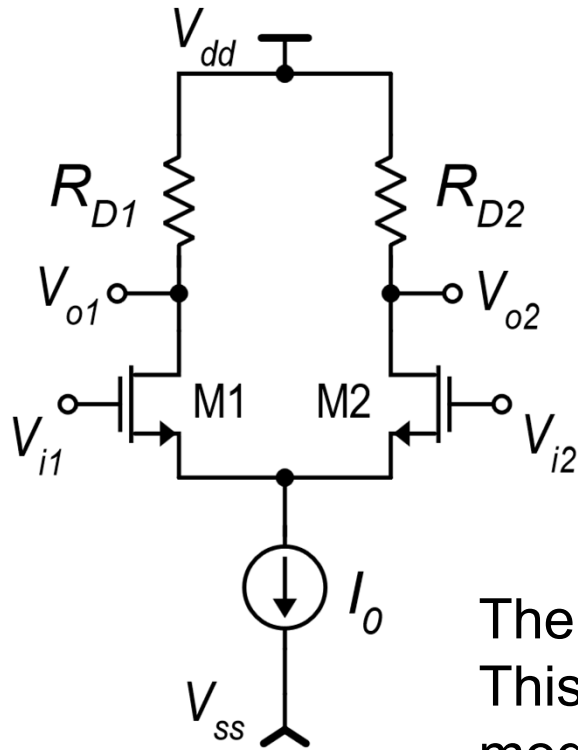


CMOS differential amplifier with resistive load: limitations



1. The S/E version has a poor CMRR (large A_c) and large input offset voltage
2. Both the S/E and fully-diff. versions reach low voltage gains at small supply voltage

Problem 1: Consider the output voltage in the fully-differential case with no resistor mismatch:

$$V_{OD} = R_D \underbrace{(I_{D2} - I_{D1})}_{\text{drain current difference}} \quad (\text{for } R_{D1} = R_{D2})$$

The drain current difference appears:

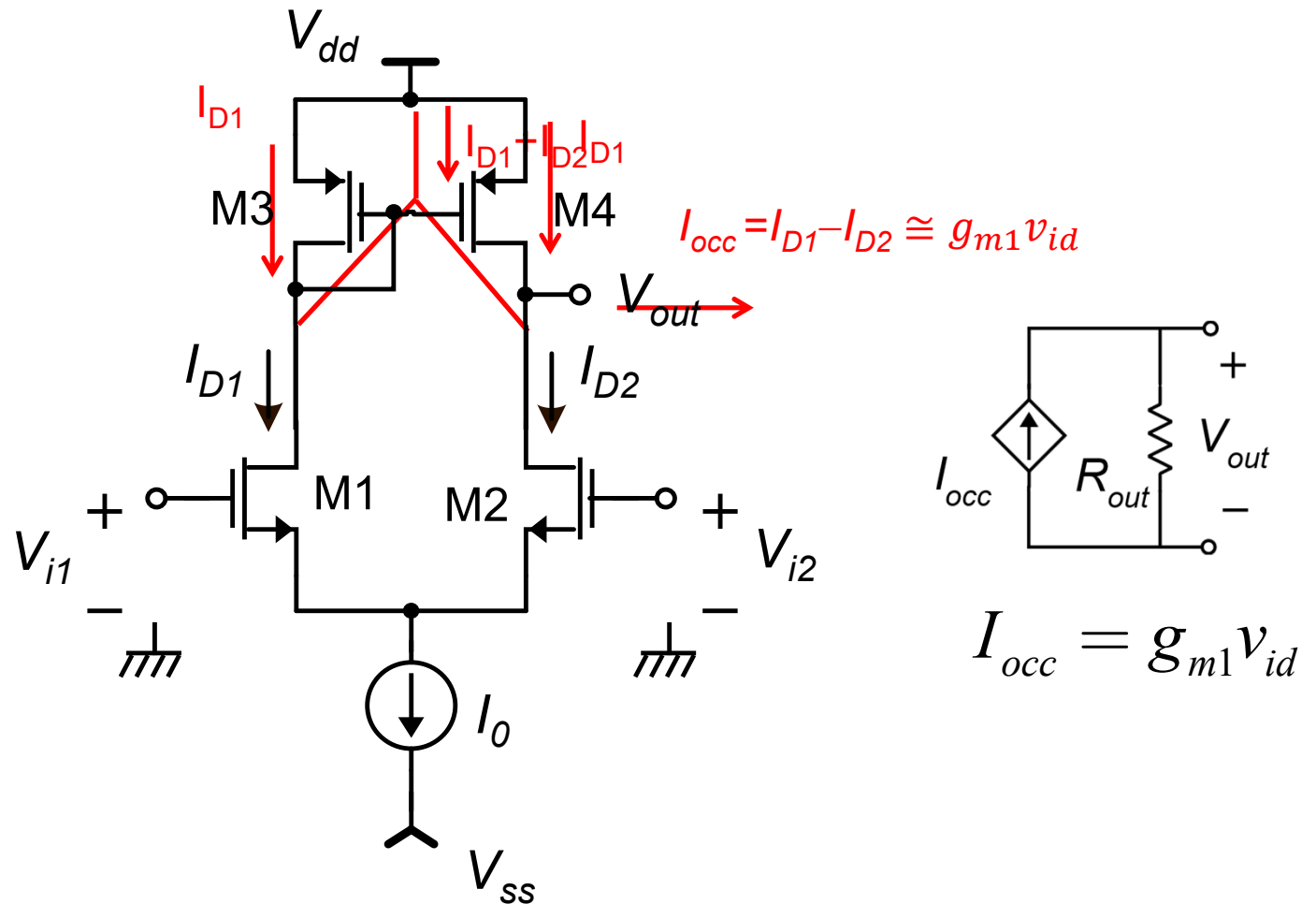
This reduces A_c (I_{D1} and I_{D2} tend to be equal for only common mode applied) and the output voltage for $V_D=0$ is affected only by matching errors.

Solution for a S/E amplifier: produce the current difference ($I_{D1}-I_{D2}$) and then put it into a single resistor. **Problem 2:** Do not use a passive component for the resistor

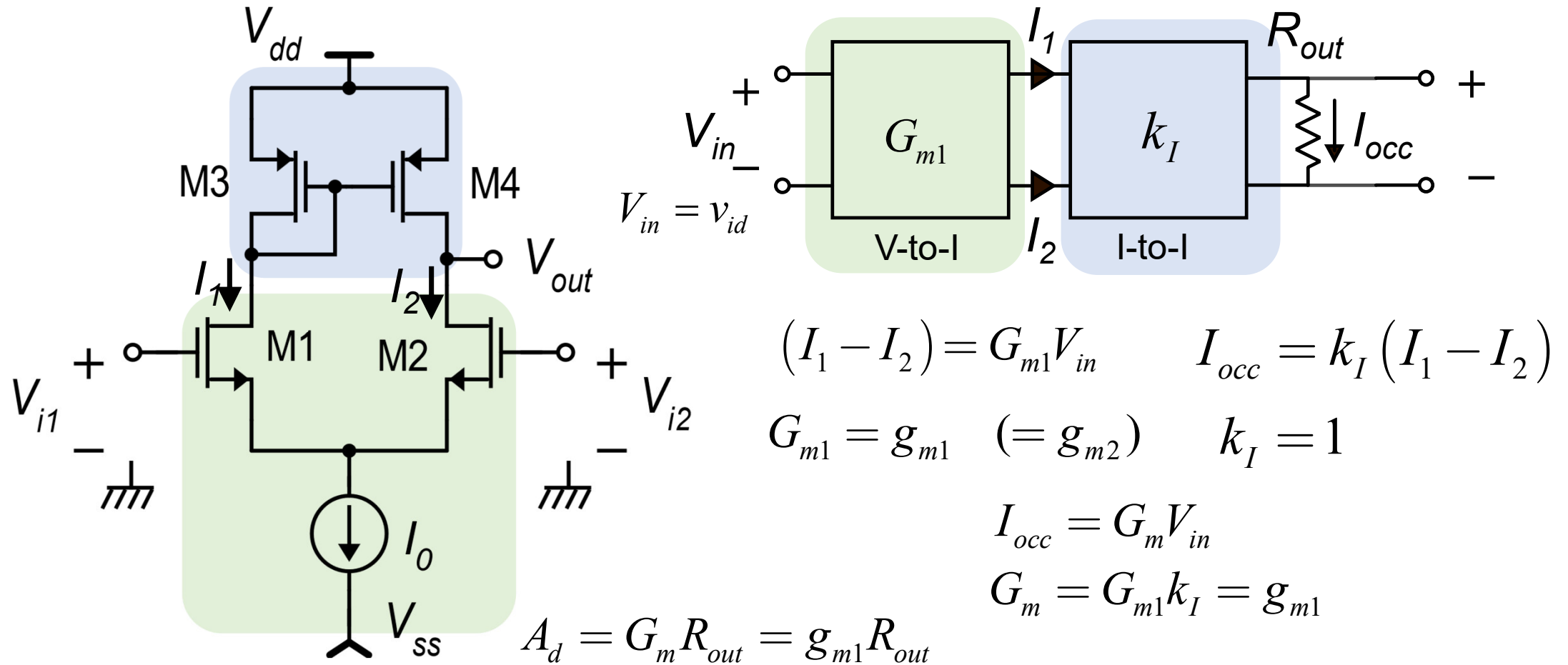
CMOS differential amplifier with current mirror load

Specifications

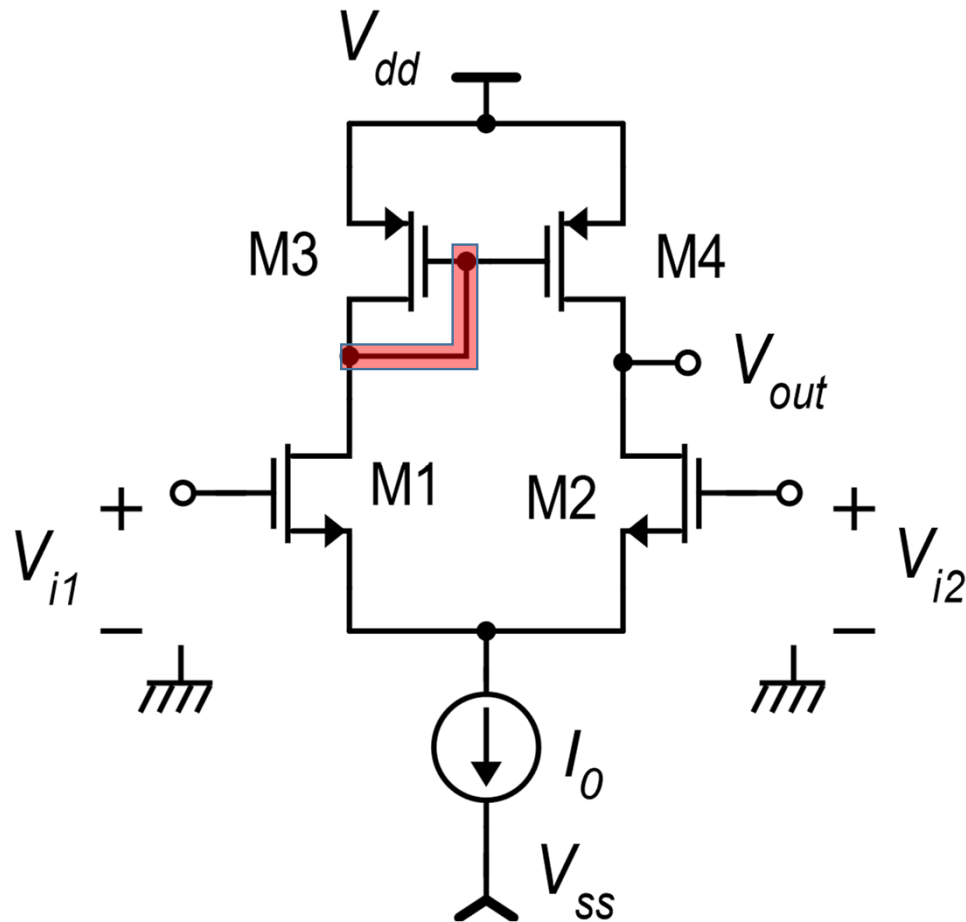
- We need a S/E output port
- High CMRR (> 80 dB)
- High gain (~ 40 dB) even at low supply voltages ($V_{dd} - V_{ss}$).



Subunits of the amplifier with mirror load



Operating point : $V_{id}=0$



$$V_{i1} = V_{i2} = V_C$$

Symmetrical stimulus, but

The circuit is not symmetrical

$$I_{D1} = I_{D2} \text{ exactly?}$$

$$I_D = f(V_{GS}, V_{BS}, V_{DS})$$

Demonstration of the exact symmetry of the electrical solution for $V_{id}=0$

With fixed V_{GS} and V_{BS} , I_D is a monotone function of V_{DS} (or $|V_{DS}|$ for a p -MOS).

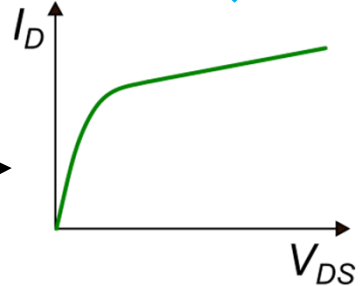
$$\begin{cases} I_{D3} = I_{D1} \\ I_{D4} = I_{D2} \end{cases}$$

M1 and M2

$$V_{iD} = 0 \Rightarrow V_{GS1} = V_{GS2}$$

$$V_{S1} = V_{S2} \Rightarrow V_{BS1} = V_{BS2}$$

$$V_{DS1} = V_{DS2} ?$$

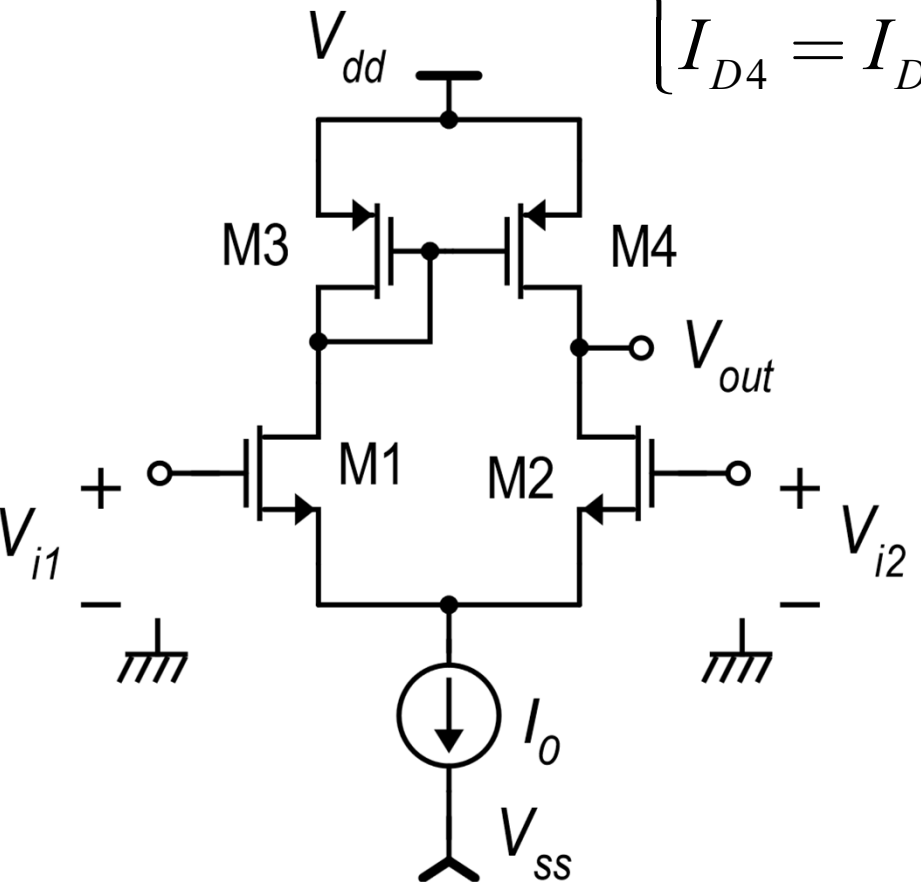
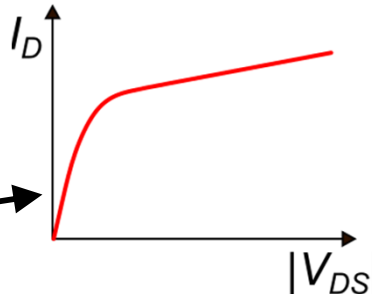


M3 and M4

$$V_{GS3} = V_{GS4}$$

$$V_{BS3} = V_{BS4}$$

$$V_{DS3} = V_{DS4} ?$$



Demonstration of the exact symmetry of the electrical solution for $V_{id}=0$

From Kirchhoff #2: $V_{DS1} + |V_{DS3}| = V_{DS2} + |V_{DS4}|$

We want to demonstrate that: $I_{D1} = I_{D2}$

Proof by contradiction:

Let us suppose: $I_{D1} > I_{D2}$

$$\begin{cases} I_{D3} = I_{D1} \\ I_{D4} = I_{D2} \end{cases}$$

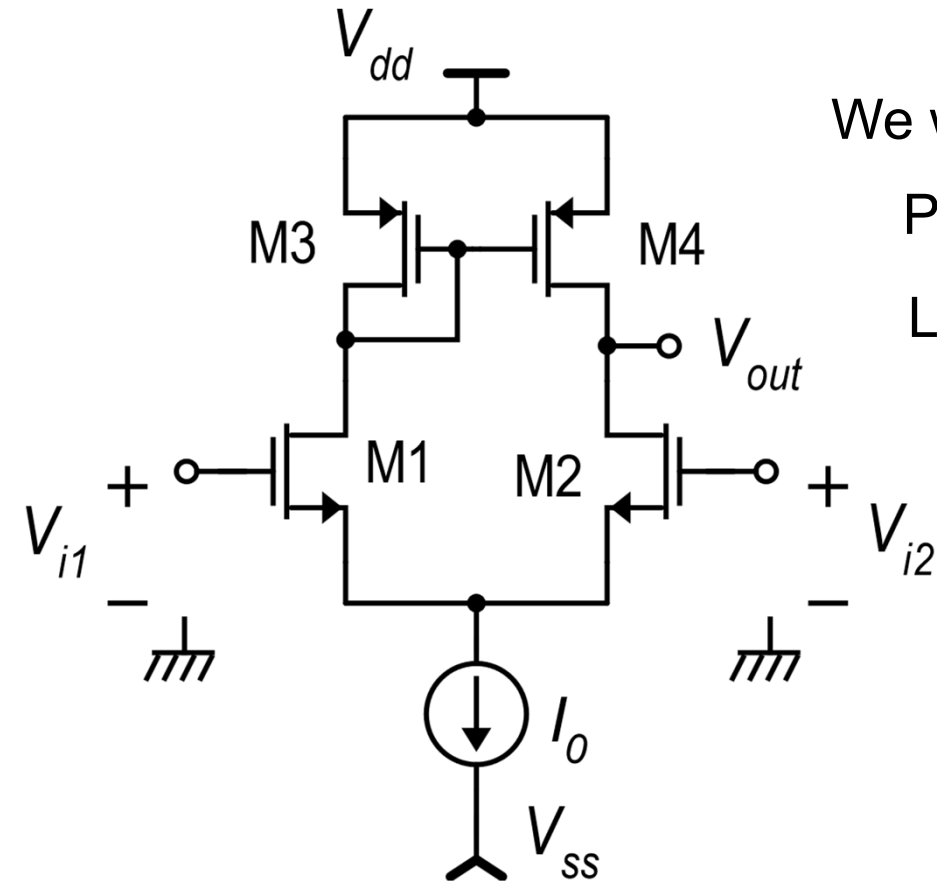
$$V_{DS1} > V_{DS2}$$

$$I_{D3} > I_{D4}$$

$$|V_{DS3}| > |V_{DS4}|$$

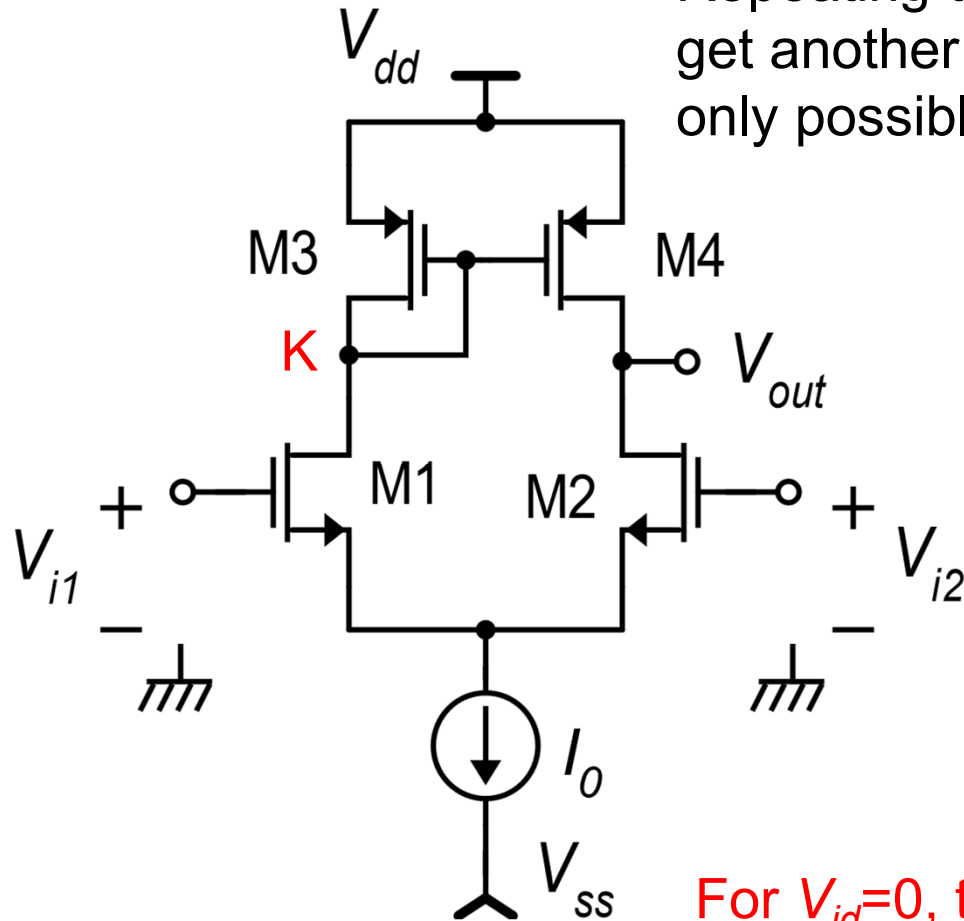
$$V_{DS1} + |V_{DS3}| > V_{DS2} + |V_{DS4}|$$

Contradiction

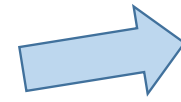


Demonstration of the exact symmetry of the electrical solution for $V_{id}=0$

Repeating the procedure for $I_{D1} < I_{D2}$ we get another contradiction; thus, the only possible solution is:



$$\begin{cases} I_{D1} = I_{D2} \\ I_{D3} = I_{D4} \end{cases}$$



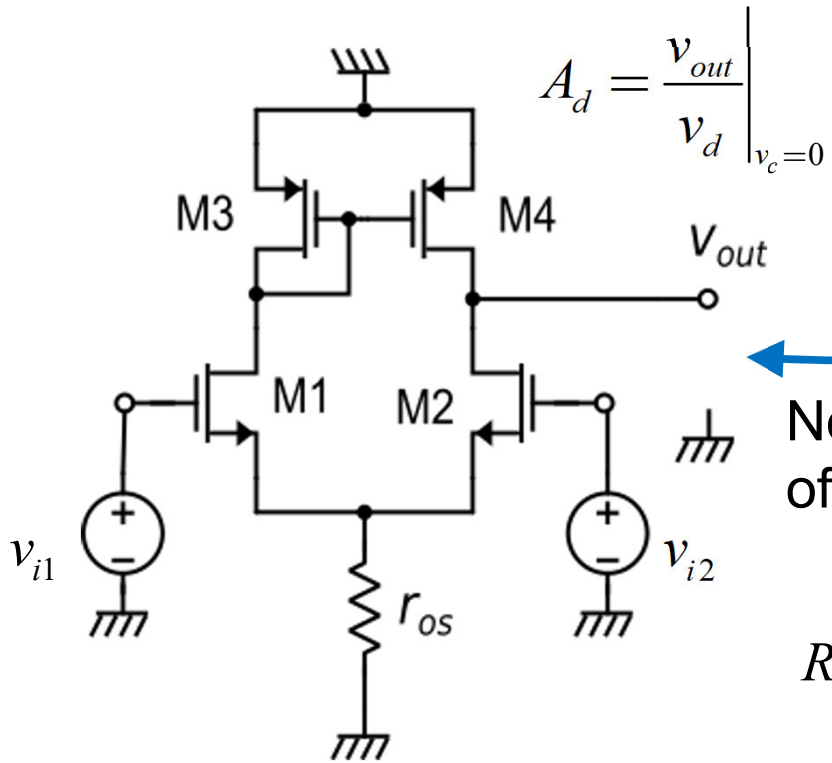
$$\begin{cases} V_{DS1} = V_{DS2} \\ |V_{DS3}| = |V_{DS4}| \end{cases}$$

$$\begin{cases} V_K = V_{dd} - |V_{DS3}| \\ V_{out} = V_{dd} - |V_{DS4}| \end{cases}$$

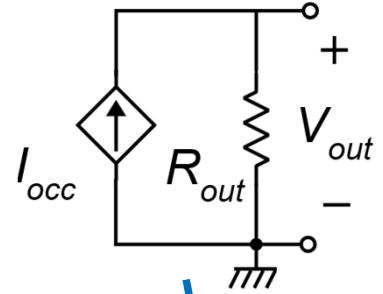
$$V_{out} = V_K$$

For $V_{id}=0$, the electrical solution is perfectly symmetrical

Differential mode gain



$$A_d = \left. \frac{v_{out}}{v_d} \right|_{v_c=0}$$



$$v_{out} = i_{occ} R_{out}$$

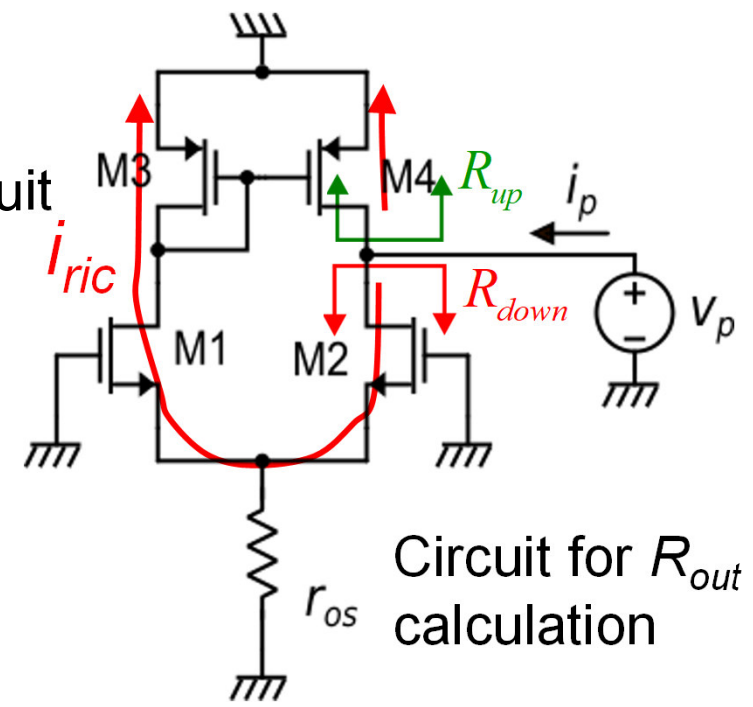
$$i_{occ} = i_{d4} - i_{d2} \cong i_{d1} - i_{d2} = g_m v_d$$

$$v_{out} = g_m v_d R_{out} \Rightarrow A_d = \underline{\underline{g_m R_{out}}}$$

Norton equivalent circuit of the output port

~~$$R_{out} = R_{up} \parallel R_{down}$$~~

$$i_p = \frac{v_p}{R_{up}} + \frac{v_p}{R_{down}} + i_{ric}$$



Circuit for R_{out} calculation

Small-signal equivalent circuit

Differential mode gain: R_{out} calculation.

Case 1: $v_{p1} = v_p, v_{p2} = 0 \rightarrow i_{p1} = i_{p1}^{(1)}, i_{p2} = i_{p2}^{(1)}$

Case 2: $v_{p1} = 0, v_{p2} = v_p \rightarrow i_{p1} = i_{p1}^{(2)}, i_{p2} = i_{p2}^{(2)}$

For the superposition theorem: $i_p = i_{p1}^{(1)} + i_{p2}^{(1)} + i_{p1}^{(2)} + i_{p2}^{(2)}$

Case 1: $i_{p1}^{(1)} = \frac{v_p}{R_{up}}, i_{p2}^{(1)} = 0, R_{up} = r_{d4}$

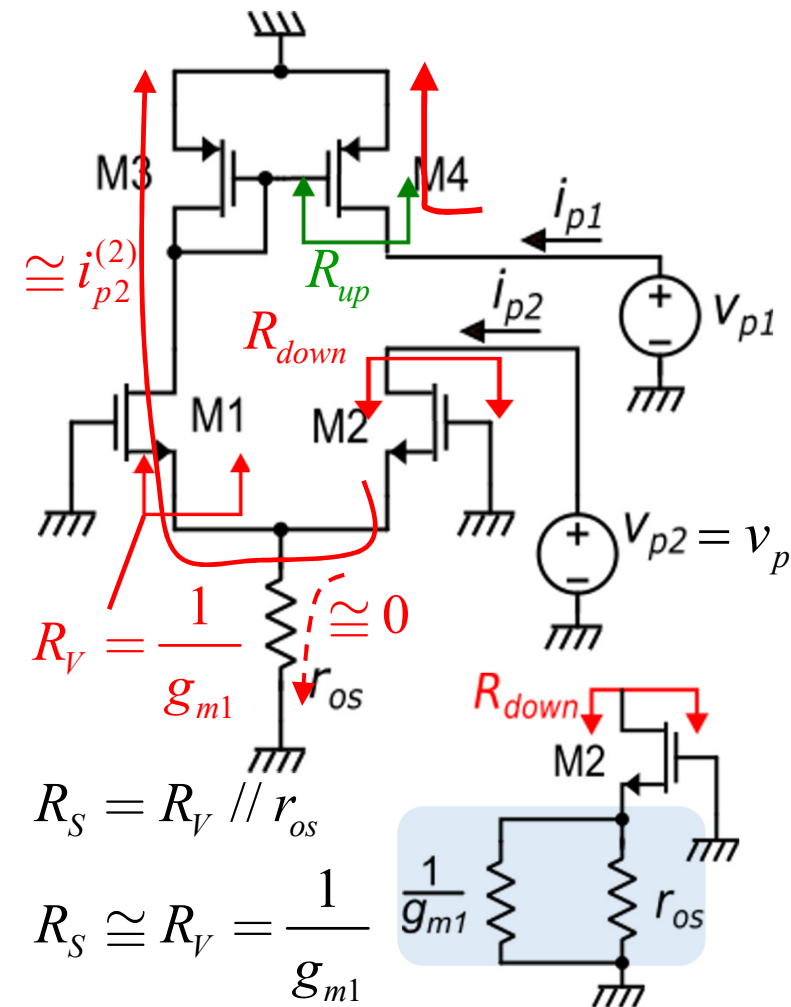
Case 2:

$i_{p2}^{(2)} = \frac{v_p}{R_{down}}, i_{p1}^{(2)} \cong i_{p2}^{(2)}$

$$R_{down} = \cancel{R_S} + r_{d2} \left(1 + \frac{g_{m2}}{g_{m1}} R_S \right) = 2r_{d2}$$

$$i_p = \frac{v_p}{r_{d4}} + 0 + \frac{v_p}{2r_{d2}} + \frac{v_p}{2r_{d2}} = v_p \left(\frac{1}{r_{d4}} + \frac{1}{r_{d2}} \right)$$

$$R_{out} = \frac{v_p}{i_p} = \left(\frac{1}{r_{d2}} + \frac{1}{r_{d4}} \right)^{-1} = \underline{r_{d2} // r_{d4}}$$



Differential mode gain

$$R_{out} = r_{d2} // r_{d4}$$

$$A_d = g_{m1} (r_{d2} // r_{d4})$$

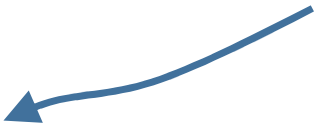
The gain is of the order of $\frac{g_m r_d}{2}$
Independently from the supply voltage

$$A_d = g_{m1} \left(\frac{1}{\frac{1}{r_{d2}} + \frac{1}{r_{d4}}} \right)$$

$$r_d = \frac{1}{\lambda I_D}$$

$$A_d = g_{m1} \left(\frac{1}{\lambda_2 I_{D2} + \lambda_4 I_{D4}} \right)$$

$$A_d = \frac{g_{m1}}{I_{D1}} \left(\frac{1}{\lambda_2 + \lambda_4} \right)$$

$$I_{D2} = I_{D4} = I_{D1}$$


Differential mode gain

$$A_d = \frac{g_{m1}}{I_{D1}} \left(\frac{1}{\lambda_2 + \lambda_4} \right)$$



$$\frac{g_m}{I_D} \triangleq \frac{1}{V_{TE}}$$

$$V_{TE} = \begin{cases} \frac{V_{GS} - V_t}{2} \\ mV_T \end{cases}$$

In strong inversion.
Minimum $V_{TE} = 50$ mV

In weak inversion.
 $V_{TE} \cong 35-40$ mV

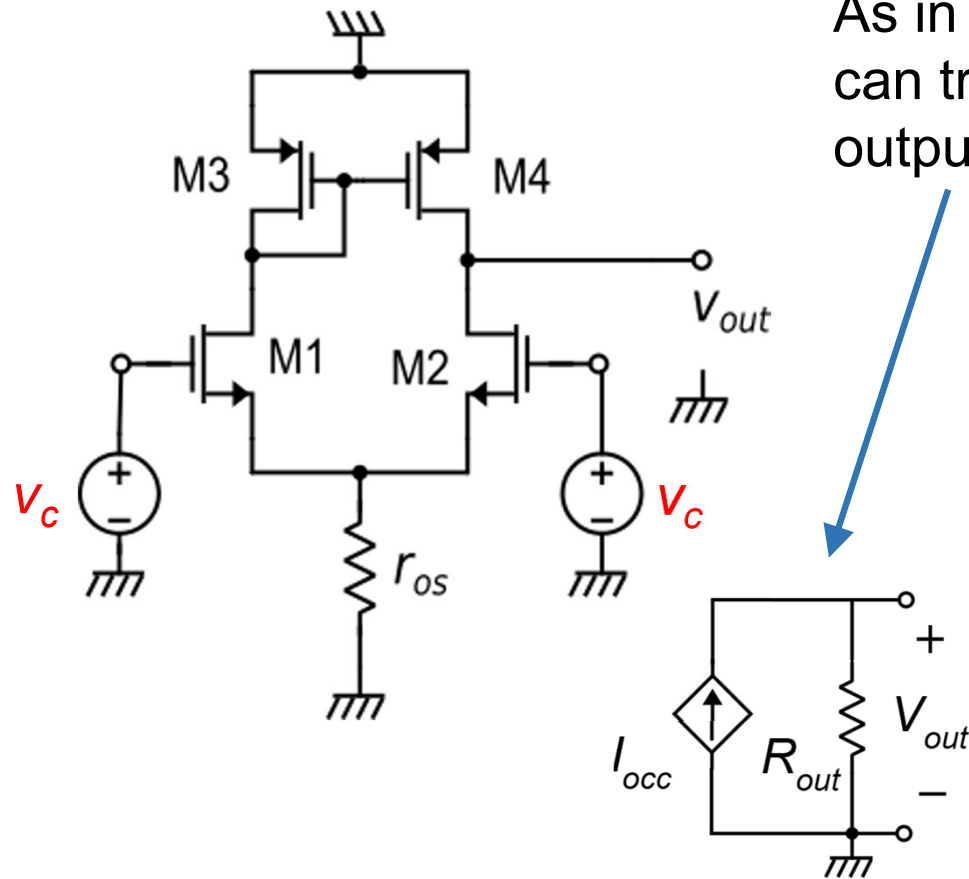
$$A_d = \frac{1}{V_{TE1}} \left(\frac{1}{\lambda_2 + \lambda_4} \right)$$

In order to obtain a large differential-mode gain it is necessary to:

- Set V_{TE} to a small value
- Use long MOSFETs (small λ)

Common mode gain

As in the case of differential input voltage, we can try to use the Norton equivalent circuit of the output port.



$$v_{out} = i_{occ} R_{out} \quad R_{out} = r_{d2} // r_{d4}$$

$$i_{occ} = i_{d1} - i_{d2}$$

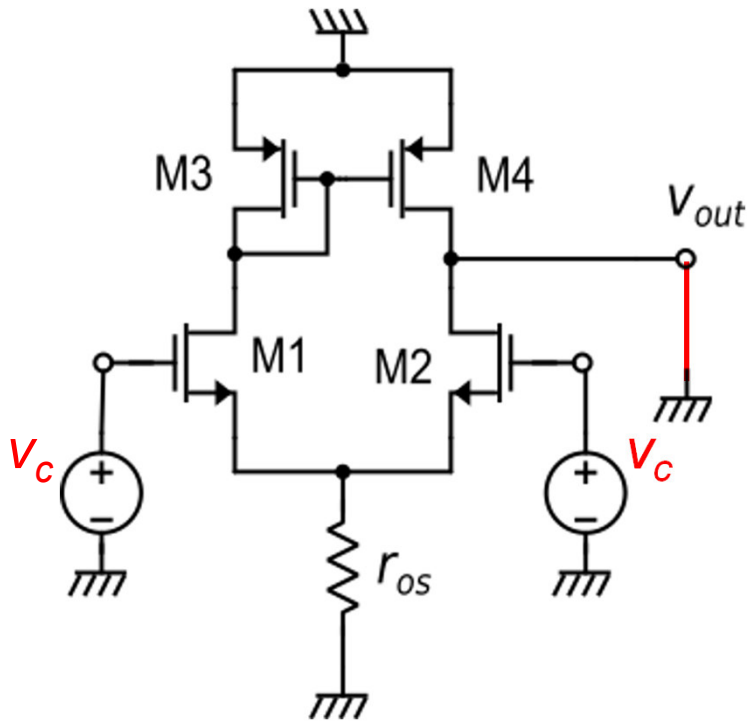
$$\text{for } v_d = 0 \quad i_{d1} = i_{d2} = \frac{v_c}{2r_{os}}$$

$$i_{occ} = 0$$

$$v_{out} = 0 \Rightarrow A_c = 0 \Rightarrow CMRR = \infty$$

Is it possible??

Common mode gain



The problem occurs when we calculate i_{occ} . Placing a short circuit across the output port, disrupts the symmetry:

$$v_{ds4} = 0 \quad v_{ds3} = -\frac{1}{g_{m3}} i_{d1} \neq 0$$

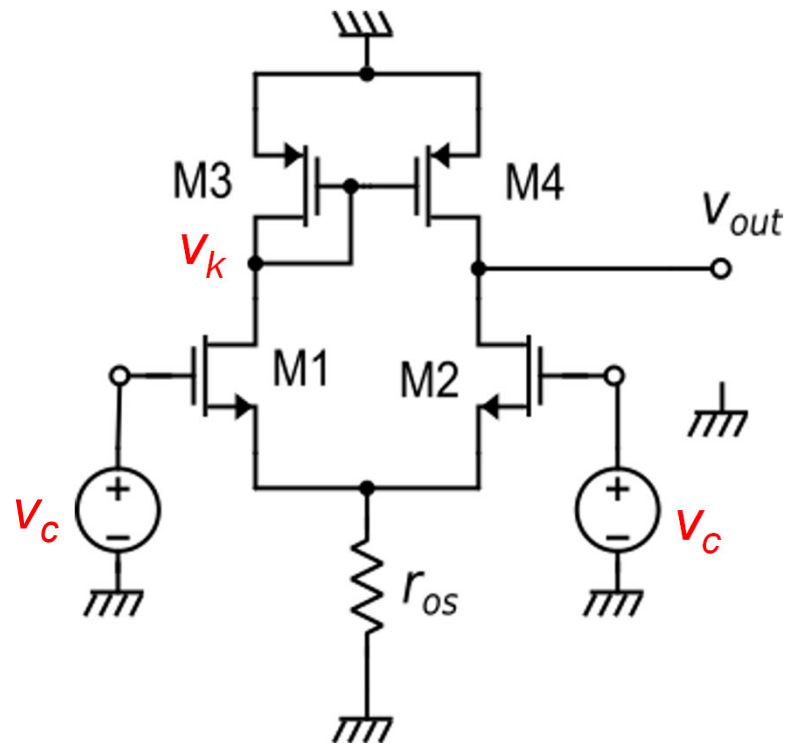
$$v_{ds1} = -\frac{1}{g_{m3}} i_{d1} - v_{s1} \quad v_{ds2} = -v_{s1} \neq v_{ds1}$$

$$i_{d1} \neq i_{d2}$$

$$i_{d3} \neq i_{d4}$$

$$i_{occ} = i_{d4} - i_{d2} \neq 0$$

Common mode gain



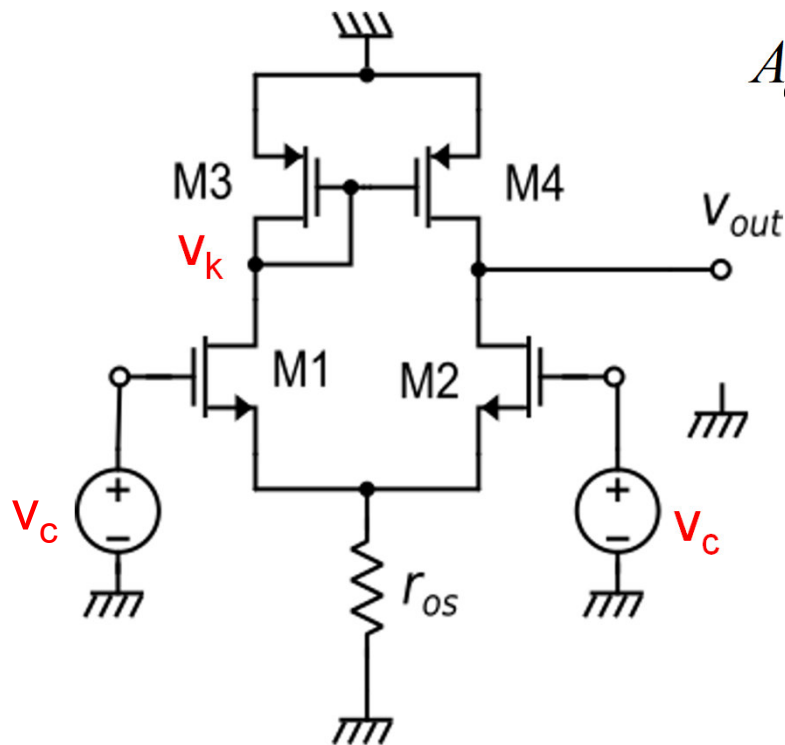
It is possible to exactly calculate i_{occ} , taking into account the actual i_{d1}/i_{d2} ratio and i_{d4}/i_{d3} ratio

... but this is a very tedious approach
There is a much simpler way:

Let us remove the short circuit and directly calculate v_{out} . Now, for $v_{id}=0$, the circuit is symmetric again and, in particular: $v_k = v_{out}$

$$v_{out} = v_k = -\frac{1}{g_{m3}} i_{d1} \cong -\frac{v_c}{2r_{os} g_{m3}}$$

Common mode gain



$$A_c = \left. \frac{v_{out}}{v_c} \right|_{v_d=0} = -\frac{1}{2g_{m3}r_{os}} \quad A_d = \left. \frac{v_{out}}{v_d} \right|_{v_c=0} = g_{m1}(r_{d2} // r_{d4})$$

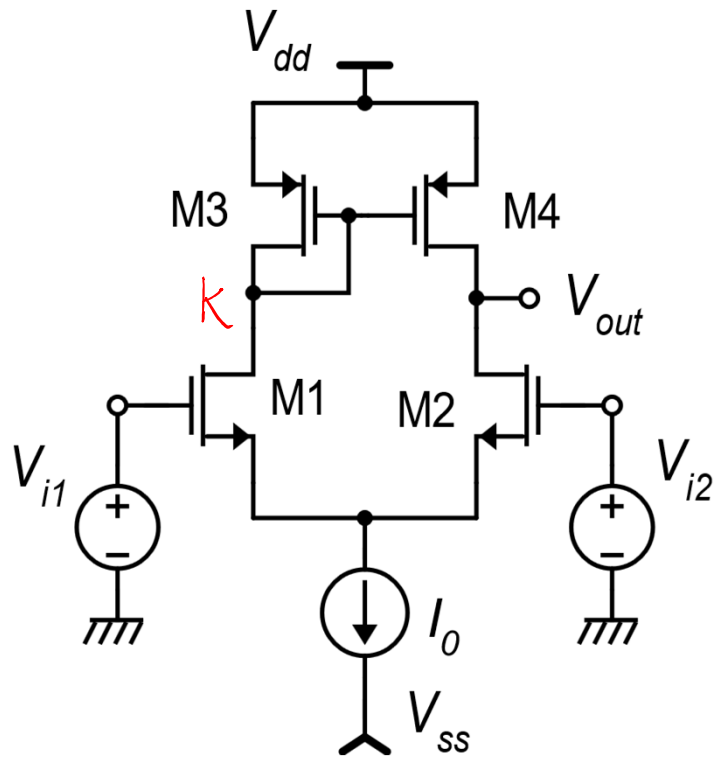
$$CMRR = \left| \frac{A_d}{A_c} \right| = 2g_{m3}r_{os}g_{m1}(r_{d2} // r_{d4}) \approx (g_m r_d)^2$$

for $r_{os}=r_d$
 (worst case: the tail is the output branch of a simple current mirror)
 and considering all g_m 's and r_d 's equal for simplicity



A CMRR of 80 dB can be easily reached

Large-signal dc transfer function



$$V_{i1} = V_C + \frac{V_D}{2}$$

$$V_{i2} = V_C - \frac{V_D}{2}$$

$$V_C = \text{constant}$$

For $V_D=0$:

$$V_{out}(0) = V_k(0) = V_{DD} - |V_{GS3}|$$

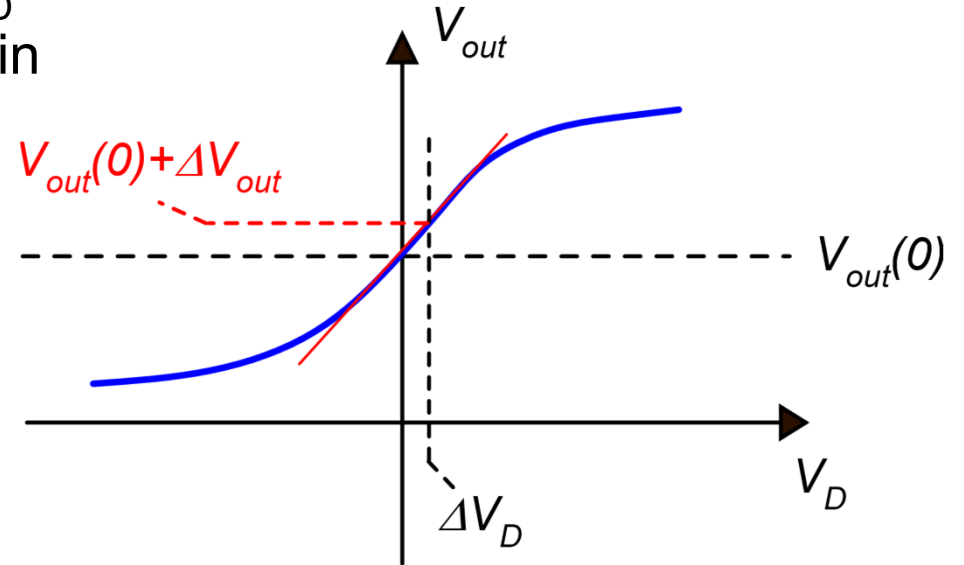
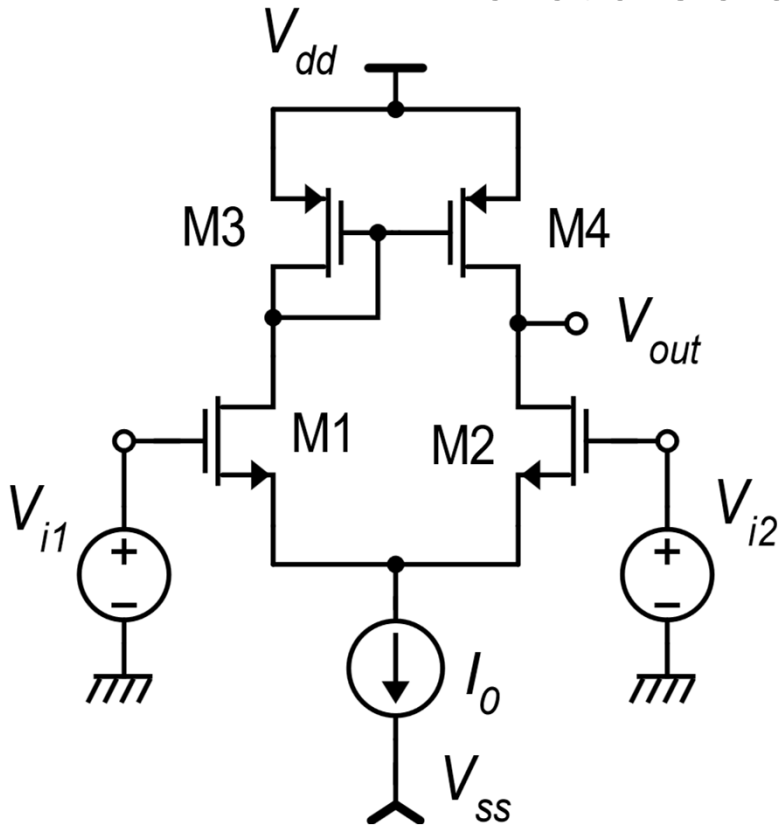
$$|V_{GS3}| = |V_{tp}| + |V_{GS} - V_{tp}|_3$$

In strong inversion

$$\sqrt{\frac{2I_{D3}}{\beta_3}}$$

Large-signal dc transfer function

V_{out} variations for small V_D variations around the origin

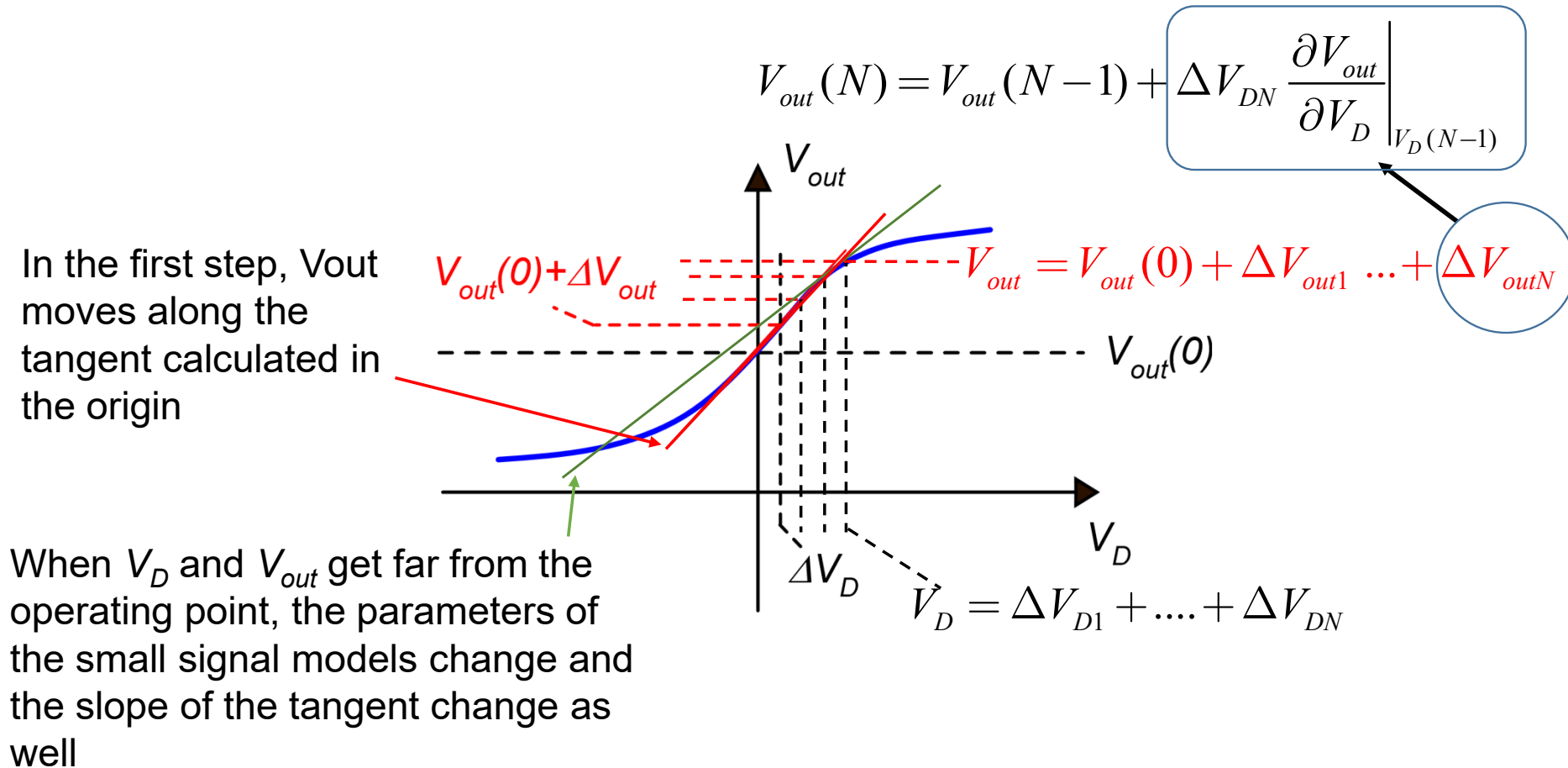


$$\Delta V_{out} = R_{out} \Delta I_{OCC}$$

$$\Delta I_{OCC} = \Delta V_D \left(\frac{dI_{occ}}{dV_D} \right)_{V_D=0}$$

g_{m1}

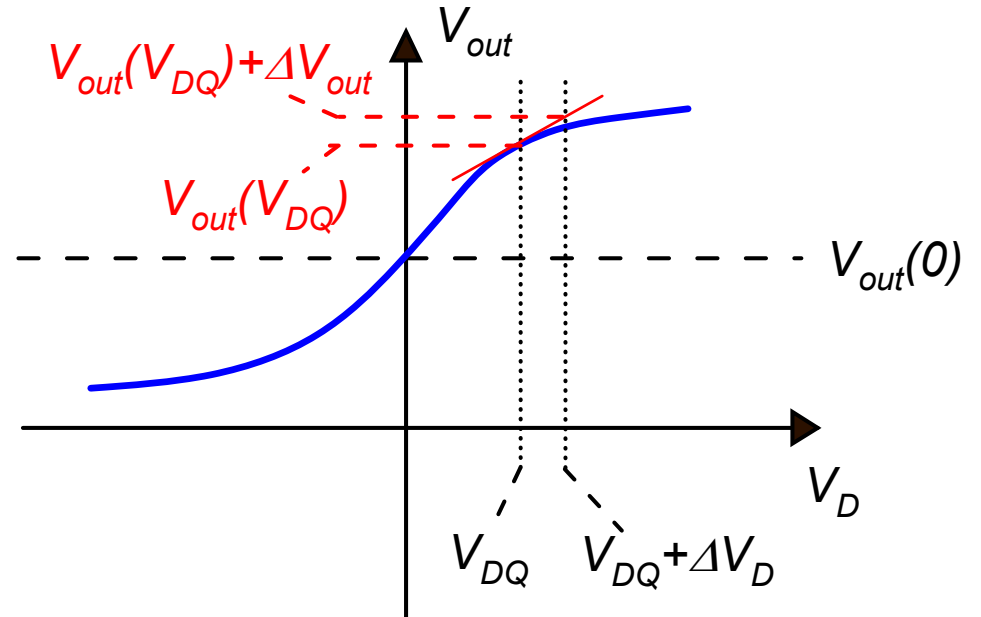
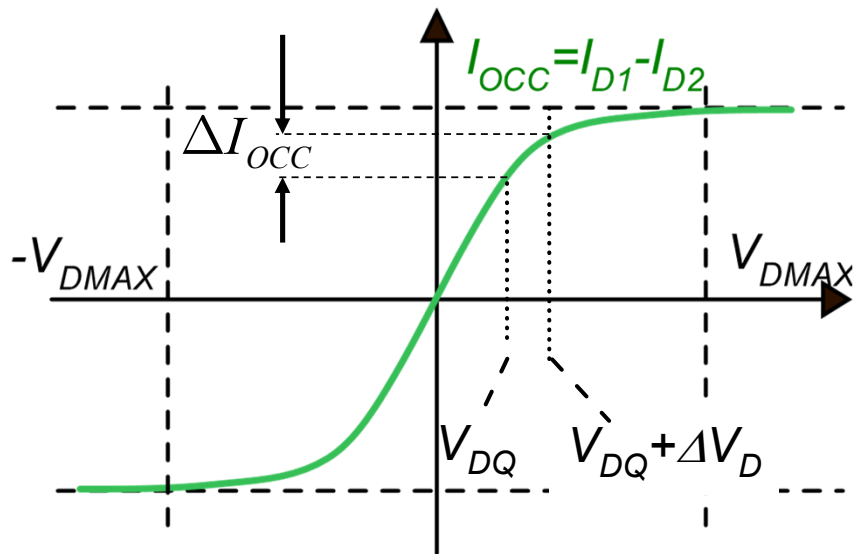
Moving away from the origin, step by step



Large-signal dc transfer function

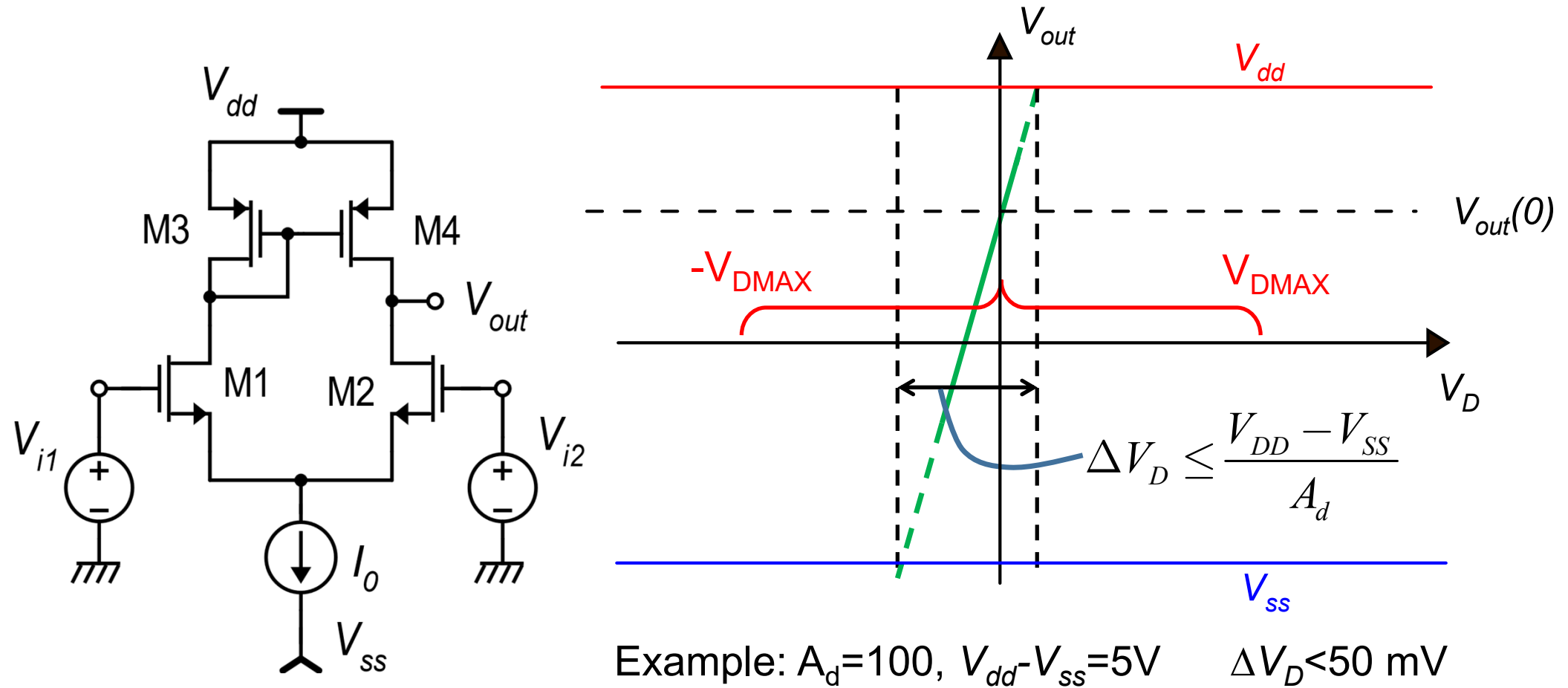
$$\Delta V_{out} = R_{out}(V_{DQ}) \Delta I_{OCC}$$

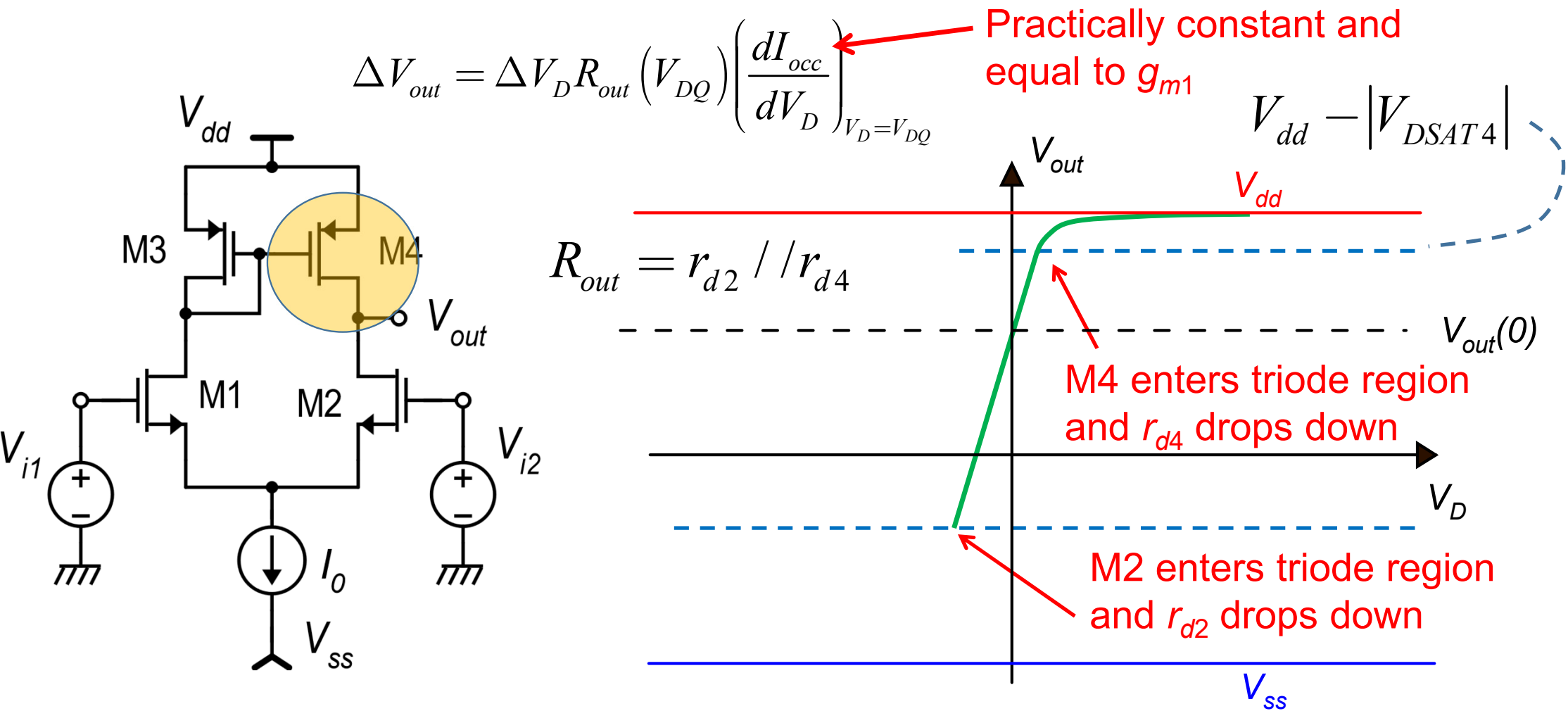
$$\Delta I_{OCC} = \Delta V_D \left(\frac{dI_{occ}}{dV_D} \right)_{V_D=V_{DQ}}$$



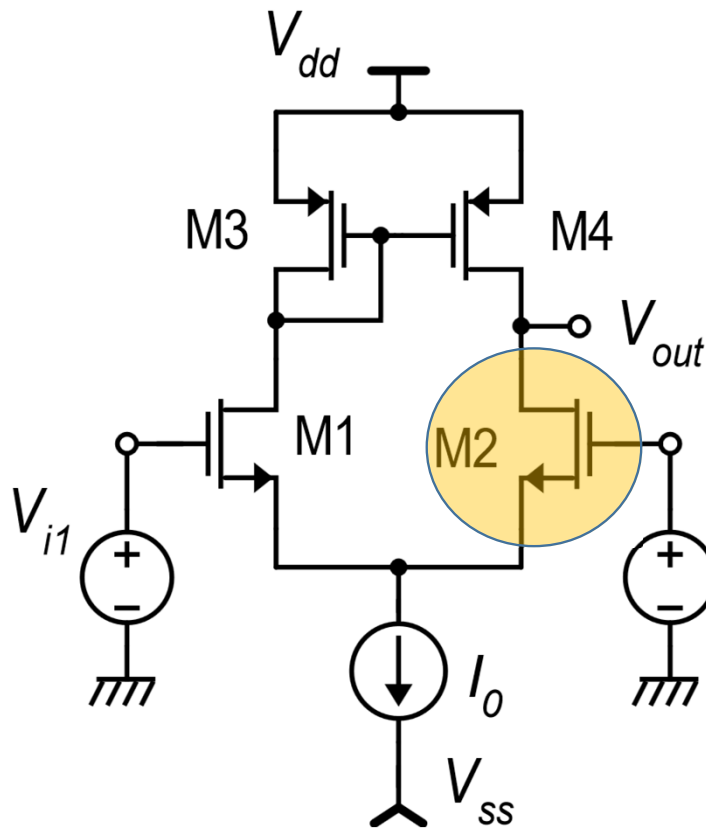
$$\Delta V_{out} = \Delta V_D R_{out}(V_{DQ}) \left(\frac{dI_{occ}}{dV_D} \right)_{V_D=V_{DQ}}$$

Approximate dc transfer characteristic





Minimum output voltage



$$V_{DS2} \geq V_{DSAT2}$$

$$V_{DS2} = V_{out} - V_{S2} = V_{out} - (V_{i2} - V_{GS2}) \geq V_{DSAT2}$$

$$V_{out} \geq V_{i2} - V_{GS2} + V_{DSAT2}$$

$$V_C - \frac{V_D}{2}$$

V_D in the linear zone is at most a few tens mV

V_C varies between V_{SS} and V_{DD}

We can neglect $V_D/2$ with respect to V_C

$$V_{out} \geq \underline{V_C - V_{GS2}} + V_{DSAT2}$$

Minimum output voltage

$$\min(V_{out}) = V_C - V_{GS2} + V_{DSAT2}$$

Strong inversion

$$V_{DSAT2} = V_{GS2} - V_{tn}$$
$$\min(V_{out}) = V_C - V_{tn}$$

Weak inversion

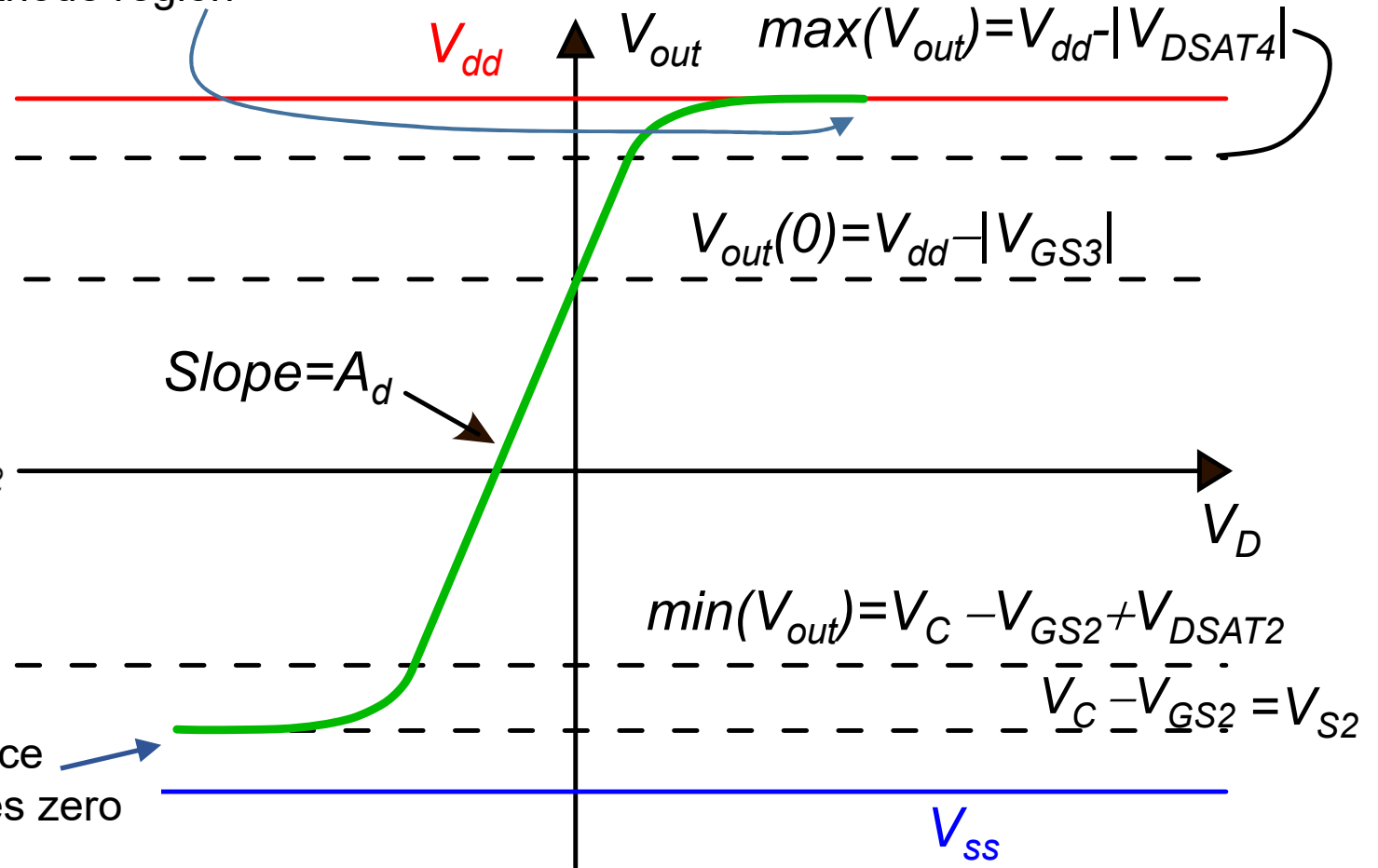
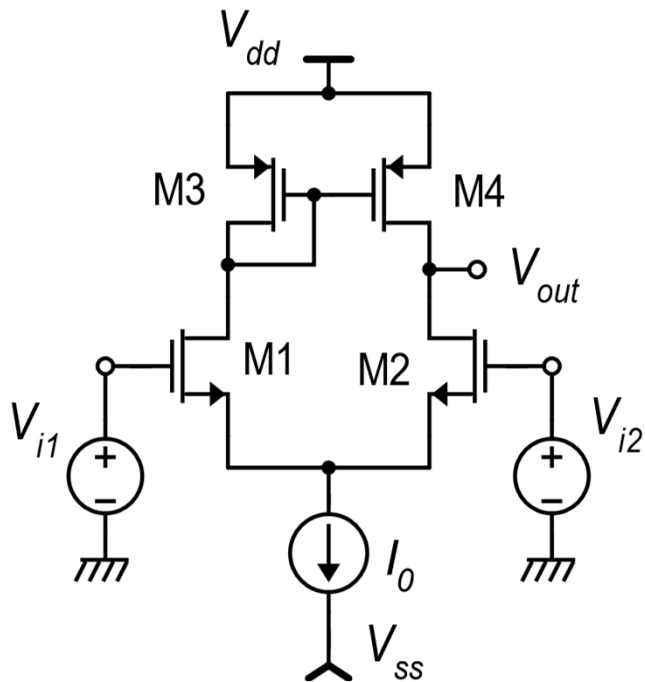
$$V_{DSAT2} \cong 100 \text{ mV}$$

$$V_{GS2} \cong V_{tn}$$

$$\min(V_{out}) \cong V_C - V_{tn} + \underline{100 \text{ mV}}$$

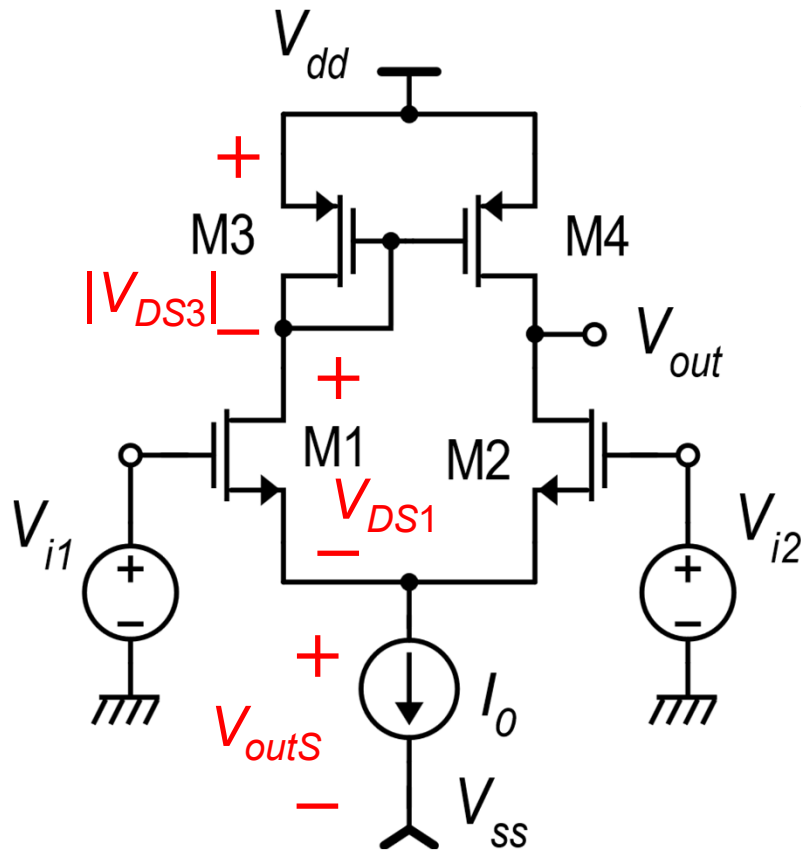
Complete dc transfer characteristic

V_{out} approaches V_{dd} when I_{D2} approaches zero and M4 gets in deep triode region



V_{out} gets close to M1, M2 source voltage when V_{DS2} approaches zero (M2 is in deep triode region)

Minimum supply voltage $V_{dd} - V_{ss}$



$$V_{dd} - V_{ss} = V_{outs} + V_{DS1} + |V_{DS3}|$$

$$V_{DS3} = V_{GS3} \quad \min(V_{outs}) = V_{MIN}$$

$$\min(V_{dd} - V_{ss}) = V_{MIN} + V_{DSAT1} + |V_{GS3}|$$

Example: $V_{MIN} = 100 \text{ mV}$

$V_{DSAT2} = 100 \text{ mV}$

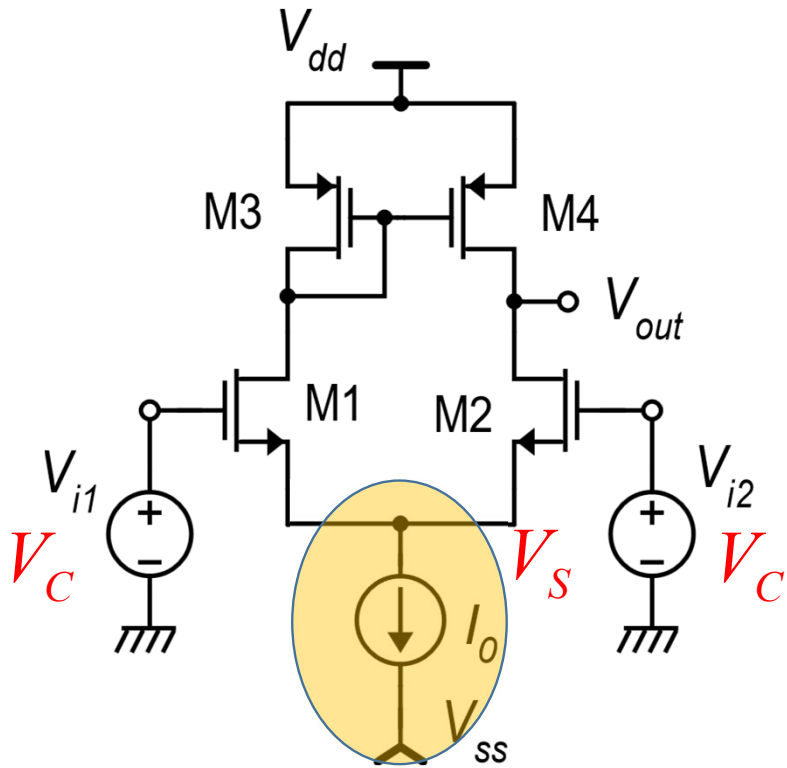
$|V_{GS3}| = 0.5 \text{ V}$

$$\min(V_{dd} - V_{ss}) \cong 0.7 \text{ V}$$

Input common mode range

Lower limit

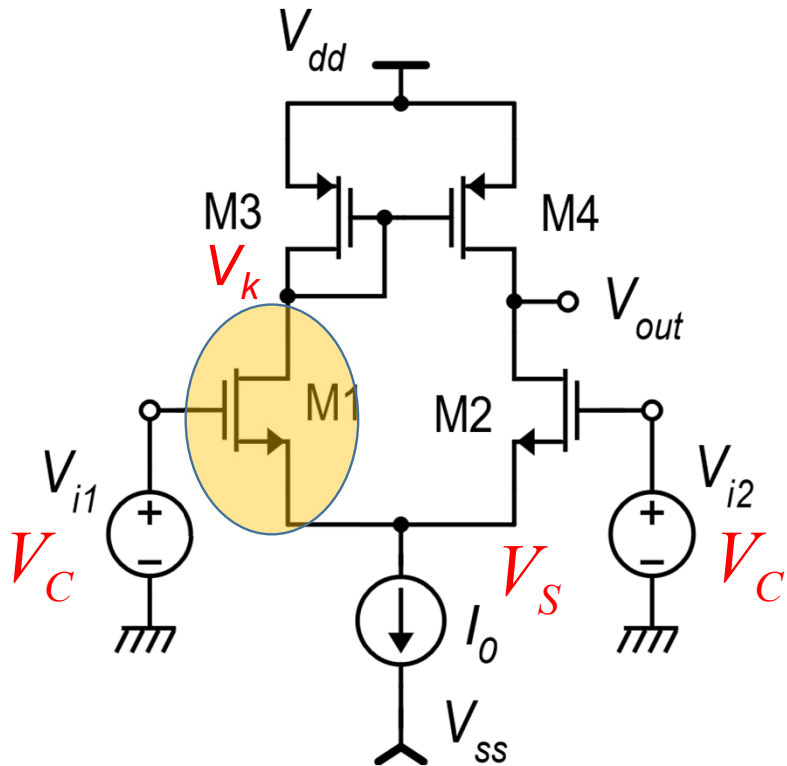
As V_C is progressively decreased, also V_S decreases at the same pace and eventually the voltage across the current source I_0 will get smaller than the minimum value V_{MIN} . From that point on, I_0 will rapidly decrease, turning off the stage.



$$\min(V_C) = V_{SS} + V_{MIN} + V_{GS1}$$

Input common mode range

Upper limit



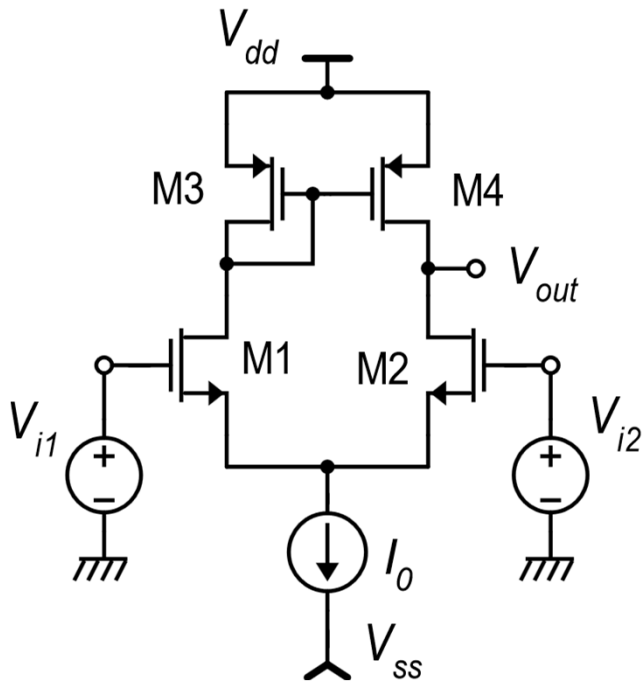
As V_C is progressively increased, also V_S increases at the same pace. Since $V_{out} = V_K = V_{D2} = V_{D1}$ is fixed, eventually V_{DS1} and V_{DS2} will drop below the saturation voltage.

$$V_{DS1} = V_K - V_{S1} \geq V_{DSAT1}$$

$$\overbrace{V_{dd} - |V_{GS3}|} - \overbrace{(V_C - V_{GS1})} \geq V_{DSAT1}$$

$$V_{dd} - |V_{GS3}| + V_{GS1} - V_{DSAT1} \geq V_C$$

Input common mode range



$$V_{dd} - |V_{GS3}| + V_{GS1} - V_{DSAT1} \geq V_C$$

$$|V_{GS3}| = |V_{tp3}| + |V_{GS3} - V_{tp3}|$$

$$V_{GS1} = V_{tn1} + (V_{GS1} - V_{tn1})$$

These overdrive voltages can be made equal by design, so they cancel each other

$$\begin{aligned} \max(V_C) &= \\ &= V_{dd} - \underbrace{|V_{tp3}| + V_{tn1}}_{\text{cancel}} - \underbrace{|V_{GS3} - V_{tp3}| + (V_{GS1} - V_{tn1})}_{\text{cancel}} - V_{DSAT1} \end{aligned}$$

This difference can be >0 because: $V_{tn1} > |V_{tp3}|$ (V_{t1} is affected by body effect if M1 and M2 body is at V_{ss})

The input common mode voltage can get even slightly higher than V_{dd}