$s(t) \in \mathbb{R}, \mathbb{C}$

$$E_s(T) \triangleq \int_{-T/2}^{T/2} |s(t)|^2 dt$$

Energia finita

se $\exists$ finito e $\neq 0$

$$E_s = \lim_{T \to \infty} \int_{-T/2}^{T/2} |s(t)|^2 dt$$

segnali periodici
costante $\Big\} \to E_s = \infty$

$$P_s(T) \triangleq \frac{1}{T}\int_{-T/2}^{T/2} |s(t)|^2 dt$$

Def. $s(t)$ potenza media finita se $\exists$ finito e $\neq 0$

$$P_s = \lim_{T\to\infty} P_s(T) =$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |s(t)|^2 dt$$

Segnali periodici

$T = N T_0 \qquad T \to \infty \equiv N \to \infty$

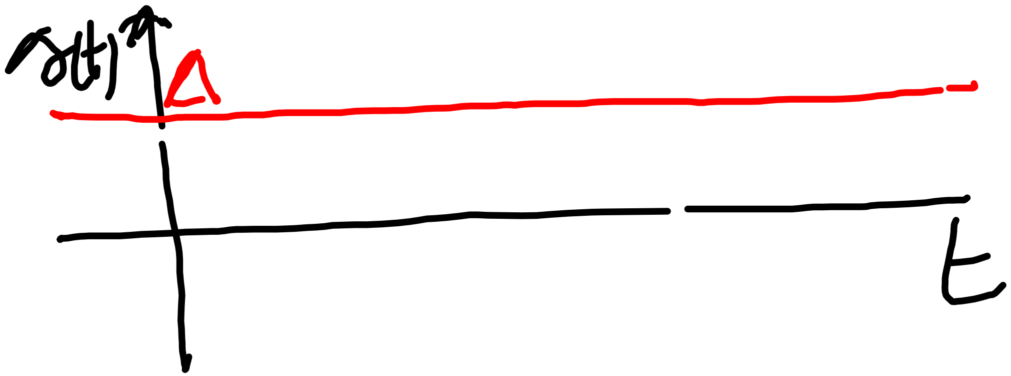
$$E_s(T) = \int_{-T/2}^{T/2} |s(t)|^2 dt = N \int_{-T_0/2}^{T_0/2} |s(t)|^2 dt$$

$$P_s = \lim_{T\to\infty} P_s(T) = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |s(t)|^2 dt =$$

$$= \lim_{N\to\infty} \frac{1}{N T_0}\int_{-\frac{N T_0}{2}}^{\frac{N T_0}{2}} |s(t)|^2 dt =$$

$$= \lim_{N\to\infty} \frac{1}{\cancel{N} T_0}\cancel{N}\int_{-T_0/2}^{T_0/2} |s(t)|^2 dt = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |s(t)|^2 dt$$

$$s(t) = A$$

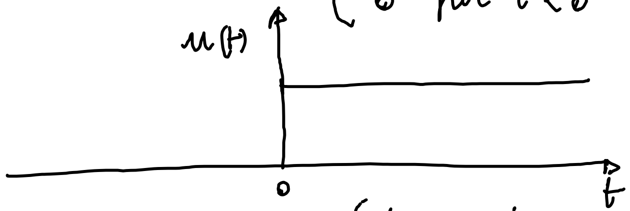


$$E_s = \lim_{T\to\infty} \int_{-T/2}^{T/2} A^2\,dt = \lim_{T\to\infty} A^2 T = \infty$$

$$P_s = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2\,dt = \lim_{T\to\infty} \frac{1}{T} A^2 T = A^2$$

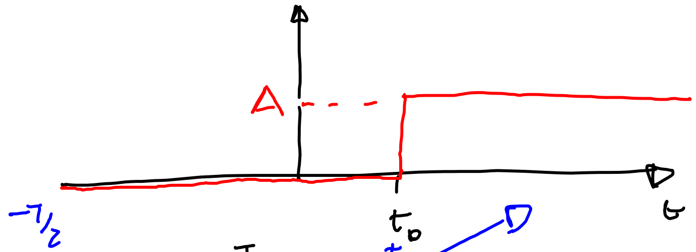
segnale a gradino unitario

$$s(t) = u(t) = \begin{cases} 1 & \text{per } t \geq 0 \\ 0 & \text{per } t < 0 \end{cases}$$



$$s_1(t) = u(t-t_0) = \begin{cases} 1 & \text{per } t-t_0 \geq 0 \Rightarrow t \geq t_0 \\ 0 & \text{per } t-t_0 < 0 \Rightarrow t < t_0 \end{cases}$$

$$s_2(t) = A\,u(t-t_0)$$



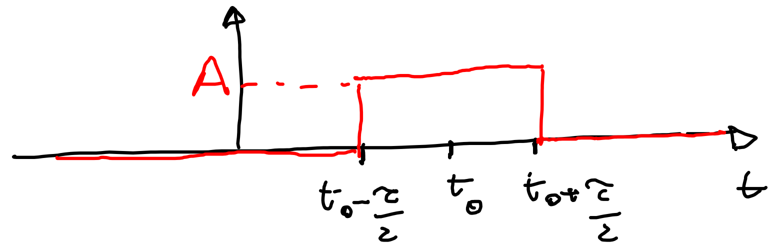
$$E_s = \lim_{T \to \infty} \int_{-T/2}^{T/2} |A\,u(t-t_0)|^2\,dt =$$

$$= \lim_{T \to \infty} \int_{t_0}^{T/2} A^2\,dt = \lim_{T \to \infty} A^2 \left(\tfrac{T}{2} - t_0\right) = \infty$$

$$P_s = \lim_{T \to \infty} \frac{1}{T} A^2 \left(\frac{T}{2} - t_0\right) = \frac{A^2}{2}$$

Impulso rettangolare

$$s(t) = \begin{cases} A & \text{per } |t - t_0| \leq \frac{\tau}{2} \\ 0 & \text{per } |t - t_0| > \frac{\tau}{2} \end{cases}$$



$A = 1 \qquad G_\tau(t) = \mathrm{rect}\left(\frac{t - t_0}{\tau}\right)$

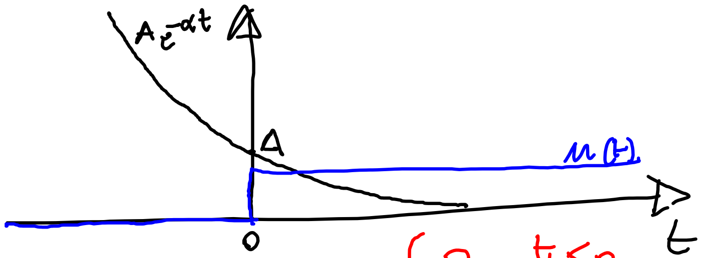
$$s(t) = A\, \mathrm{rect}\left(\frac{t - t_0}{\tau}\right)$$

$$E_s = \lim_{T \to \infty} \int_{-T/2}^{T/2} A^2\, \mathrm{rect}\left(\frac{t - t_0}{\tau}\right)^2 dt =$$

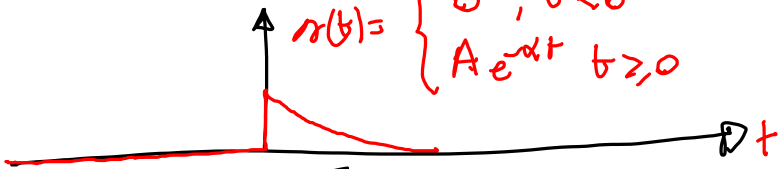
$$= \lim_{T \to \infty} \int_{t_0 - \frac{\tau}{2}}^{t_0 + \frac{\tau}{2}} A^2\, dt = \lim_{T \to \infty} A^2 \tau = A^2 \tau$$

$$P_s = \lim_{T \to \infty} \frac{1}{T} A^2 \tau = 0$$

$$s(t) = A e^{-\alpha t} u(t) \qquad \alpha > 0, \in \mathbb{R}$$



$$s(t) = \begin{cases} 0 & , t < 0 \\ A e^{-\alpha t} & t \geq 0 \end{cases}$$



$$E_s = \lim_{T\to\infty} \int_{-T/2}^{T/2} |A e^{-\alpha t} u(t)|^2 dt =$$

$$= \lim_{T\to\infty} \int_{-T/2}^{T/2} A^2 e^{-2\alpha t} u^2(t)\, dt =$$

$$= \lim_{T\to\infty} \int_{0}^{T/2} A^2 e^{-2\alpha t} dt = \lim_{T\to\infty} A^2 \frac{1}{(-2\alpha)} \left(e^{-2\alpha t} \Big|_0^{T/2} \right) =$$

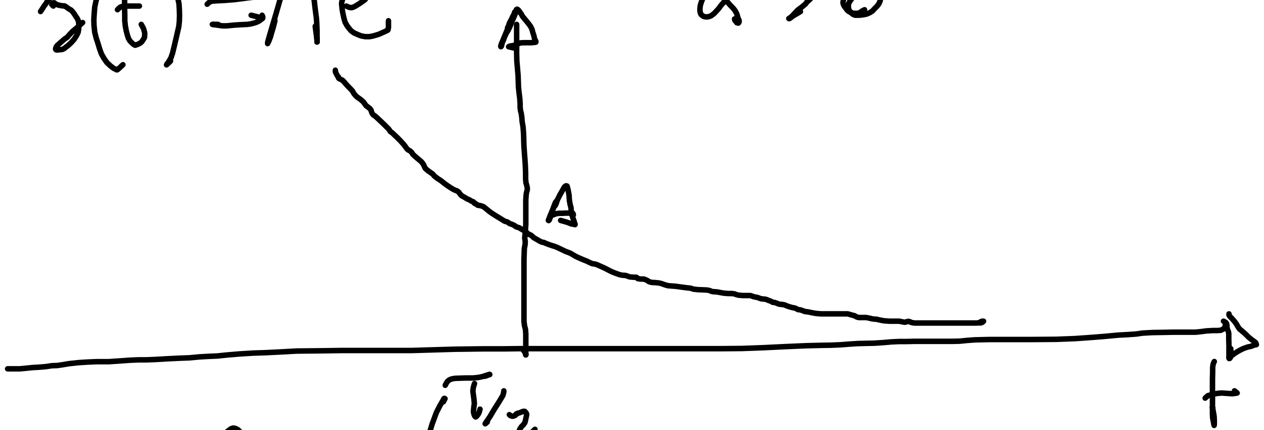
$$= \lim_{T\to\infty} \frac{A^2}{(-2\alpha)} \left(e^{-2\alpha T/2} - 1 \right) = \frac{A^2}{2\alpha}$$

$$\Downarrow$$

$$\alpha > 0 \to 0$$

$s(t)$ E_s finita $\Rightarrow P_s = 0$

$x(t) = Ae^{-\alpha t} \qquad \alpha > 0$

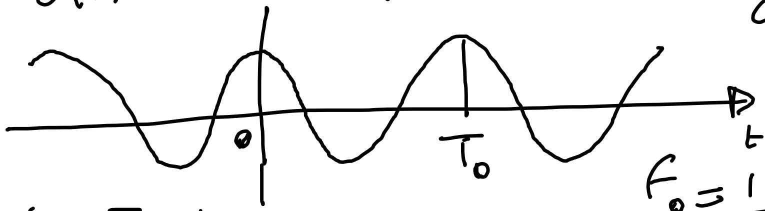


$$E_x = \lim_{T \to \infty} \int_{-T/2}^{T/2} A^2 e^{-2\alpha t}\, dt =$$

$$= \lim_{T \to \infty} A^2 \frac{1}{(-2\alpha)} \left(e^{-2\alpha t} \Big|_{-T/2}^{T/2} \right) =$$

$$= \lim_{T \to \infty} \frac{A^2}{(-2\alpha)} \left(\underset{\substack{\parallel \\ 0}}{\cancel{e^{-2\alpha T/2}}} - \underbrace{e^{+2\alpha T/2}}_{\to\, \infty} \right) = \infty$$

$s(t) = A\cos(\omega t) \qquad T_0 = \frac{2\pi}{\omega}$



$f_0 = \frac{1}{T_0} = \frac{\omega}{2\pi}$

ω [rad/sec]

f [1/sec]

$$P_s = \frac{1}{T_0}\int_0^{T_0} A^2\cos^2(\omega t)\,dt =$$

$$\left(\cos^2(\alpha) = \frac{1+\cos 2\alpha}{2}\right)$$

$$= \frac{1}{T_0}\int_0^{T_0} A^2\left(\frac{1+\cos 2\omega t}{2}\right)dt = \frac{1}{T_0}\int_0^{T_0}\frac{A^2}{2}\,dt +$$

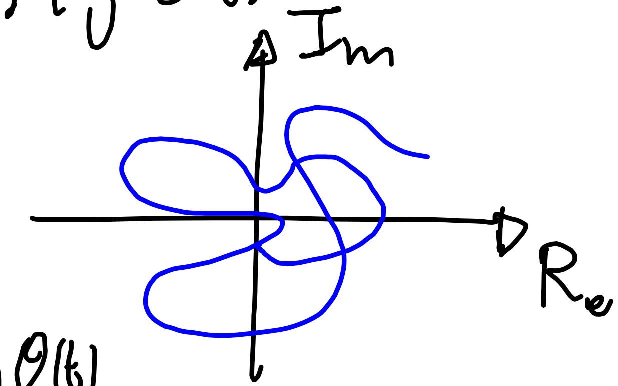
$$+\frac{A^2}{2T_0}\int_0^{T_0}\cos 2\omega t\,dt = \frac{1}{T_0}\frac{A^2}{2}T_0 + \frac{A^2}{2T_0}\frac{1}{2\omega}(\sin 2\omega t)\Big|_0^{T_0}$$

$$= \frac{A^2}{2} + \cancel{\frac{A^2}{2T_0}\frac{1}{2\omega}\sin 2\omega T_0} = 0 = \frac{A^2}{2}$$

$2\cancel{\omega}\frac{2\pi}{\cancel{\omega}}$

$$s(t) = R_e\{s(t)\} + j\, I_m\{s(t)\} =$$

$$= a(t) + j\, b(t)$$



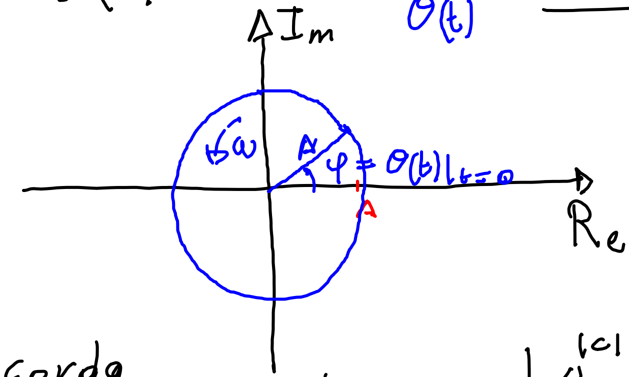
$$s(t) = A(t)\, e^{j\theta(t)}$$

$$A(t) = \sqrt{R_e^2\{s(t)\} + I_m^2\{s(t)\}}$$

$$\theta(t) = \operatorname{atg} \frac{I_m\{s(t)\}}{R_e\{s(t)\}}$$

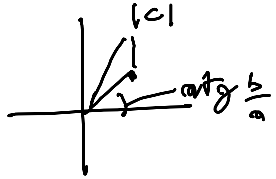
$$s(t) = A e^{j\underbrace{(\omega t+\varphi)}_{\theta(t)}}$$

Fasore



R. corda

$$C = a + jb = |C| e^{j\,\mathrm{atg}\frac{b}{a}}$$



$$\mathrm{Re}\{s(t)\} = \frac{s(t) + s^*(t)}{2} =$$

$$= \frac{A e^{j(\omega t+\varphi)} + A e^{-j(\omega t+\varphi)}}{2} =$$

$$= \frac{A[\cos(\omega t+\varphi) + j\cancel{\sin(\omega t+\varphi)}]}{2} +$$

$$+ \frac{A[\cos(\omega t+\varphi) - j\cancel{\sin(\omega t+\varphi)}]}{2} =$$

$$= A\cos(\omega t+\varphi)$$

$$\mathrm{Im}\{s(t)\} = \frac{s(t) - s^*(t)}{2j} = A\sin(\omega t+\varphi)$$

$y(t) = A\cos(\omega t + \varphi)$