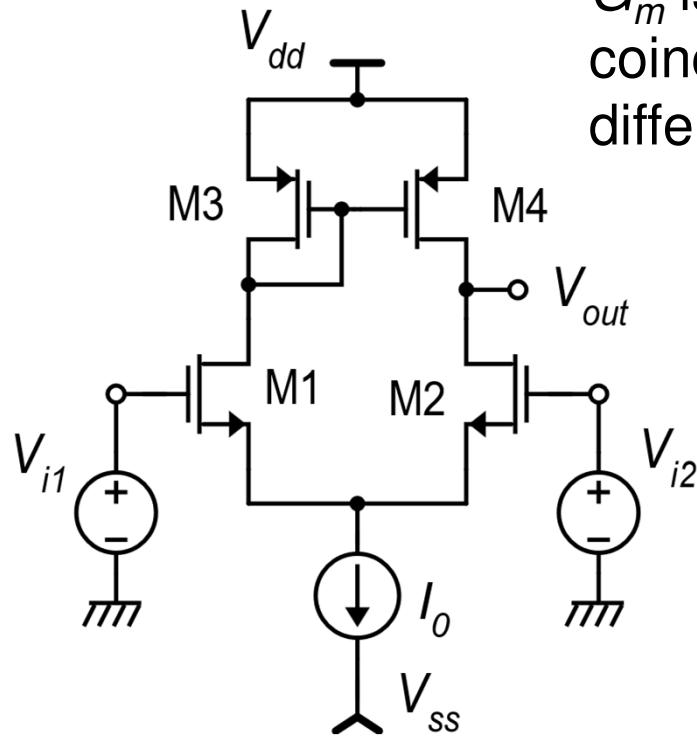


## Increasing the gain of the simple differential amplifier with mirror load

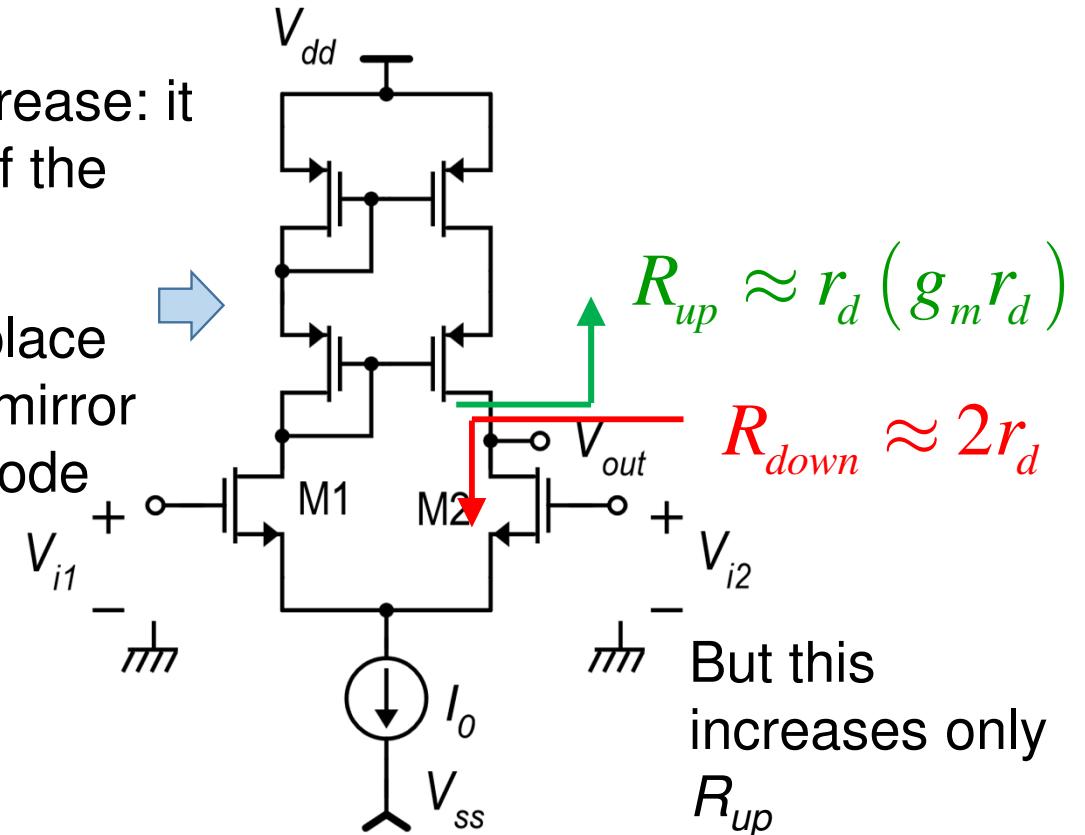
$$R_{out} = r_{d2} // r_{d4}$$

We need to increase the output resistance  
 $A_d = G_m R_{out}$

$G_m$  is difficult to increase: it coincides with  $g_m$  of the differential pair

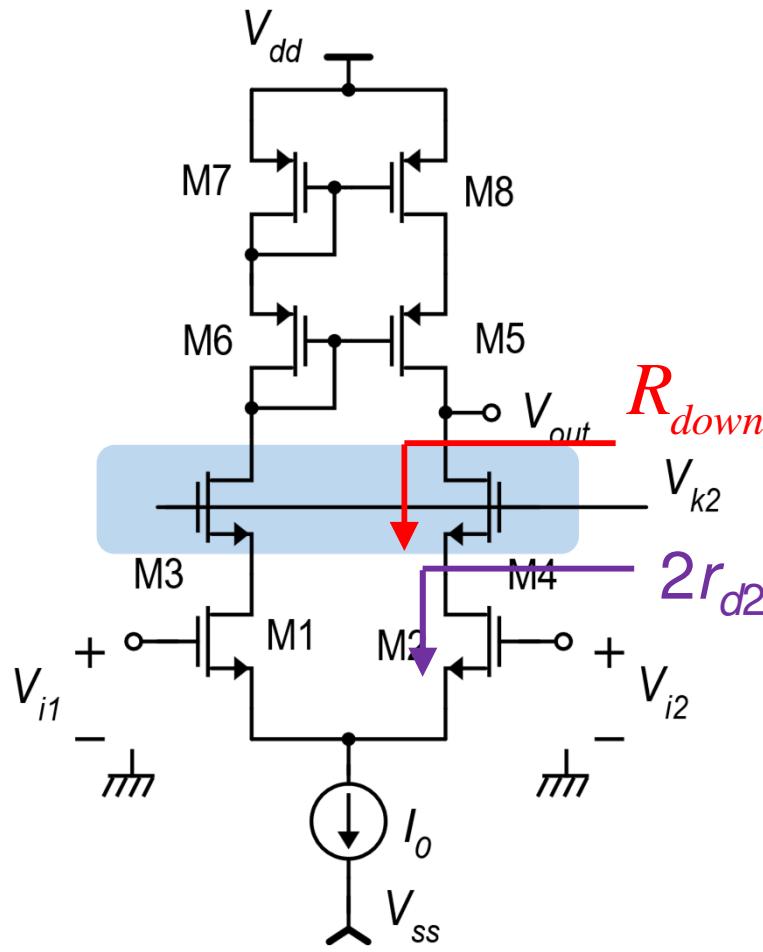


We can replace the simple mirror with a cascode mirror



But this increases only  
 $R_{up}$

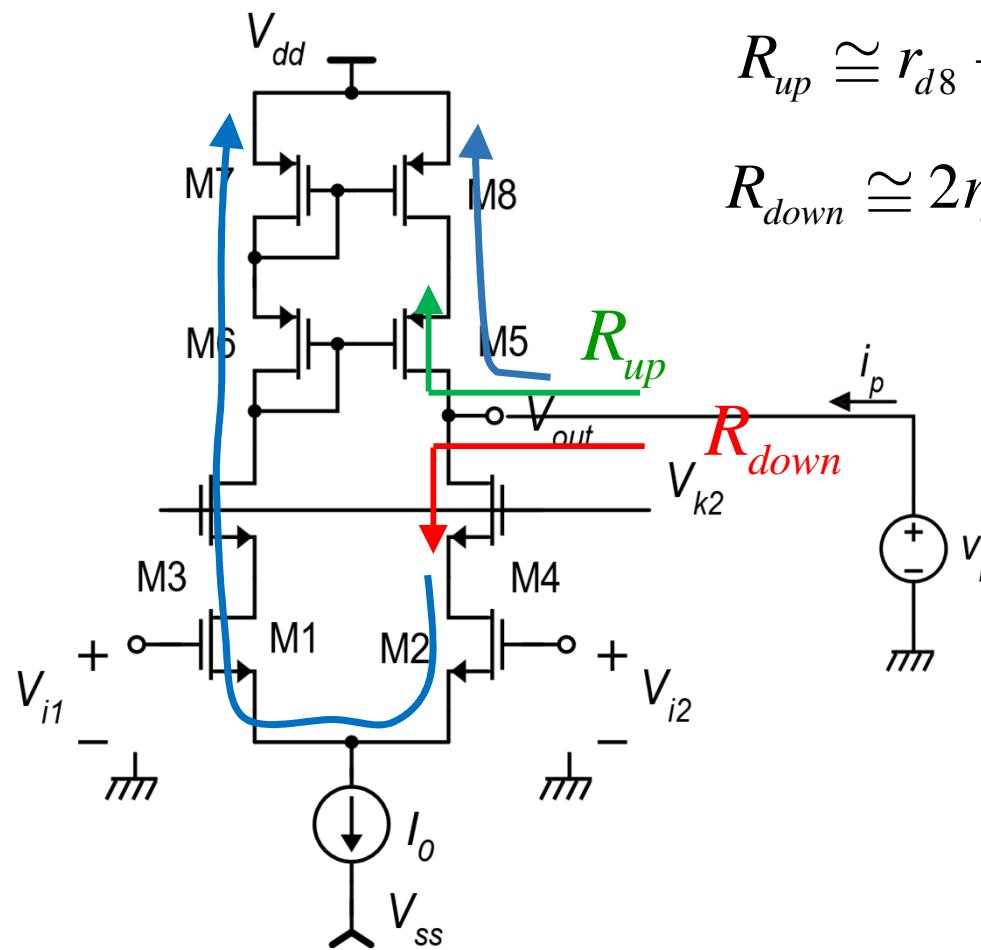
## Cascode differential amplifier



$$R_{down} \cong 2r_{d2} + r_{d4} (1 + g_{m4} 2r_{d2})$$

We add a differential common gate stage (M3, M4) to the differential pair: forming a differential cascode stage

# Cascode differential amplifier: $R_{out}$



$$R_{up} \cong r_{d8} + r_{d5}(1 + r_{d8}g_{m5}) \cong r_{d5}(r_{d8}g_{m5})$$

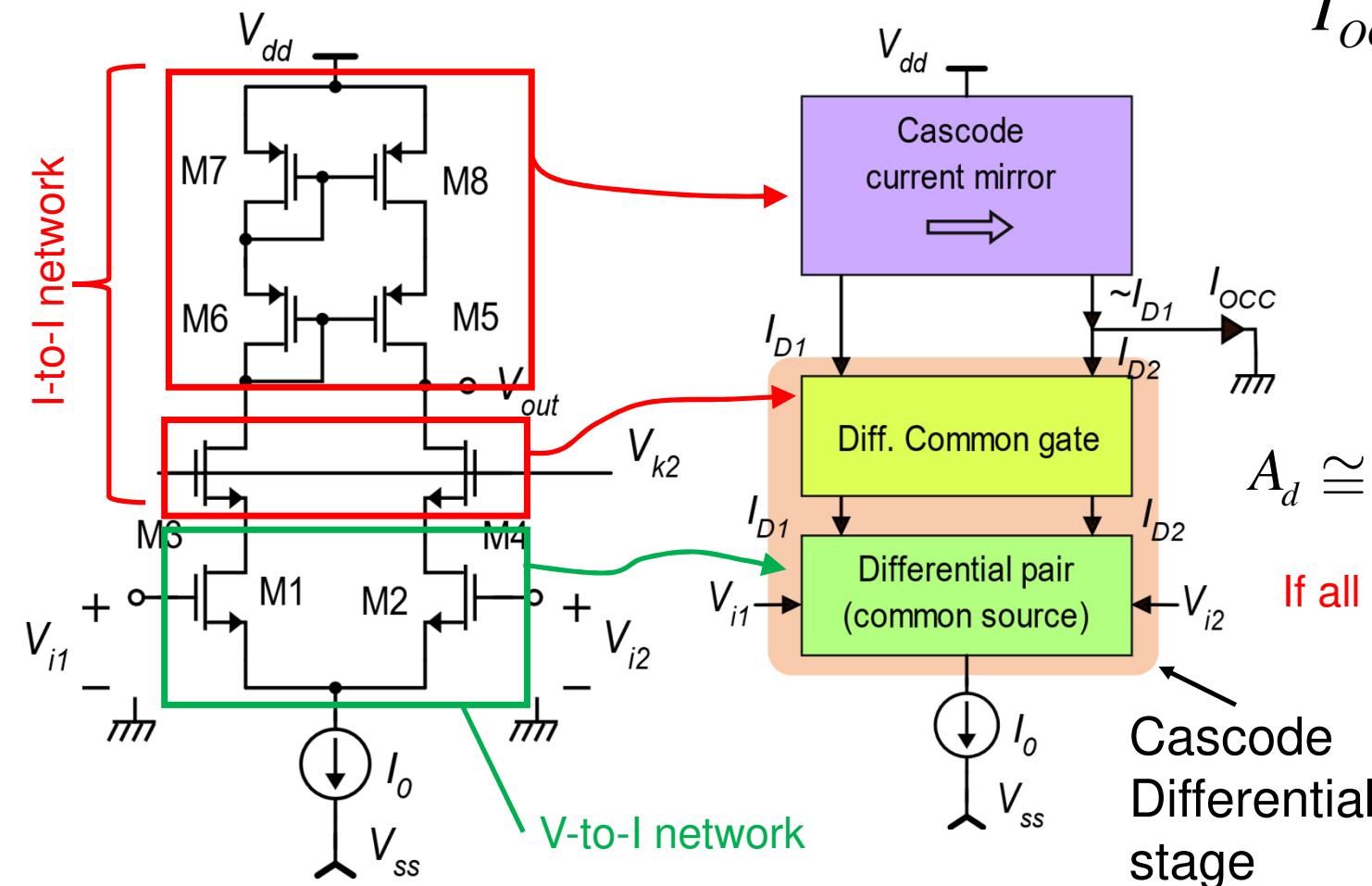
$$R_{down} \cong 2r_{d2} + r_{d4}(1 + g_{m4} \cdot 2r_{d2}) \cong 2r_{d4}(r_{d2}g_{m4})$$

$$i_p = \frac{v_p}{R_{up}} + 2 \frac{v_p}{R_{down}}$$

$$R_{out} = R_{up} // \left( \frac{R_{down}}{2} \right)$$

$$R_{out} \cong r_{d5}(r_{d8}g_{m5}) // r_{d4}(r_{d2}g_{m4})$$

# Cascode differential amplifier



$$I_{OCC} \cong I_{D1} - I_{D2} \cong g_{m1} v_d$$

$$I_{OCC} = G_m v_d$$

$$G_m = g_{m1}$$

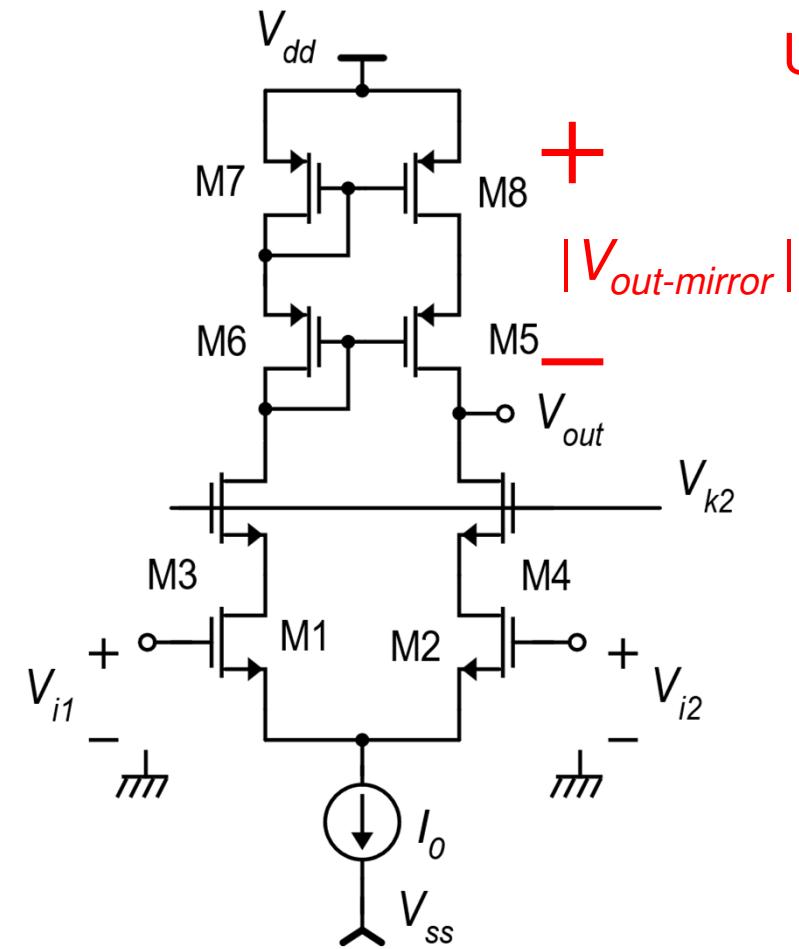
$$A_d = G_m R_{out}$$

$$A_d \cong g_{m1} \left[ (r_{d2} g_{m4} r_{d4}) // (r_{d8} g_{m5} r_{d5}) \right]$$

If all devices have the same  $r_d$  and  $g_m$ :

$$A_d \cong \frac{(g_m r_d)^2}{2}$$

# CMOS cascode amplifier: output range



Upper limit

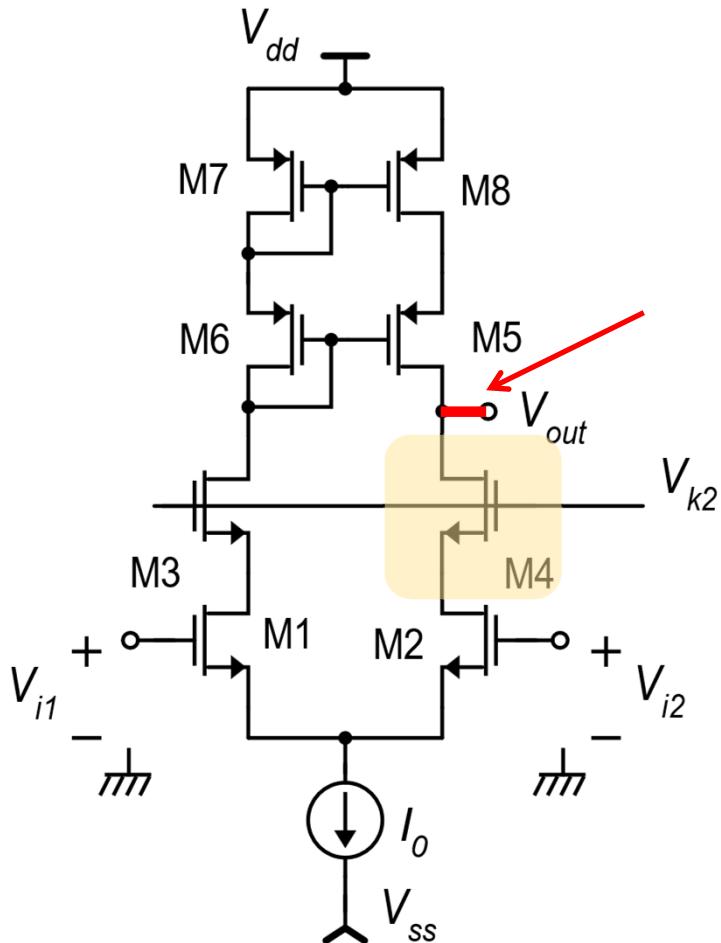
$$|V_{out-mirror}| = V_{dd} - V_{out} \geq V_{MIN-cascode}$$

$$V_{dd} - V_{MIN-cascode} \geq V_{out}$$

$$\max(V_{out}) = V_{dd} - V_{MIN-cascode}$$

As  $V_{out}$  gets higher than  $\max(V_{out})$ ,  $R_{up}$  decreases making the gain decrease

# CMOS cascode amplifier: output range



Lower limit

$$V_{DS4} = V_{out} - V_{S4} \geq V_{DSAT4}$$

$$V_{S4} = V_{k2} - V_{GS4}$$

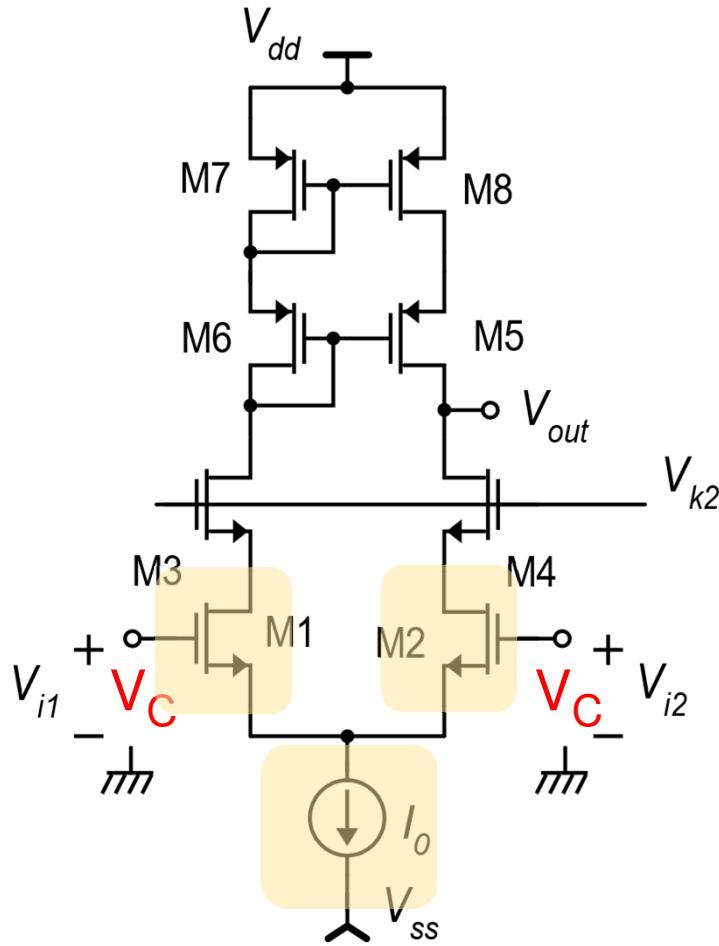
$$V_{out} - V_{k2} + V_{GS4} \geq V_{DSAT4}$$

$$\min(V_{out}) = V_{k2} - V_{GS4} + V_{DSAT4}$$

$$\text{In strong inversion: } \min(V_{out}) = V_{k2} - V_{t4}$$

For an approximation of  $\min(V_{out})$  in weak inversion just add 100 mV

## Input common mode range



$$\min(V_{iC}) = V_{SS} + V_{MIN-tail} + V_{GS1}$$

$$V_{DS1} = V_{D1} - V_{S1} \geq V_{DSAT1}$$

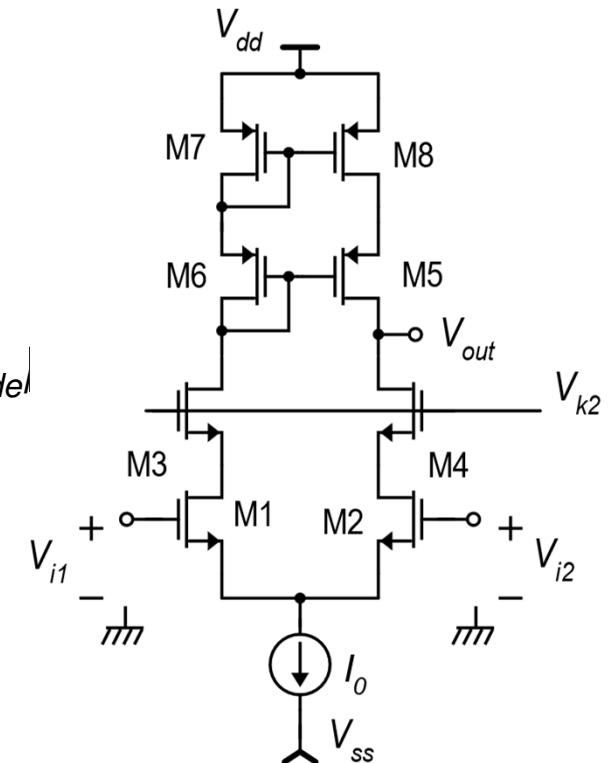
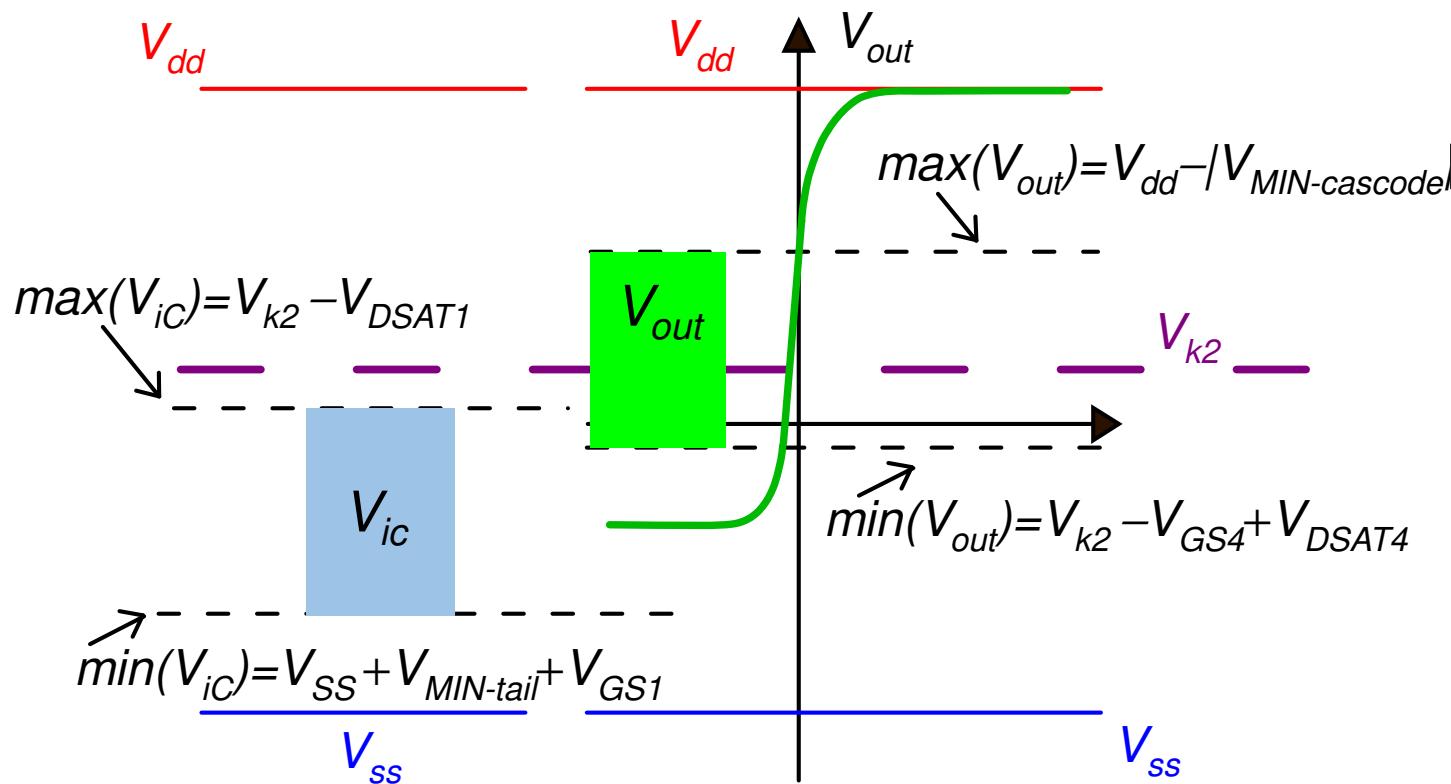
$$\begin{cases} V_{D1} = V_{k2} - V_{GS3} \\ V_{S1} = V_{iC} - V_{GS1} \end{cases}$$

$$V_{k2} - V_{GS3} - V_{iC} + V_{GS1} \geq V_{DSAT1}$$

$$\max(V_{iC}) = V_{k2} - V_{GS3} + V_{GS1} - V_{DSAT1}$$

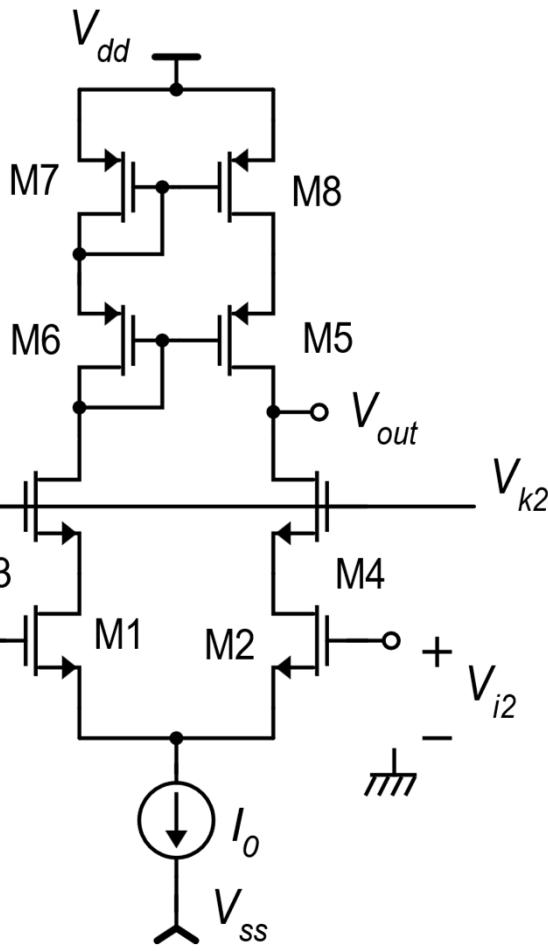
$$\max(V_{iC}) \cong V_{k2} - V_{DSAT1}$$

## Cascode amplifier: all ranges



$V_{k2}$  determines a trade-off between the input common mode range and the output swing

## Adaptive $V_{k2}$

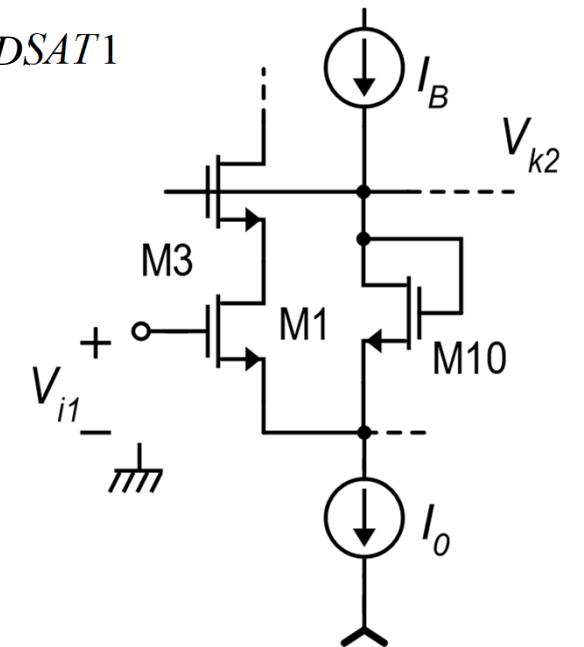


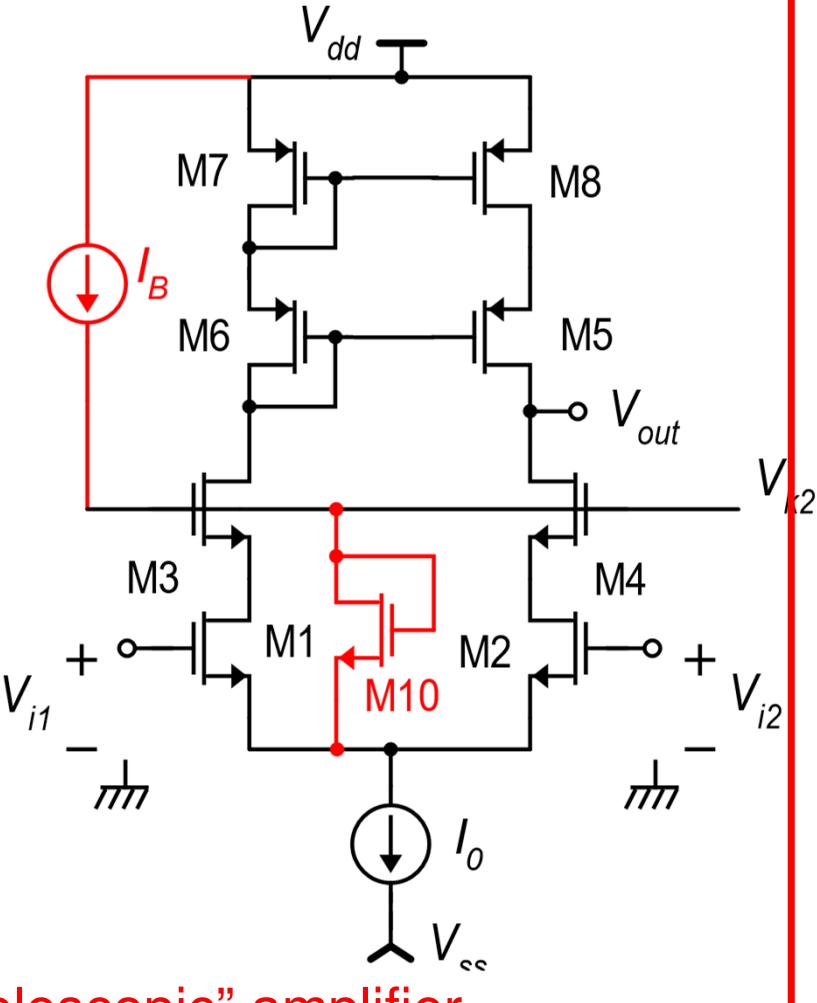
$$V_{k2} - V_{GS3} - V_{iC} + V_{GS1} \geq V_{DSAT1}$$

$$V_{k2} \geq \underbrace{V_{iC} - V_{GS1}}_{V_{S1}} + V_{GS3} + V_{DSAT1}$$

$$V_{k2} = V_{S1} + \underbrace{V_{GS3} + V_{DSAT1}}_{V_{GS10}}$$

$$V_{GS10} = V_{GS3} + V_{DSAT1}$$





Adaptive  $V_{K2}$

Same as in the case of a general cascode structure

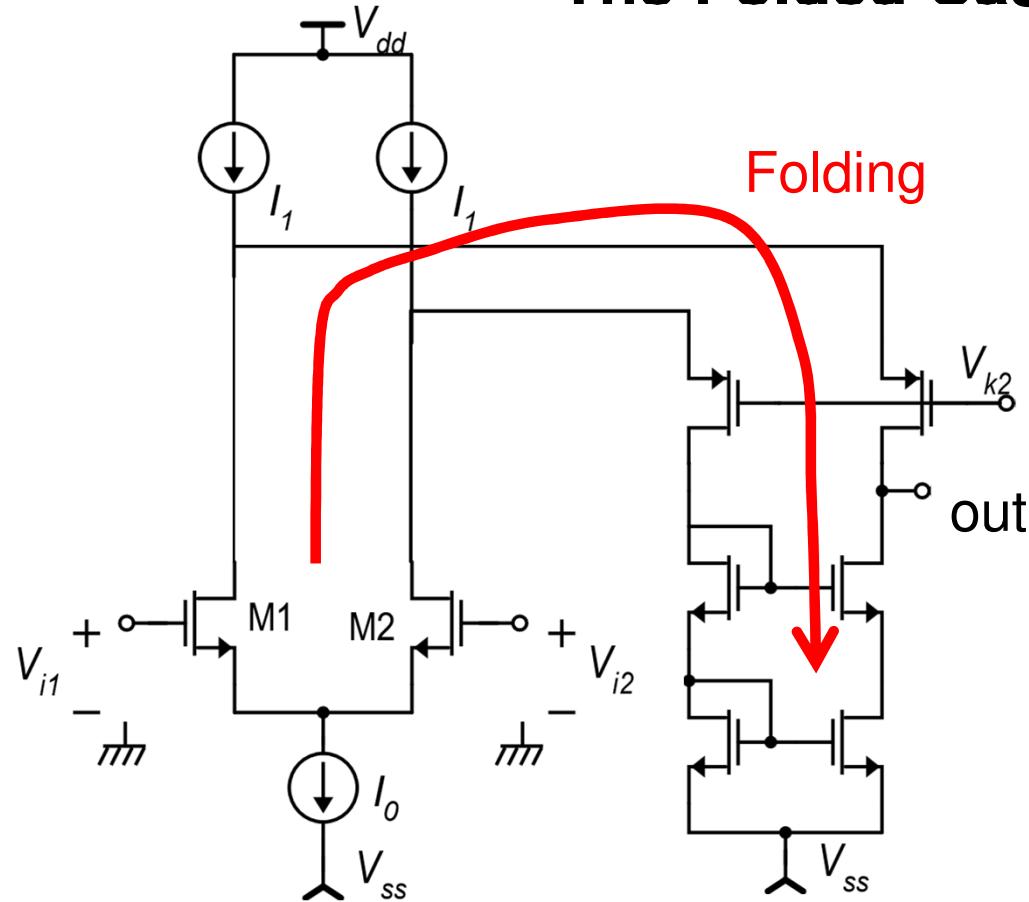
$$V_{GS10} = V_{GS3} + V_{DSAT1}$$

$$(V_{GS} - V_t)_{10} = m_3 V_{DSAT1} + (V_{GS} - V_t)_3$$

$$I_{0-eff} = I_0 - I_B$$

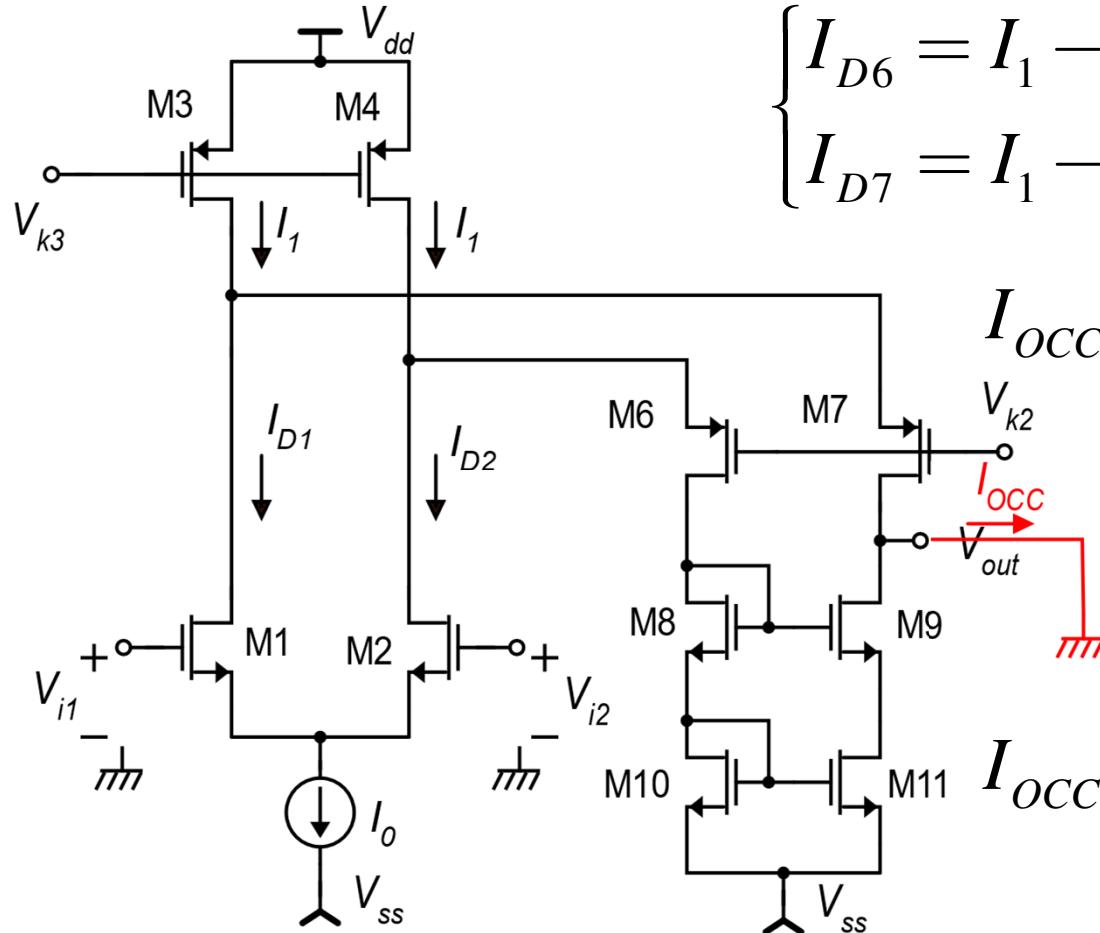
The adaptive  $V_{K2}$  does not solve the problem that, as  $V_{ic}$  increases, the output swing gets smaller. The advantage is that the amplifier is more flexible and no trade-off on  $V_{K2}$  should be made in the design phase.

# Removing the interaction between input and output range: The Folded Cascode



According to the folding mechanism, the signal (variations) initially travels from one rail to the other and then invert direction and goes back to the initial rail

## Folded cascode amplifier: device currents



$$\begin{cases} I_{D6} = I_1 - I_{D2} \\ I_{D7} = I_1 - I_{D1} \end{cases}$$

Condition:

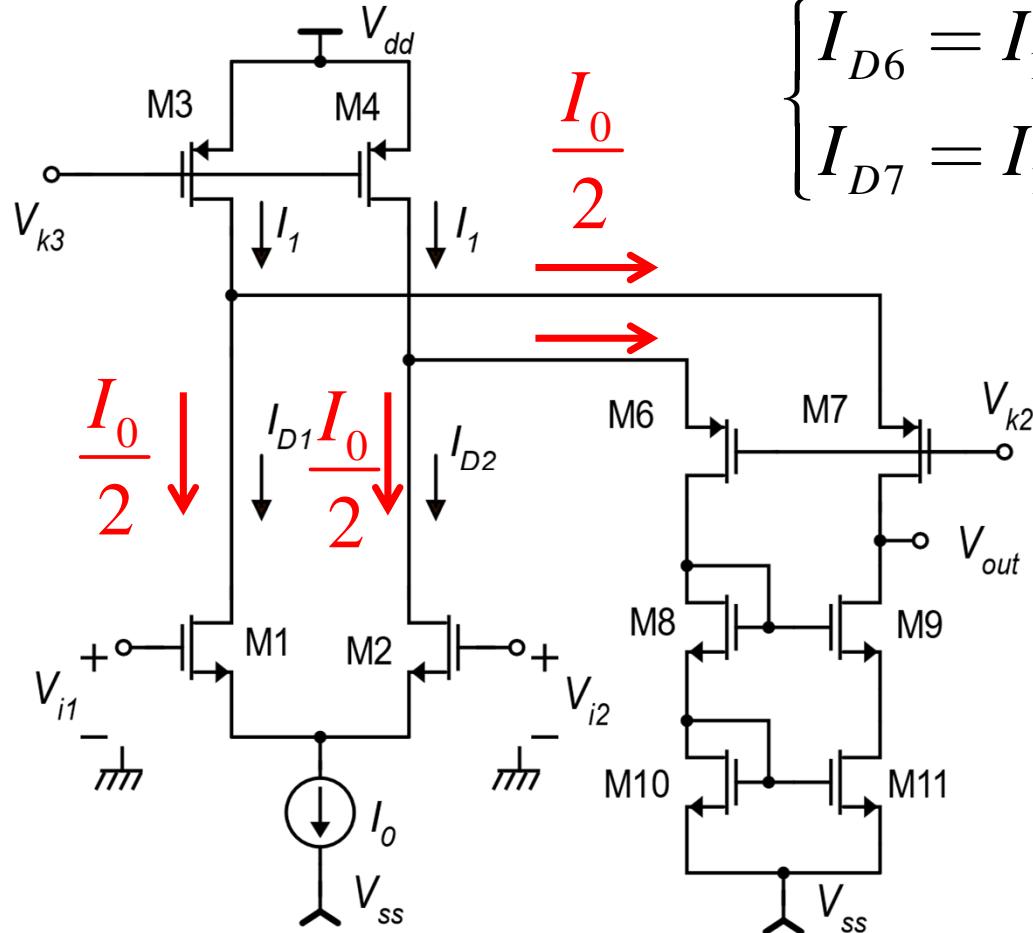
$$\begin{cases} I_{D6} \geq 0 \\ I_{D7} \geq 0 \end{cases}$$

$$I_{OCC} = I_{D7} - I_{D9} \cong I_{D7} - I_{D6}$$

$$I_{OCC} = (I_1 - I_{D1}) - (I_1 - I_{D2})$$

$$I_{OCC} = -(I_{D1} - I_{D2}) \cong -g_{m1}v_d$$

## Folded cascode: setting the correct $I_1$ value



$$\begin{cases} I_{D6} = I_1 - I_{D2} \geq 0 \\ I_{D7} = I_1 - I_{D1} \geq 0 \end{cases}$$

Quiescent point:

$$I_{D1} = I_{D2} = \frac{I_0}{2}$$

$$I_1 > \frac{I_0}{2}$$

With a large input signal ( $|V_{id}| > V_{DMAX}$ ):

$$V_{id} > V_{DMAX} : I_{D1} = I_0$$

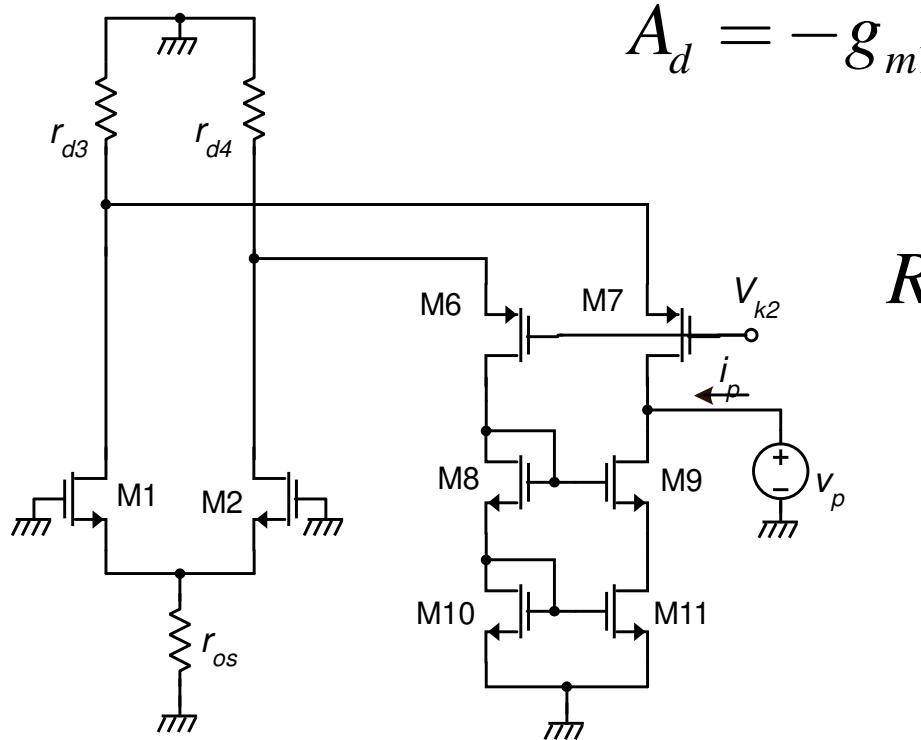
$$V_{id} < -V_{DMAX} : I_{D2} = I_0$$

$$I_1 \geq I_0 \quad \text{usually : } I_1 = I_0$$

## Folded cascode: differential mode gain

$$A_d = G_m R_{out}$$

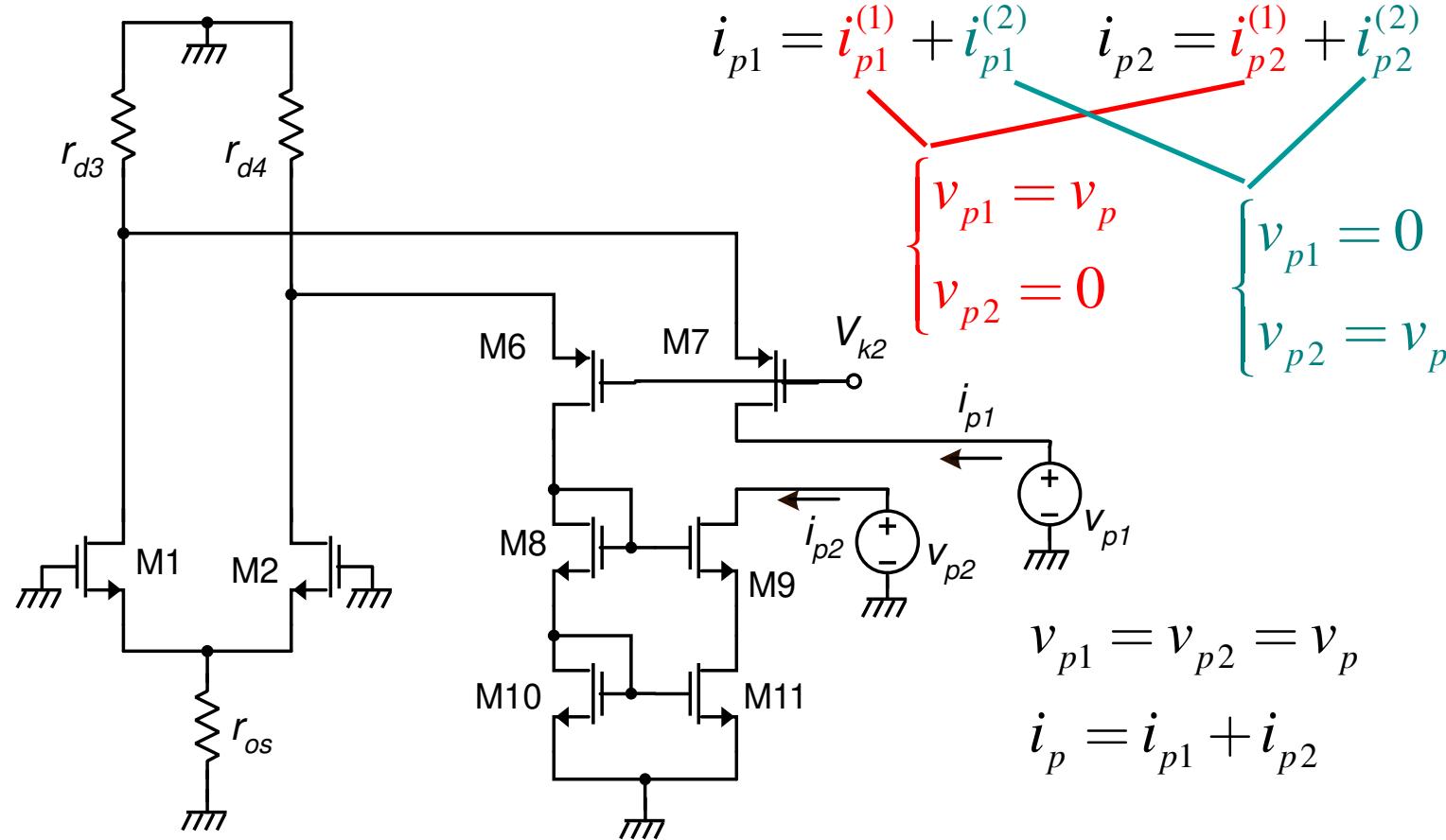
$$I_{OCC} \cong -g_{m1} v_d \Rightarrow G_m = -g_{m1}$$



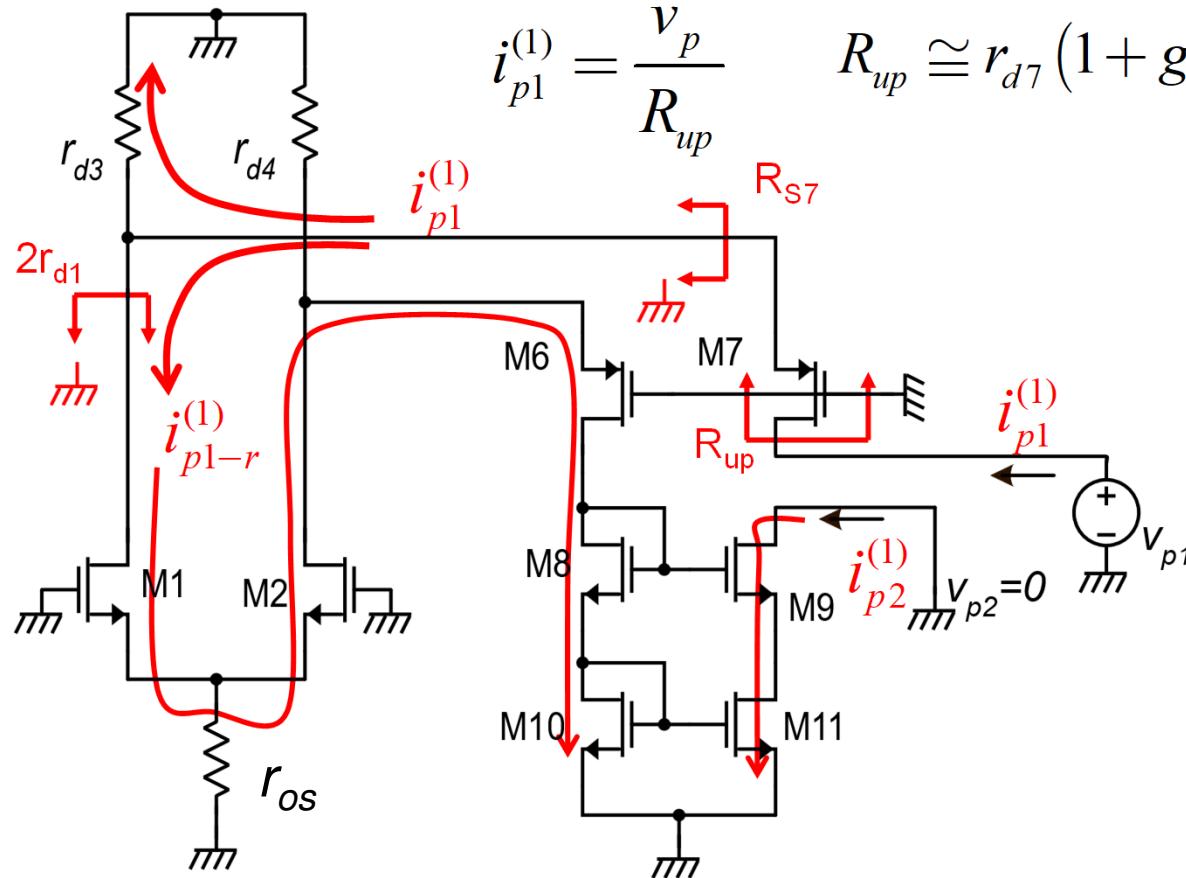
$$A_d = -g_{m1} R_{out}$$

$$R_{out} = \frac{v_p}{i_p}$$

## Folded cascode: output resistance (2)



## Folded cascode: output resistance (2)



$$i_{p1}^{(1)} = \frac{V_p}{R_{up}}$$

$$R_{up} \cong r_{d7} (1 + g_{m7} R_{s7})$$

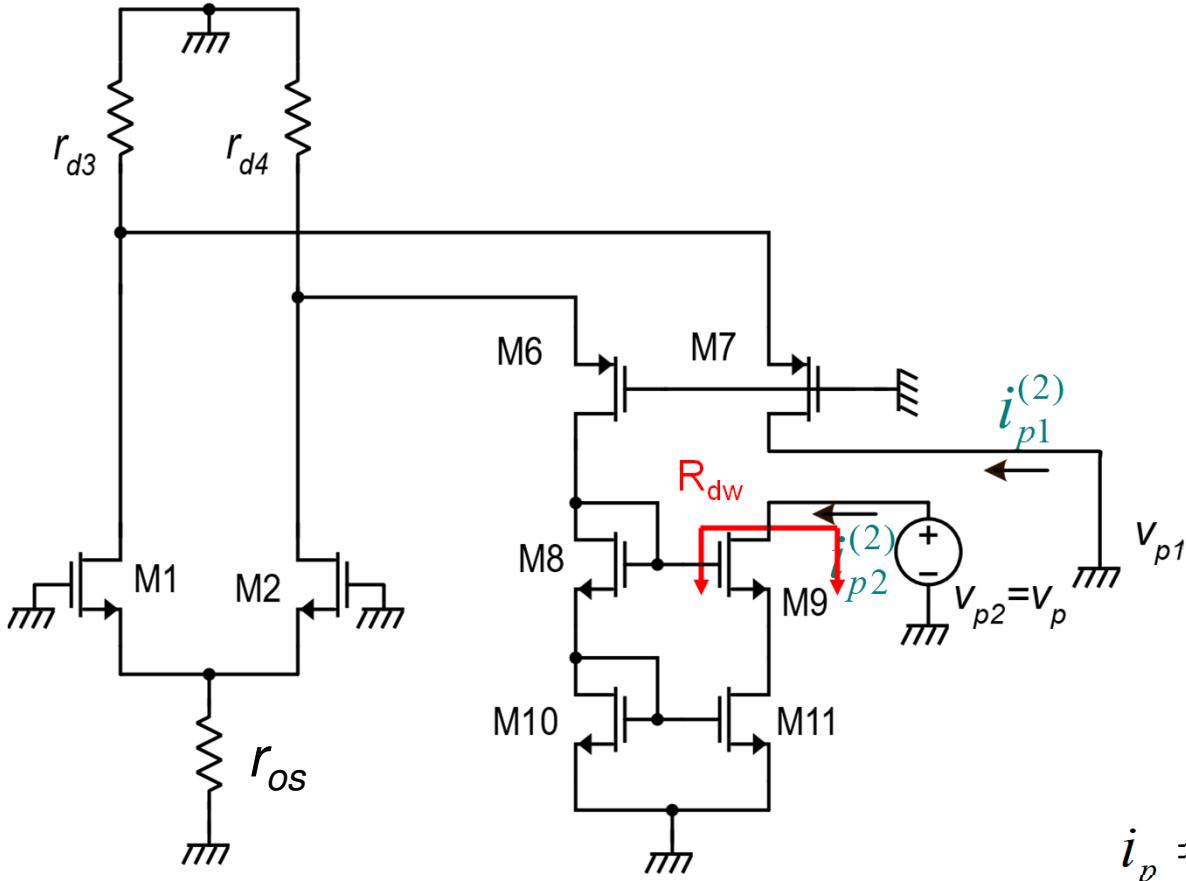
$$R_{s7} = r_{d3} // 2r_{d1}$$

$$R_{up} \cong r_{d7} g_{m7} (r_{d3} // 2r_{d1})$$

$$i_{p1-r}^{(1)} = i_{p1}^{(1)} \frac{r_{d3}}{r_{d3} + 2r_{d1}}$$

$$i_{p2}^{(1)} = i_{p1-r}^{(1)} = i_{p1}^{(1)} \frac{r_{d3}}{r_{d3} + 2r_{d1}}$$

## Folded cascode: output resistance (2)



$$R_{dw} \cong r_{d9} g_{m9} r_{d11}$$

$$i_{p2}^{(2)} = \frac{v_p}{R_{dw}}$$

$$i_{p1}^{(2)} = 0$$

$$i_{p1}^{(1)} = \frac{v_p}{R_{up}}$$

$$i_{p2}^{(1)} = i_{p1}^{(1)} \frac{r_{d3}}{r_{d3} + 2r_{d1}}$$

$$i_p = \frac{v_p}{R_{up}} \left( 1 + \frac{r_{d3}}{r_{d3} + 2r_{d1}} \right) + \frac{v_p}{R_{dw}}$$

## Folded cascode: output resistance (3)

$$i_p = v_p \left( \frac{1}{R_{up}} \left( 1 + \frac{r_{d3}}{r_{d3} + 2r_{d1}} \right) + \frac{1}{R_{dw}} \right)$$

$$R_{out} = \left( \frac{1}{R_{up}} \left( 1 + \frac{r_{d3}}{r_{d3} + 2r_{d2}} \right) + \frac{1}{R_{dw}} \right)^{-1}$$

$$R_{out} = \left( \frac{1}{\frac{R_{up}}{\left( 1 + \frac{r_{d3}}{r_{d3} + 2r_{d1}} \right)}} + \frac{1}{R_{dw}} \right)^{-1}$$

If  $r_{d2}=r_{d3}=r_{d7}=r_{d9}=r_{d11}=r_d$   
 $g_{m7}=g_{m9}=g_m$

$$R_{out} = R_{dw} // R_{up-r}$$

$$R_{dw} = r_{d9} g_{m9} r_{d11}$$

$$R_{up-r} = \frac{r_{d7} g_{m7} (r_{d3} / 2r_{d1})}{\left( 1 + \frac{r_{d3}}{r_{d3} + 2r_{d1}} \right)}$$

$$\frac{2}{3} r_d$$

$$\frac{4}{3}$$

$$R_{dw} = r_d (g_m r_d)$$

$$R_{up-r} = \frac{r_d (g_m r_d)}{2}$$

$$R_{out} = \frac{r_d (g_m r_d)}{3}$$

## Folded cascode: differential mode gain

$$A_d = -g_{m1} R_{out}$$

If  $r_{d2}=r_{d3}=r_{d7}=r_{d9}=r_{d11}=r_d$   
 $g_{m7}=g_{m9}=g_{m1}=g_m$

$$A_d = -g_m \frac{r_d (g_m r_d)}{3} = -\frac{(g_m r_d)^2}{3}$$

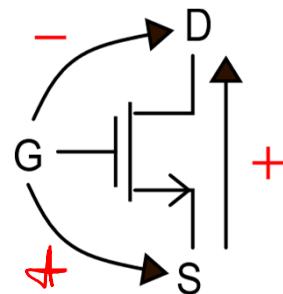
Compare with the  
“Telescopic” amplifier

$$A_d = \frac{(g_m r_d)^2}{2}$$

... and with the simple  
amplifier with mirror  
load (non-cascode)

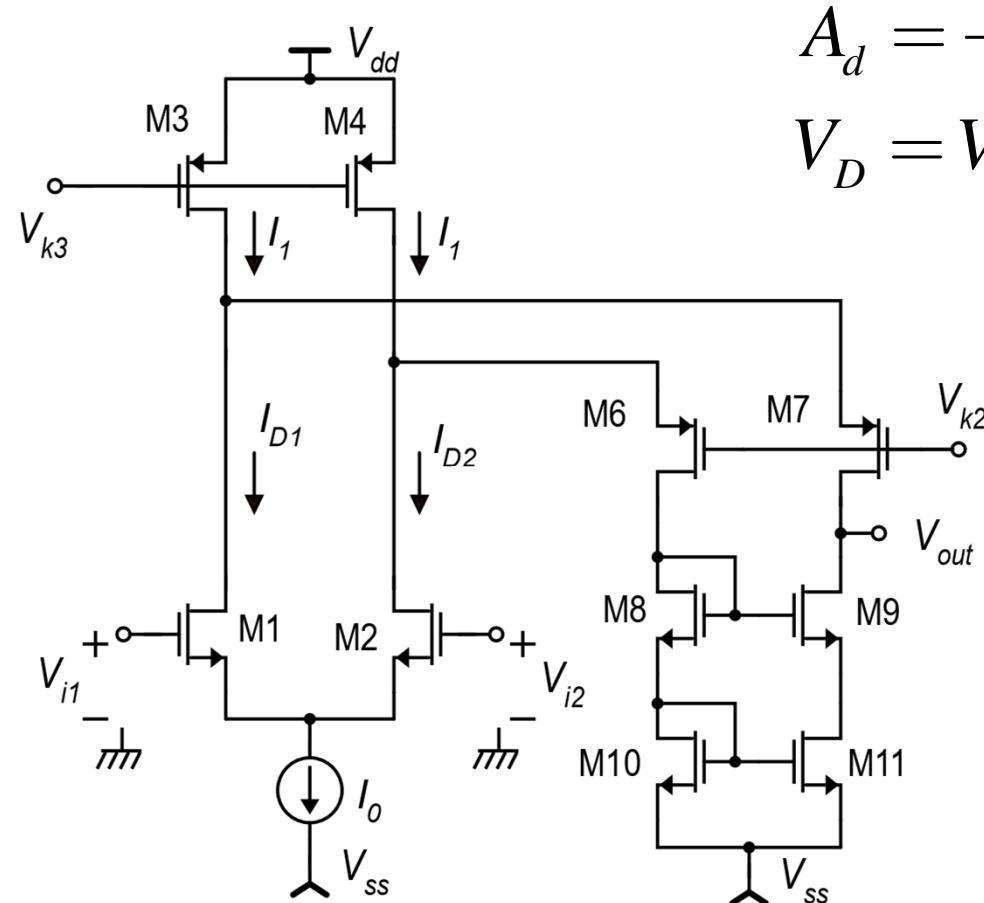
$$A_d = \frac{(g_m r_d)}{2}$$

## Simple method to find if a terminal is inverting / non-inverting



Signal paths:

1. From G to D: inversion
2. From G to S: no inversion
3. From S to D: no inversion

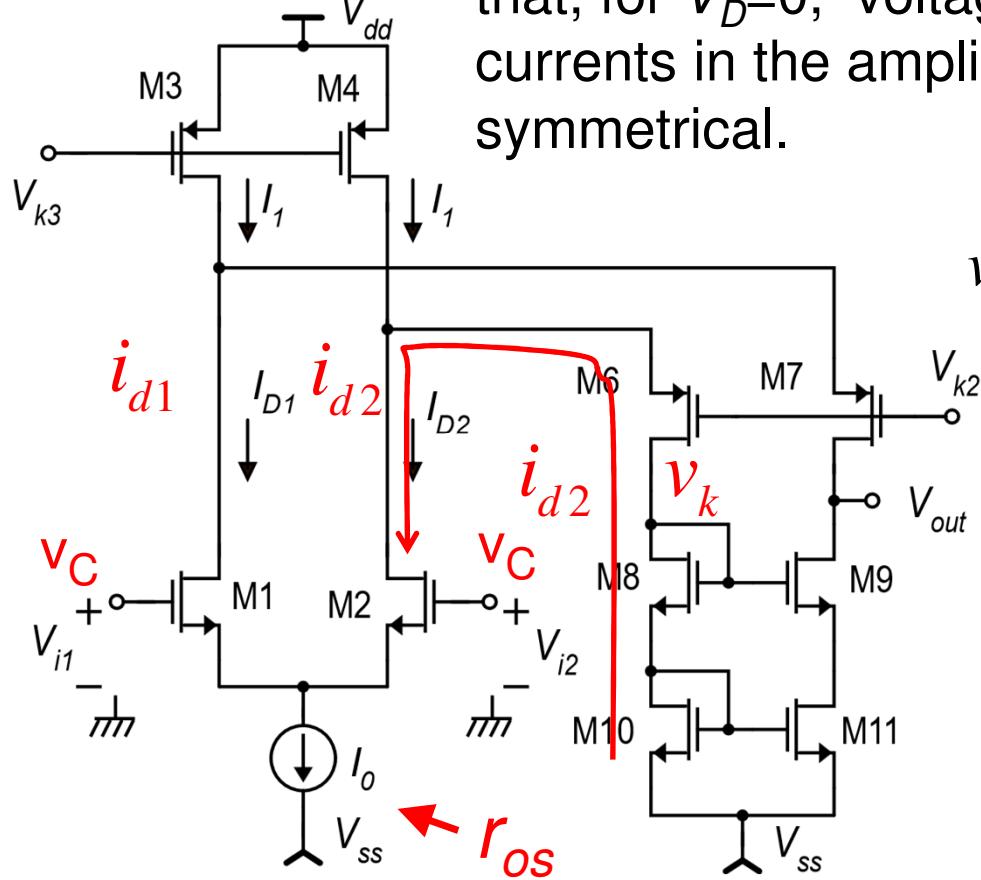


$$A_d = -g_{m1} R_{out}$$

$$V_D = V_{i1} - V_{i2}$$

## Ac, CMRR

It is possible to demonstrate that, for  $V_D=0$ , voltage and currents in the amplifiers are symmetrical.



Then: 
$$\begin{cases} i_{d1} = i_{d2} \cong \frac{v_c}{2r_{os}} \\ v_{out} = v_k \end{cases}$$

$$v_{out} = v_k = -i_{d2} \left( \frac{1}{g_{m8}} + \frac{1}{g_{m10}} \right)$$

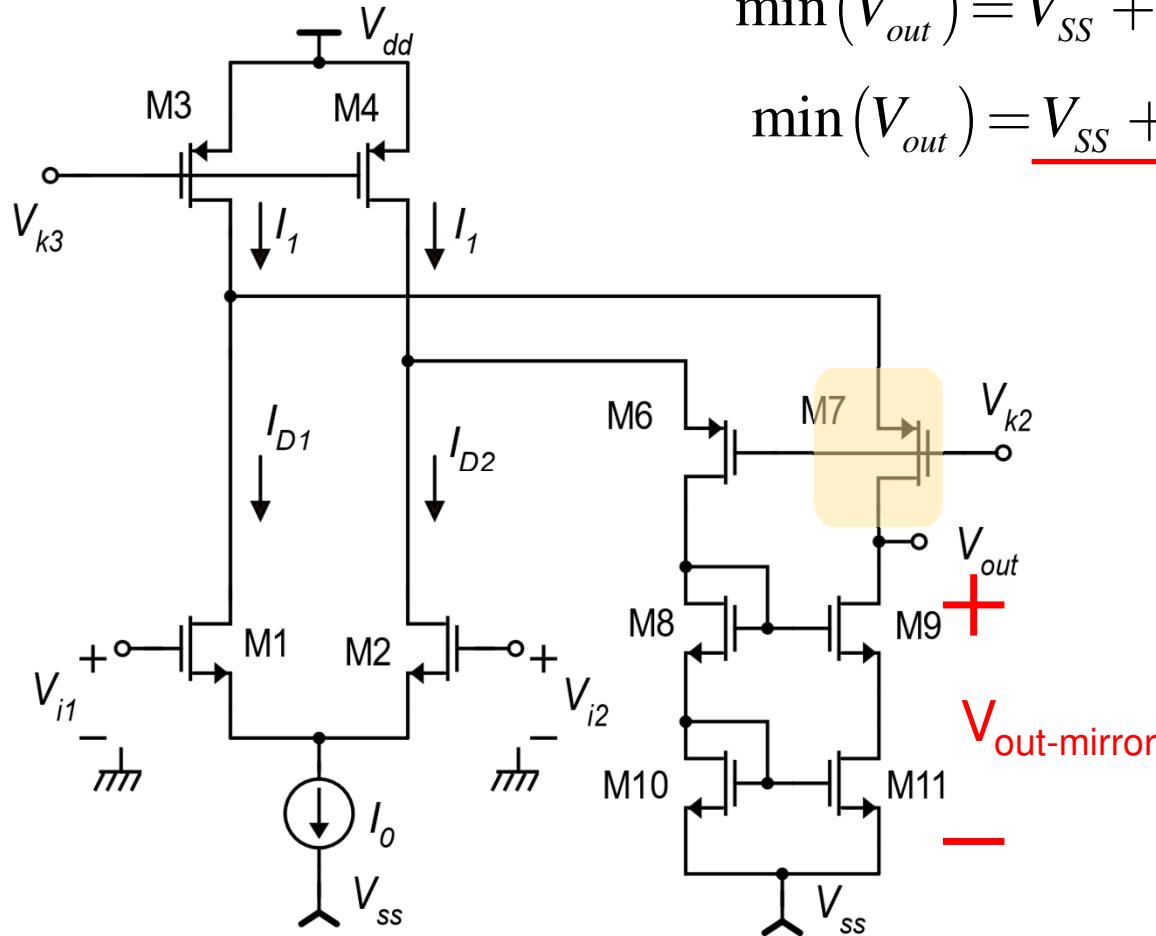
$$v_{out} = -\frac{v_c}{2r_{os}} \left( \frac{1}{g_{m8}} + \frac{1}{g_{m10}} \right) = -\frac{v_c}{g_m r_{os}}$$

for  $g_{m8} = g_{m10} = g_m$

$$A_C = -\frac{1}{g_m r_{os}}$$

$$CMRR \approx \frac{(g_m r_d)^2}{3} g_m r_{os}$$

## Folded cascode: output range



$$\min(V_{out}) = V_{ss} + V_{MIN-cascode} \quad \text{Lower limit}$$

$$\min(V_{out}) = V_{ss} + V_{GS11} + V_{DSAT9} \quad \text{Upper limit}$$

$$|V_{DS7}| = V_{k2} + |V_{GS7}| - V_{out} \geq |V_{DSAT7}|$$

$$V_{k2} + |V_{GS7}| - |V_{DSAT7}| \geq V_{out}$$

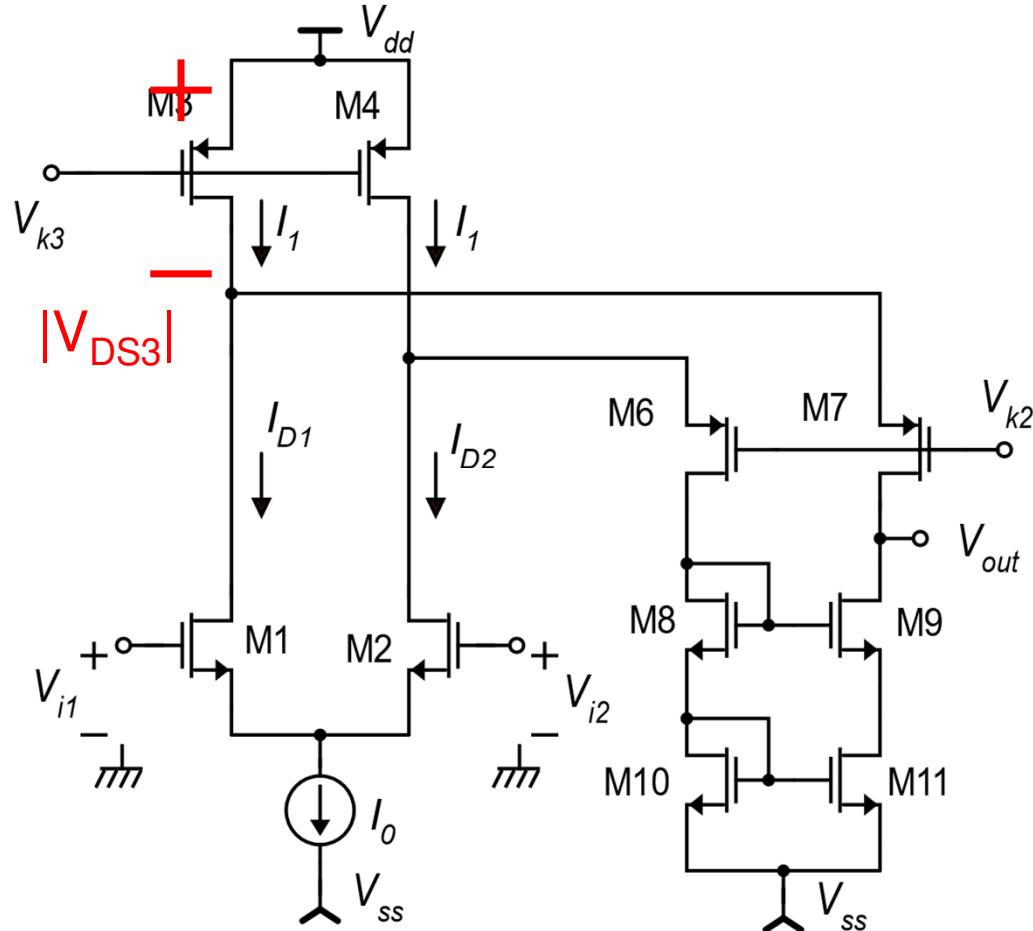
$$\max(V_{out}) = V_{k2} + |V_{GS7}| - |V_{DSAT7}|$$

In strong inversion:

$$|V_{DSAT7}| = |V_{GS7}| - |V_{t7}|$$

$$\max(V_{out}) = V_{k2} + |V_{t7}|$$

## Maximum $V_{k2}$



$$\max(V_{out}) = V_{k2} + |V_{GS7}| - |V_{DSAT7}|$$

$$V_{D3} = V_{D1} = V_{k2} + |V_{GS7}|$$

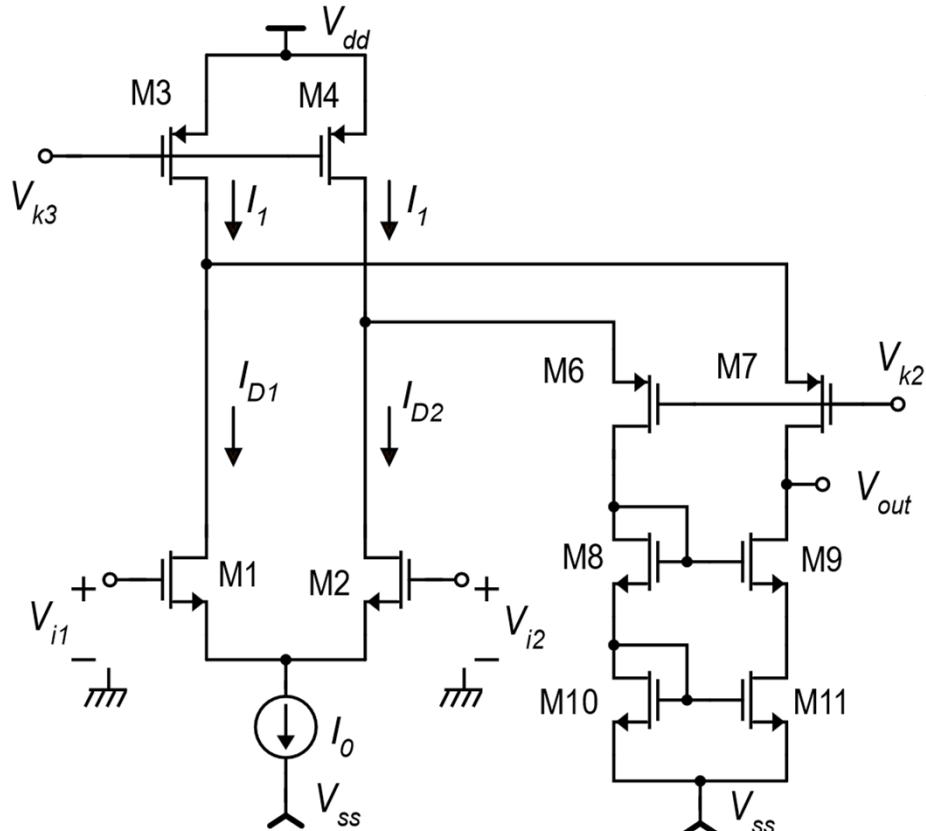
$V_{k2}$  sets the voltage of M1-M3 and M2-M4 drains

$$|V_{DS3}| = V_{dd} - V_{D3} = V_{dd} - V_{k2} - |V_{GS7}| \geq |V_{DSAT3}|$$

$$\max(V_{k2}) = V_{dd} - |V_{DSAT3}| - |V_{GS7}|$$

$$\max(V_{out}) = V_{dd} - |V_{DSAT3}| - |V_{DSAT7}|$$

## Input common mode range



$$\min(V_{iC}) = V_{SS} + V_{MIN-tail} + V_{GS1}$$

$$V_{D3} = V_{D1} = V_{k2} + |V_{GS7}|$$

M1 (M2) has the drain at a fixed voltage. If the gate voltage increases, M1 (M2) will eventually leave saturation

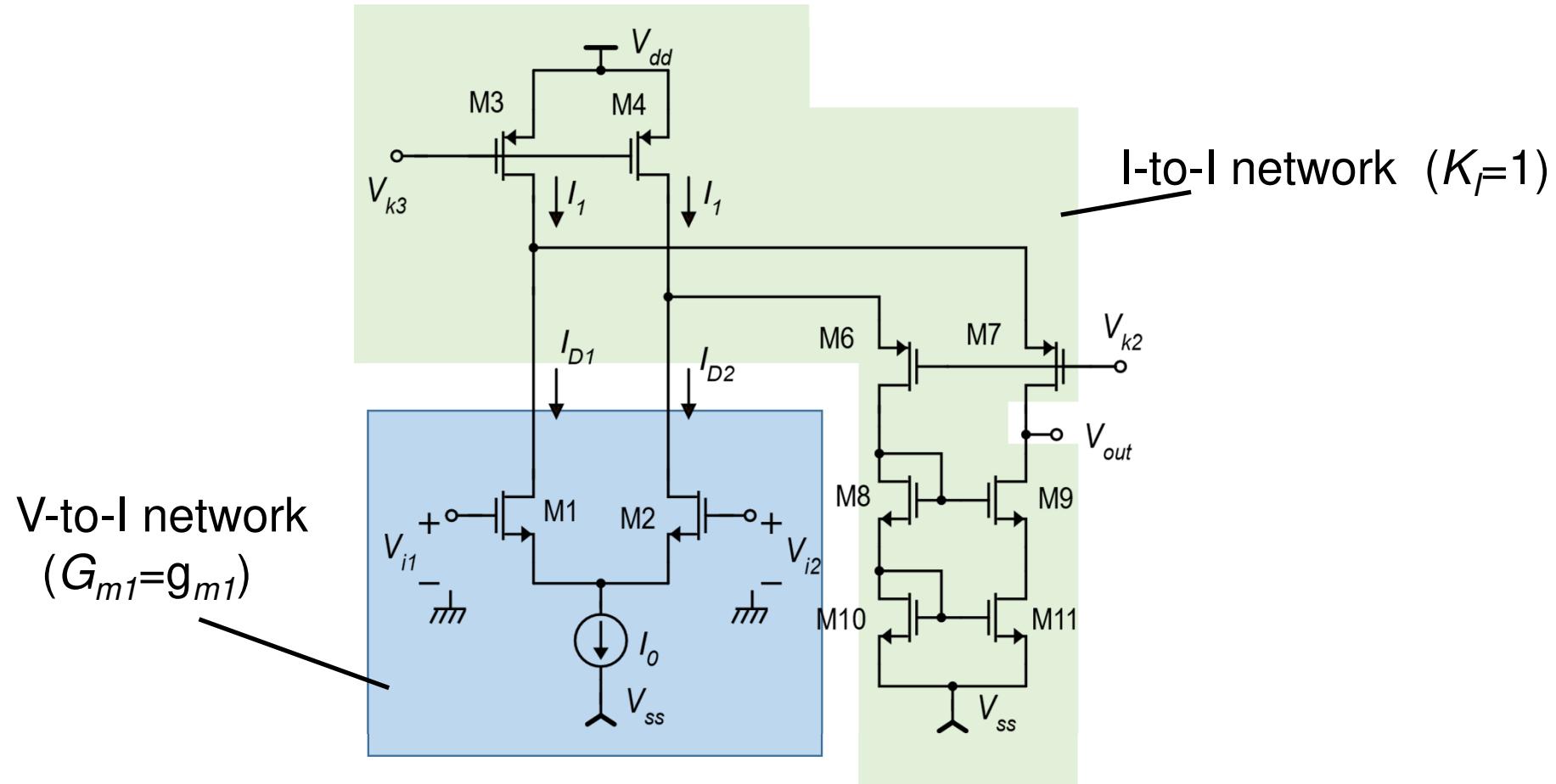
$$\begin{aligned} V_{DS1} &= V_{D1} - V_{S1} = \\ &= V_{k2} + |V_{GS7}| - (V_C - V_{GS1}) \geq V_{DSAT1} \end{aligned}$$

$$\max(V_C) = V_{k2} + |V_{GS7}| + V_{GS1} - V_{DSAT1}$$

$$\max(V_{k2}) = V_{dd} - |V_{DSAT3}| - |V_{GS7}|$$

$$\max(V_C) = V_{dd} - |V_{DSAT3}| + V_{GS1} - V_{DSAT1}$$

# The folded cascode as a single stage amplifier



## Folded cascode: summary of properties

**DC Gain:** Slightly smaller than the telescopic amplifier (non-folded cascode) gain. May reach several thousands or even  $10^4$  (80 dB) with long mosfets. Larger than the gain of the cascade of two common source stages.

**Output swing:**

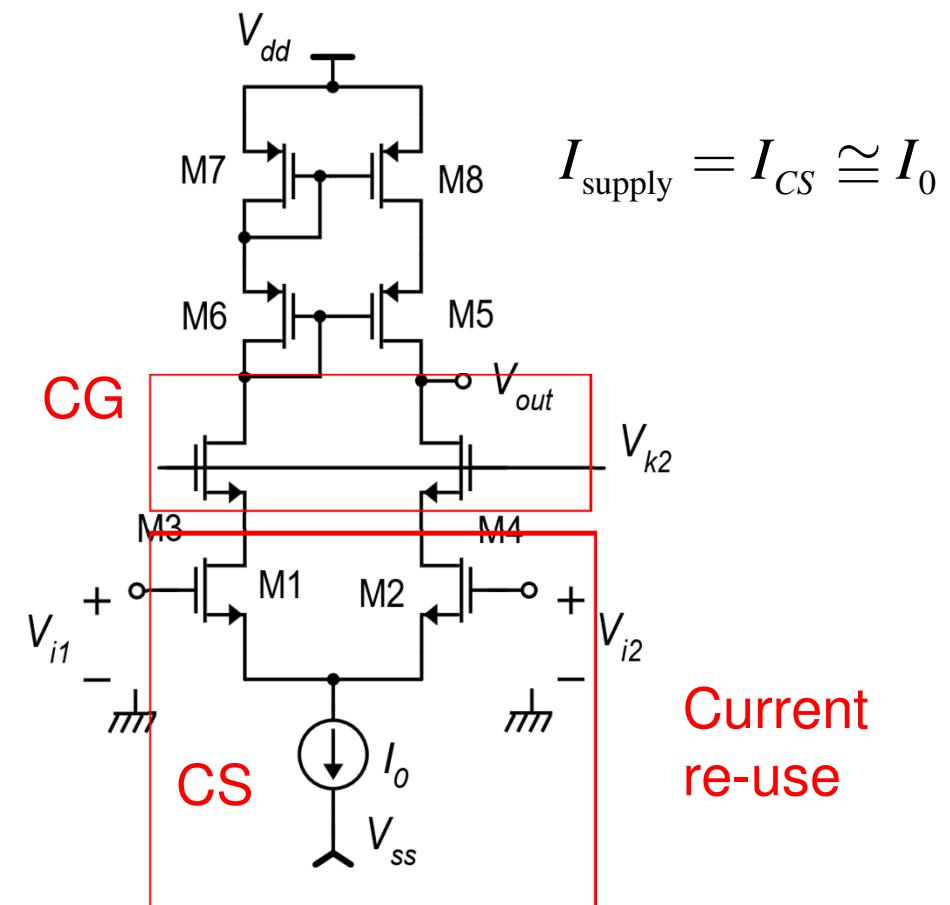
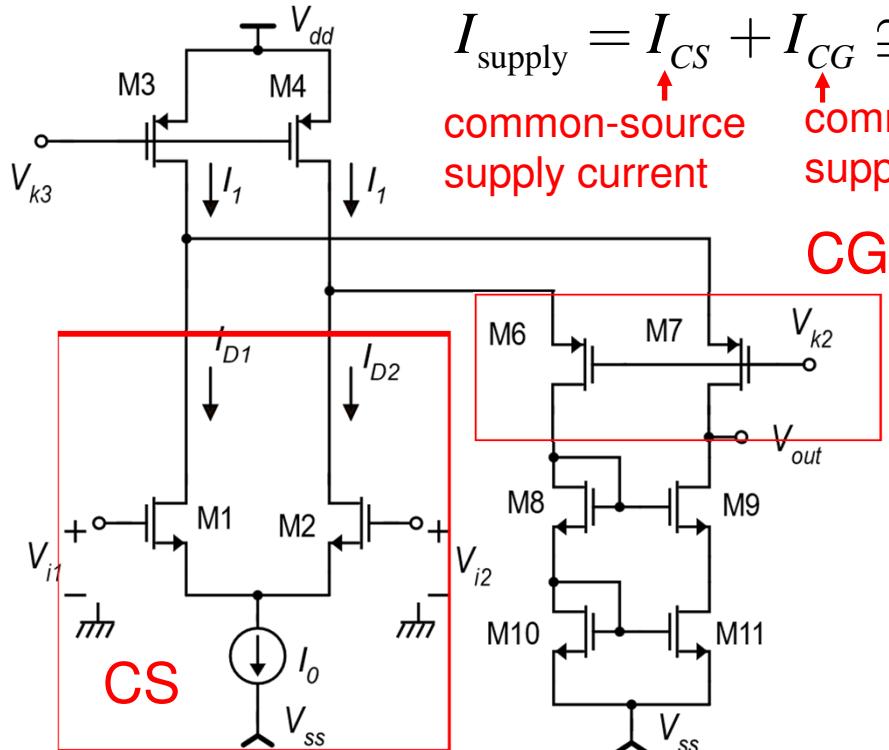
$$\max(V_{out}) = V_{dd} - |V_{DSAT3}| - |V_{DSAT7}|$$
$$\min(V_{out}) = V_{ss} + V_{GS11} + V_{DSAT9}$$

**Input CM range:**

$$\min(V_{iC}) = V_{ss} + V_{MIN-tail} + V_{GS1}$$
$$\max(V_C) = V_{dd} - |V_{DSAT3}| + V_{GS1} - V_{DSAT1}$$

# The only true advantage of the telescopic amplifier

Current consumption ( $I_{\text{supply}}$ )



# Example of BJT Op-Amp with a Folded cascode input stage

THS4031-EP  
THS4032-EP

SLOS610–NOVEMBER 2008



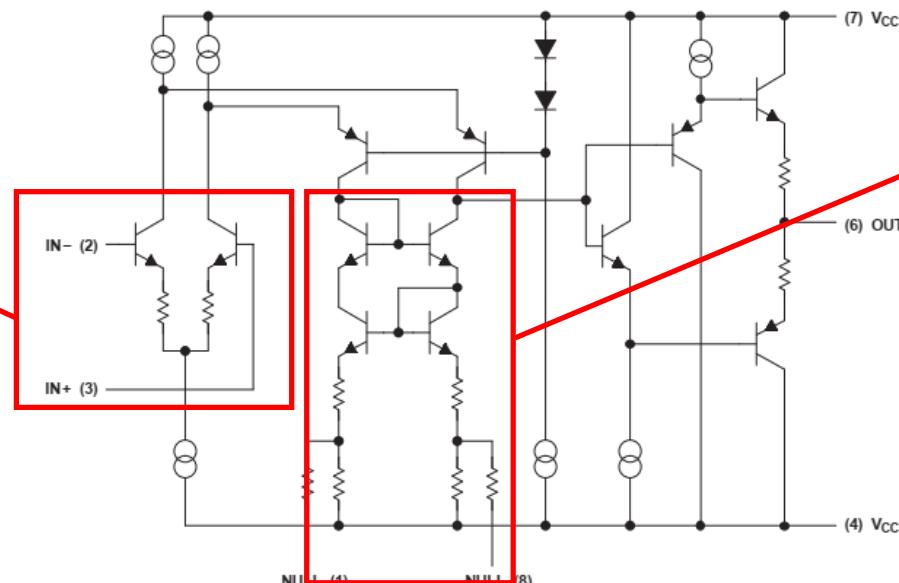
www.ti.com

## APPLICATION INFORMATION

### THEORY OF OPERATION

The THS403x is a high-speed operational amplifier configured in a voltage feedback architecture. It is built using a 30-V, dielectrically isolated, complementary bipolar process with NPN and PNP transistors possessing  $f_T$ s of several GHz. This results in an exceptionally high-performance amplifier that has wide bandwidth, high slew rate, fast settling time, and low distortion. A simplified schematic is shown in Figure 51.

Emitter-degenerated  
input pair  
(improves input  
differential range, with  
benefits in terms of  
Slew-Rate)



Wilson current mirror  
with emitter  
degeneration.

Figure 51. THS4031 Simplified Schematic