Quantitative evaluation of Dependability

Quantitative evaluation of Dependability

- Faults are the cause of errors and failures. Does the arrival time of faults fit a **probability distribution**? If so, what are the parameters of that distribution?

- Consider the time to failure of a system or component. It is not exactly predictable - **random variable**.

probability theory

Quantitative evaluation of failure rate, Mean Time To Failure (MTTF), Mean Time To Repair (MTTR), Reliability function (R(t)), Availability function (A(t)) and Safety function (S(t))

Random variable

Random variable

a random variable X *is a function from a sample space (Ω) to reals numbers*

Let us consider the random experiment of **tossing a die.**

Let X be the random variable defined as the face you obtain

Sample space Ω : faces of the die (1, 2, 3, 4, 5, 6)
Real numbers S: 1, 2, 3, 4, 5, 6

Any element in the sample space Ω has a well defined probability distribution.

The probability assigned to each output of the experiment is 1/6.

If the set of values the variable can assume (S) is finite then
X is a **discrete random variable**

Random variable

We define the **probability distribution function** of a discrete random variable: a mapping of all possible values of the random variable (S) to their corresponfing probabilities for the given sample space Ω

$$f(x) = P(X=x)$$

$$f(x) = \begin{cases} 1/6 & \text{for all } i=1, \ldots, 6 \\ 0 & \text{otherwise} \end{cases}$$

P(X=1)=1/6
P(X=2)=1/6
…….

An order relation can be defined on Λ. The probability of the following sets can be computed:

$$P\{X <= x\} \text{ for x in S}$$

We define the **cumulative distribution function** of X

$$F(x) = P\{X <= x\}$$

F is a non-decreasing function, if $x_1 <= x_2$, then $F(x_1) <= F(x_2)$

$$F(3) = P\{X<=3\} = P\{X=1\}+P\{X=2\}+P\{X=3\} = 1/6 +1/6+1/6 =1/2$$

Random variable

Let us consider the random experiment of the measuring the temperature in a region.

Let X be the random variable defined as the temperature you obtain.

Sample space Ω : Real numbers
Real numbers S: Real numbers

By definition, the probability of any real number is zero. The random variable can be infinitely divided into smaller parts such that the probability of selecting a real integer value x is zero.

$$P(X=x) = 0$$

Probability is compiuted as:

$P(X <= x)$ $P(X >= x)$ $P(x_1 <= x <= x_2)$

Random variable

We define the **probability density function**:

$$P(x_1 < X < x_2) = \int_{x_1}^{x_2} f(x)\mathrm{dx}$$

probability that a given output will occur at a given point

An example of probability density function :

$$f(x) = \begin{cases} 3x^{-4}, & x > 1 \\ 0, & elsewhere \end{cases}$$

Cumulative distribution function for a continuos random variable:

$$F(x) = P(X \leq x)$$

which is the same as

$$F(x) = \int_{-\infty}^{x} f(t)\mathrm{dt}, \ \text{for} \ -\infty < x < \infty$$

The probability density function can be computed by the cumulative distribution function if the derivative exists:

$$f(x) = \frac{\mathrm{d}F(x)}{\mathrm{dx}}$$

Quantitative definition of dependability attributes

Reliability - R(t)

conditional probability that the system performs correctly throughout the *interval of time* [t0, t], given that the system was performing correctly at the *instant* of time t0

Availability - A(t)

the probability that the system is operating correctly and is available to perform its functions at the *instant* of time t

Safety – **S(t)**

the probability that the system either behaves correctly or will discontinue its functions in a manner that causes no harm throughout the *interval of time* [t0, t], given that the system was performing correctly at the *instant* of time t0

Definitions

Reliability R(t)

$$R(0) = 1 \quad R(\infty) = 0$$

Failure probability Q(t)

$$Q(t) = 1 - R(t)$$

Failure probability density function f(t)

the failure density function f(t) at time t is the number of failures in Δt

$$f(t) = \frac{dQ(t)}{dt} = \frac{-\,dR(t)}{dt}$$

Failure rate function $\lambda(t)$

the failure rate $\lambda(t)$ at time *t* is defined by the number of failures during Δt in relation to the number of correct components at time *t*

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{-\,dR(t)}{dt}\frac{1}{R(t)}$$

Hardware Reliability

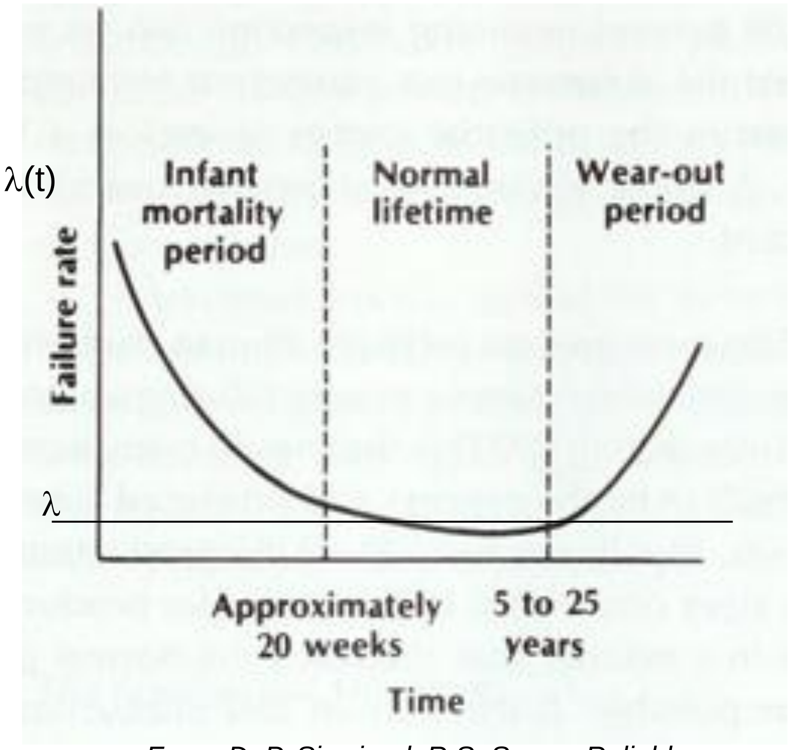
λ(t) is a function of time
(bathtub-shaped curve)

λ(t) constant > 0 in the useful life period

Constant failure rate λ

(usually expressed in number of failures for million hours)

λ = 1/2000
one failure every 2000 hours



From: D. P. Siewiorek R.S. Swarz, Reliable Computer Systems, Prentice Hall, 1992

Early life phase: there is a higher failure rate, calleld infant mortality, due to the failures of weaker components. Often these infant mortalities result from defetct or stress introduced in the manufacturing process.

Operational life phase: the failure rate is approximately constant.

Wear-out phase: time and use cause the failure rate to increase.

Hardware Reliability

Constant failure rate

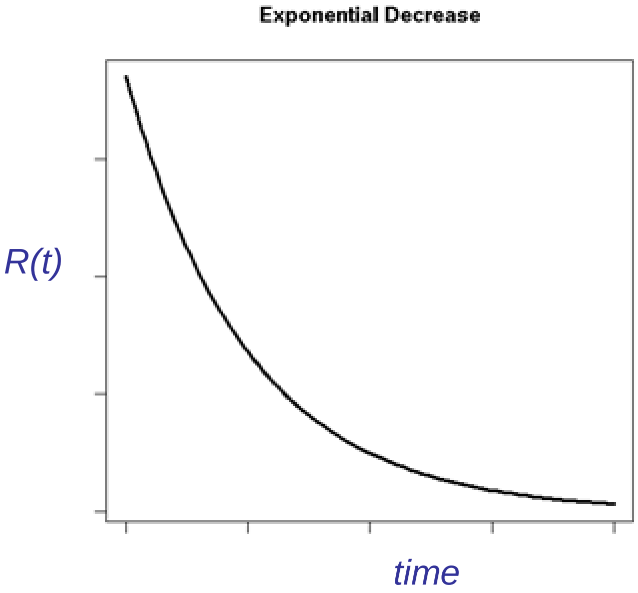
$\lambda(t) = \lambda$

Reliability function

$R(t) = e^{-\lambda t}$

Probability density function

$f(t) = \lambda e^{-\lambda t}$



the exponential relation between reliability and time is known as ***exponential failure law***

Time to failure of a component

Time to failure of a component can be modeled by a **random variable X**

$f_X(t)$ probability density function $P[X=t]$ (X discrete)
$F_X(t)$ cumulative distribution function $P[X<=t]$

Unreliability of the component at time t is given by

$$Q(t) = P[X <= t] = F_X(t)$$

Reliability of the component at time t is given by

$$R(t) = P[X > t] = 1 - P[X <= t] = 1 - F_X(t)$$ **reliability function**

R(t) is the probability of not observing any failure before time t

Hardware Reliability

Mean time to failure (MTTF)

is the expected time that a system will operate before the first failure occurs (e.g., 2000 hours)

$$MTTF = \int_0^\infty t\, f(t)\, dt = \int_0^\infty t \lambda e^{-\lambda t}\, dt = \frac{1}{\lambda}$$

λ = 1/2000	0.0005 per hour
MTTF = 2000	time to the first failure 2000 hours

Failure in time (FIT)

measure of failure rate in 10^9 device hours

1 FIT means 1 failure in 10^9 device hours

Failure Rate

- Handbooks of failure rate data for various components are available from government and commercial sources.

- Reliability Data Sheet of product

- **Commercially available databases**
  - Military Handbook MIL-HDBK-217F
  - Telcordia,
  - PRISM User's Manual,
  - International Eletrotechnical Commission (IEC) Standard 61508
  - …

Databases used to obtain reliability parameters in
''Traditional Probabilistic Risk Assessment Methods
for Digital Systems'',
U.S. Nuclear Regulatory Commission,
NUREG/CR-6962, October 2008

Distribution model for permanent faults

MIL-HBDK-217 (*Reliability Prediction of Electronic Equipment* -Department of Defence) is a model for chip failure. Statistics on electronic components failures are studied since 1965 (periodically updated).

Typical component failure rates in the range 0.01-1.0 per million hours.

Failure rate for a single chip :

$$\lambda = \tau_L \tau_Q (C_1 \tau_T \tau_V + C_2 \tau_E)$$

τ_L = learning factor, based on the maturity of the fabrication process

τ_Q = quality factor, based on incoming screening of components

τ_T = temperature factor, based on the ambient operating temperature and the type of semiconductor process

τ_E = environmental factor, based on the operating environment

τ_V = voltage stress derating factor for CMOS devices

C_1, C_2 = complexity factors, based on the number of gates, or bits for memories in the component and number of pins.

From Reliable Computer Systems.D. P. Siewiorek R.S. Swarz, Prentice Hall, 1992

Model-based evaluation of dependability

MODEL-BASED evaluation of dependability
(a model is an abstraction of the system that highlights the important features for the objective of the study)

Dependability of a system is calculated in terms of the dependability of individual components

“divide And conquer approach”: the solution of the entire model is constructed on the basis of the solutions of individual sub-models

Methodologies that employ combinatorial models
Reliability Block Diagrams, Fault tree, ….

State space representation methodologies
Markov chains, Petri-nets, SANs, …

Model-based evaluation of dependability

Combinatorial methods

offer simple and intuitive methods of the construction and solutions of models

independent components

each component is associated a failure rate

model construction is based on the structure of the systems (series/parallel connections of components)

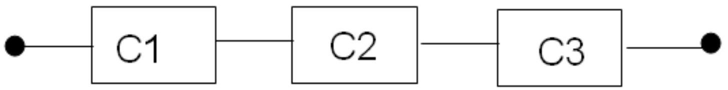
inadequate to deal with systems that exhibits complex dependencies among components and repairable systems

Combinatorial models

Series: all components must be operational (a)

$R_i(t)$ reliability of module i at time t

$R_{series}(t) = \Pi_{i=1}^{n} R_i(t)$
where Π is the product



(a)

If each individual component i satisfies the exponential failure law with constant failure rate λ_i:

$R_{series}(t) = e^{-\lambda_1 t} ... e^{-\lambda_n t} = e^{-\sum_{i=1}^{n} \lambda_i t}$

Unreliability function

$Q_{series}(t) = 1 - R_{series}(t) = 1 - \Pi_{i=1}^{n} R_i(t) = 1 - \Pi_{i=1}^{n} [1 - Q_i(t)]$

Combinatorial models

If the system does not contain any redundancy, that is any component must function properly for the system to work, and if component failures are independent, then

- the **system reliability** is the product of the component reliability, and it is exponential

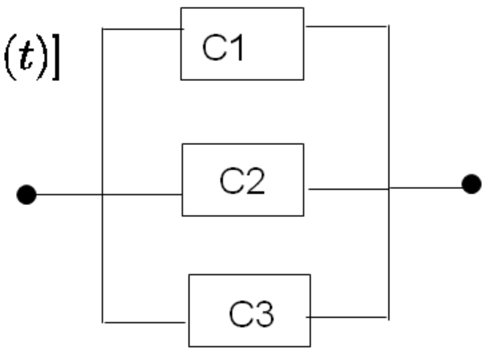
- the **failure rate of the system** is the sum of the failure rates of the individual components

Combinatorial models

Parallel: at least one of the components must be operational (b)

$$Q_{parallel}(t) = \Pi_{i=1}^{n} Q_i(t)$$
$$R_{parallel}(t) = 1 - Q_{parallel}(t) = 1 - \Pi_{i=1}^{n} Q_i(t) = 1 - \Pi_{i=1}^{n}[1 - R_i(t)]$$

Note the duality between Q and R in the two cases



(b)

M-of-N systems - a generalisation of parallel model
at least M modules of N are required to function

Assume N identical modules and M of those are required for the system to function properly, the expression for reliability of M-of-N substems can be written as:

$$R_{M-of-N}(t) = \Sigma_{i=0}^{N-M} \frac{N!}{(N-i)!i!} R^{N-i}(t)(1 - R(t))^i$$

i number of faulty components

$$\binom{N}{i} = \frac{N!}{(N-i)!\ i!}$$

Binomial coefficient

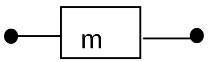
Combinatorial models

If the system contain redundancy, that is a subset of components must function properly for the system to work, and if component failures are independent, then

- the **system reliability** is the reliability of a series/parallel combinatorial model

TMR

Simplex system



λ **failure rate of module m**

$R_m = e^{-\lambda t}$

$R_{simplex} = e^{-\lambda t}$

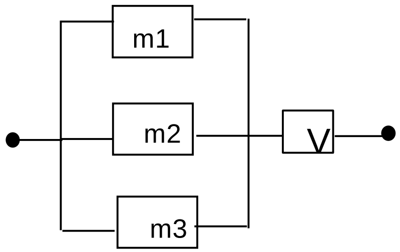
TMR system

$R_V(t) = 1$

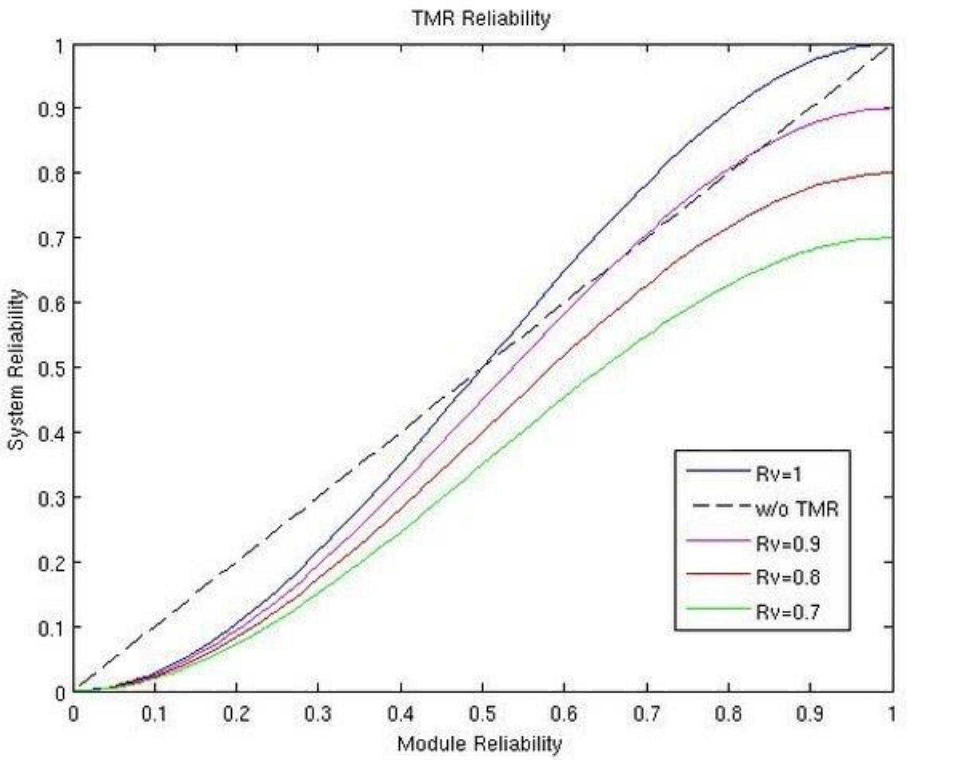
$R_{TMR} = \sum_{i=0}^{1} \binom{3}{i} (e^{-\lambda t})^{3-i} (1- e^{-\lambda t})^i$

$= (e^{-\lambda t})^3 + 3(e^{-\lambda t})^2 (1- e^{-\lambda t})$

$R_{TMR} > R_m$ if $R_m > 0.5$



2 of 3



From www.google.com

TMR: reliability function and mission time

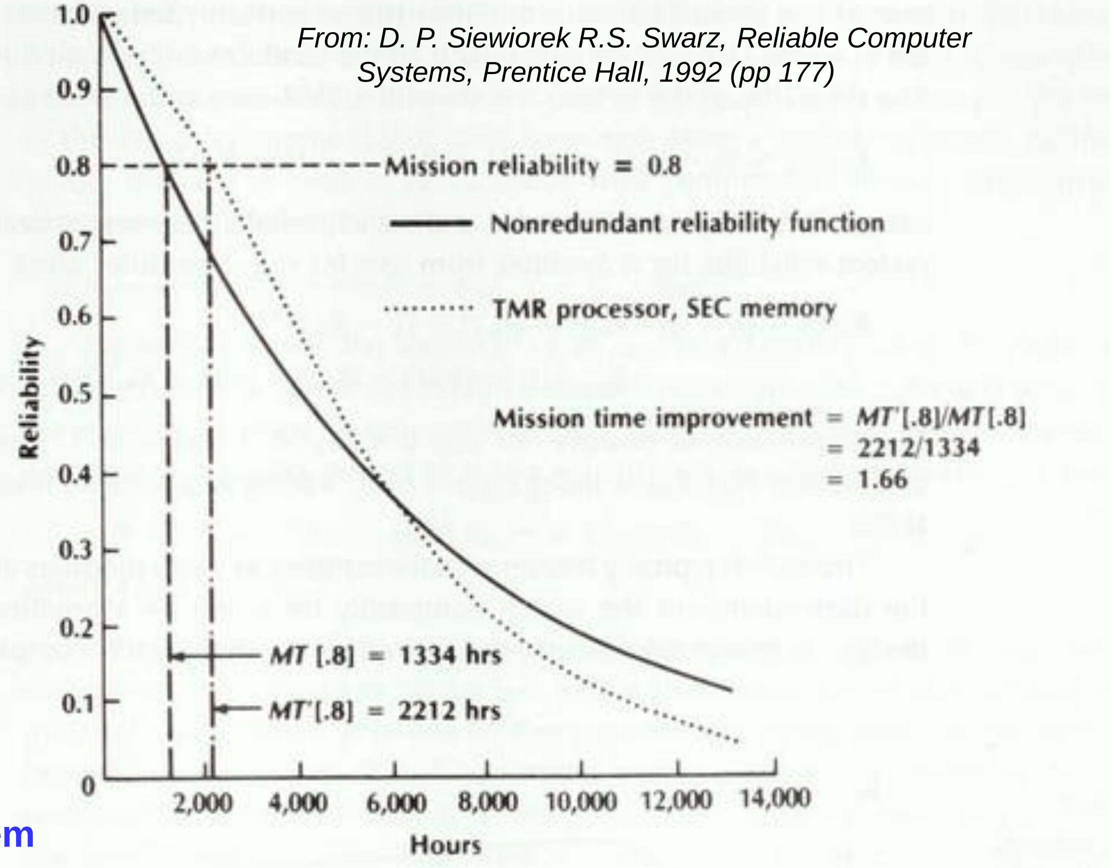
$$R_{simplex} = e^{-\lambda t}$$

$$MTTF_{simplex} = \frac{1}{\lambda}$$

TMR system

$$R_{TMR} = 3e^{-2\lambda t} - 2e^{-3\lambda t}$$

$$MTTF_{TMR} = \frac{3}{2\lambda} - \frac{2}{3\lambda} = \frac{5}{6\lambda} < \frac{1}{\lambda}$$



From: D. P. Siewiorek R.S. Swarz, Reliable Computer Systems, Prentice Hall, 1992 (pp 177)

TMR worse than a simplex system !

TMR has a higher reliability for the first 6.000 hours of system life

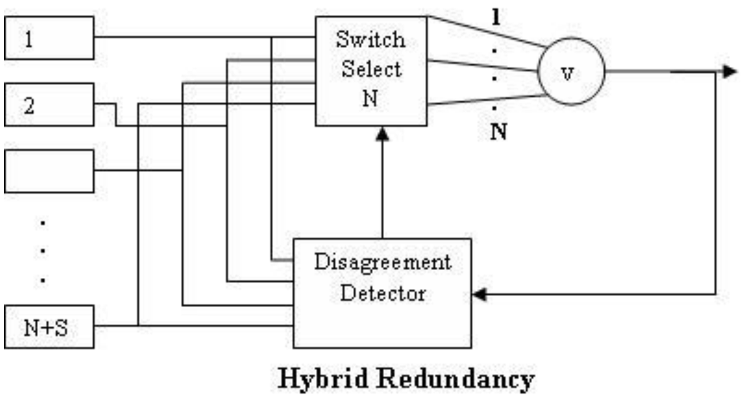
TMR operates at or above 0.8 reliability 66 percent longer than the simplex system

- **S shape curve is typical of redundant systems (there is the well known knee): above the knee the redundant system has components that tolerate failures; after the knee there is a sharper decrease of the reliability function in the redundant system (the system has exhausted redundancy, there is more hardware to fail than in the non redundant system)**

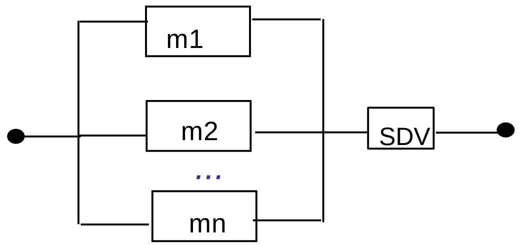
Hybrid redundancy with TMR

Symplex system
λ failure rate m
$R_m = e^{-\lambda t}$
$R_{sys} = e^{-\lambda t}$

Hybrid system
n=N+S total number of components
S number of spares



Let N = 3 **$R_{SDV}(t) = 1$**
λ failure rate of on line comp
λ failure rate of spare comp



The first system failure occurs if 1) all the modules fail; 2) all but one modules fail

$R_{Hybrid} = R_{SDV}(1- Q_{Hybrid})$

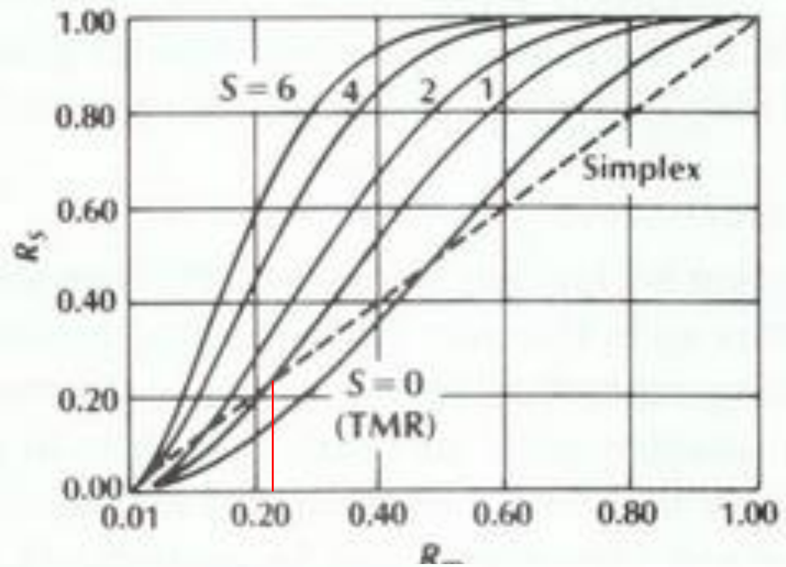
$R_{Hybrid} = (1 – ((1-R_m)^n + n(R_m)(1-R_m)^{n-1}))$

$R_{Hybrid(n+1)} – R_{Hybrid(n)} > 0$

adding modules increases the system reliability under the assumption R_{SDV} *independent of n*

Hybrid redundancy with TMR

Hybrid TMR system reliability R_S vs individual module reliability R_m



S is the number of spares

R_{SDV} =1

Figure 1. system with standby failure rate equal to on-line failure rate

the TMR with one spare is more reliable than simplex system if R_m>0.23

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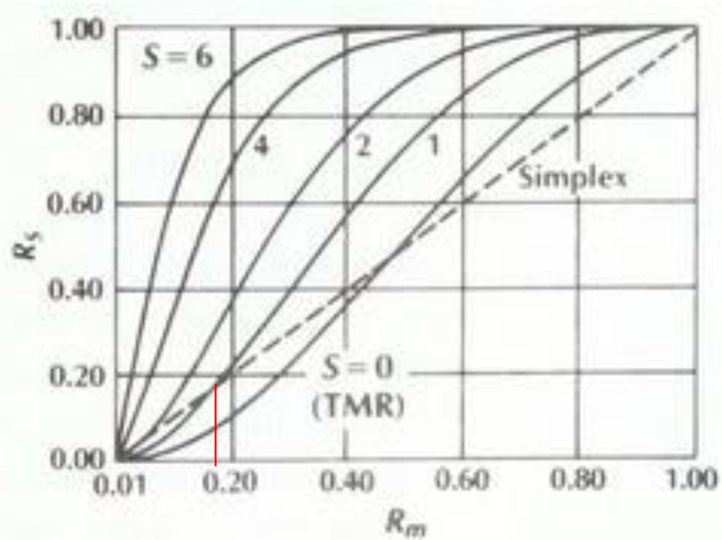

Figure 2. system with standby failure rate equal to 10% of on line failure rate

the TMR with one spare is more reliable than simplex system if R_m>0.17

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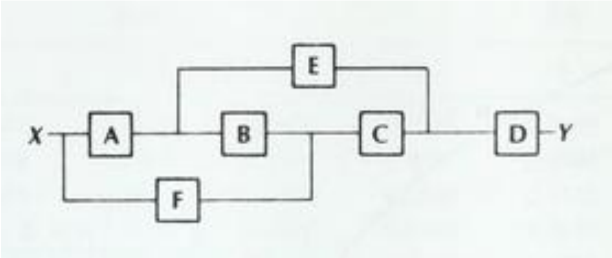
Non-series/nonparallel models

Succes diagram



System successfully operational
for each path from X to Y

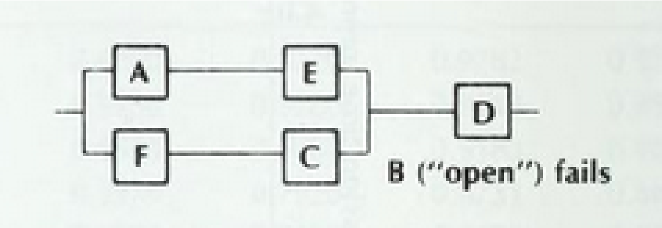
From: D. P. Siewiorek R.S. Swarz, Reliable Computer Systems, Prentice Hall, 1992

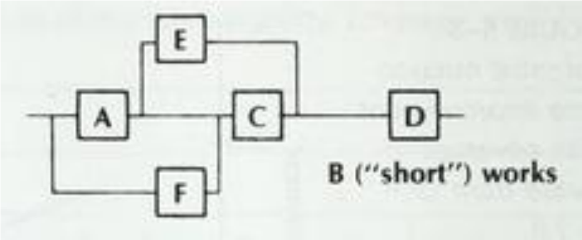
Reliability computed expanding around one module m:

R_{sys} = R_m x P(system works | m works) + (1- R_m) x P(system works | m fails)

Let m = B

R_{sys} = R_B x P(system works | B works) + (1- R_B) x P(system works | B fails)





P(system works | B fails) =
{ R_D [1 – (1- $R_A R_E$) (1- $R_F R_C$)]}

P(system works | B works)

must be further reduced

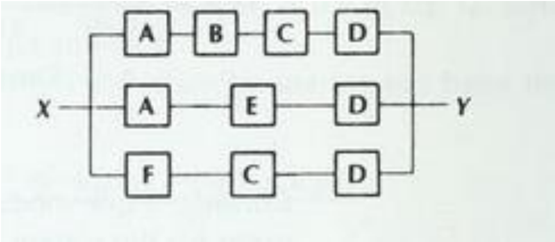
………………………………

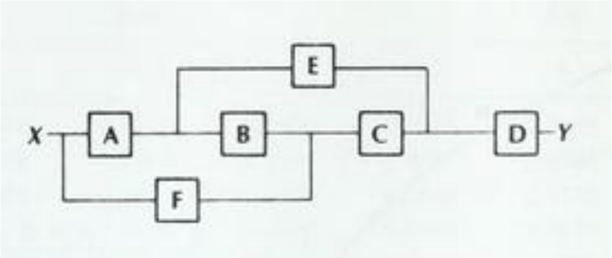
R_i=R_m

$R_{Sys} <= (R_m)^6 - 3 (R_m)^5 + (R_m)^4 + 2(R_m)^3$

Non-series/nonparallel upper-limit

Reliability Block Diagram: all path in parallel





From: D. P. Siewiorek R.S. Swarz, Reliable Computer Systems, Prentice Hall, 1992

Upper-bound:

$R_{Sys} \le 1 - \Pi_i (1 - R_{path\ i})$

Upper-bound because paths are not independent, the faiure of a single module affects more than one path (close approximation if paths are small)

Upper-bound:

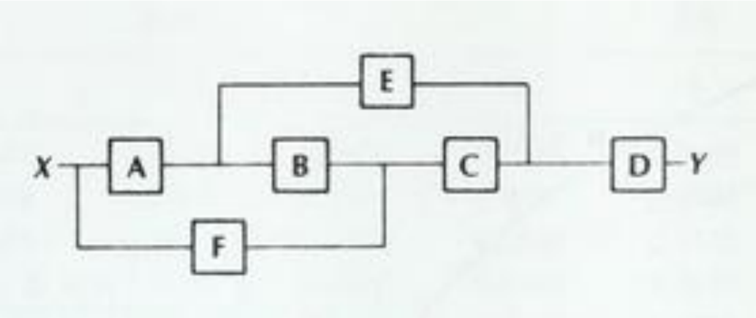
$R_{Sys} \le 1 - (1 - R_A R_B R_C R_D)(1 - R_A R_E R_D)(1 - R_F R_C R_D)$

Let R_m be the reliability of a component

$R_{Sys} \le 2 (R_m)^3 + (R_m)^4 - (R_m)^6 - 2 (R_m)^7 + (R_m)^{10}$

Non-series/nonparallel lower-limit

Minimal cut set : is a list of sets of components such that every operational path includes at least one component from each element the list



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Minimal cut sets of the system:
{D}{A,F}{E,C}{A,C}{BEF}

Lower-bound:

$R_{Sys} >= \Pi_i R_{cut\ i}$ **reliability of the series of cut sets**

where $R_{cut\ i}$ is the reliability of cut i (parallel of components)

Let Rm be the reliability of a component

R({D}) = Rm R({A,F})= R({E,C})= R({A,C}) })= 1–(1 - Rm)^2 R({B,E,F})= 1–(1 - Rm)^3

Lower-bound:

$R_{Sys} >= Rm (1- (1-Rm)^2)^3 (1- (1-Rm)^3)$

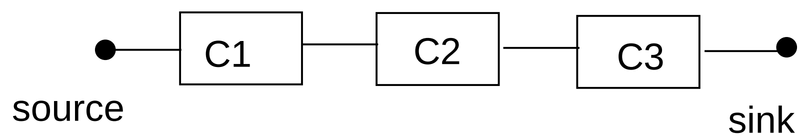
$R_{Sys} >= 24 R_m^5 -60 R_m^6 +62 R_m^7 - 33 R_m^8 + 9 R_m^9 - R_m^{10}$

SHARPE tool
Reliability Blocks diagrams

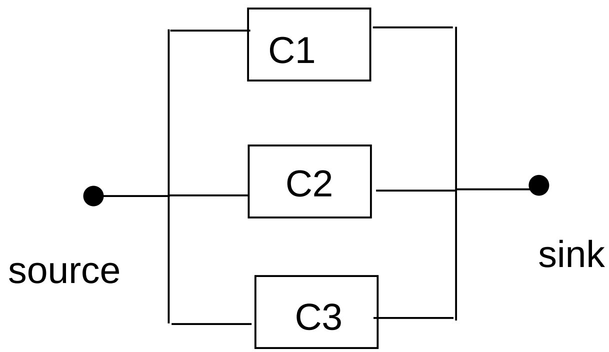
- Blocks are components
  connected among them to represent the temporal order with which
  the system uses components, or the management of redundancy schemes
  or the success critera of the system

- System failure occurs if there is no path from source to sink

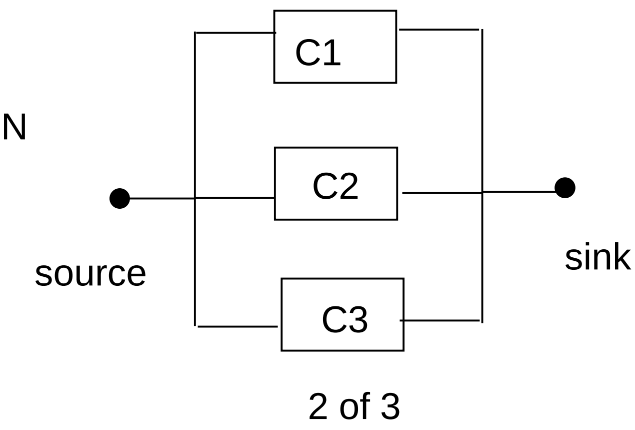
Series



Parallel



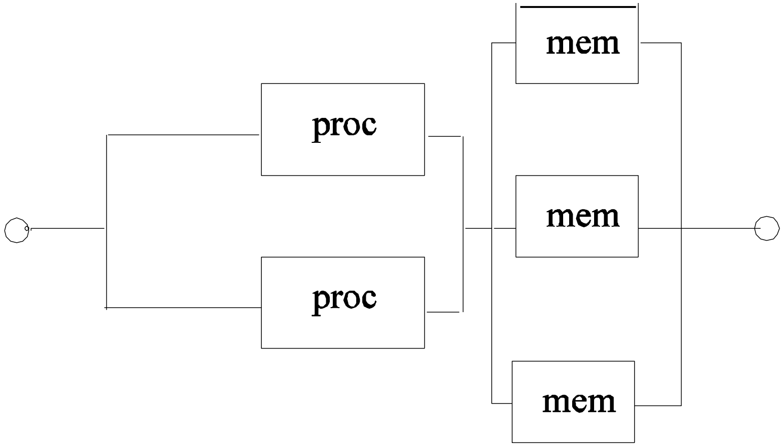
M of N



Example

Multiprocessor with 2 processors and three shared memories

-> analysis under different conditions



Series/Parallel