PROPRIETÀ TCF

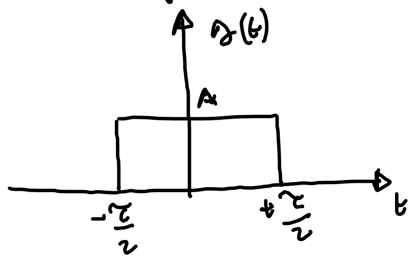
Dualità

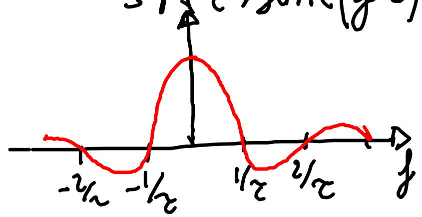
$$s(t) \overset{\mathcal{F}}{\Longleftrightarrow} S(f)$$

$$S(t) \Longleftrightarrow s(-f)$$

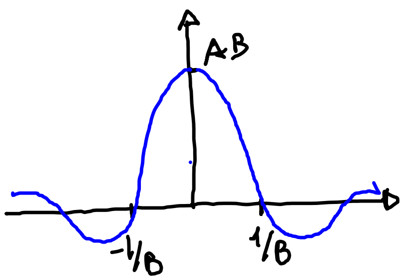
ES. 1

$$s(t) = A\,\mathrm{rect}\left(\frac{t}{\tau}\right) \Longleftrightarrow S(f) = $$
$$= A\tau\,\mathrm{sinc}(f\tau)$$

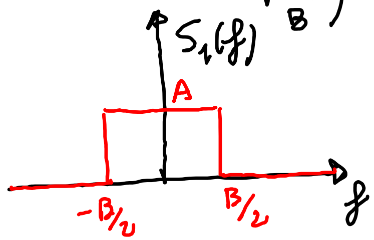




$$s_1(t) = AB\,\mathrm{sinc}(Bt)$$



$$S_1(f) = A\,\mathrm{rect}\left(\frac{-f}{B}\right) =$$
$$= A\,\mathrm{rect}\left(\frac{f}{B}\right)$$



Traslazione in Frequenza

$$s(t) \Longleftrightarrow S(f)$$

$$s(t)\, e^{j2\pi f_0 t} \Longleftrightarrow S(f-f_0)$$

Convoluzione temporale

$x(t) \Longleftrightarrow X(f) \quad y(t) \Longleftrightarrow Y(f)$

$$s(t) = x(t) \otimes y(t) = \int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau$$

$$s(t) \Longleftrightarrow S(f) = X(f)\, Y(f)$$

□ ES.

$S(f) = \mathrm{sinc}^2(f\tau) \qquad \mathcal{F}_c^{-1}[S(f)] = ?$

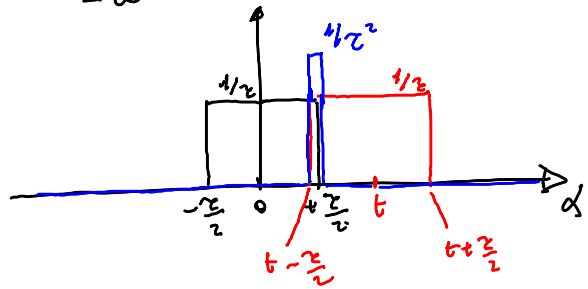
$$S(f) = \mathrm{sinc}(f\tau)\, \mathrm{sinc}(f\tau)$$

$S_1(f) = \mathrm{sinc}(f\tau)$

$s_1(t) = \frac{1}{\tau}\mathrm{rect}\left(\frac{t}{\tau}\right) \qquad s_2(t) = s_1(t)$

$$s(t) = \frac{1}{\tau}\mathrm{rect}\left(\frac{t}{\tau}\right) \otimes \frac{1}{\tau}\mathrm{rect}\left(\frac{t}{\tau}\right)$$

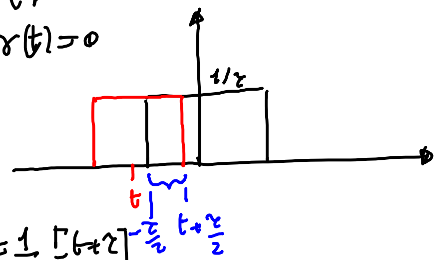
$$r(t) = \int_{-\infty}^{\infty} \underbrace{\frac{1}{\tau}\,\mathrm{rect}\left(\frac{\alpha}{\tau}\right)\frac{1}{\tau}\,\mathrm{rect}\left(\frac{t-\alpha}{\tau}\right)}\,d\alpha$$



$t < -\tau \Rightarrow r(t) = 0$

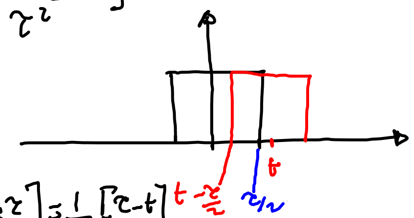
$t > +\tau \Rightarrow r(t) = 0$

$-\tau \leq t < 0$

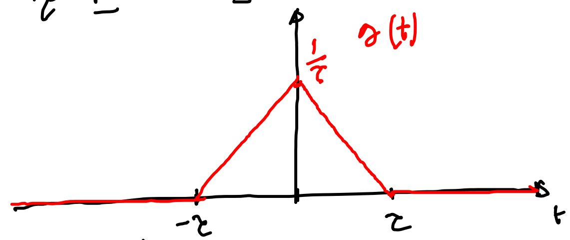


$$r(t) = \frac{1}{\tau^2}\left[t + \frac{\tau}{2} + \frac{\tau}{2}\right] = \frac{1}{\tau^2}\left[t + \tau\right]$$

$0 < t \leq \tau$



$$r(t) = \frac{1}{\tau^2}\left[\frac{\tau}{2} - t + \frac{\tau}{2}\right] = \frac{1}{\tau^2}\left[\tau - t\right]$$

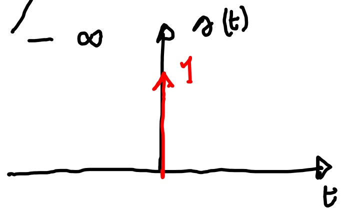


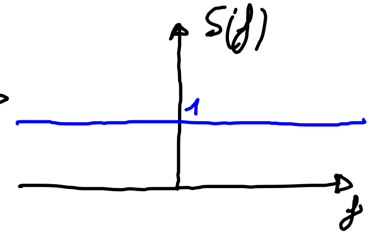
$$r(t) = \begin{cases} 0 & t < -\tau,\ t > \tau \\ \frac{1}{\tau^2}\left[\tau - |t|\right] & -\tau \leq t \leq \tau \end{cases}$$

$$s(t) = \delta(t) \Longleftrightarrow S(f) = ?$$

$$S(f) = \int_{-\infty}^{\infty} s(t)\, e^{-j2\pi f t}\, dt =$$

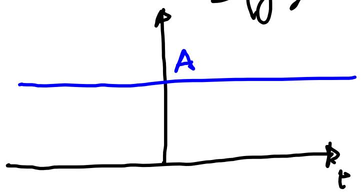
$$= \int_{-\infty}^{+\infty} \delta(t)\, e^{-j2\pi f t}\, dt = 1$$

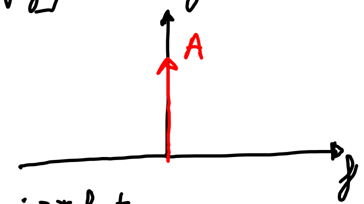




$\rightarrow$ $s(t) = A \Longleftrightarrow S(f)$

$$S(f) = A\,\delta(-f) = A\,\delta(f)$$

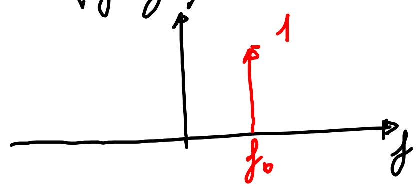




$\rightarrow$ $s(t) = e^{j2\pi f_0 t} = 1 \cdot e^{j2\pi f_0 t}$

$s_1(t) \Longleftrightarrow S_1(f)$ $\qquad$ $s_1(t)\, e^{j2\pi f_0 t} \Longleftrightarrow S_1(f - f_0)$

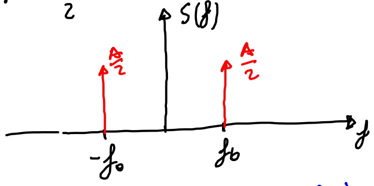
$$S(f) = \delta(f - f_0)$$

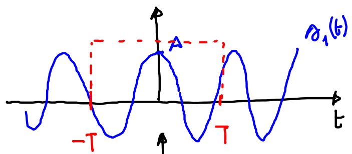


$\rightarrow s(t) = A\cos(2\pi f_0 t)$

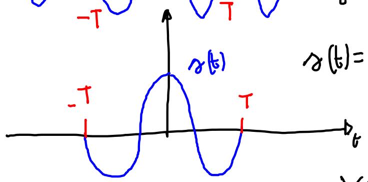
$$s(t) = A\left(\frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2}\right)$$

$$S(f) = \frac{A}{2}\left(\delta(f-f_0) + \delta(f+f_0)\right)$$





$s_1(t) = A\cos(2\pi f_0 t)$



$s(t) = s_1(t)\,\mathrm{rect}\left(\frac{t}{2T}\right)$

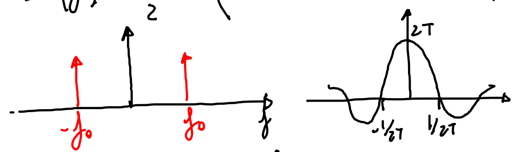
$x(t) \Leftrightarrow X(f) \qquad y(t) \Leftrightarrow Y(f)$

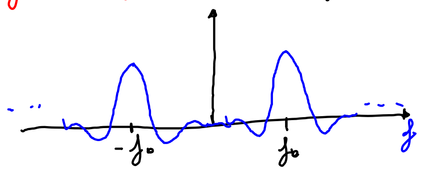
$s(t) = x(t)\,y(t) \Longleftrightarrow S(f) = X(f) \otimes Y(f)$

$$S(f) = \frac{A}{2}\left(\delta(f+f_0) + \delta(f-f_0)\right) \otimes 2T\,\mathrm{sinc}(2Tf)$$

$$\left[\; f(t) \otimes \delta(t-t_0) = \int_{-\infty}^{\infty} f(\alpha)\,\delta(t-t_0-\alpha)\,d\alpha = f(t-t_0) \;\right]$$

$$S(f) = \frac{A}{2}\cdot 2T\left(\mathrm{sinc}(2T(f+f_0)) + \mathrm{sinc}(2T(f-f_0))\right)$$





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$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} e^{j2\pi n \frac{t}{T_0}}$$

DA TRASL. IN FREQ. . . . .

$$S(f) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \delta(f - nf_0)$$