Exercise (B+-tree index)

Suppose we have a relation r = (A,B,C), with A primary key.
Assume

nr = 100.000	number of records in the relation
Lr = 50 byte	size of a record (fixed length record)
LA = 6 byte	size of attribute A
Lp = 4 byte	size of a pointer
Lb = 1000 byte	size of a block

1. Show the number of leaves of a B+-tree index on search-key A, in the case of
   a. heap file organization
   b. sequential file organization on A

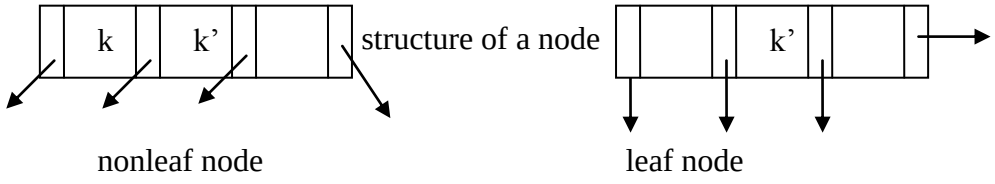
2. Outline the steps in answering the following queries and the cost in terms of number of block transfers from disk, assuming nodes with fill factor 70%:

   1) select * from r where A=xxx;

   2) select * from r where 2.000 <= A < 3.000;
   assuming A uniformly distributed on the interval [1; 500.000]

   3) select * from r where B = xxxx;
   where B is not a key

Point 1



a) Heap file organization

We have a B+-tree secondary index. The index is dense, with an entry in the leaves for every search-key value in the file.
Since A is a key of the relation, the number of search-key values in the leaves of the B+tree is equal to the number of records in the file (100.000).

We evaluate the maximum number of (key, point) in a node (blocking factor of the index, named f_I)

$$f_I = \left\lfloor \frac{Lb - Lp}{(LA+Lp)} \right\rfloor = \left\lfloor \frac{1000-4}{6+4} \right\rfloor = 99$$

m = 100	fanout of the nodes: max number of pointers in a node
m-1 = 99	number of search-key values

$$\left\lceil \frac{100.000}{99} \right\rceil = 1011$$ Minimum number of leaf nodes in the B+-tree

$\lceil m/2 \rceil = 50$ minimum number of pointers in a node
$\lceil m/2 \rceil - 1 = 49$ number of search-key values

$$\left\lceil \frac{100.000}{49} \right\rceil = 2041$$ Maximum number of leaf nodes in the B+-tree

b) Sequential file organization

We have a B+-tree primary index. The index is sparse, with an entry in the leaves for every block of the file.
The number of search-key values in the leaves of the B+tree is equal to the number of blocks in the file.

We evaluate the number of blocks in the file.

$$f_r = \left\lfloor \frac{Lb}{Lr} \right\rfloor \qquad f_r = \left\lfloor \frac{1000}{50} \right\rfloor = 20$$ *blocking factor of the relation r max number of records in a block of the file*

$$n_b = \left\lceil \frac{nr}{f_r} \right\rceil \qquad n_b = \left\lceil \frac{100.000}{20} \right\rceil = 5.000$$ *number of blocks of the file*

$$\left\lceil \frac{5.000}{99} \right\rceil = 51$$ Minimum number of leaf nodes in the B+-tree

$$\left\lceil \frac{5.000}{49} \right\rceil = 103$$ Maximum number of leaf nodes in the B+-tree

Point 2

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Let h be the height of a B+-tree, it can be shown that

Full nodes:

1	level 1
|	
m	level 2
|	
m*m	level 3
.....	
m*m … *m => m^{h-1}	level h

- number of blocks (nodes) is:

$$1 + m + m^2 + ... + m^{h-1} = (m^h - 1) / (m-1)$$

- number of search-key values is:

$m^h - 1$ (number of nodes * number of values in the node)

Given the number of leaves, the height of the B+tree can be computed as follows:

$$n_{leaves} = m^{h-1}$$

$$h-1 = log_m (n_{leaves})$$

$$h = 1 + log_m (n_{leaves})$$

Half full nodes:

- number of blocks (nodes) is:

$$1 + 2 + 2\lceil m/2 \rceil + ... + 2\lceil m/2 \rceil^{h-2} =$$

$$= 1 + 2\frac{\lceil m/2 \rceil^{h-1} - 1}{\lceil m/2 \rceil - 1}$$

- number of search-key values is:

$2\lceil m/2 \rceil^{h-1} - 1$ (number of nodes * min number of values in the node)

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Nodes with fill factor 70%.

m' = 70% m
m' = 70

Heap file organization

$$n_{leaves} = \left\lceil \frac{100.000}{69} \right\rceil = 1450$$

$$h = 1 + \lceil \log_{70}(1450) \rceil = 3$$

Point 2.1

select * from R where A=xxx

Cost of the query:

C = height of the B+-tree + 1 block for the file
C = 3 + 1 = 4

Point 2.2

select * from R where 2.000 <=A<3.000

- Cost of the query using the index
fs = 1.000/500.000 = 1/500 selectivity factor of the query

Let h be the height of the B+-tree

$$C = (h-1) + \lceil fs * n_{leaves} \rceil + \lceil fs * n_r \rceil$$

Number of leaf node transfers:

$$\lceil fs * n_{leaves} \rceil = \lceil 1/500 * 1450 \rceil = 3$$

Number of file block transfers:

$\lceil fs * n_r \rceil = \lceil 1/500 * 100.000 \rceil = 200$ (heap file organization, a block transfer for each record)

C = 2 + 3 + 200 = 205

- Cost of sequential scan of the file
Number of blocks of the file: 5000

The worst case cost is 5000 and the best case cost is 1. On average, we have:$(n_b + 1)/2$

$$C' = (n_b + 1)/2 = 2.500$$

Cost of the query: min(C, C') = min(205, 2.500) = 205

Point 2.3

select * from r where B = xxxx;
No index on B. Moreover B is not a key. We estimate $C = n_b$

C = 5.000

Sequential file organization

$$n_{leaves} = \left\lceil \frac{5.000}{69} \right\rceil = 73$$

$h = 1 + \lceil \log_{70}(73) \rceil = 3$

Point 2.1

select * from R where A=xxx

- Cost of the query using the index
  C = height of the B+-tree + 1 block for the file
  C = 3 + 1 = 4

- Cost of the query using binary search
  $C' = \lceil \log_2 n_b \rceil = \lceil \log_2 5.000 \rceil = 13$

Cost of the query: $\min(C, C') = \min(4, 13) = 4$

Point 2.2

select * from R where 2.000 <=A<3.000

- Cost using the index:
  fs = 1/500

$$C = (h-1) + \lceil fs * n_{leaves} \rceil + \lceil fs * n_b \rceil$$

Number of leaves transfers:

$$\lceil fs * n_{leaves} \rceil = \lceil 1/500 * 73 \rceil = 1$$

Number of file block transfers:

$$\lceil fs * n_b \rceil = \lceil 1/500 * 5000 \rceil = 10$$

(sequential file organization, records are stored in search-key order in the blocks)

$$C = 2 + 1 + 10 = 13$$

Point 2.3

select * from r where B = xxxx;

No index on B. Moreover B is not a key. We estimate $C = n_b$
C = 5.000