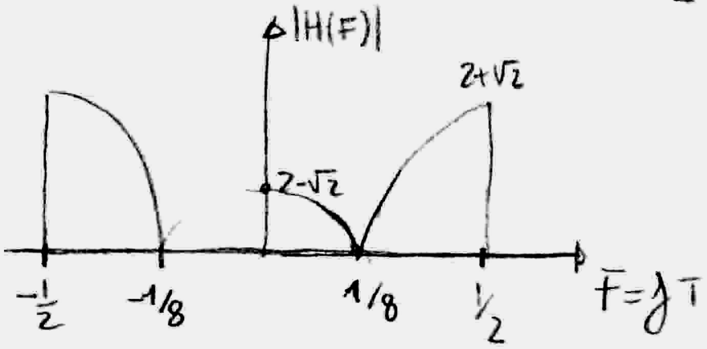
$$Y(z) = X(z) - b\,X(z)\,z^{-1} + z^{-2}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = 1 - \sqrt{2}\,z^{-1} + z^{-2} = \frac{z^2 - \sqrt{2}\,z + 1}{z^2}$$

$\Delta = 2 - 4$ $\qquad$ $z_{1,2} = \frac{\sqrt{2}}{2} \pm j\frac{\sqrt{2}}{2}$



$$H(f) = \sum_{n=-\infty}^{\infty} h[n]\,e^{-j2\pi nfT} = 1 - \sqrt{2}\,e^{-j2\pi fT} + e^{-j4\pi fT} = e^{-j2\pi fT}\left(2\cos 2\pi fT - \sqrt{2}\right)$$

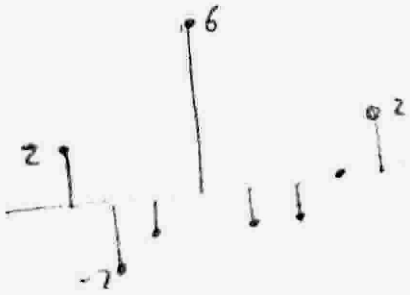


per ottenere $df = 0{,}1$ deve essere $df = \frac{1}{NT} \Rightarrow N = \frac{1}{T}\frac{1}{df} = \frac{10}{0{,}5} = 20$

bisogna eseguire uno zero padding di $N = 20 - 3 = 17$ campioni

ESERCIZIO 3

I)



II) $x(t) = 1 + \sin\left(\frac{\pi}{3}t\right) + \sin\left(\frac{\pi}{4}t\right)$

tre componenti

1) $f = 0$

2) $f = 1/6$

3) $f = \frac{1}{8}$

$f_{max} = 1/6 \quad f_c \geq \frac{1}{3}$

$T_c \leq 3$

III)

$f_{min} = 27$

$f_{max} = 35$

$B = 8 \qquad \frac{f_{max}}{B} = \frac{35}{8} \Rightarrow m = 4 \qquad f_c = \frac{2 \cdot 35}{4} = 17{,}5\ Hz$

IV)

si richiede $df = 0{,}2\ Hz \Rightarrow T_{oss} \geq \frac{1}{df} = 5s$

- eseguire la TDF di un segmento del segnale lungo 20s