1 Output voltage of BJT Cascode and Wilson current mirrors

1.1 Cascode mirror

The topology of the cascode current mirror and the small signal equivalent circuit used to calculate the output resistance are shown in Fig. 1.1. The circuit is probed by the voltage source v_p, so that the output resistance is v_p/i_p. In order to simplify the calculations, the simple mirror formed by Q_1 and Q_2 is modeled with its small signal equivalent circuit.

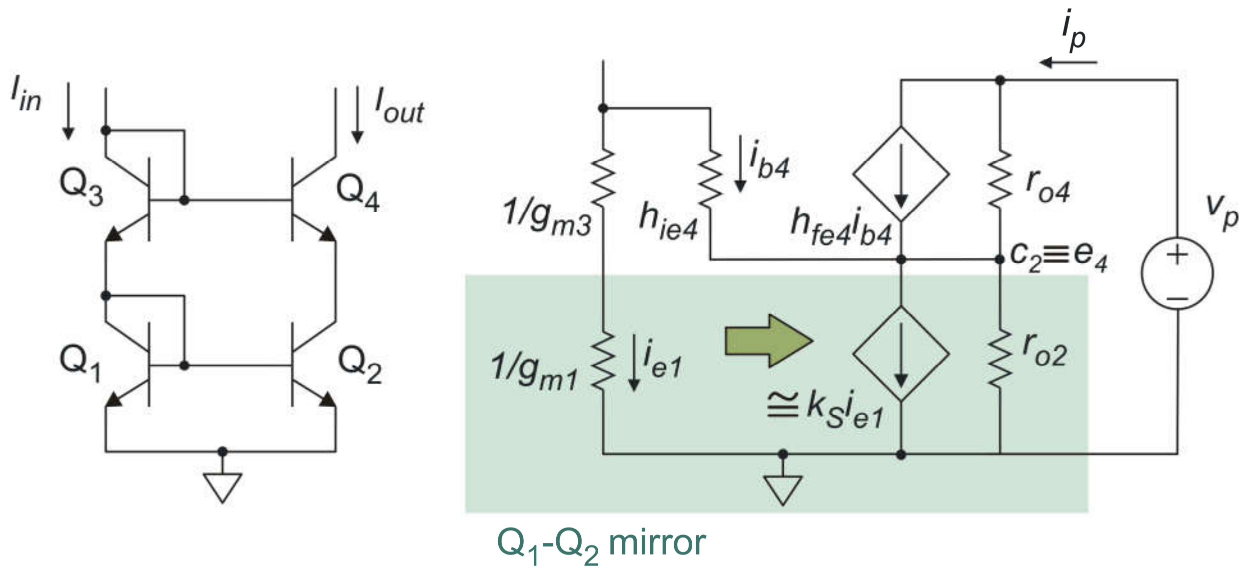

Fig.1.1. BJT Cascode mirror topology (left); Small signal equivalent circuit for output resistance calculation (right).The arrow indicates the input-to-output direction of the Q_1-Q_2 simple current mirror.

Current balance at Q_2 collector node (Q_4 emitter) gives:

$$i_{b4} + i_p = k_S i_{e1} + \frac{v_{c2}}{r_{o2}} = -k_S i_{b4} + \frac{v_{c2}}{r_{o2}} \tag{1.1}$$

with v_{c2} given by:

$$v_{c2} = v_{e4} = -i_{b4}\left(\frac{1}{g_{m1}} + \frac{1}{g_{m3}} + h_{ie4}\right) \cong -i_{b4}h_{ie4} \tag{1.2}$$

Substituting v_{c2} from (1.2) into (1.1) we find:

$$i_p \cong -b i_{b4} \tag{1.3}$$

Where coefficient b is defined as:

$$b = \left(1 + k_S + \frac{h_{ie4}}{r_{o2}}\right) \tag{1.4}$$

Current balance at the output node of the mirror (Q_4 collector) gives:

$$i_p = h_{fe} i_{b4} + \frac{v_p - v_{e4}}{r_{o4}} \tag{1.5}$$

Substituting v_{e4} from (1.2) into (1.5) we get:

$$i_p = h_{fe} i_{b4} + \frac{v_p + i_{b4} h_{ie4}}{r_{o4}} \tag{1.6}$$

Using (1.3):

$$i_p = -h_{fe} \frac{i_p}{b} + \frac{v_p}{r_{o4}} - \frac{i_p}{b} \frac{h_{ie4}}{r_{o4}} \tag{1.7}$$

From (1.7) , the output resistance of the cascode mirror is given by:

$$R_{out} = \frac{v_p}{i_p} = r_{o4}\left(1 + \frac{h_{fe}}{b} + \frac{1}{b}\frac{h_{ie4}}{r_{o4}}\right) \tag{1.8}$$

Considering that, generally, for a BJT operating in active region, $h_{ie} << r_o$, we find the approximation:

$$R_{out} = \frac{v_p}{i_p} \cong r_{o4}\left(1 + \frac{h_{fe}}{k_S + 1}\right) \tag{1.9}$$

For a current mirror designed to provide a unity gain (k_S=1), we get:

$$R_{out} \cong r_{o4}\left(1 + \frac{h_{fe}}{2}\right) \tag{1.10}$$

1.2 Wilson Current Mirror

The topology of a 4-transistor current mirror and the small-signal equivalent circuit used for calculation of the output resistance is shown in Fig. 1.2. The circuit is probed by the voltage source v_p. The output resistance is v_p/i_p. In order to simplify the calculations, the simple mirror formed by Q_1 and Q_2 is modeled with its small signal equivalent circuit. Note that the Q_2-Q_1 mirror transmits the currents in the opposite direction with respect to the case of the cascode mirror in Fig. 1.1 (see the arrow in both figures). The current gain of the Q_2-Q_1 mirror is equal to $1/k_S$, which is the inverse of the overall current gain of the Wilson mirror (k_S). Calculations will be carried out considering a generic k_S. However, it should be observed that the Wilson current mirror provides base current compensation only for k_S=1.

Neglecting the current into r_{o1} resistor, it is possible to find that $i_{b4}= -i_{e2}/k_S$. This approximation can be easily motivated considering the mesh formed by resistors $1/g_{m2}$, h_{ie4}, $1/g_{m3}$ and r_{o1}. Voltage balance gives:

$$i_{e2}\frac{1}{g_{m2}} = -i_{b4}\left(h_{ie4}+\frac{1}{g_{m3}}\right)+\left(-i_{b4}-\frac{i_{e2}}{k_S}\right)r_{o1} \tag{1.11}$$

The resulting i_{e2} vs i_{b4} relationship is:

$$i_{e2} = -i_{b4}\frac{\left(h_{ie4}+\frac{1}{g_{m3}}+r_{o1}\right)}{\left(\frac{1}{g_{m2}}+\frac{r_{o1}}{k_S}\right)} = -i_{b4}\frac{\left(1+\frac{h_{ie4}}{r_{o1}}+\frac{1}{r_{o1}g_{m3}}\right)}{\left(\frac{1}{k_S}+\frac{1}{r_{o1}g_{m2}}\right)} \cong -k_S i_{b4} \;\Rightarrow\; i_{b4} = -\frac{1}{k_s}i_{e2} \tag{1.12}$$

The approximation is due to the fact that, generally, for BJTs operating in active region, $h_{ie} << r_o$, and $g_m r_o >> 1$.

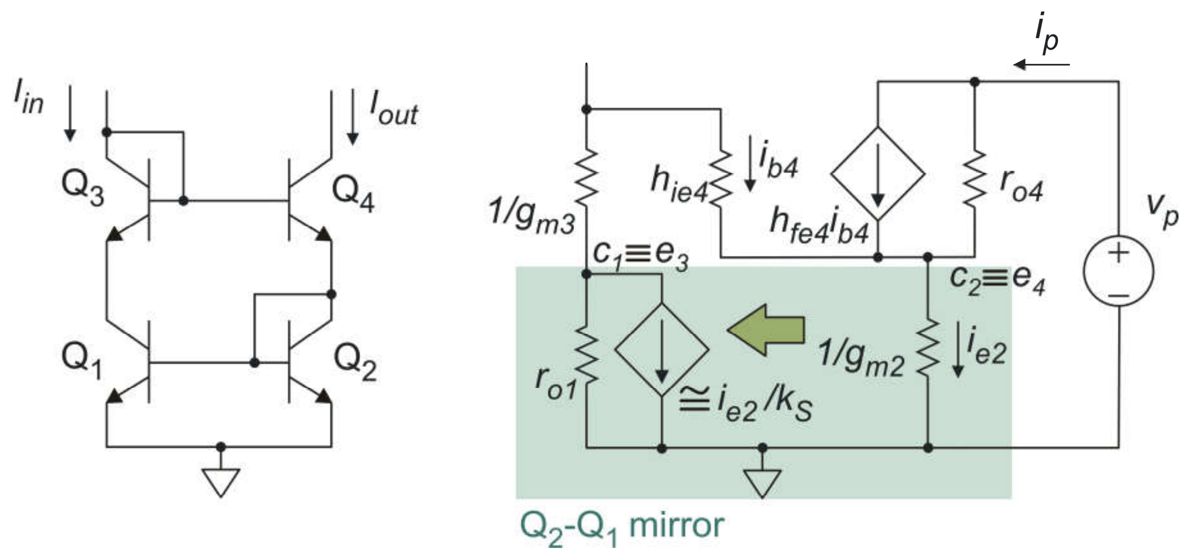

Fig. 1.2. Topology of a 4-BJT Wilson current mirror (left) and small signal equivalent circuit used for calculation of the output resistance. The arrow indicates the input-to-output direction of the Q_1-Q_2 simple current mirror.

Current balance at node c_2, gives:

$$i_p = i_{e2} - i_{b4} \cong -\left(k_S+1\right)i_{b4} \cong \left(1+\frac{1}{k_S}\right)i_{e2} \tag{1.13}$$

Current balance at the output node gives:

$$i_p = h_{fe4}i_{b4}+\frac{v_p - v_{e4}}{r_{o4}} = h_{fe4}i_{b4}+\frac{v_p-\frac{i_{e2}}{g_{m2}}}{r_{o4}} \tag{1.14}$$

Using (1.13) for i_{e2} and i_{b4}, we finally find an equation where only variables i_p and v_p are present.

$$i_p = -h_{fe4}\frac{i_p}{1+k_S} + \frac{v_p}{r_{o4}} - \frac{i_p}{1+k_S^{-1}}\frac{1}{g_{m2}r_{o4}} \tag{1.15}$$

which gives the output resistance:

$$R_{out} = \frac{v_p}{i_p} = r_{o4}\left(1+\frac{h_{fe4}}{1+k_S}+\frac{1}{\left(1+k_S^{-1}\right)g_{m2}r_{o4}}\right) \cong r_{o4}\left(1+\frac{h_{fe4}}{1+k_S}\right) \tag{1.16}$$

Since the Wilson current mirror is generally used with $k_S=1$, R_{out} becomes:

$$R_{out} \cong r_{o4}\left(1+\frac{h_{fe4}}{2}\right) \quad \text{(for } k_S = 1\text{)} \tag{1.17}$$