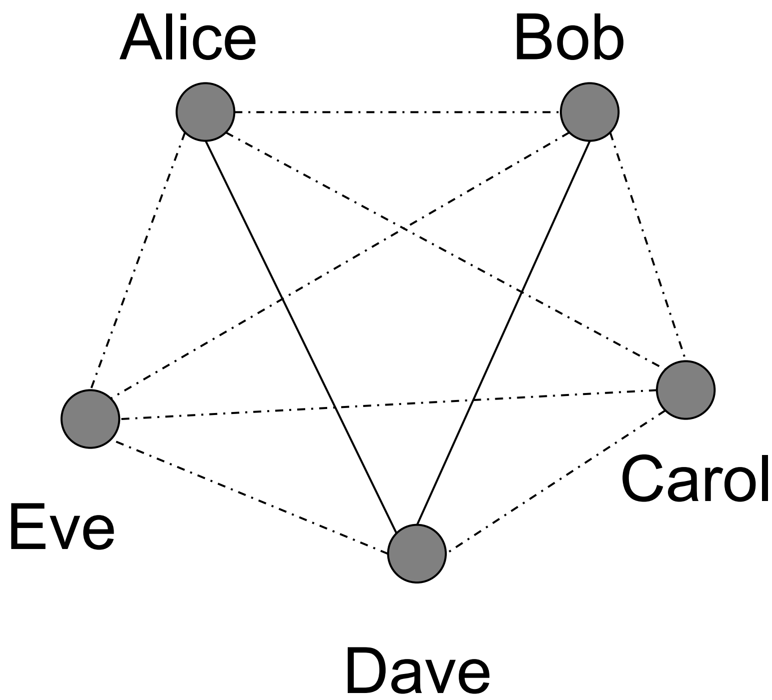
Elements of Applied Cryptography

Key Distribution

- Trusted third party: KDC, KTC
- Diffie-Helmann protocol
- The man-in-the-middle attack

Point-to-point key establishment



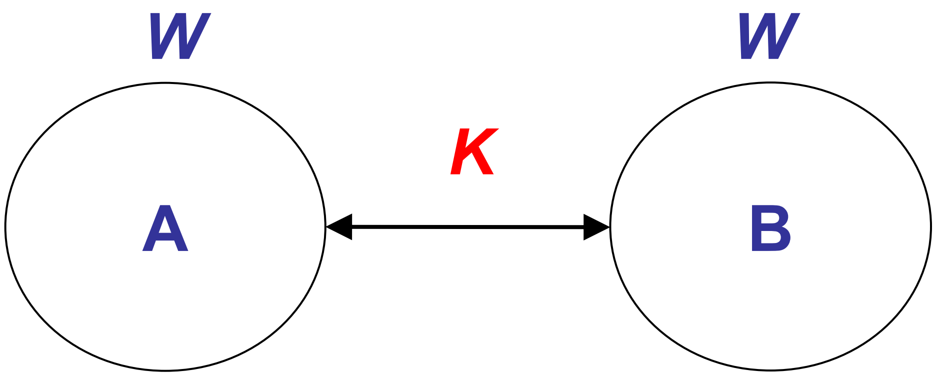
- Each pair of users must share an *a priori*, long-term secret key
- Each user has $(n - 1)$ keys
- The overall number of keys is

$$\frac{n \times (n-1)}{2} \cong \frac{n^2}{2} \quad \text{or} \quad O(n^2)$$

Point-to-point key establishment

- **PROS**
  - **Security**. If a subject is compromised only its communications are compromised; communications between two other subjects are not compromised
- **CONS**
  - **Poor scalability**.
    - The number of keys is quadratic in the number of subjects
    - New member's joining and a member's leaving affect all current members

Session/ephemeral key



- Parties know each other
  e.g., a client *A* has an account on server *B*
- *A* and *B a priori* share a long term ***key W***
- *A* and *B* wants to establish a ***session key K***

- Session key is used for a communication session
- Session key is used for bulk encryption
- Long term key is used for key establishment

Session/ephemeral key

one-pass

$$M1 \quad A \rightarrow B: \quad E_W \; t_A, B, K$$

- t_A is a **timestamp** (a "**fresh**" quantity) requires **synchronized** clocks

with challenge-response

$$M1 \quad A \leftarrow B: \quad n_B$$
$$M2 \quad A \rightarrow B: \quad E_W \; n_B, B, K$$

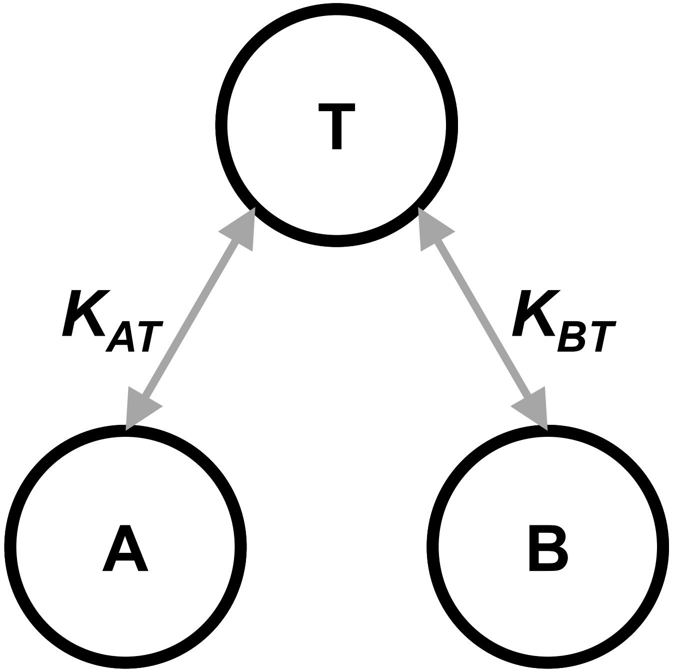
- n_B is a **nonce** (a "**fresh**" quantity)

both parties contribute to the session key

$$M1 \quad A \leftarrow B: \quad n_B$$
$$M2 \quad A \rightarrow B: \quad E_W \; K_A, n_B, n_A, B$$
$$M3 \quad A \leftarrow B \quad E_W \; K_B, n_A, n_B, A$$

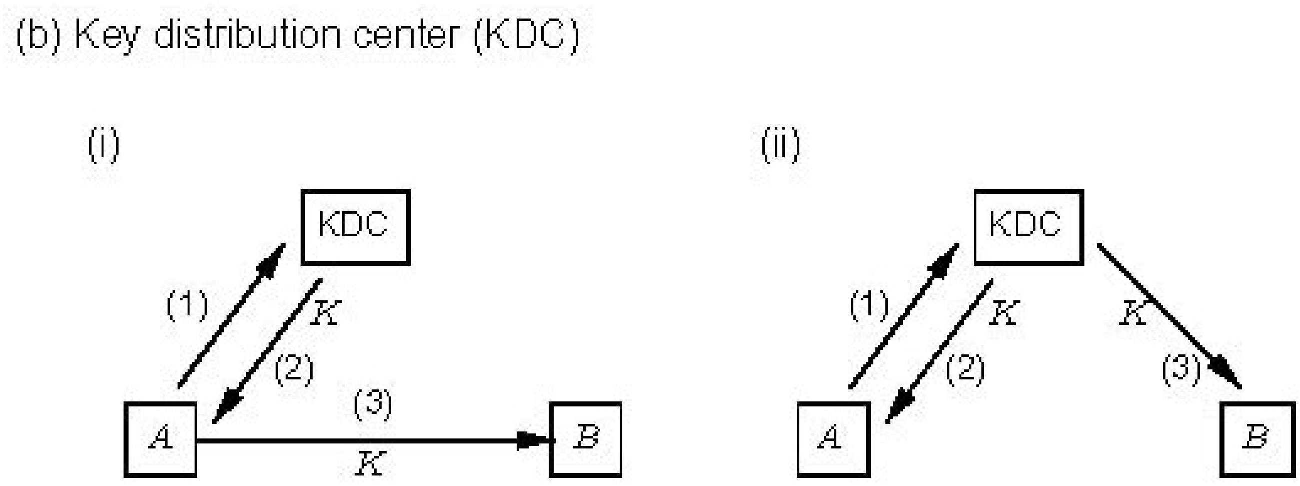
- n_A and n_B are **nonces**
- K_A and K_B are keying materiale
- $K = f(K_A, K_B)$

Key distribution with Trusted Third Party



- ***T*** allows pair of users to establish a session key
- Each user shares a long-term, a priori key with ***T***
- The overall number of long-term keys is ***O***(***n***)
- ***T*** is a trusted third-party
  - Maintain a database **<*U*, K_{TU}>**
  - Guarantee integrity and secrecy of the database
  - Correctly play the key distribution protocol

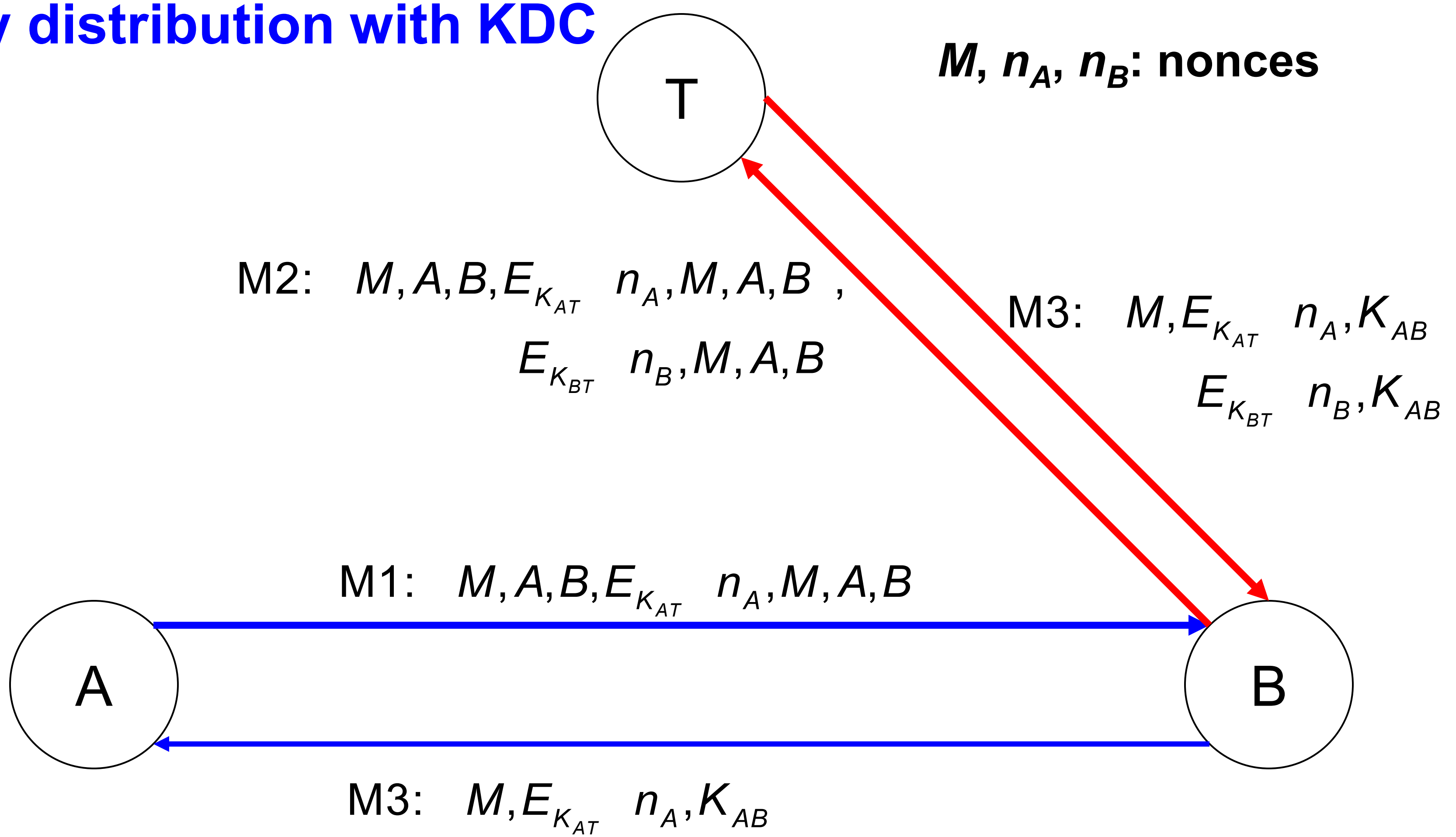
Key Distribution Center



- *A* and *B* share distinct secret keys, K_{AT} and K_{BT}, with KDC
- *KDC* generates the *session key K* and distributes it to *A* and *B*
- *KDC* is trusted to correctly generate the key

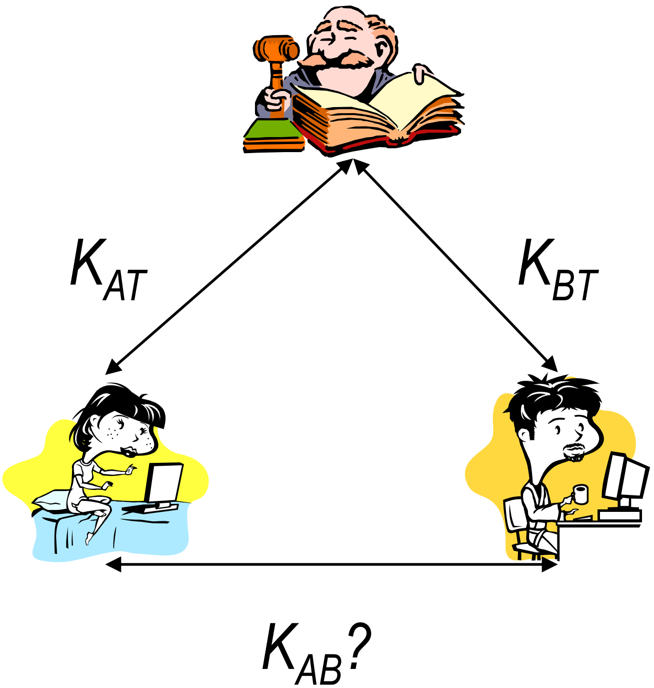
The Otway-Rees protocol (1987)

Key distribution with KDC



Trusted Third Party

Kerberos (*Unix*, *Active directory*)

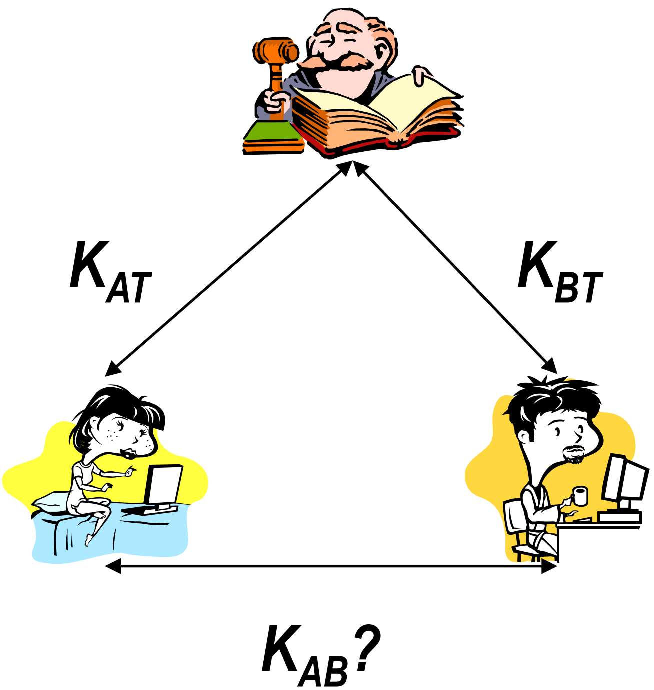


K_{AT}: *shared key between* Trent and Alice

K_{BT}: *shared key between* Trent and Bob

Objective: Alice and Bob establish a secret shared session key K_{AB}

Trusted Third Party: il protocollo



$M1\ A \rightarrow T: A, B$

$M2\ T \rightarrow A:\ E((T, L, K_{AB}, B), K_{AT}),$
$E(T, L, K_{AB}, A), K_{BT})$

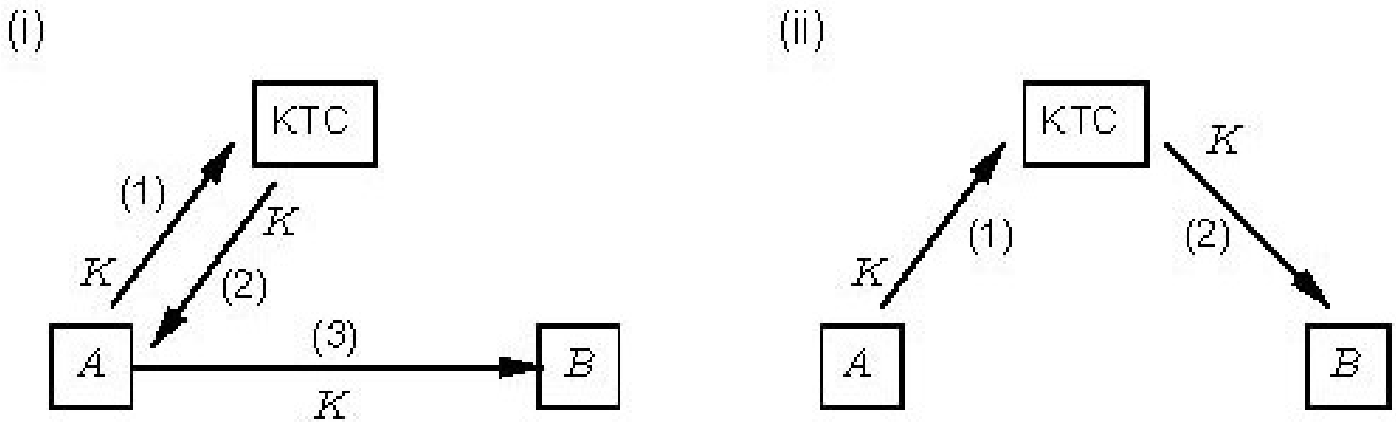
$M3\ A \rightarrow B:\ E((A, T), K_{AB}),$
$E(T, L, K_{AB}, A), K_{BT})$

$M3\ B \rightarrow A: E(T{+}1, K_{AB})$

T: timestamp (nonce)
L: lifetime di K_{AB}

Decentralized key management

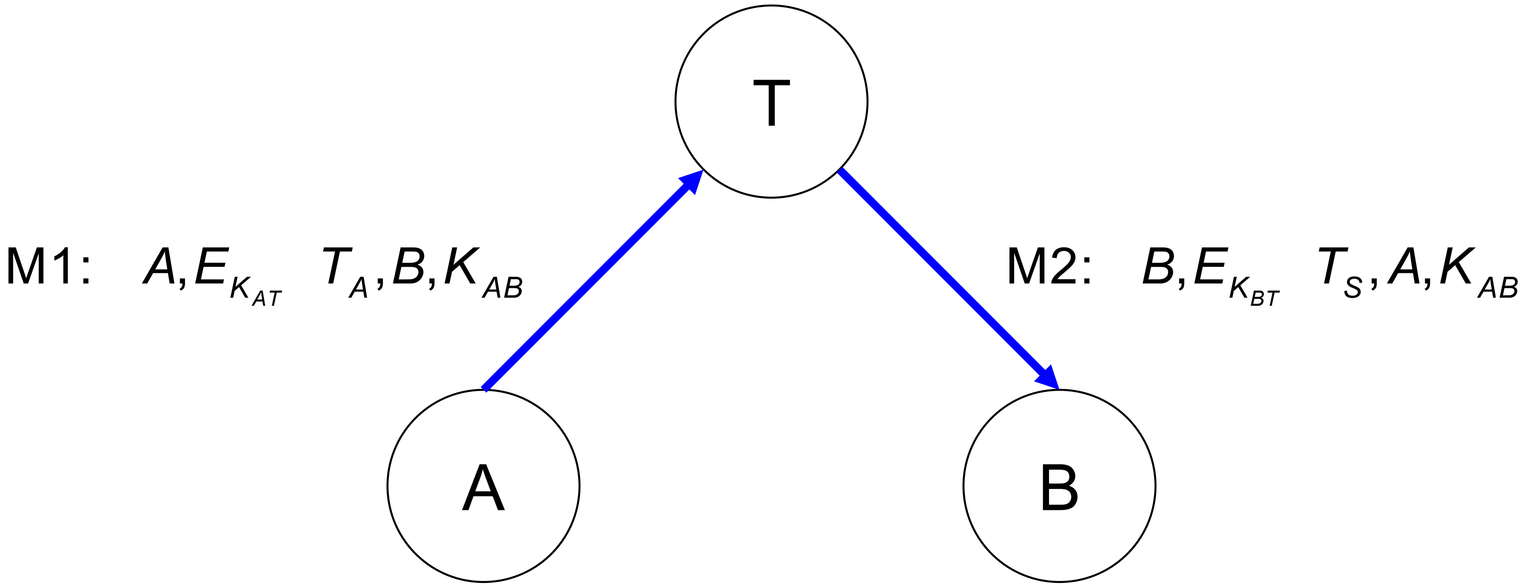
(c) Key translation center (KTC)



- *A* and *B* share distinct secret keys, K_{AT} and K_{BT}, with *KTC*
- One of the parties generates the session key *K*;
  KTC transmits that key to the other peer
- The party is trusted to correctly generate the key
  KTC is trusted to correctly relay the key

The Wide-mouthed frog protocol

Key distribution with KTC



- Synchronized clocks
- Bob trusts Alice to be competent in generating keys

Key distribution with trusted third party

Pros

- It is easy to add and remove entities from the network
- Each entity needs to store only one long-term secret key

Cons

- All communication require initial interaction with the TTP
  - *TTP must be always online (availability)*
  - *TTP is a performance bootleneck (efficiency)*
- The TTP must be trusted
  - *TTP must store n long-terms keys*
  - *TTP has the ability to read and forge all messages*
  - *If the TTP is compromised, all communications are insecure*

Public key distribution system

A **public key distribution systems** allows two users to securely exchange a key over ***an insecure channel***

Whitfield Diffie and Martin Hellman, “New directions in cryptography,” IEEE Transactions on Information Theory, Vol. 22, no. 6, pages 644-654, November 1976.

Whitfield Diffie, “The first ten years of public key cryptography,” Proceedings of IEEE, Vol. 76, no. 5, May 1988.

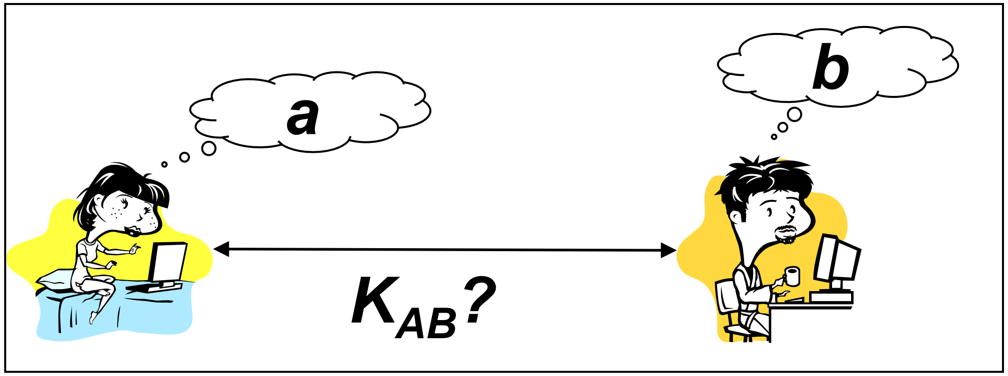
The discrete logarithm problem

- Let p be prime
- Let $1 \leq g < p$ be a generator, i.e.,
  $\forall\ 1 \leq n < p$, $\exists\ t$ s.t. $g^t \bmod p = n$
- **DISCRETE EXPONENTIATION**

  Given g and x,
  computing $y = g^x \bmod p$ is *computationally easy*
- **DISCRETE LOGARITHM** $x \hat{=}$ d p

  Given g, $1 \leq y \leq p-1$,
  it is *computationally difficult* to determine x $(0 \leq x \leq p - 2)$ s.t.
  $y = g^x \bmod p$

Diffie-Hellman protocol: scenario



- Let p be a large prime
- Let $1 \leq g < p$
- Let p e g publicly known

Alice chooses a random number a
Bob chooses a random number b

M1 A $\rightarrow$ B: A, $Y_A = g^a \bmod p$
M2 B $\rightarrow$ A: B, $Y_B = g^b \bmod p$

Alice computes $K_{AB} = (Y_B)^a \bmod p = g^{ab} \bmod p$
Bob computes $K_{AB} = (Y_A)^b \bmod p = g^{ab} \bmod p$

Security of Diffie-Hellman

- An adversary can compute K_{AB} from Y_A and Y_B by computing, for example,

$$K_{AB} = \qquad \bmod p$$

- If logs mod *p* are easily computed then the system can be broken
- There is no proof of the converse, i.e., if logs mod *p* are difficult to compute then the system is secure
- We don't see any way to compute K_{AB} from Y_A and Y_B without first obtaining either *a* or *b*

Security of Diffie-Hellman

- Let p be a prime, $p < 2^n$, then

  All quantities are representable as n-bit numbers

  Exponentiation takes at most $2 \times \log_2 p = 2n$ multiplications mod p

  Taking logs mod p requires $p^{½} = 2^{n/2}$ operations

- **Example n = 512**

  Exponentiation requires at most 1024 multiplications

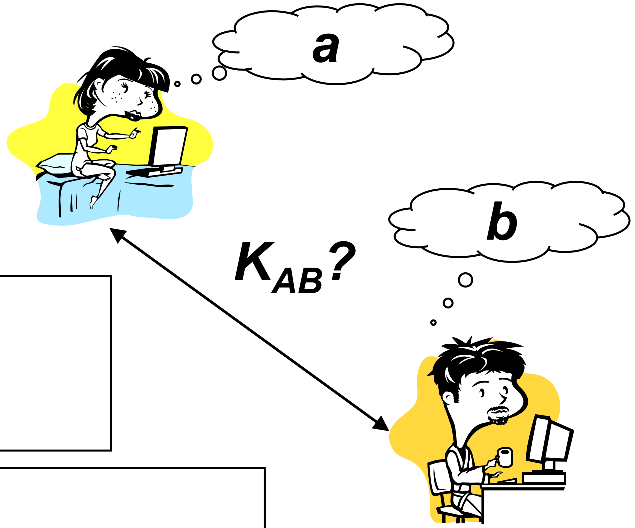
  Taking logs mod p requires $2^{256} = 10^{77}$ operations

Diffie-Hellman protocol: an example

Let p = 11, g = 7

Alice chooses a = 3 and computes $Y_A = g^a \bmod p = 7^3 \bmod 11 = 343 \bmod 11 = 2$

Bob chooses b = 6 and computes $Y_B = g^b \bmod p = 7^6 \bmod 11 = 117649 \bmod 11 = 4$



$A \rightarrow B$: 2

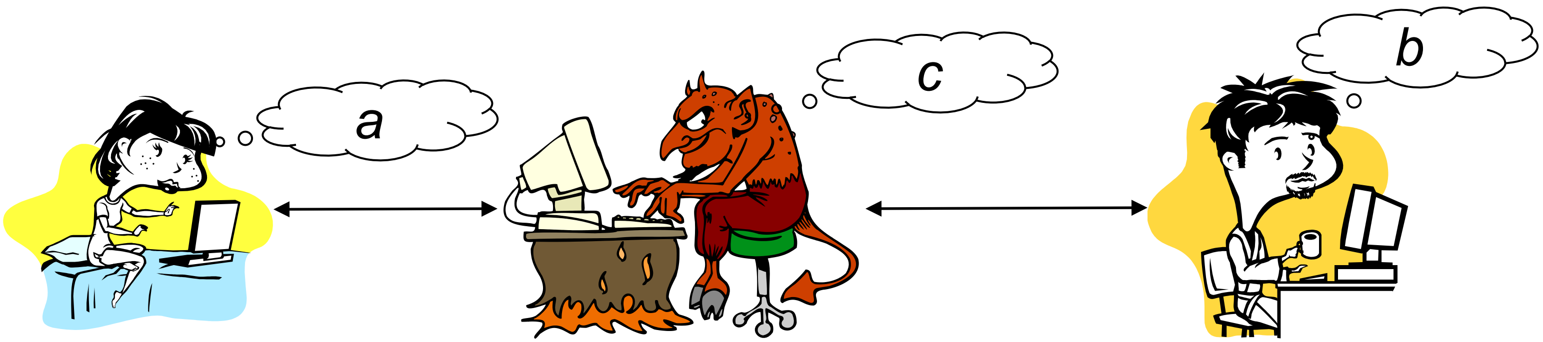
$B \rightarrow A$: 4

Alice receives 4 and computes $K_{AB} = (Y_B)^a \bmod p = 4^3 \bmod 11 = 9$

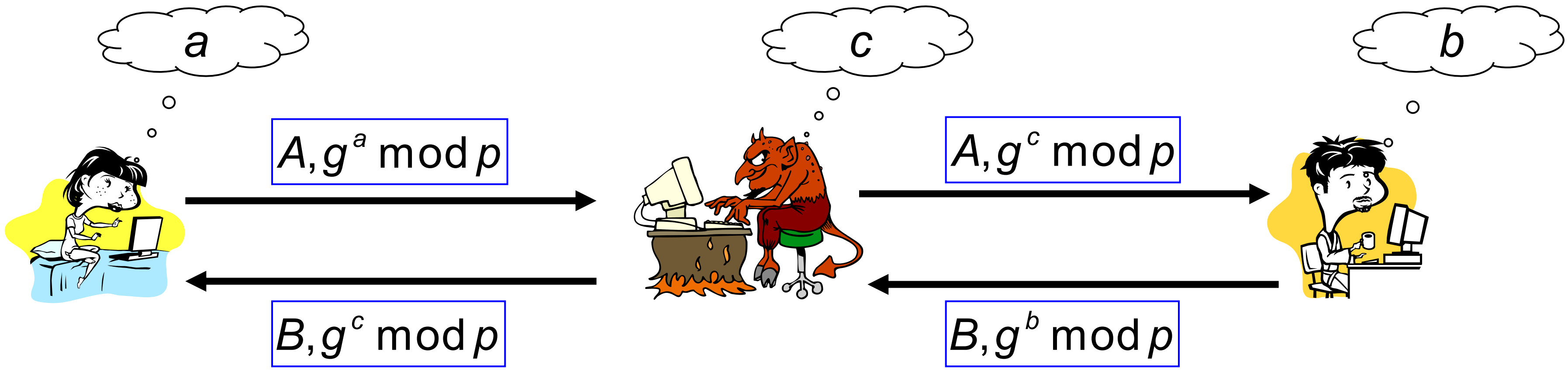
Bob receives 2 and computes $K_{AB} = (Y_A)^b \bmod p = 2^6 \bmod 11 = 9$

The man-in-the-middle attack

Alice has no guarantee that she is actually talking with Bob and vice versa



The man-in-the-middle



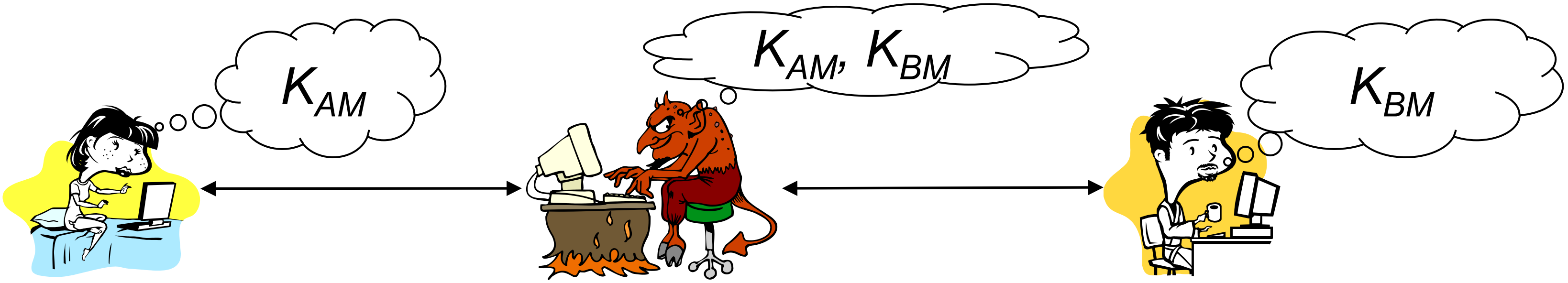
$K_{AM} = g^{ac} \bmod p$

$K_{AM} = g^{ac} \bmod p$, e

$K_{BM} = g^{bc} \bmod p$

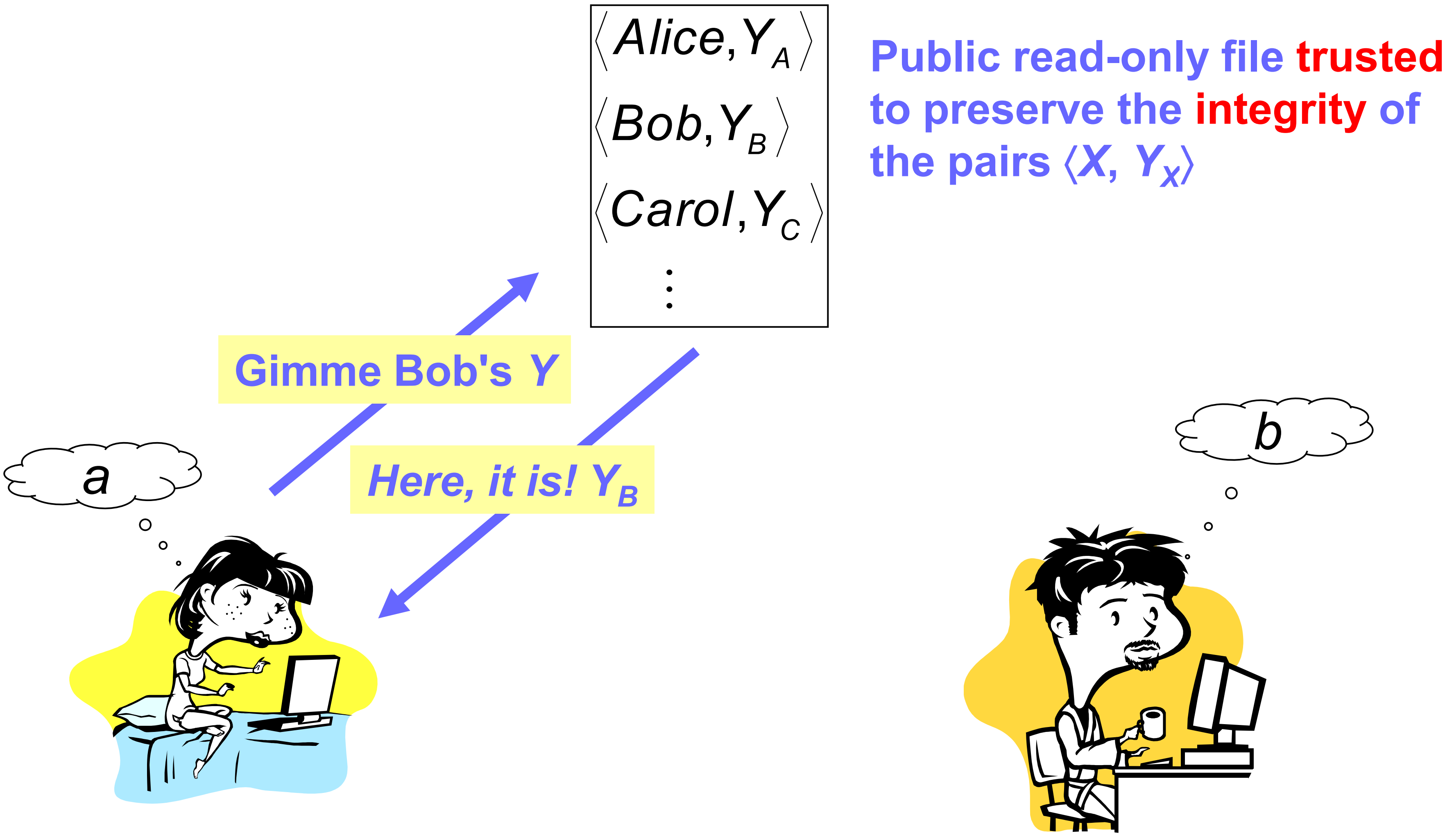
$K_{BM} = g^{bc} \bmod p$

The man-in-the-middle

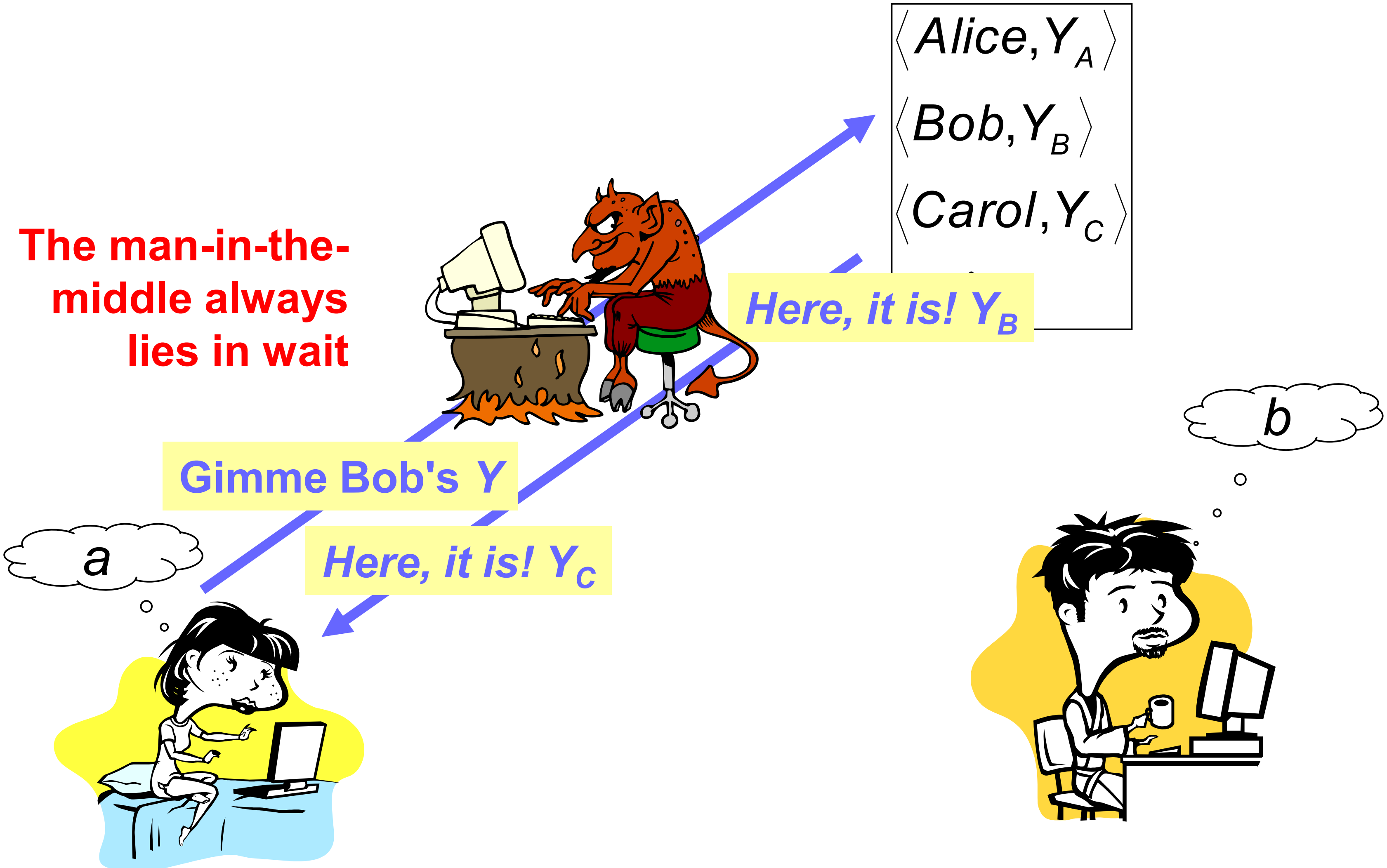


- Alice believes to communicate with Bob by means of K_{AM}
- Bob believes to communicate with Alice by means of K_{BM}
- The adversary can
  - read messages between Alice and Bob
  - inject messages between Alice and Bob (impersonate Alice and Bob)

Diffie-Hellman protocol



Diffie-Hellman protocol

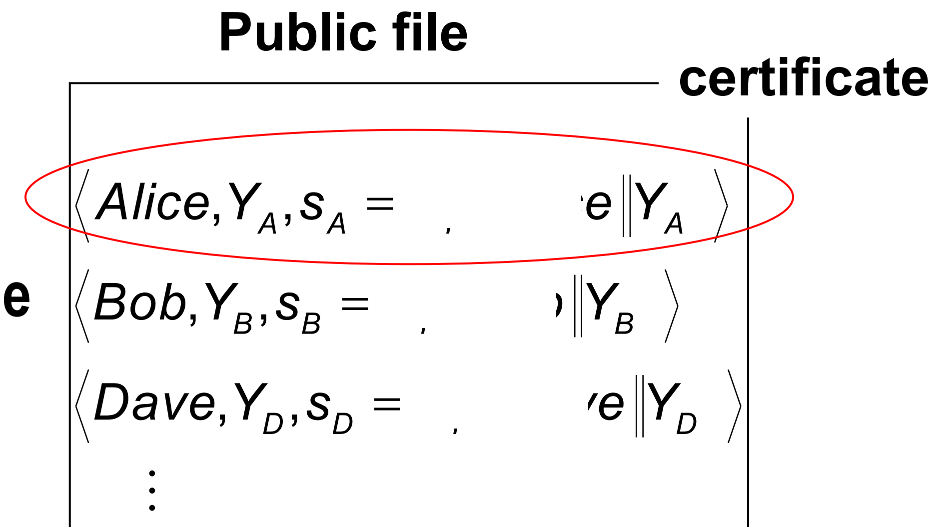


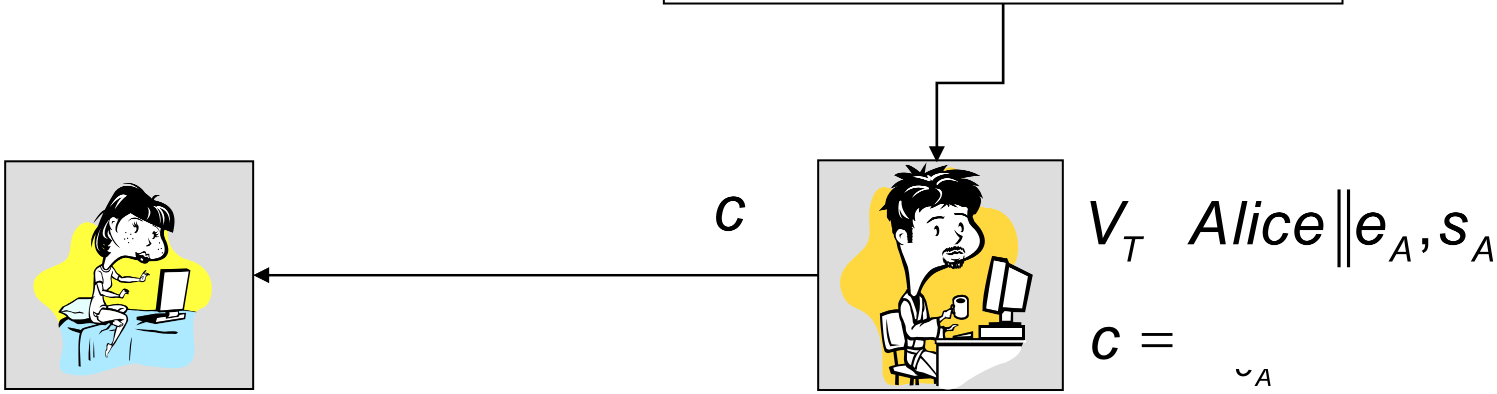
Key distribution with public encryption

- **Pros**
  - No TTP is required
  - The public file could reside with each entity
  - Only *n* public keys need to be stored to allow secure communications between any pair of entities, assuming that the only attack is that by a passive adversary
- **Cons**
  - Key management becomes difficult in the presence of an active adversary (man-in-the-middle)

Key distribution with public keys

- A TTP certifies the public key of each entity
- S_T and V_T are the signing and verification keys of the TTP





Key distribution with certificates

- **Pros**
  - Prevent an active adversary from impersonation
  - Entities need to trust the TTP only to bind identities to public keys properly
  - Certicates can be stored locally so eliminating per-communication interaction with the public file
    - Uhmmm…not really!

- **Disadvantages**
  - if the signing key of TTP is compromised, all communications become insecure
  - All trust is placed with one entity

Thanks for your attention