An Introduction to Mobility of Cooperating Robots with Unactuated Joints and Closed-Chain Mechanisms

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References

Referenced articles

[1] Antonio Bicchi. "On the problem of decomposing grasp and manipulation forces in multiple whole limb manipulation". In: *Int. Journal of Robotics and Autonomous Systems vol. 13 (1994), pp. 127–147.Elsevier Science, Oxford, UK.* 3 (1994).

[2] Antonio Bicchi and Domenico Prattichizzo. "Manipulability of Cooperating Robots with Unactuated Joints and Closed-Chain Mechanisms". In: *IEEE Transactions on Robotics and Automation*. 2000.

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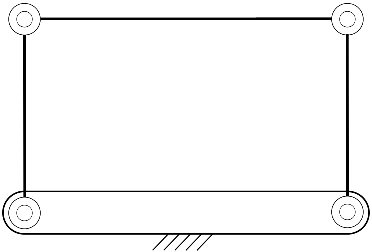
Results proposed in the above articles apply directly to the analysis of cooperating robots, parallel robots, dextrous robotic hands and legged vehicles, and, in general, to closed kinematic chains.

System definition

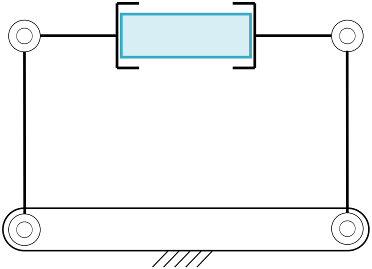
MANIPULATION

The approach we follow to analyze kinematics and statics of closed-chain mechanical system is to consider them as **embodiments of a cooperative manipulation paradigm**, where multiple robotic limbs (or fingers) interact with an object at a number of contacts. The object is the reference member of the mechanism, whose motions and forces are the ultimate goal of analysis. **Contacts represent in fact unactuated kinematic pairs of different nature between the object and the contacting link, that restrict some or all the components of the relative velocities of the two bodies**

System definition example



(a) Four bar linkeage



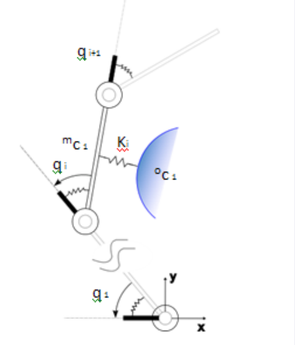
(b) Equivalent multiple cooperating limbs system system

Figure: From closed chain mechanisms to multiple limbs cooperative robots : an illustrative example

Mobility analysis equation system

Recalling what stated in [1] :

$$\begin{cases} H(\Delta^m x - \Delta^o x) = 0 \\ \Delta^o x = \tilde{G}^T \Delta u \quad \Rightarrow \quad {}^o\dot{x} = \tilde{G}^T \dot{u} \\ \Delta^m x = \tilde{J} \Delta q \quad \Rightarrow \quad {}^m\dot{x} = \tilde{J} \dot{q} \end{cases}$$



Passive joints

Distinction had to be made between active and passive joint variables. A suitable permutation matrix P can be found that reorders joint angular/position variables q to have actuated joints on top , and unactuated joints at bottom:

$$\delta\tilde{q}^T = (\delta q_A^T, \delta q_P^T)^T = \delta q^T P^T$$

where δq_A (respectively, δq_P) is the $q_A(q_P)$-vector of actuated (unactuated) joint velocities. Correspondingly, the Jacobian matrix is partitioned as

$$J\delta q = JP^{-1}\delta\tilde{q} = \begin{bmatrix} J_A & J_P \end{bmatrix} \begin{bmatrix} \delta q_A \\ \delta q_P \end{bmatrix}$$

- *In order to avoid more complicate notation in the following analysis $\tilde{q}$ will be indicated as q.*

Mobility analysis formulation

Defining

$$G = \tilde{G} H^T$$
$$J = H \tilde{J}$$
$$\tilde{t} = H^T t$$

The mobility of the system is then studied by analyzing the constraint equation

Mobility analysis system

$$\begin{bmatrix} J_A & J_P & -G^T \end{bmatrix} \begin{bmatrix} \dot{q}_A \\ \dot{q}_P \\ \dot{u} \end{bmatrix} = 0$$

All possible motions of the system belong to the nullspace (or kernel) of the constraint matrix $\begin{bmatrix} J_A & J_P & -G^T \end{bmatrix}$ and, hence, can be rewritten as linear combinations of vectors forming a basis of the nullspace.

Mobility matrix parametrization

By suitable linear algebra operations, such a basis can always be written in a block-partitioned form

$$\begin{bmatrix} \dot{q}_A \\ \hline \dot{q}_P \\ \dot{u} \end{bmatrix} = Cx = \left[\begin{array}{c|cc} C_{11} & C_{12} & 0 \\ \hline 0 & C_{22} & C_{23} \end{array}\right] \begin{bmatrix} x_1 \\ \hline x_2 \\ x_3 \end{bmatrix}$$

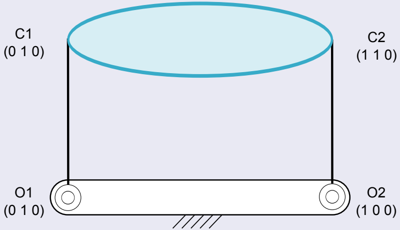
$$\begin{bmatrix} \dot{q}_A \\ \hline \dot{q}_P \\ \dot{u} \end{bmatrix} = Cx = \left[\begin{array}{c|cc} C_{11} & C_{12} & 0 \\ \hline 0 & C_{22} & C_{23} \end{array}\right] \begin{bmatrix} x_1 \\ \hline x_2 \\ x_3 \end{bmatrix}$$

- redundancy
- indeterminancy
- coordinated motions
- C_{23} ulterior partition
- $\left[C_{12}^T, C_{22}^T\right]^T$ ulterior partition

A simple, simple example

Structure

Let 's examine the example reported below



We have to find:

- C_{11}
- $[C_{12}^T, C_{22}^T]^T$
- C_{23}

A simple, simple example

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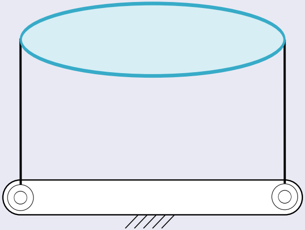

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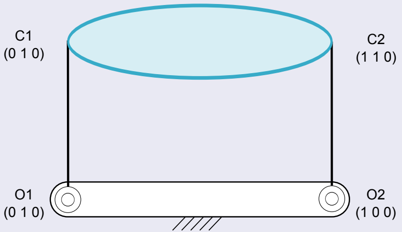

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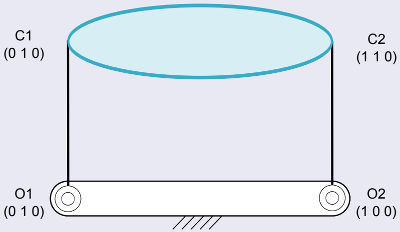


We have to find:

- C_{11}
- $\left[C_{12}^T, C_{22}^T\right]^T$
- C_{23}

According with the type of contact (hf w/h friction or soft finger)

$$C_{1,1} = /$$

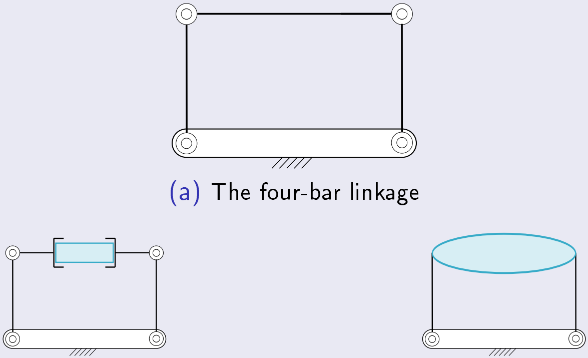
$$C_{1,2} = \begin{bmatrix} -0.3 \\ -0.3 \end{bmatrix}$$

$$C_{22} = \begin{bmatrix} 0.3 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}$$

$$C_{2,3} = / \text{ or } C_{2,3} = \begin{bmatrix} 0.0 \\ 0.0 \\ -0.5 \\ 0.5 \\ 0.0 \\ 0.0 \end{bmatrix}$$

Parallel structure equivalence

The analyzed example can be considered a 4 bar linkage "translation"



(a) The four-bar linkage

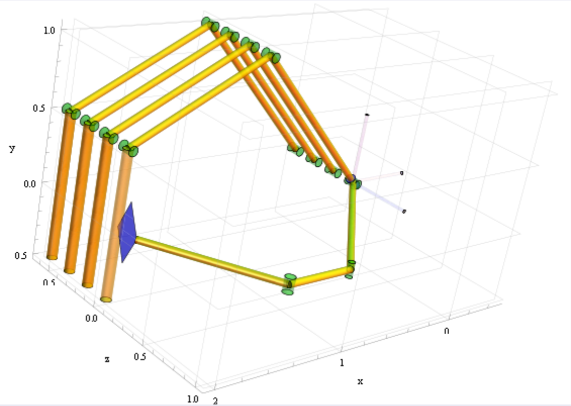
(b)

(c)

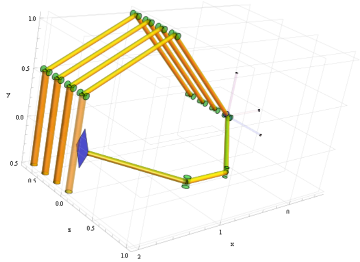
Figure: Parallel structure analysis

An interesting example

Santello's zip postures



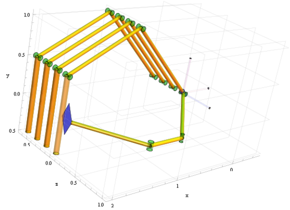
An interesting example



Santello's zip postures

$$C_{11} = /$$

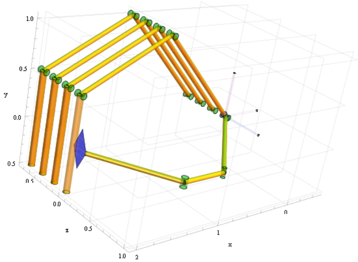
An interesting example



Santello's zip postures

$$C_{22purged} = \begin{bmatrix} -15.021 & 33.0 & 0.0 & 0.0 \\ -13.018 & 2.7925 & 1.0 & -1.0 \\ -2.0 & 2.0 & -33.288 & 0.0 \\ -59.611 & 0.0 & 0.0 & 0.0 \\ 1.0 & -1.0 & 16.644 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

An interesting example



rank$(C_{22}) = 4$.
No permitted motions:

$$\begin{bmatrix} 0.0 \\ 0.0 \\ 1.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{bmatrix} \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 1.0 \\ 0.0 \end{bmatrix}$$