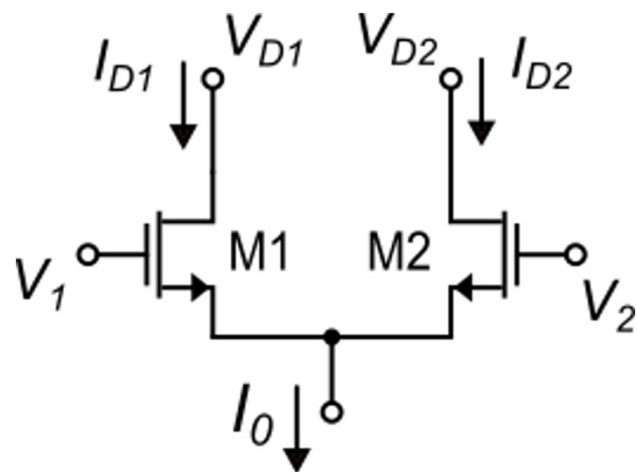


Source coupled MOSFET pair (differential MOSFET pair)

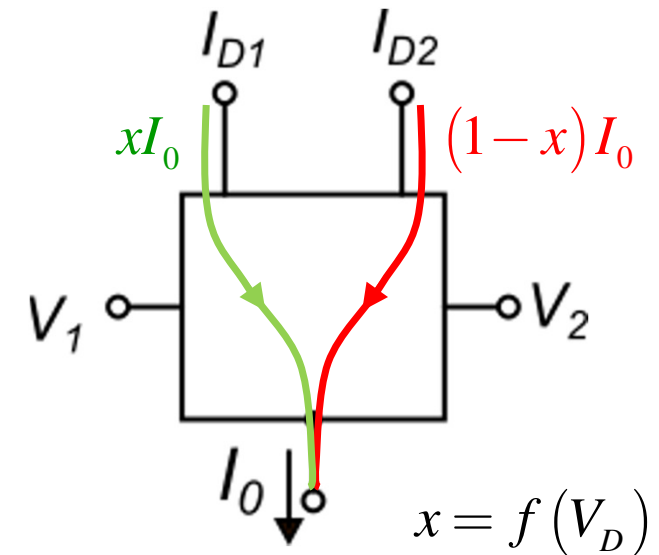


Inputs: V_1, V_2 (effective input signal: $V_D = V_1 - V_2$)
 I_0 ("Tail current")

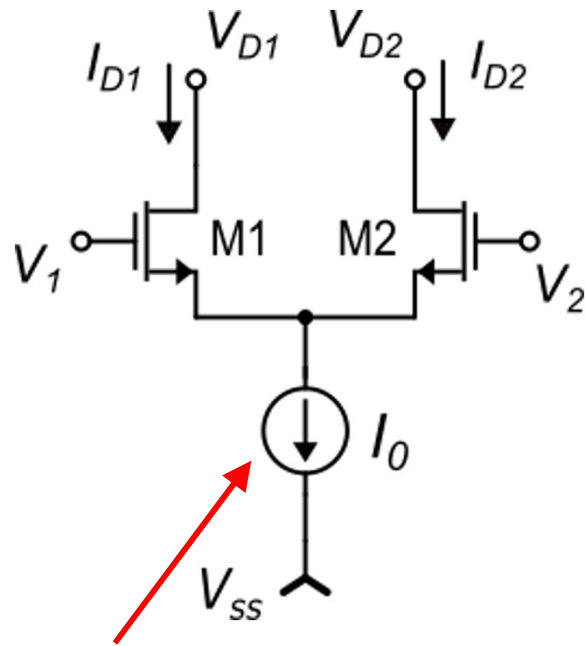
Outputs: I_{D1}, I_{D2} ($I_{D1} - I_{D2}$)

Function

- The input current I_0 is split into I_{D1} and I_{D2} , according to fractions x and $(1-x)$
- x depends on V_D ($x=0.5$ for $V_D=0$)



Analysis of the source coupled MOSFET pair (differential MOSFET pair)



Target: obtain the relationship between the ratio x and the input differential voltage V_D

$$I_{D1} = xI_0 \quad I_{D2} = (1-x)I_0 \quad \Rightarrow \quad x \triangleq \frac{I_{D1}}{I_0}$$

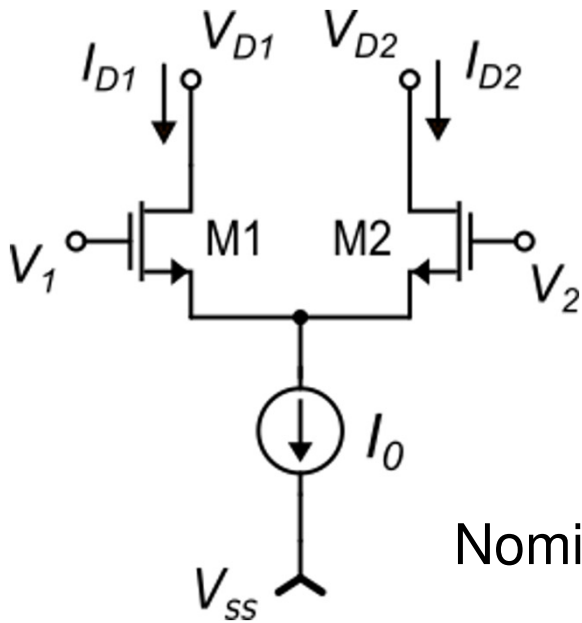
Hypotheses

- M1 and M2 work in saturation and the effect of V_{DS} can be neglected
- Strong inversion equation can be applied
- I_0 does not depend on V_1, V_2

I_0 has been represented as an ideal current source placed between M1, M2 sources and V_{SS} : this is just an example.

$$I_D = \frac{\beta}{2}(V_{GS} - V_t)^2 \Rightarrow V_{GS} = V_t + \sqrt{\frac{2I_D}{\beta}}$$

Analysis of the source coupled MOSFET pair (differential MOSFET pair)



$$\begin{cases} V_1 = V_{S1} + V_{GS1} \\ V_2 = V_{S2} + V_{GS2} \end{cases} \quad V_{S1} = V_{S2} \triangleq V_S$$

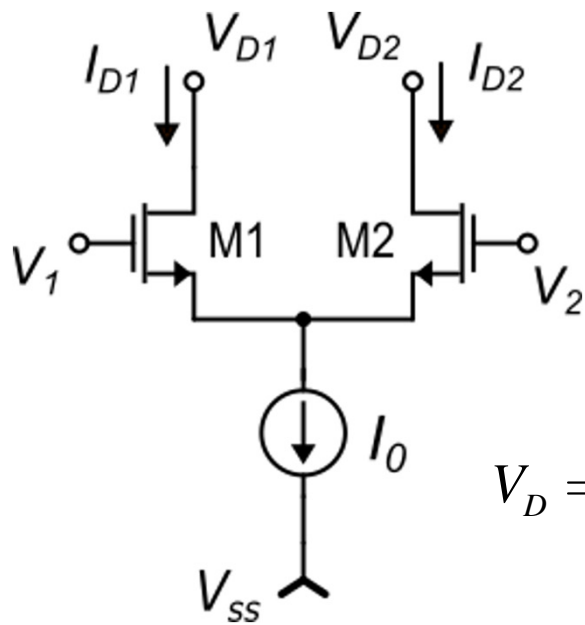
$$V_D = V_1 - V_2 = V_{GS1} - V_{GS2}$$

$$V_D = V_{t1} + \sqrt{\frac{2I_{D1}}{\beta_1}} - \left(V_{t2} + \sqrt{\frac{2I_{D2}}{\beta_2}} \right)$$

Nominal conditions: $M1=M2$: $\beta_1=\beta_2=\beta$ and $V_{t1}=V_{t2}$ ($V_{BS1}=V_{BS2}$).

$$V_D = \sqrt{\frac{2I_{D1}}{\beta}} - \sqrt{\frac{2I_{D2}}{\beta}} = \sqrt{\frac{2}{\beta}} \cdot (\sqrt{I_{D1}} - \sqrt{I_{D2}})$$

Analysis of the source coupled MOSFET pair (differential MOSFET pair)



$$V_D = \sqrt{\frac{2I_{D1}}{\beta}} - \sqrt{\frac{2I_{D2}}{\beta}} = \sqrt{\frac{2}{\beta}} \cdot (\sqrt{I_{D1}} - \sqrt{I_{D2}})$$

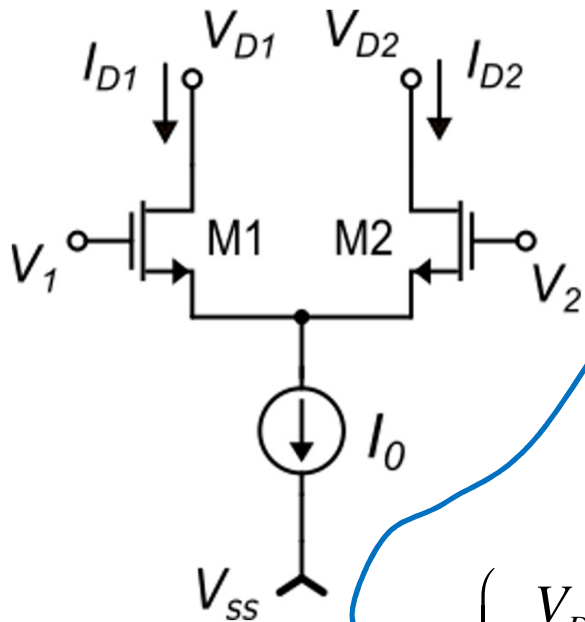
$$I_{D1} = xI_0 \quad I_{D2} = (1-x)I_0$$

$$V_D = \sqrt{\frac{2}{\beta}} \cdot [\sqrt{xI_0} - \sqrt{(1-x)I_0}] = \boxed{\sqrt{\frac{2I_0}{\beta}}} \cdot [\sqrt{x} - \sqrt{(1-x)}]$$

$\sqrt{\frac{2I_0}{\beta}} \triangleq V_{DMAX}$

$$\frac{V_D}{V_{DMAX}} = [\sqrt{x} - \sqrt{(1-x)}]$$

Analysis of the source coupled MOSFET pair (differential MOSFET pair)



$$\frac{V_D}{V_{DMAX}} = \left[\sqrt{x} - \sqrt{(1-x)} \right]$$

To solve this equation, we have to square both hands. In this way we add new solutions that are not valid.

Before squaring, we have to note that:

$$V_D > 0 \Rightarrow \sqrt{x} > \sqrt{(1-x)}$$

$$V_D > 0 \Rightarrow x > (1-x)$$

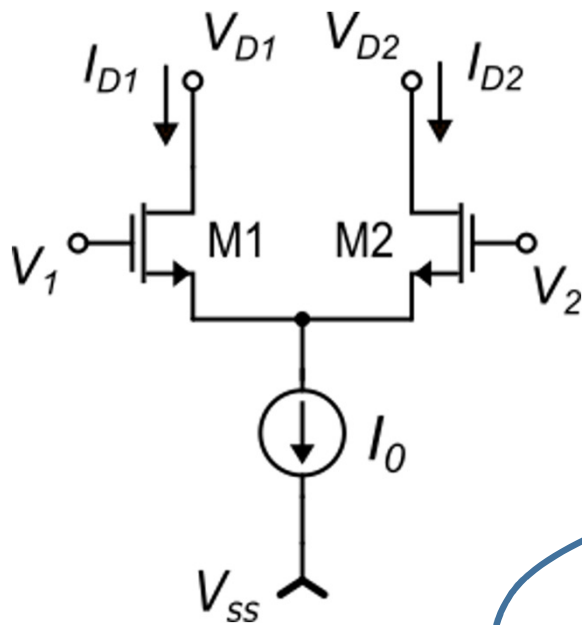
$$V_D > 0 \Rightarrow x > \frac{1}{2}$$

condition 1

$$\left(\frac{V_D}{V_{DMAX}} \right)^2 = x + (1-x) - 2\sqrt{x(1-x)}$$

$$\left(\frac{V_D}{V_{DMAX}} \right)^2 - 1 = -2\sqrt{x(1-x)}$$

Analysis of the source coupled MOSFET pair (differential MOSFET pair)



$$\left(\frac{V_D}{V_{DMAX}}\right)^2 - 1 = -2\sqrt{x(1-x)}$$

We have to square this equation again.

This time, the condition is:

$$\left[\left(\frac{V_D}{V_{DMAX}}\right)^2 - 1\right]^2 = 4x(1-x)$$

$$V_D > 0 \Rightarrow x > \frac{1}{2}$$

condition 1

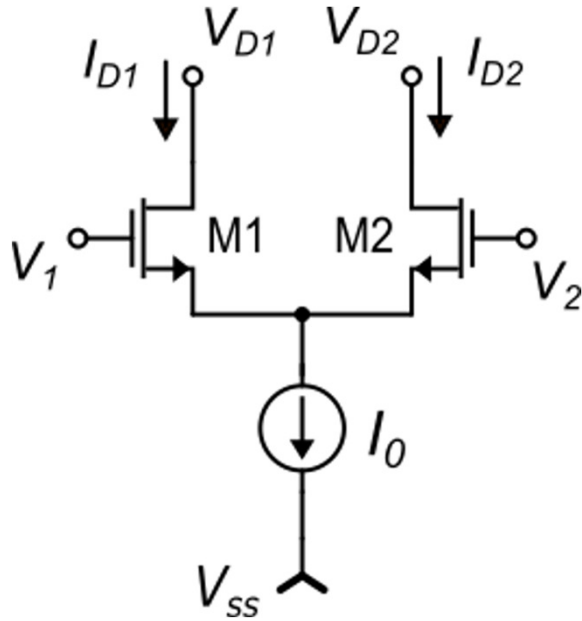
$$\left[\left(\frac{V_D}{V_{DMAX}}\right)^2 - 1\right]^2 \leq 0$$

condition 2

$$\frac{1}{4} \left[\left(\frac{V_D}{V_{DMAX}}\right)^2 - 1\right]^2 = x - x^2$$

$$x^2 - x + \frac{1}{4} \left[\left(\frac{V_D}{V_{DMAX}}\right)^2 - 1\right]^2 = 0$$

Analysis of the source coupled MOSFET pair (differential MOSFET pair)



$$V_D > 0 \Rightarrow x > \frac{1}{2}$$

condition 1

$$\left(\frac{V_D}{V_{DMAX}} \right)^2 - 1 \leq 0$$

condition 2

$$x^2 - x + \frac{1}{4} \left[\left(\frac{V_D}{V_{DMAX}} \right)^2 - 1 \right]^2 = 0$$

$$x = -\frac{b}{2a} \pm \frac{1}{2a} \sqrt{b^2 - 4ac}$$

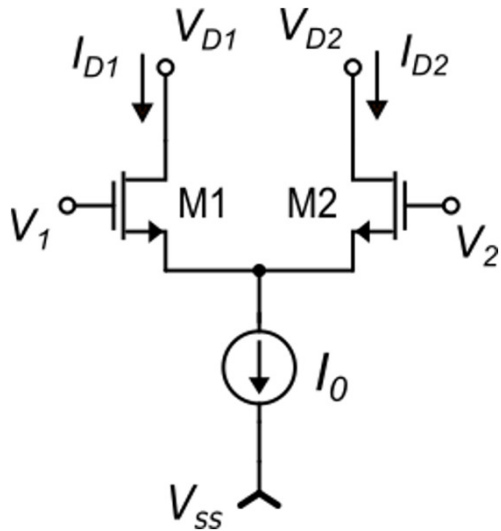
$$ax^2 + bx + c$$

$$a = 1, b = -1, c = \frac{1}{4} \left[\left(\frac{V_D}{V_{DMAX}} \right)^2 - 1 \right]^2$$

$$x = \frac{1}{2} \pm \frac{1}{2} \sqrt{1 - \left[\left(\frac{V_D}{V_{DMAX}} \right)^2 - 1 \right]^2}$$

$$x = \frac{1}{2} \pm \frac{1}{2} \sqrt{1 - \left[\left(\frac{V_D}{V_{DMAX}} \right)^4 - 2 \left(\frac{V_D}{V_{DMAX}} \right)^2 + 1 \right]}$$

Analysis of the source coupled MOSFET pair (differential MOSFET pair)



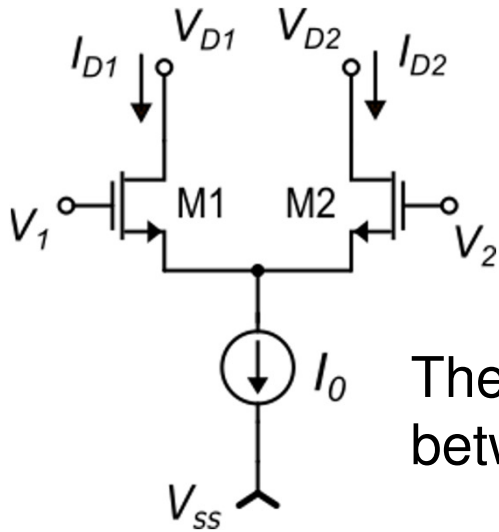
$$x = \frac{1}{2} \pm \frac{1}{2} \sqrt{2 \left(\frac{V_D}{V_{DMAX}} \right)^2 - \left(\frac{V_D}{V_{DMAX}} \right)^4} = \frac{1}{2} \pm \frac{1}{2} \left(\frac{V_D}{V_{DMAX}} \right) \sqrt{2 - \left(\frac{V_D}{V_{DMAX}} \right)^2}$$

condition 1

$$V_D > 0 \Rightarrow x > \frac{1}{2}$$

+

$$\left\{ \begin{array}{l} x = \frac{I_{D1}}{I_0} = \frac{1}{2} + \frac{1}{2} \left(\frac{V_D}{V_{DMAX}} \right) \sqrt{2 - \left(\frac{V_D}{V_{DMAX}} \right)^2} \\ 1 - x = \frac{I_{D2}}{I_0} = \frac{1}{2} - \frac{1}{2} \left(\frac{V_D}{V_{DMAX}} \right) \sqrt{2 - \left(\frac{V_D}{V_{DMAX}} \right)^2} \end{array} \right. \Rightarrow \left\{ \begin{array}{l} I_{D1} = \frac{I_0}{2} + \frac{I_0}{2} \left(\frac{V_D}{V_{DMAX}} \right) \sqrt{2 - \left(\frac{V_D}{V_{DMAX}} \right)^2} \\ I_{D2} = \frac{I_0}{2} - \frac{I_0}{2} \left(\frac{V_D}{V_{DMAX}} \right) \sqrt{2 - \left(\frac{V_D}{V_{DMAX}} \right)^2} \end{array} \right.$$



$$\left(\frac{V_D}{V_{DMAX}} \right)^2 - 1 \leq 0$$

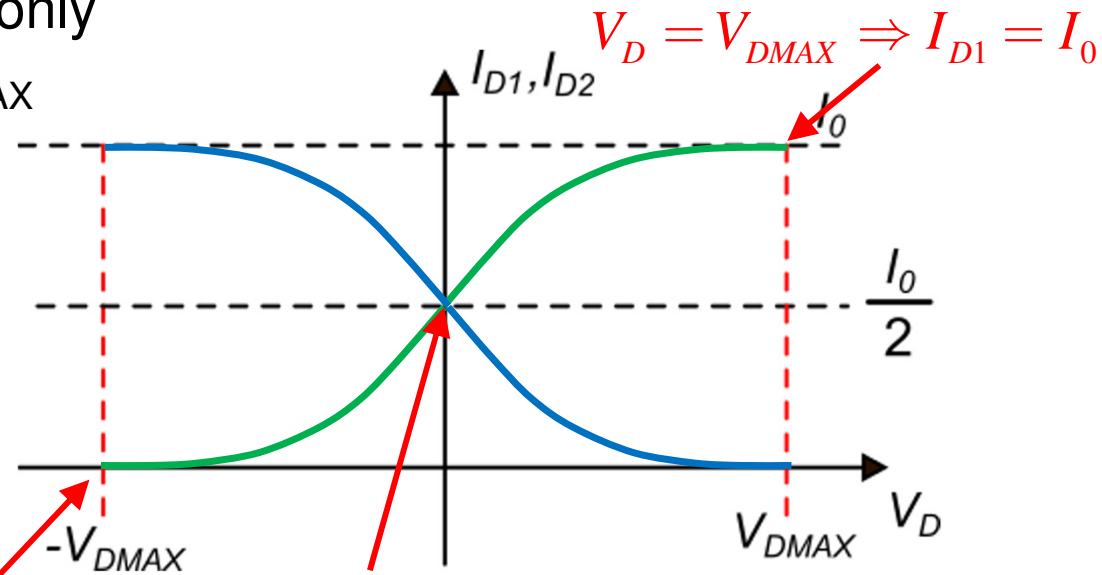
condition 2

$$\left| \frac{V_D}{V_{DMAX}} \right| \leq 1 \Rightarrow -V_{DMAX} \leq V_D \leq V_{DMAX}$$

The analysis is applicable only between $-V_{DMAX}$ and $+V_{DMAX}$

$$I_{D1} = \frac{I_0}{2} + \frac{I_0}{2} \left(\frac{V_D}{V_{DMAX}} \right) \sqrt{2 - \left(\frac{V_D}{V_{DMAX}} \right)^2}$$

$$I_{D2} = \frac{I_0}{2} - \frac{I_0}{2} \left(\frac{V_D}{V_{DMAX}} \right) \sqrt{2 - \left(\frac{V_D}{V_{DMAX}} \right)^2}$$



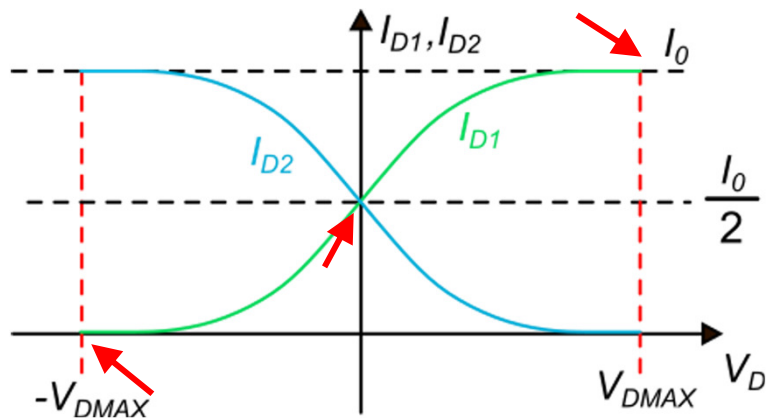
$$V_D = -V_{DMAX} \Rightarrow I_{D1} = 0 \qquad V_D = 0 \Rightarrow I_{D1} = \frac{I_0}{2}$$

$$V_D = V_{DMAX} \Rightarrow I_{D1} = I_0$$

Derivative of the differential pair input-output curves (optional)

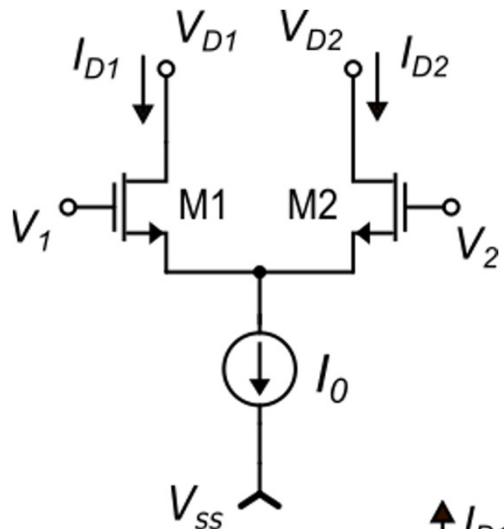
$$I_{D1} = \frac{I_0}{2} + \frac{I_0}{2} \left(\frac{V_D}{V_{DMAX}} \right) \sqrt{2 - \left(\frac{V_D}{V_{DMAX}} \right)^2} = \frac{I_0}{2} \left[1 + z \sqrt{2 - z^2} \right] \quad \text{with } z = \left(\frac{V_D}{V_{DMAX}} \right)$$

$$\frac{dI_{D1}}{dV_D} = \frac{I_0}{2V_{DMAX}} \left[\sqrt{2 - z^2} - \frac{2z^2}{2\sqrt{2 - z^2}} \right] = \frac{I_0}{2V_{DMAX}} \frac{2 - z^2 - z^2}{\sqrt{2 - z^2}} = \frac{I_0}{V_{DMAX}} \frac{1 - z^2}{\sqrt{2 - z^2}}$$



$$\left\{ \begin{array}{l} V_D = \pm V_{DMAX} \Rightarrow \frac{dI_{D1}}{dV_D} = 0 \quad \sqrt{\frac{2I_0}{\beta}} \triangleq V_{DMAX} \\ V_D = 0 \Rightarrow \frac{dI_{D1}}{dV_D} = \frac{I_0}{\sqrt{2}V_{DMAX}} = \sqrt{\frac{I_0^2 \beta}{4I_0}} = \frac{1}{2} \sqrt{\beta I_0} \end{array} \right.$$

Extrapolation outside the $-V_{DMAX} \leq V_D \leq V_{DMAX}$ region

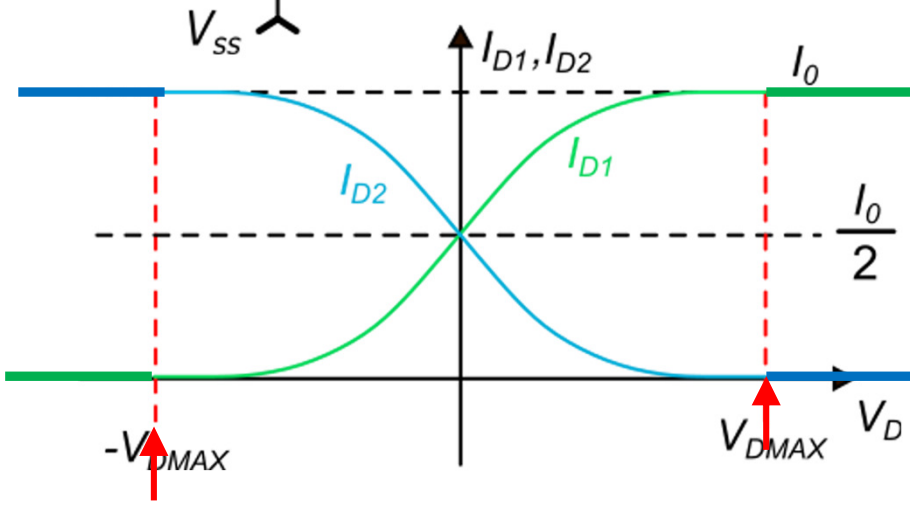


$$V_D = V_{GS1} - V_{GS2} \Rightarrow V_{GS2} = V_{GS1} - V_D$$

Considering the boundary:

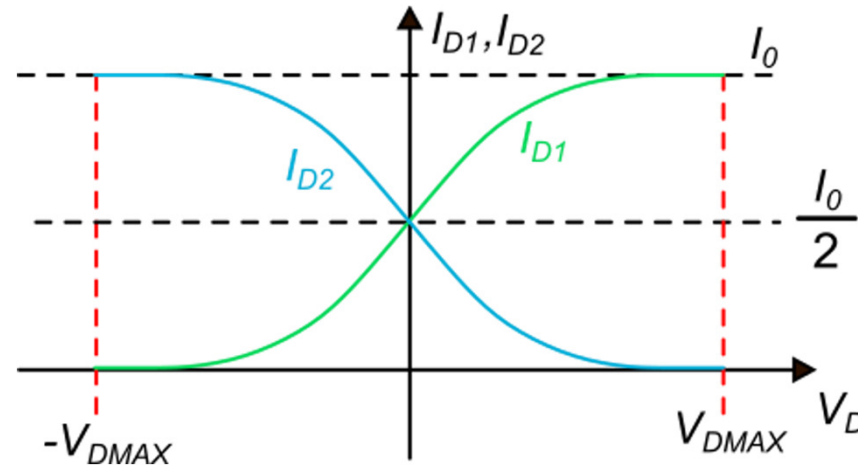
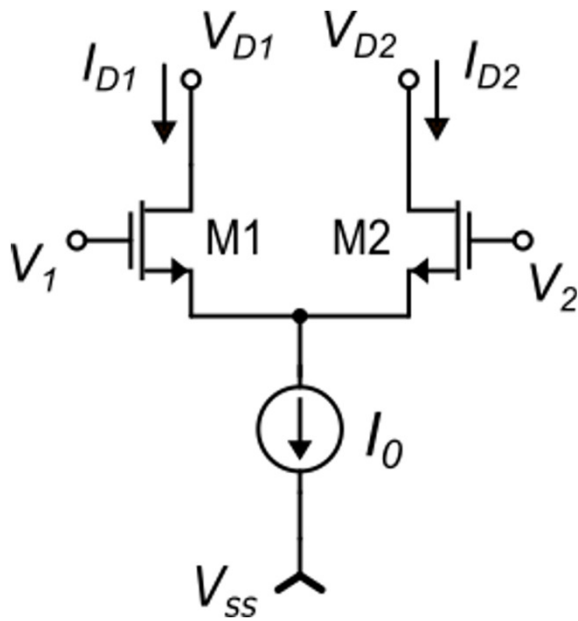
$$V_D = V_{DMAX} \quad I_{D1} = I_0 \Rightarrow V_{GS1} = V_t + \sqrt{\frac{2I_0}{\beta}} \quad V_{DMAX}$$

$$\begin{cases} V_{GS1} = V_t + V_{DMAX} \\ V_{GS2} = V_{GS1} - V_D = V_t \end{cases}$$



If V_D increases over V_{DMAX} :
 V_{GS1} cannot increase because I_{D1} would become $> I_0$, which is impossible.
 Then V_{GS2} gets smaller than V_t
 I_{D2} keeps being $=0$
 I_{D1} keeps being $= I_0$
 The opposite occurs when V_D decreases below V_{DMAX} :

Mosfet differential pair: parameters



$$V_{D\text{MAX}} = \sqrt{\frac{2I_0}{\beta}}$$

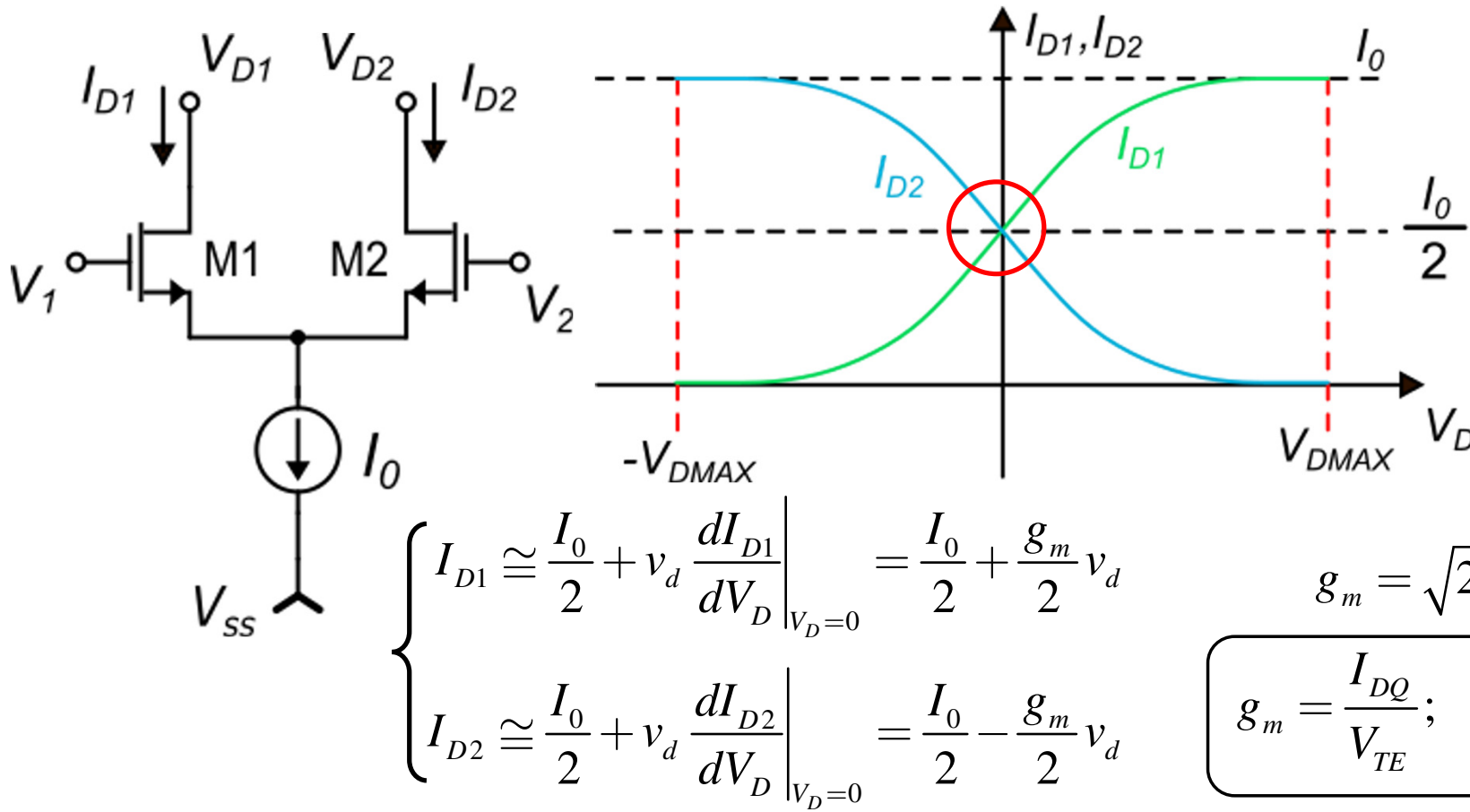
For $V_D=0$ (typical operating point): $I_{D1} = I_{D2} \triangleq I_{DQ} = \frac{I_0}{2}$

$$V_{D\text{MAX}} = \sqrt{\frac{2 \cdot 2I_{DQ}}{\beta}} = \sqrt{2} \sqrt{\frac{2I_{DQ}}{\beta}} = \underline{\underline{\sqrt{2} (V_{GS} - V_t)_Q}}$$

$$\left. \frac{dI_{D1}}{dV_D} \right|_{V_D=0} = \frac{1}{2} \sqrt{2\beta I_{DQ}} = \frac{g_m}{2}$$

Small signal behavior

For small variations (v_d) of V_D around 0:



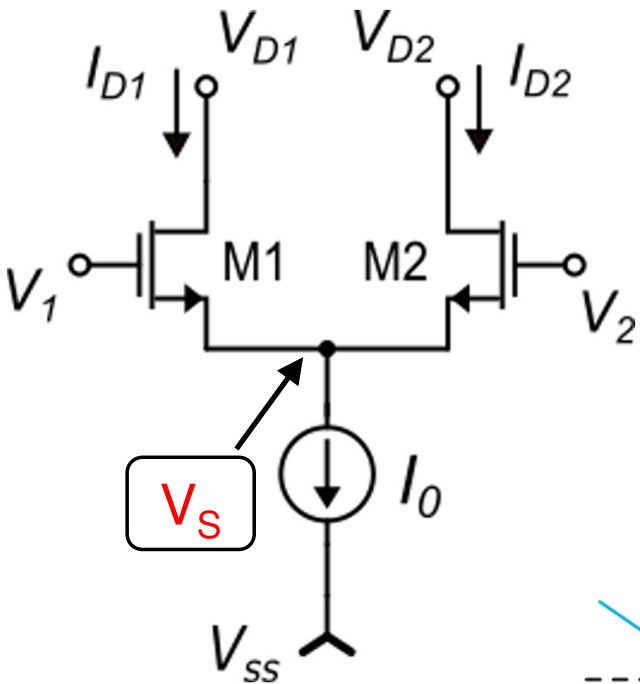
Check that this is consistent with other expressions of g_m

$$\begin{cases} I_{D1} \approx \frac{I_0}{2} + v_d \left. \frac{dI_{D1}}{dV_D} \right|_{V_D=0} = \frac{I_0}{2} + \frac{g_m}{2} v_d \\ I_{D2} \approx \frac{I_0}{2} + v_d \left. \frac{dI_{D2}}{dV_D} \right|_{V_D=0} = \frac{I_0}{2} - \frac{g_m}{2} v_d \end{cases}$$

$$g_m = \sqrt{2\beta I_{DQ}} = \sqrt{\beta I_0}$$

$$g_m = \frac{I_{DQ}}{V_{TE}}; \quad V_{TE} = \frac{V_{GS} - V_t}{2} = \frac{V_{DMAX}}{2\sqrt{2}}$$

Large-signal dependence of the source voltage on V_D



$$V_1 = V_C + \frac{V_D}{2}; \quad V_2 = V_C - \frac{V_D}{2}$$

V_C : common mode voltage = constant

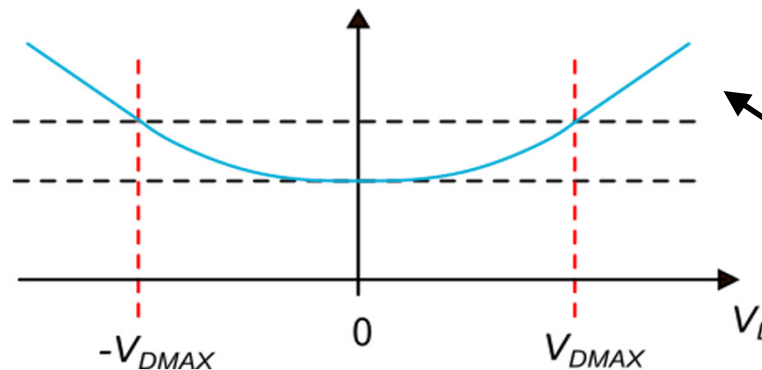
$$V_S = V_1 - V_{GS1} = V_2 - V_{GS2}$$

$$V_S|_{V_D=0} = V_C - V_t - (V_{GS} - V_t)_Q$$

$$V_{DMAX} = \sqrt{2} (V_{GS} - V_t)_Q$$

$$V_S|_{V_D=0} = V_C - V_t - \frac{V_{DMAX}}{\sqrt{2}} <$$

$$V_S|_{V_D=V_{DMAX}} = V_C - \frac{V_{DMAX}}{2} - V_t$$

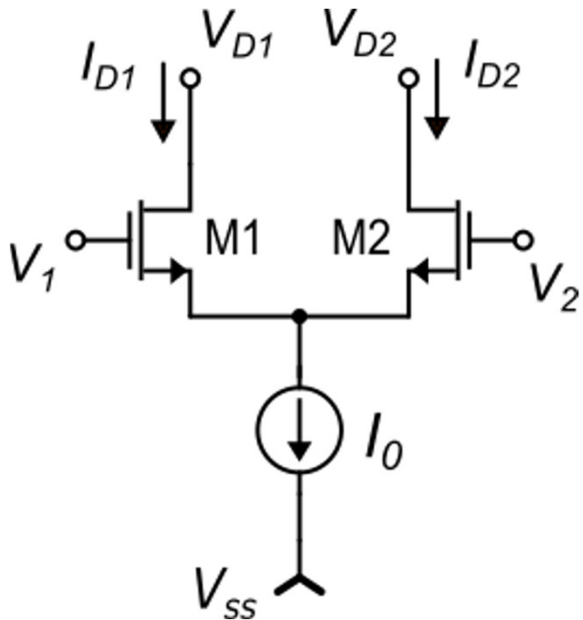


for $V_D > V_{DMAX}$

$$V_S = V_C + \frac{V_D}{2} - \underbrace{V_{GS1}}_{\text{constant}}$$

$$V_S = V_C + \frac{V_D}{2} - V_{DMAX} - V_t$$

Differential output current

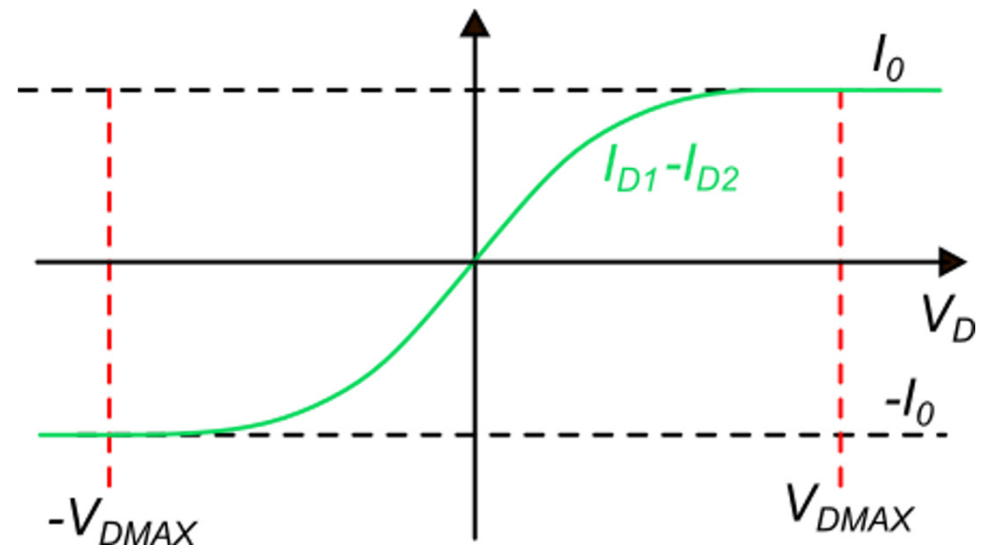


$$I_{D1} = \frac{I_0}{2} + \frac{I_0}{2} \left(\frac{V_D}{V_{DMAX}} \right) \sqrt{2 - \left(\frac{V_D}{V_{DMAX}} \right)^2}$$

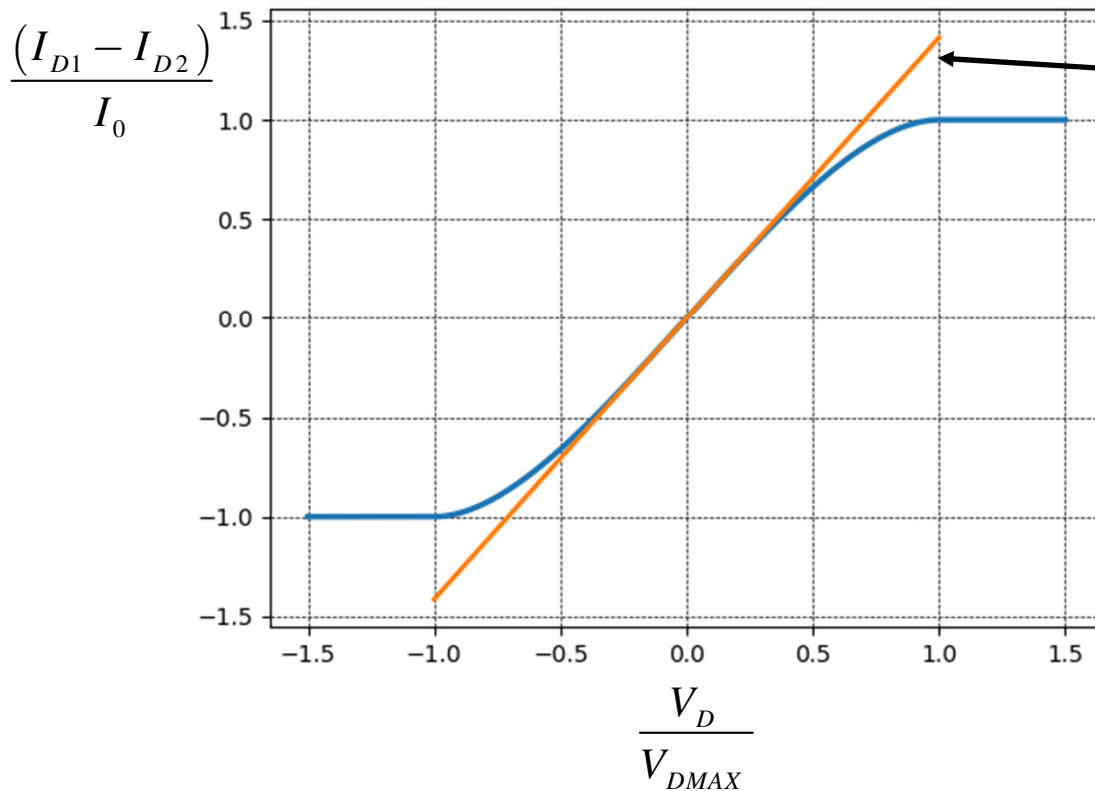
$$I_{D2} = \frac{I_0}{2} - \frac{I_0}{2} \left(\frac{V_D}{V_{DMAX}} \right) \sqrt{2 - \left(\frac{V_D}{V_{DMAX}} \right)^2}$$

$$I_{D1} - I_{D2} = I_0 \left(\frac{V_D}{V_{DMAX}} \right) \sqrt{2 - \left(\frac{V_D}{V_{DMAX}} \right)^2}$$

$$i_d = i_{d1} - i_{d2} = g_m v_d$$



Mosfet differential pair: real curves (calculated) and linearity



linear approximation
calculated around origin

Non-linearity error:

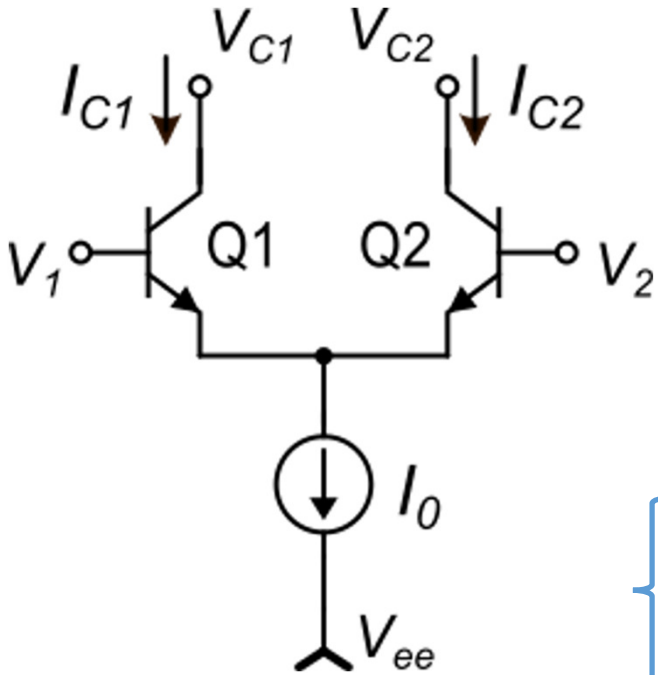
$$|V_D| < \frac{V_{DMAX}}{5} : e_R < 1\%$$

$$|V_D| < \frac{V_{DMAX}}{2\sqrt{2}} = \frac{V_{GS} - V_t}{2} : e_R < 3\%$$

$$|V_D| < \frac{V_{DMAX}}{2} : e_R < 7\%$$

$$|V_D| < \frac{V_{DMAX}}{\sqrt{2}} = (V_{GS} - V_t) : e_R < 15\%$$

BJT differential pair



$$I_{E1} + I_{E2} = I_0 \quad \Rightarrow \quad \frac{\beta + 1}{\beta} I_{C1} + \frac{\beta + 1}{\beta} I_{C2} = I_0$$

$$V_D = V_{BE1} - V_{BE2} \quad I_{C1} + I_{C2} = \frac{\beta}{\beta + 1} I_0 \cong I_0$$

$$\begin{cases} I_{C1} = I_S e^{\frac{V_{BE1}}{V_T}} \\ I_{C2} = I_S e^{\frac{V_{BE2}}{V_T}} \end{cases}$$

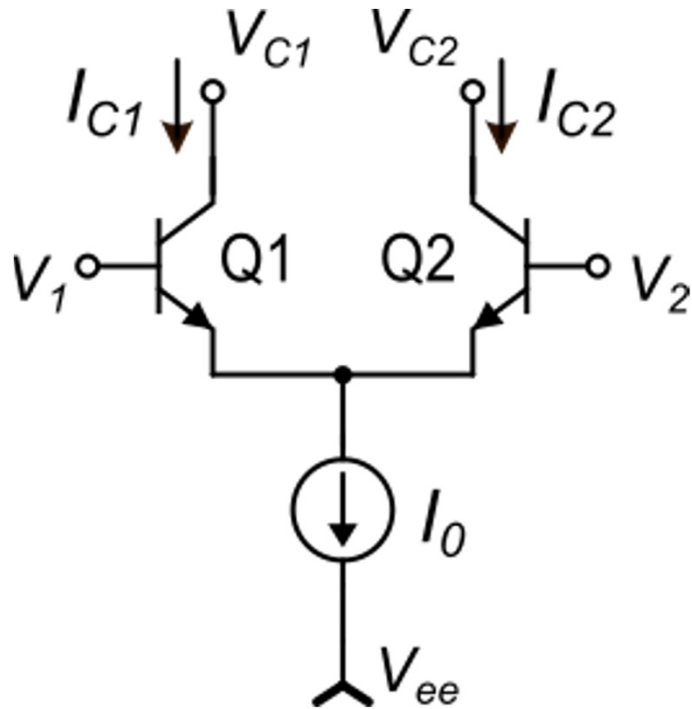
$$I_{C1} \left(1 + e^{\frac{-V_D}{V_T}} \right) = I_0$$

$$I_{C1} = \frac{I_0}{1 + e^{\frac{-V_D}{V_T}}}$$

$$I_{C2} = \frac{I_0 e^{\frac{-V_D}{V_T}}}{1 + e^{\frac{-V_D}{V_T}}}$$

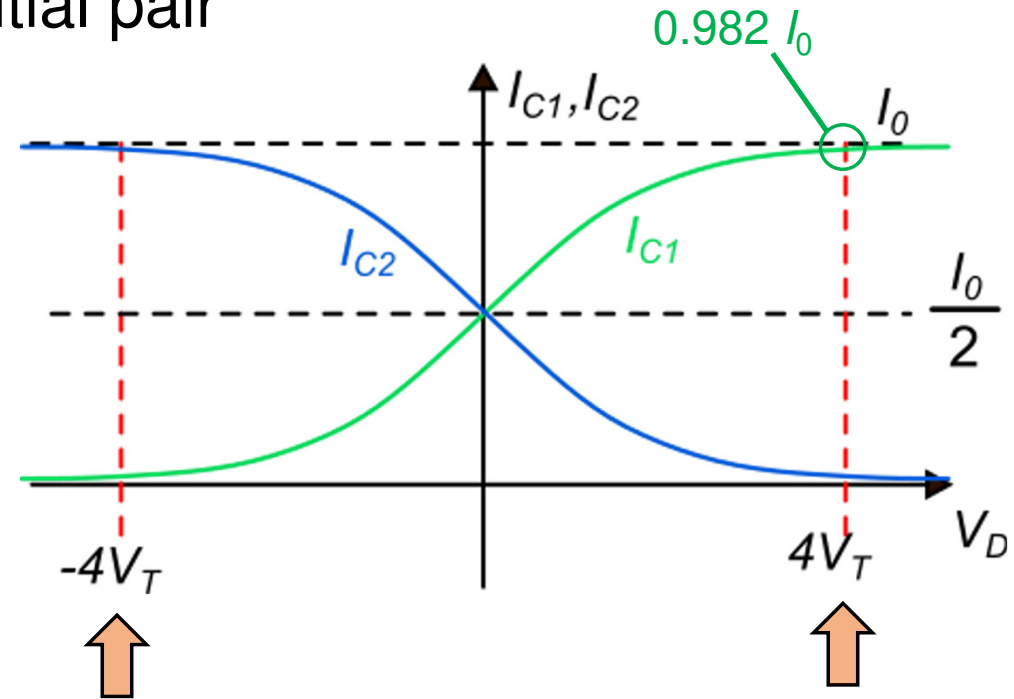
$$\frac{I_{C2}}{I_{C1}} = e^{\frac{V_{BE2} - V_{BE1}}{V_T}} = e^{\frac{-V_D}{V_T}}$$

BJT differential pair



$$I_{C1} = \frac{I_0}{1 + e^{\frac{-V_D}{V_T}}}$$

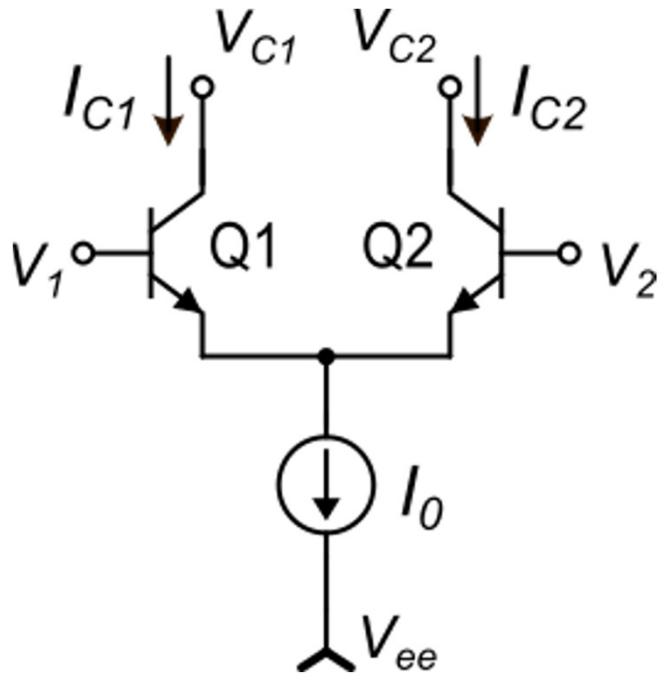
$$I_{C2} = \frac{I_0 e^{\frac{-V_D}{V_T}}}{1 + e^{\frac{-V_D}{V_T}}}$$



$$I_{C1} - I_{C2} = I_0 \frac{1 - e^{\frac{-V_D}{V_T}}}{1 + e^{\frac{-V_D}{V_T}}}$$

$$I_{C1} - I_{C2} = I_0 \frac{e^{\frac{-V_D}{2V_T}} (e^{\frac{V_D}{2V_T}} - e^{\frac{-V_D}{2V_T}})}{e^{\frac{-V_D}{2V_T}} (e^{\frac{V_D}{2V_T}} + e^{\frac{-V_D}{2V_T}})} = I_0 \tanh\left(\frac{V_D}{2V_T}\right)$$

BJT differential pair - small signal currents



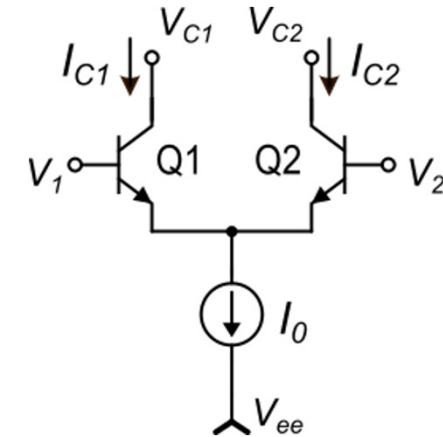
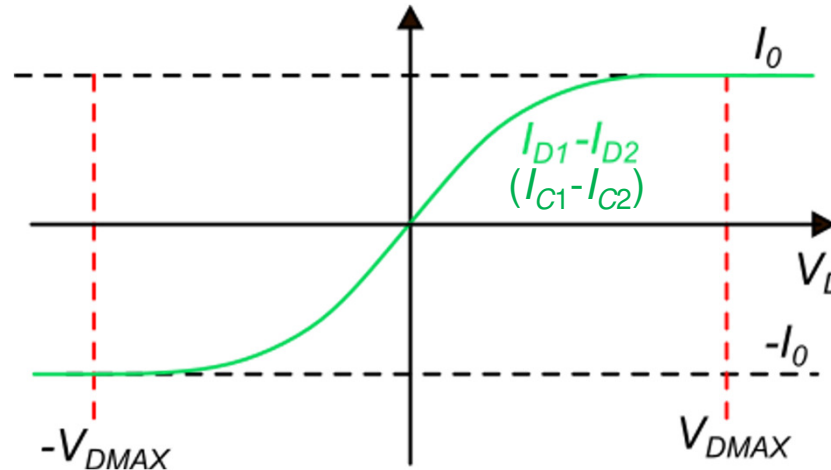
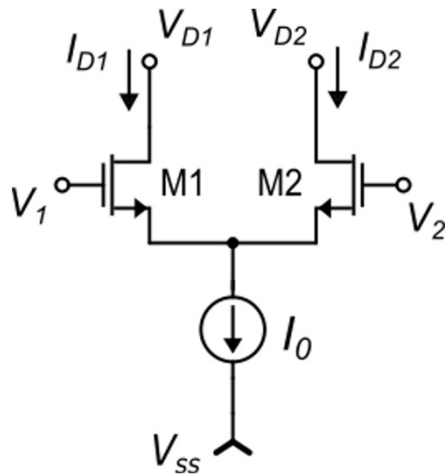
$$I_{C1} = \frac{I_0}{2} + \frac{g_m}{2} v_d$$

$$I_{C2} = \frac{I_0}{2} - \frac{g_m}{2} v_d$$

$$I_{C1} - I_{C2} = g_m v_d$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{I_0}{2V_T}$$

MOSFET and BJT differential pairs compared



In the mosfet pair, V_{DMAX} can be varied by modifying β and I_0

Parameter g_m depends on both I_0 and β :

$$g_m = \sqrt{\beta I_0} \quad V_{DMAX} = \sqrt{\frac{2I_0}{\beta}}$$

Note: a mosfet pair in subthreshold region behaves like a BJT pair with the substitution:

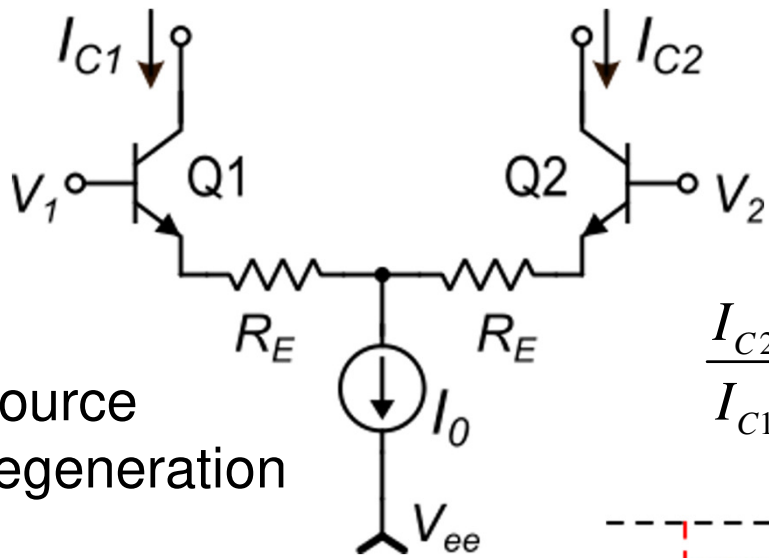
$$mV_T \rightarrow V_T$$

In the BJT pair, V_{DMAX} is fixed to around $4V_T$

Parameter g_m depends only on I_0

$$g_m = \frac{I_{CQ}}{V_T} = \frac{I_0}{2V_T}$$

How to increase the V_{DMAX} of a BJT pair



Source degeneration

$$V_{DMAX} = V'_{DMAX} + I_0 R_E$$

↑

$$\cong 4V_T$$

To demonstrate this, let us write V_D :

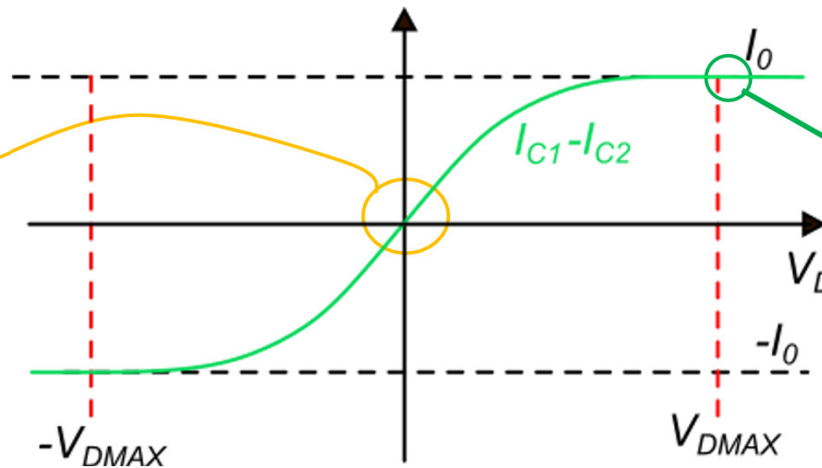
$$V_D = V_{BE1} + I_{C1} R_E - (V_{BE2} + I_{C2} R_E)$$

$$V_D = V_{BE1} - V_{BE2} + R_E (I_{C1} - I_{C2})$$

$$\frac{I_{C2}}{I_{C1}} = e^{\frac{V_{BE2} - V_{BE1}}{V_T}}$$

$$g_{m-rid} = \left. \frac{d(I_{C1} - I_{C2})}{dV_D} \right|_{V_D=0}$$

$$g_{m-rid} = \frac{g_m}{1 + g_m R_E} < g_m$$



$0.964 I_0$

$$V_{BE1} - V_{BE2} \cong 4V_T$$

$$I_{D1} - I_{D2} \cong I_{D1} \cong I_0$$