PECSN, 28/7/2020

Exercise 1

Consider a MAC-layer protocol where a transmitter sends frames whose length is L bits over a line whose transmission speed is C bits per second. The receiver acknowledges the frames by sending back an ACK, if the frame has been received correctly, or a NACK, if the frame is corrupted. A frame is corrupted if at least one bit is corrupted, and the link's *bit error rate* (i.e., the probability that a *single bit* is corrupted) is constant and equal to p. The transmitter retransmits the same frame until it receives an ACK.
Assume that:

- all transmitted bits are independent of each other;
- an ACK/NACK never gets corrupted;
- the propagation time along both directions of the link is constant and equal to t;
- the time it takes to transmit an ACK/NACK is equal to t'.

1) Compute the probability p_{err} that a frame is corrupted;
2) Compute the values that the RV T, "correct transmission time of a frame" can assume;
3) Compute the PMF of the above RV;
4) Compute the mean time of correct transmission of a frame;
5) Assuming that a frame has been corrupted $k-1$ consecutive times, find the conditional probability that it will be corrupted at the subsequent transmission attempt. Justify the result.

Exercise 2

Consider a switch, whose output link has a capacity of C bits per second, connected to 50 input terminals. Each terminal sends packets whose length is exponentially distributed, with a mean of 1000 bits. The interarrival times of packets *from a terminal* are exponentially distributed. For half of the terminals, the mean interarrival time is 10 seconds. For the other half, it is 5 seconds.

1) Compute the value of C so that the mean number of packets in the switch is equal to 5
2) Compute the value of C so that the 99th percentile of the number of packets in the switch is equal to 5, call it C'.
3) *This point differs depending on the year when you attended the course*
    a. 2019 (last edition): assume that the length of packets is *constant*. Solve again point 1). Comment on your findings.
    b. Previous editions: Find the condition on C by which the standard deviation of the number of jobs in the system is larger than the mean.
4) Compute the 99th percentile of the response time of a packet as a function of C.

Exercise 1 - Solution

1) $p_{err} = 1 - \binom{L}{0} p^0 \cdot (1-p)^{L-0} = 1 - (1-p)^L$

2) T can be equal to $t_k = k \cdot \left(2t + t' + \frac{L}{C}\right)$, $k \geq 1$ being the number of required transmissions before an ACK is received back.

3) Call $P_j = P\{T = t_j\}$. It is:

$$P_j = \left(\prod_{i=1}^{j-1} p_{err,i}\right) \cdot \left(1 - p_{err,j}\right) = {p_{err}}^{j-1} \cdot (1 - p_{err})$$

4) It can be easily seen that T is a (scaled) geometric variable, hence its mean value is

$$E[T] = \frac{(2t + t' + L/C)}{(1 - p_{err})}$$

5) The conditional probability is

$$P\{\text{corrupted k-th time} \mid \text{corrupted k-1 times}\} = \frac{P\{\text{corrupted k consecutive times}\}}{P\{\text{corrupted k-1 times}\}}$$

$$= \frac{{p_{err}}^k}{{p_{err}}^{k-1}} = p_{err} = P\{\text{corrupted}\}$$

This is obvious, since retransmissions are independent of each other.

Exercise 2 - Solution

The system can be modeled as an M/M/1 queueing system where:

- $\mu = \frac{C}{E[L]} = \frac{C}{1000}$
- $\lambda = 25 \cdot \frac{1}{10} + 25 \cdot \frac{1}{5} = 7.5$

1) The mean number of jobs in an M/M/1 is given by Kleinrock's formula, i.e. $E[N] = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda}$. Imposing $E[N] = 5$ and solving the latter for C yields $C = 9000$.

2) We also know that, in an M/M/1, SS probabilities are $p_k = \rho^k \cdot (1-\rho)$, hence we need to impose that:

$$\sum_{k=0}^{5} p_k = (1-\rho) \cdot \frac{1-\rho^6}{1-\rho} = 0.99$$

This yields $\rho^6 = 0.01$, i.e. $\rho = \frac{1}{\sqrt[3]{10}} \cong 0.464$. Solving this equality for C yields

$$C' \cong 16163$$

3) The variance of the number of jobs in the system is the variance of a geometric RV, i.e. $Var(N) = \frac{\rho}{(1-\rho)^2}$. Hence the inequality that checks if the std is larger than the mean is the following:

$$\sqrt{\frac{\rho}{(1-\rho)^2}} > \frac{\rho}{1-\rho}$$

Which quickly boils down to $\rho < 1$. Therefore, the required condition is the same as the stability condition, i.e. $C > 7500$.

On the other hand, if the length of the packets is constant, the system is an M/D/1, for which PK's formula yields the mean number of jobs in the system:

$$E[N] = \rho + \frac{\rho^2}{2(1-\rho)}$$

Imposing $E[N] = 5$ and solving the latter for ρ yields the following: $\rho = 6 \pm \sqrt{26}$. Since stability requires $\rho < 1$, the only good solution is $\rho = 6 - \sqrt{26} \cong 0.901$. Solving the latter for C yields $C \cong 8324$.
The value required is *smaller* than at point 1), since in this case there is no randomness in the transmission. Randomness increases queueing, as PK's formula shows.

4) In an M/M/1 system, the response time is an exponential RV, whose CDF is the following:

$$R(t) = 1 - e^{-\mu(1-\rho)t}$$

Hence the required equation is $1 - e^{-(\mu-\lambda)\pi_{99}} = 0.99$, to be solved for π_{99}. After a few straightforward computations, one gets:

$$\pi_{99} = \frac{2log10 \cdot E[L]}{C - 7.5 \cdot E[L]}$$