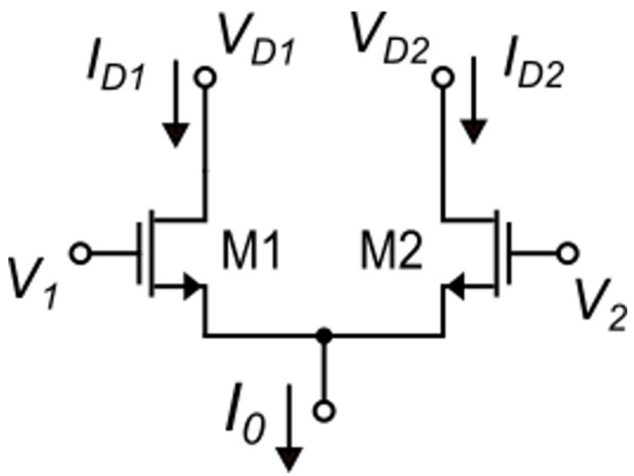
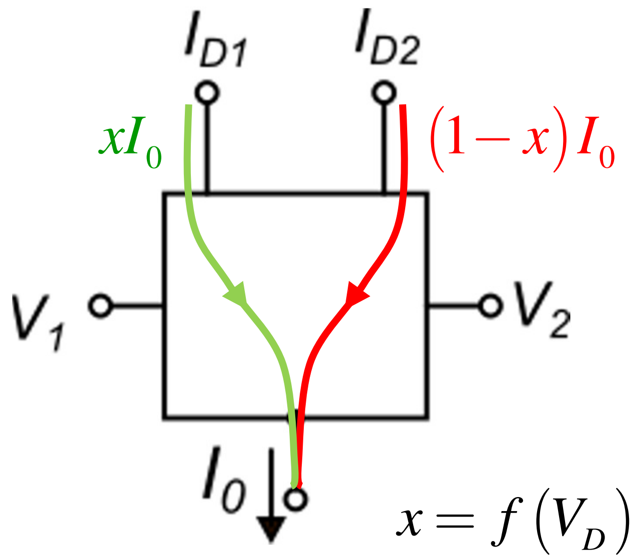
Source coupled MOSFET pair (differential MOSFET pair)



Inputs: V_1, V_2 (effective input signal: $V_D = V_1 - V_2$)
I_0 ("Tail current")

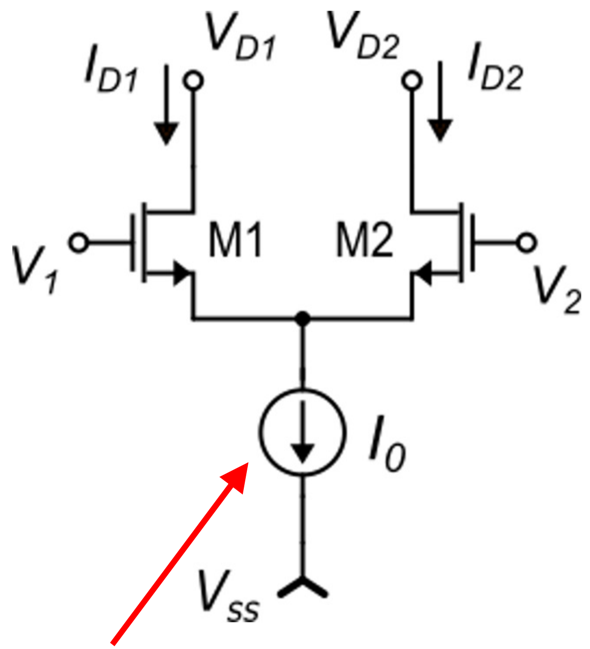
Outputs: I_{D1}, I_{D2} ($I_{D1} - I_{D2}$)



Function

- The input current I_0 is split into I_{D1} and I_{D2}, according to fractions x and (1-x)
- x depends on V_D (x=0.5 for V_D=0)

Analysis of the source coupled MOSFET pair (differential MOSFET pair)



I_0 has been represented as an ideal current source placed between M1,M2 sources and V_{SS}: this is just an example.

Target: obtain the relationship between the ratio *x* and the input differential voltage V_D

$$I_{D1} = xI_0 \quad I_{D2} = (1-x)I_0 \quad \Rightarrow \quad x \triangleq \frac{I_{D1}}{I_0}$$

Hypotheses

- M1 and M2 work in saturation and the effect of V_{DS} can be neglected
- Strong inversion equation can be applied
- I_0 does not depend on V_1, V_2

$$I_D = \frac{\beta}{2}\left(V_{GS} - V_t\right)^2 \Rightarrow V_{GS} = V_t + \sqrt{\frac{2I_D}{\beta}}$$

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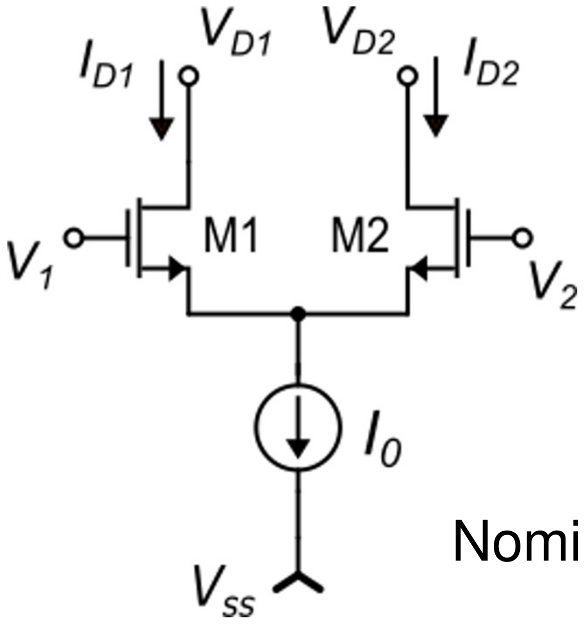


$$\begin{cases} V_1 = V_{S1} + V_{GS1} \\ V_2 = V_{S2} + V_{GS2} \end{cases} \qquad V_{S1} = V_{S2} \triangleq V_S$$

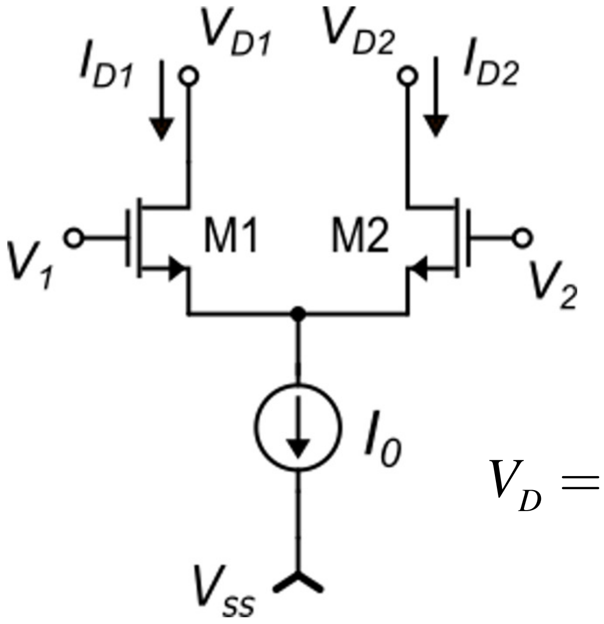
$$V_D = V_1 - V_2 = V_{GS1} - V_{GS2}$$

$$V_D = V_{t1} + \sqrt{\frac{2I_{D1}}{\beta_1}} - \left(V_{t2} + \sqrt{\frac{2I_{D2}}{\beta_2}} \right)$$

Nominal conditions: M1=M2: $\beta_1=\beta_2=\beta$ and $V_{t1}=V_{t2}$ $(V_{BS1}=V_{BS2})$.

$$V_D = \sqrt{\frac{2I_{D1}}{\beta}} - \sqrt{\frac{2I_{D2}}{\beta}} = \sqrt{\frac{2}{\beta}} \cdot \left(\sqrt{I_{D1}} - \sqrt{I_{D2}} \right)$$

Analysis of the source coupled MOSFET pair (differential MOSFET pair)



$$V_D = \sqrt{\frac{2I_{D1}}{\beta}} - \sqrt{\frac{2I_{D2}}{\beta}} = \sqrt{\frac{2}{\beta}} \cdot \left(\sqrt{I_{D1}} - \sqrt{I_{D2}}\right)$$

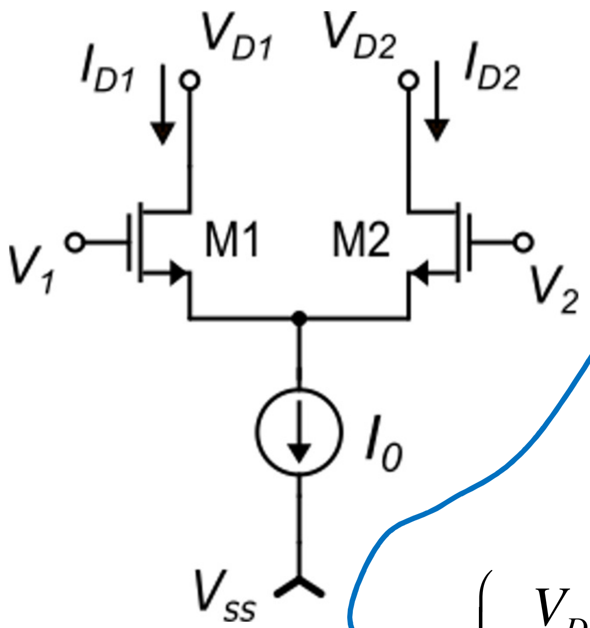
$$I_{D1} = xI_0 \quad I_{D2} = (1-x)I_0$$

$$V_D = \sqrt{\frac{2}{\beta}} \cdot \left[\sqrt{xI_0} - \sqrt{(1-x)I_0}\right] = \boxed{\sqrt{\frac{2I_0}{\beta}}} \cdot \left[\sqrt{x} - \sqrt{(1-x)}\right]$$

$$\sqrt{\frac{2I_0}{\beta}} \triangleq V_{DMAX}$$

$$\frac{V_D}{V_{DMAX}} = \left[\sqrt{x} - \sqrt{(1-x)}\right]$$

Analysis of the source coupled MOSFET pair (differential MOSFET pair)



$$\frac{V_D}{V_{DMAX}} = \left[\sqrt{x} - \sqrt{(1-x)}\right]$$

To solve this equation, we have to square both hands. In this way we add new solutions that are not valid.

Before squaring, we have to note that:

$$V_D > 0 \quad \Rightarrow \sqrt{x} > \sqrt{(1-x)}$$

$$V_D > 0 \quad \Rightarrow x > (1-x)$$

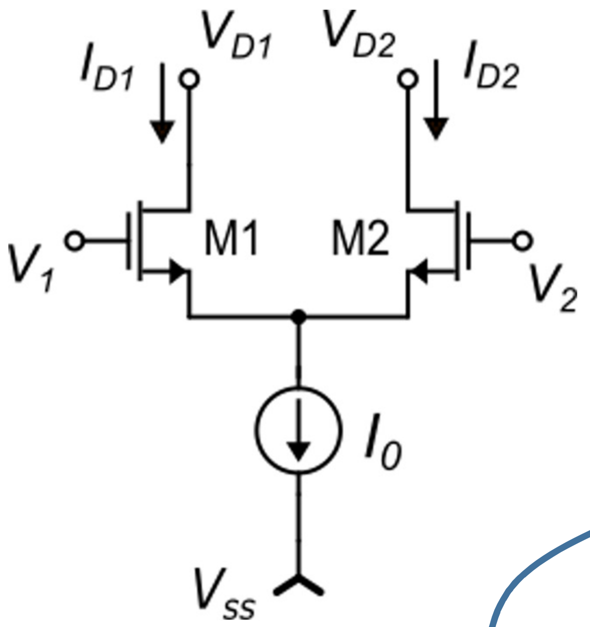
$$\boxed{V_D > 0 \quad \Rightarrow x > \frac{1}{2}}$$

condition 1

$$\left(\frac{V_D}{V_{DMAX}}\right)^2 = x + (1-x) - 2\sqrt{x(1-x)}$$

$$\left(\frac{V_D}{V_{DMAX}}\right)^2 - 1 = -2\sqrt{x(1-x)}$$

Analysis of the source coupled MOSFET pair (differential MOSFET pair)



$$\left(\frac{V_D}{V_{DMAX}}\right)^2 - 1 = -2\sqrt{x(1-x)}$$

We have to square this equation again.
This time, the condition is:

$$\boxed{\left(\frac{V_D}{V_{DMAX}}\right)^2 - 1 \le 0}$$

condition 2

$$\boxed{V_D > 0 \quad \Rightarrow x > \frac{1}{2}}$$

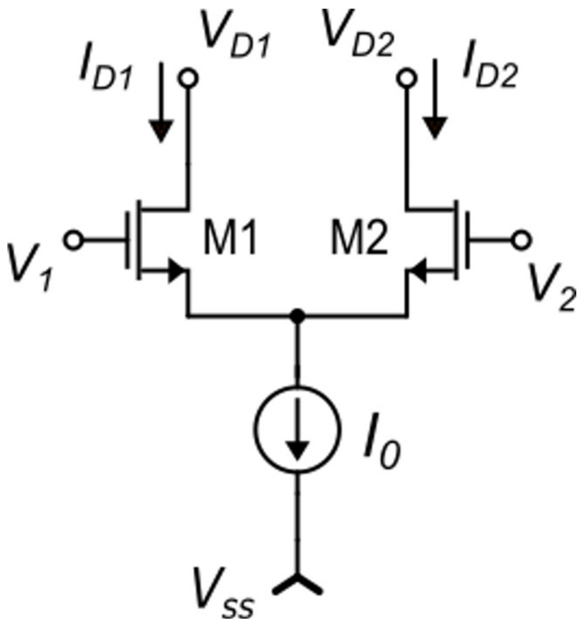
condition 1

$$\left[\left(\frac{V_D}{V_{DMAX}}\right)^2 - 1\right]^2 = 4x(1-x)$$

$$\frac{1}{4}\left[\left(\frac{V_D}{V_{DMAX}}\right)^2 - 1\right]^2 = x - x^2$$

$$x^2 - x + \frac{1}{4}\left[\left(\frac{V_D}{V_{DMAX}}\right)^2 - 1\right]^2 = 0$$

Analysis of the source coupled MOSFET pair (differential MOSFET pair)



$$\boxed{V_D > 0 \quad \Rightarrow x > \frac{1}{2}}$$

condition 1

$$\boxed{\left(\frac{V_D}{V_{DMAX}}\right)^2 - 1 \leq 0}$$

condition 2

$$x^2 - x + \frac{1}{4}\left[\left(\frac{V_D}{V_{DMAX}}\right)^2 - 1\right]^2 = 0$$

$$ax^2 + bx + c$$

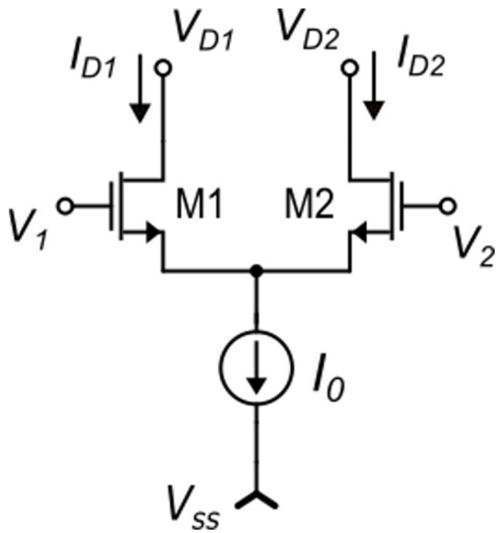
$$a = 1,\ b = -1,\ c = \frac{1}{4}\left[\left(\frac{V_D}{V_{DMAX}}\right)^2 - 1\right]^2$$

$$x = -\frac{b}{2a} \pm \frac{1}{2a}\sqrt{b^2 - 4ac}$$

$$x = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - \left[\left(\frac{V_D}{V_{DMAX}}\right)^2 - 1\right]^2}$$

$$x = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - \left[\left(\frac{V_D}{V_{DMAX}}\right)^4 - 2\left(\frac{V_D}{V_{DMAX}}\right)^2 + 1\right]}$$

Analysis of the source coupled MOSFET pair (differential MOSFET pair)

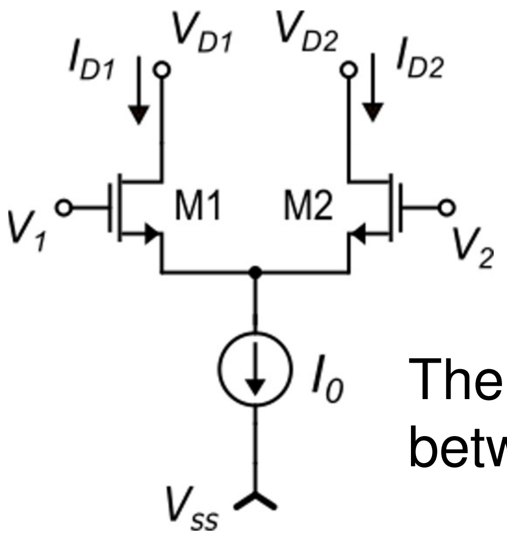


$$x = \frac{1}{2} \pm \frac{1}{2}\sqrt{2\left(\frac{V_D}{V_{DMAX}}\right)^2 - \left(\frac{V_D}{V_{DMAX}}\right)^4} = \frac{1}{2} \pm \frac{1}{2}\left(\frac{V_D}{V_{DMAX}}\right)\sqrt{2 - \left(\frac{V_D}{V_{DMAX}}\right)^2}$$

+

condition 1 $\boxed{V_D > 0 \quad \Rightarrow x > \frac{1}{2}}$

$$\begin{cases} x = \dfrac{I_{D1}}{I_0} = \dfrac{1}{2} + \dfrac{1}{2}\left(\dfrac{V_D}{V_{DMAX}}\right)\sqrt{2 - \left(\dfrac{V_D}{V_{DMAX}}\right)^2} \\ 1 - x = \dfrac{I_{D2}}{I_0} = \dfrac{1}{2} - \dfrac{1}{2}\left(\dfrac{V_D}{V_{DMAX}}\right)\sqrt{2 - \left(\dfrac{V_D}{V_{DMAX}}\right)^2} \end{cases} \Rightarrow \begin{cases} I_{D1} = \dfrac{I_0}{2} + \dfrac{I_0}{2}\left(\dfrac{V_D}{V_{DMAX}}\right)\sqrt{2 - \left(\dfrac{V_D}{V_{DMAX}}\right)^2} \\ I_{D2} = \dfrac{I_0}{2} - \dfrac{I_0}{2}\left(\dfrac{V_D}{V_{DMAX}}\right)\sqrt{2 - \left(\dfrac{V_D}{V_{DMAX}}\right)^2} \end{cases}$$



$$\boxed{\left(\frac{V_D}{V_{DMAX}}\right)^2 - 1 \le 0}$$

condition 2

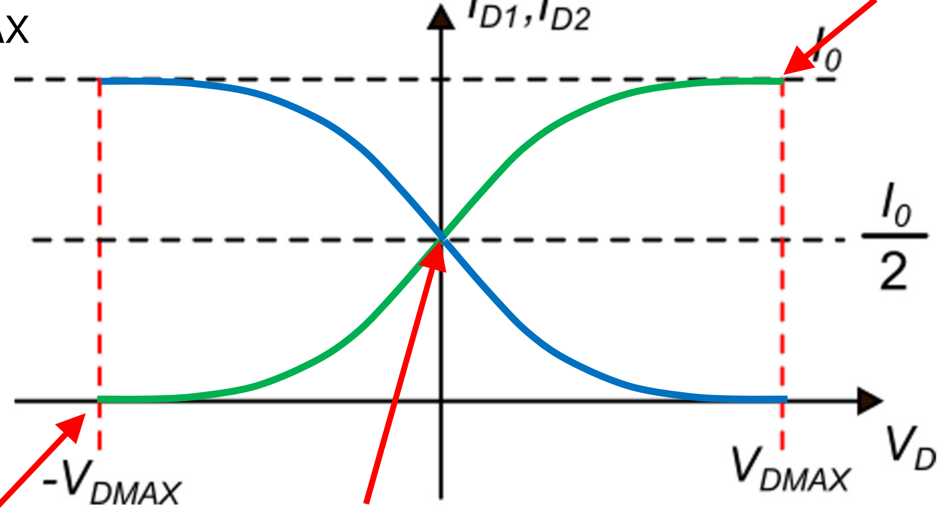
$$\left|\frac{V_D}{V_{DMAX}}\right| \le 1 \;\Rightarrow\; -V_{DMAX} \le V_D \le V_{DMAX}$$

The analysis is applicable only between -V_{DMAX} and +V_{DMAX}

$$I_{D1} = \frac{I_0}{2} + \frac{I_0}{2}\left(\frac{V_D}{V_{DMAX}}\right)\sqrt{2-\left(\frac{V_D}{V_{DMAX}}\right)^2}$$

$$I_{D2} = \frac{I_0}{2} - \frac{I_0}{2}\left(\frac{V_D}{V_{DMAX}}\right)\sqrt{2-\left(\frac{V_D}{V_{DMAX}}\right)^2}$$

$$V_D = V_{DMAX} \Rightarrow I_{D1} = I_0$$



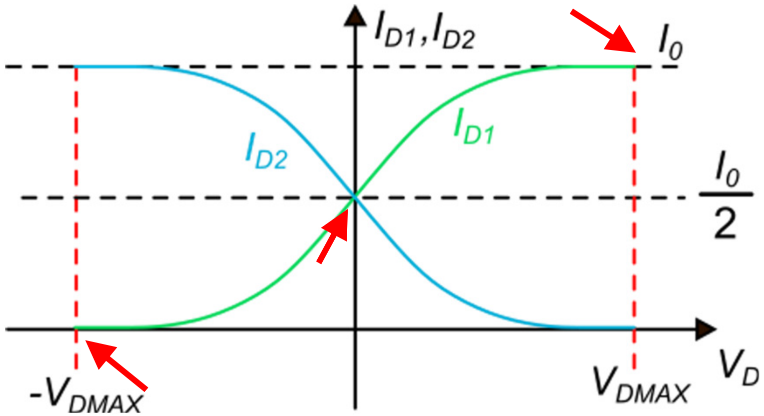
$$V_D = -V_{DMAX} \Rightarrow I_{D1} = 0$$

$$V_D = 0 \Rightarrow I_{D1} = \frac{I_0}{2}$$

Derivative of the differential pair input-output curves (optional)

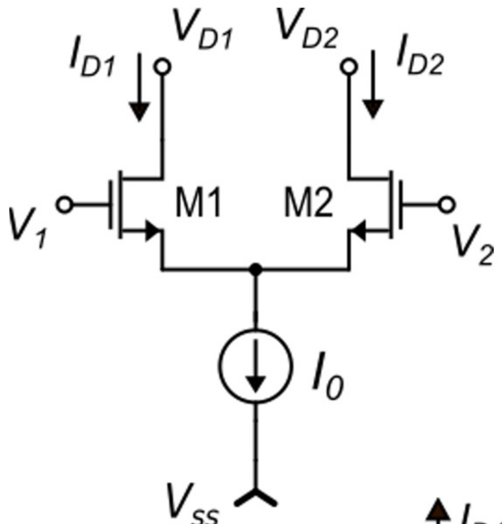
$$I_{D1} = \frac{I_0}{2} + \frac{I_0}{2}\left(\frac{V_D}{V_{DMAX}}\right)\sqrt{2-\left(\frac{V_D}{V_{DMAX}}\right)^2} = \frac{I_0}{2}\left[1+z\sqrt{2-z^2}\right] \quad with\ z = \left(\frac{V_D}{V_{DMAX}}\right)$$

$$\frac{dI_{D1}}{dV_D} = \frac{I_0}{2V_{DMAX}}\left[\sqrt{2-z^2} - \frac{2z^2}{2\sqrt{2-z^2}}\right] = \frac{I_0}{2V_{DMAX}}\frac{2-z^2-z^2}{\sqrt{2-z^2}} = \frac{I_0}{V_{DMAX}}\frac{1-z^2}{\sqrt{2-z^2}}$$



$$\begin{cases} V_D = \pm V_{DMAX} \Rightarrow \dfrac{dI_{D1}}{dV_D} = 0 & \sqrt{\dfrac{2I_0}{\beta}} \triangleq V_{DMAX} \\ V_D = 0 \Rightarrow \dfrac{dI_{D1}}{dV_D} = \dfrac{I_0}{\sqrt{2}V_{DMAX}} = \sqrt{\dfrac{I_0^2\beta}{4I_0}} = \dfrac{1}{2}\sqrt{\beta I_0} & \end{cases}$$

Extrapolation outside the $-V_{DMAX} \le V_D \le V_{DMAX}$ region

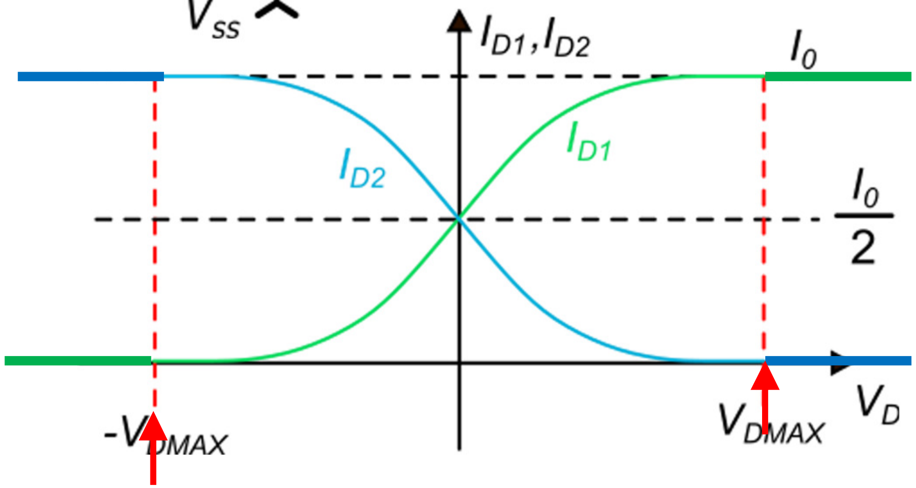


$$V_D = V_{GS1} - V_{GS2} \Rightarrow V_{GS2} = V_{GS1} - V_D$$

Considering the boundary:

$$V_D = V_{DMAX} \qquad I_{D1} = I_0 \Rightarrow V_{GS1} = V_t + \boxed{\sqrt{\frac{2I_0}{\beta}}} \quad V_{DMAX}$$

$$\begin{cases} V_{GS1} = V_t + V_{DMAX} \\ V_{GS2} = V_{GS1} - V_D = V_t \end{cases}$$



If V_D increases over V_{DMAX} :

V_{GS1} cannot increase because I_{D1} would become > I_0, which is impossible.

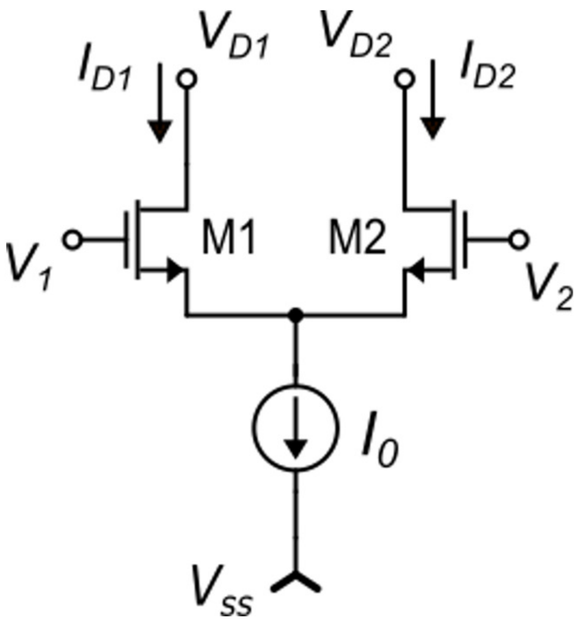
Then V_{GS2} gets smaller than V_t

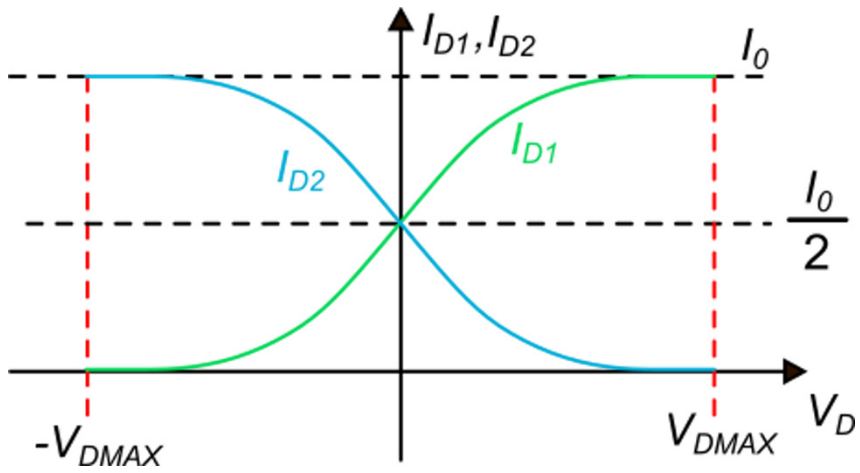
I_{D2} keeps being =0

I_{D1} keeps being = I_0

The opposite occurs when V_D decreases below V_{DMAX}:

Mosfet differential pair: parameters





$$V_{DMAX} = \sqrt{\frac{2I_0}{\beta}}$$

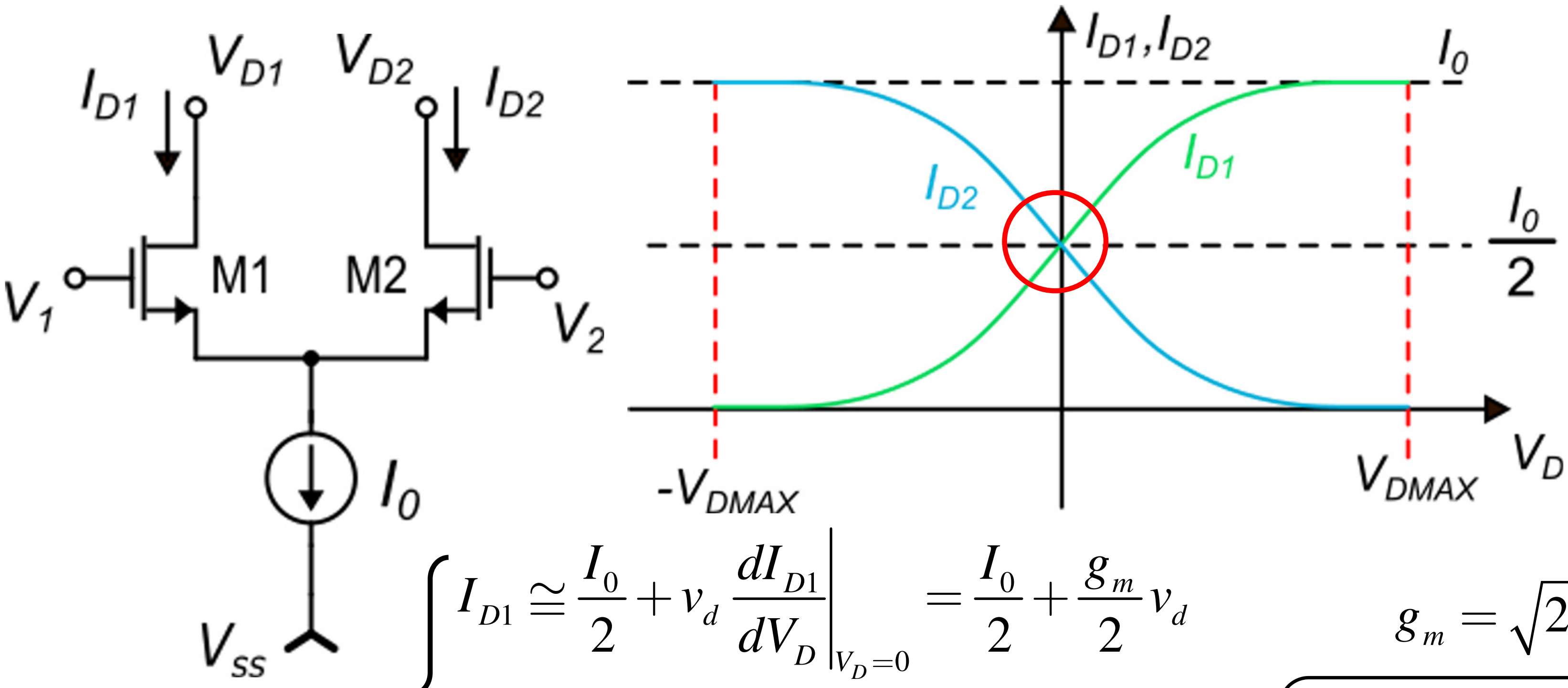
For V_D=0 (typical operating point): $I_{D1} = I_{D2} \triangleq I_{DQ} = \frac{I_0}{2}$

$$V_{DMAX} = \sqrt{\frac{2 \cdot 2I_{DQ}}{\beta}} = \sqrt{2}\sqrt{\frac{2I_{DQ}}{\beta}} = \sqrt{2}\left(V_{GS} - V_t\right)_Q$$

$$\left.\frac{dI_{D1}}{dV_D}\right|_{V_D=0} = \frac{1}{2}\sqrt{2\beta I_{DQ}} = \frac{g_m}{2}$$

Small signal behavior

For small variations (v_d) of V_D around 0:



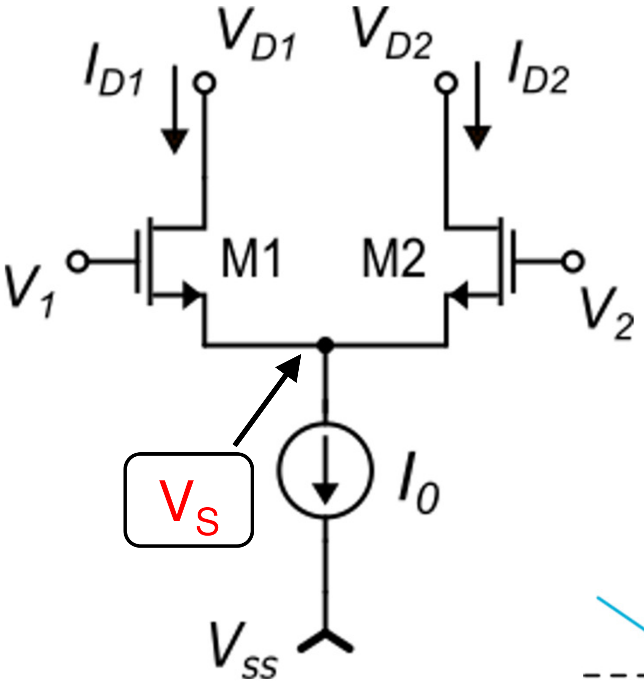
Check that this is consistent with other expressions of gm

$$\begin{cases} I_{D1} \cong \dfrac{I_0}{2} + v_d \left.\dfrac{dI_{D1}}{dV_D}\right|_{V_D=0} = \dfrac{I_0}{2} + \dfrac{g_m}{2} v_d \\ I_{D2} \cong \dfrac{I_0}{2} + v_d \left.\dfrac{dI_{D2}}{dV_D}\right|_{V_D=0} = \dfrac{I_0}{2} - \dfrac{g_m}{2} v_d \end{cases}$$

$$g_m = \sqrt{2\beta I_{DQ}} = \sqrt{\beta I_0}$$

$$\boxed{g_m = \frac{I_{DQ}}{V_{TE}}; \quad V_{TE} = \frac{V_{GS} - V_t}{2} = \frac{V_{DMAX}}{2\sqrt{2}}}$$

Large-signal dependence of the source voltage on V_D



$$V_1 = V_C + \frac{V_D}{2}; \quad V_2 = V_C - \frac{V_D}{2}$$

V_C: common mode voltage=*constant*

$$V_S = V_1 - V_{GS1} = V_2 - V_{GS2}$$

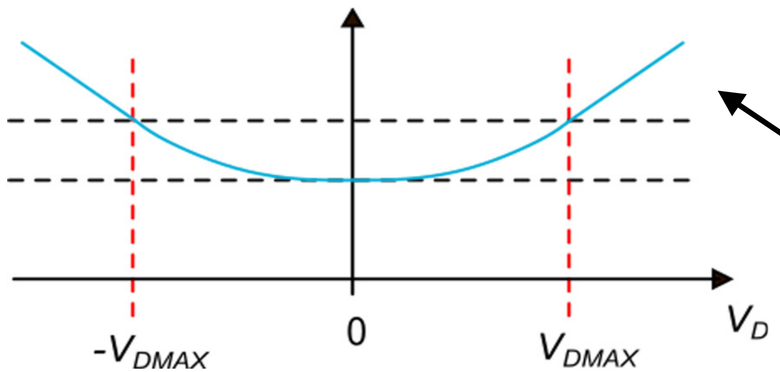
$$V_S\big|_{V_D=0} = V_C - V_t - \left(V_{GS} - V_t\right)_Q \qquad V_{DMAX} = \sqrt{2}\left(V_{GS} - V_t\right)_Q$$

$$V_S\big|_{V_D=0} = V_C - V_t - \frac{V_{DMAX}}{\sqrt{2}} < V_S\big|_{V_D=V_{DMAX}} = V_C - \frac{V_{DMAX}}{2} - V_t$$

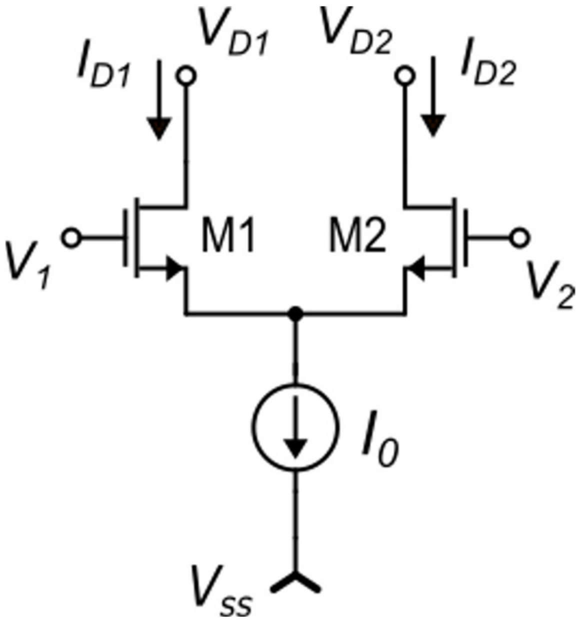
for $V_D > V_{DMAX}$

$$V_S = V_C + \frac{V_D}{2} - V_{GS1}$$

$$V_S = V_C + \frac{V_D}{2} \underbrace{- V_{DMAX} - V_t}_{\text{constant}}$$



Differential output current

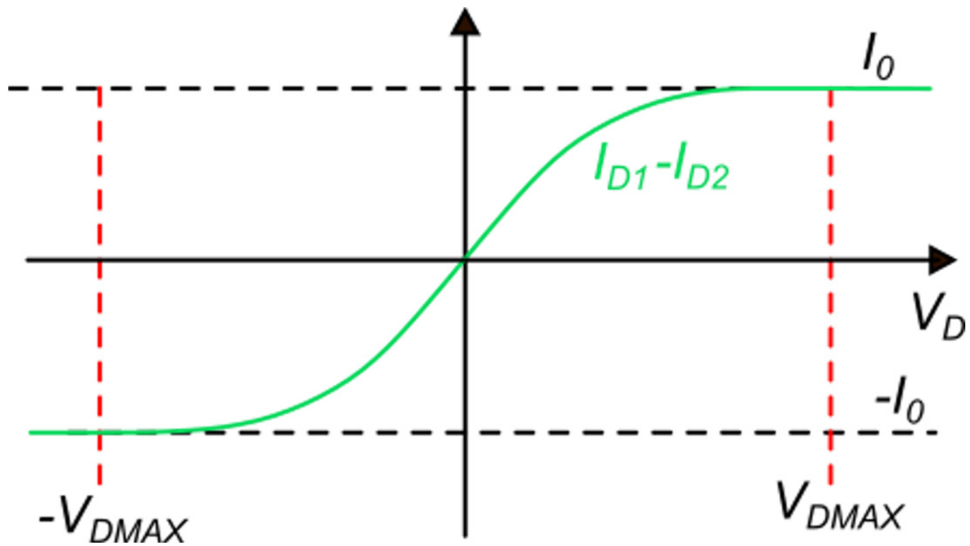


$$I_{D1} = \frac{I_0}{2} + \frac{I_0}{2}\left(\frac{V_D}{V_{DMAX}}\right)\sqrt{2 - \left(\frac{V_D}{V_{DMAX}}\right)^2}$$

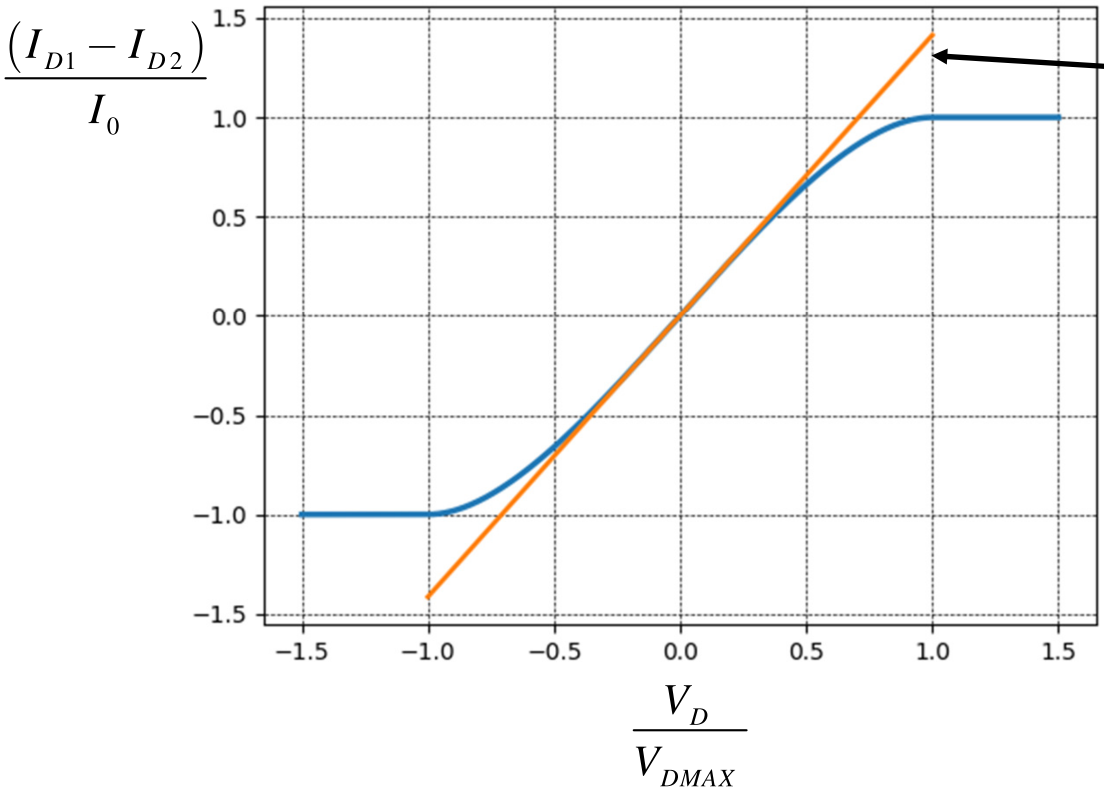
$$I_{D2} = \frac{I_0}{2} - \frac{I_0}{2}\left(\frac{V_D}{V_{DMAX}}\right)\sqrt{2 - \left(\frac{V_D}{V_{DMAX}}\right)^2}$$

$$I_{D1} - I_{D2} = I_0\left(\frac{V_D}{V_{DMAX}}\right)\sqrt{2 - \left(\frac{V_D}{V_{DMAX}}\right)^2}$$

$$i_d = i_{d1} - i_{d2} = g_m v_d$$



Mosfet differential pair: real curves (calculated) and linearity



linear approximation calculated around origin

Non-linearity error:

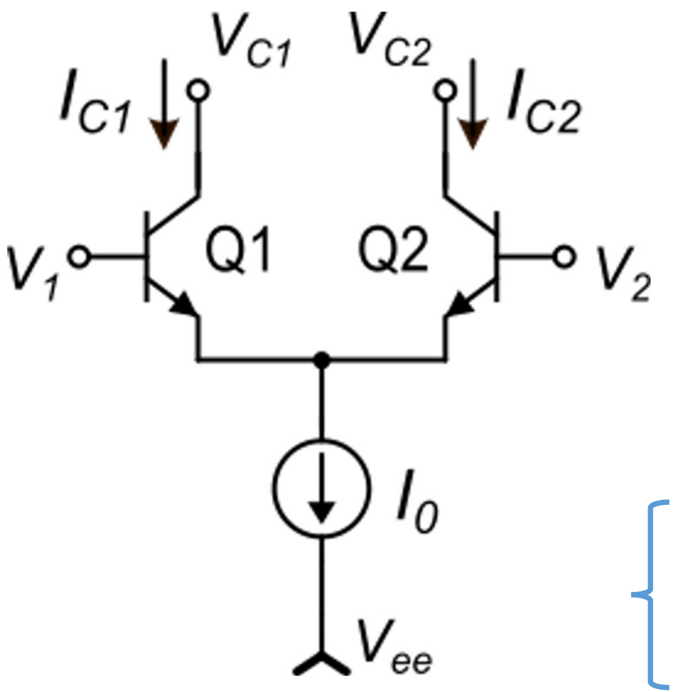
$$|V_D| < \frac{V_{DMAX}}{5} : e_R < 1\%$$

$$|V_D| < \frac{V_{DMAX}}{2\sqrt{2}} = \frac{V_{GS} - V_t}{2} : e_R < 3\%$$

$$|V_D| < \frac{V_{DMAX}}{2} : \; e_R < 7\%$$

$$|V_D| < \frac{V_{DMAX}}{\sqrt{2}} = (V_{GS} - V_t) : e_R < 15\%$$

BJT differential pair



$$I_{E1} + I_{E2} = I_0 \Rightarrow \frac{\beta+1}{\beta} I_{C1} + \frac{\beta+1}{\beta} I_{C2} = I_0$$

$$V_D = V_{BE1} - V_{BE2}$$

$$I_{C1} + I_{C2} = \frac{\beta}{\beta+1} I_0 \cong I_0$$

$$\begin{cases} I_{C1} = I_S e^{\frac{V_{BE1}}{V_T}} \\ I_{C2} = I_S e^{\frac{V_{BE2}}{V_T}} \end{cases}$$

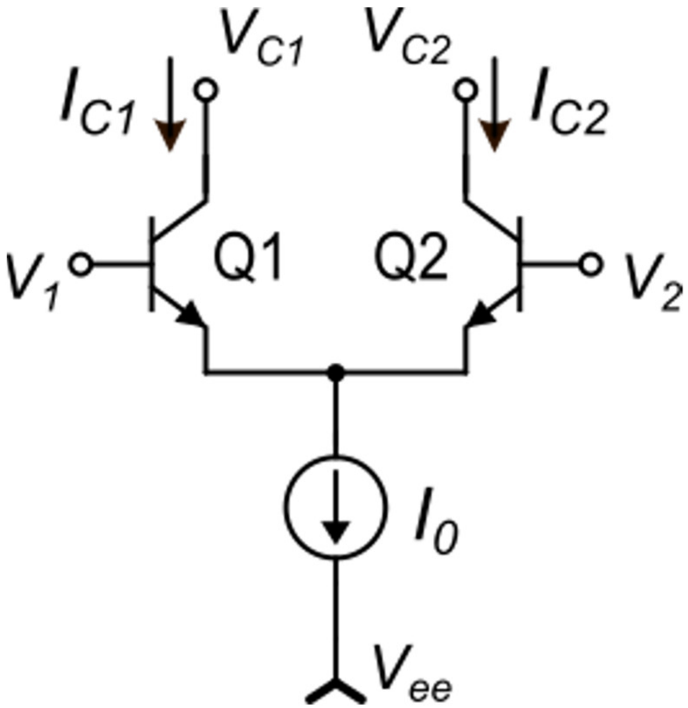
$$\Downarrow$$

$$\frac{I_{C2}}{I_{C1}} = e^{\frac{V_{BE2}-V_{BE1}}{V_T}} = e^{\frac{-V_D}{V_T}}$$

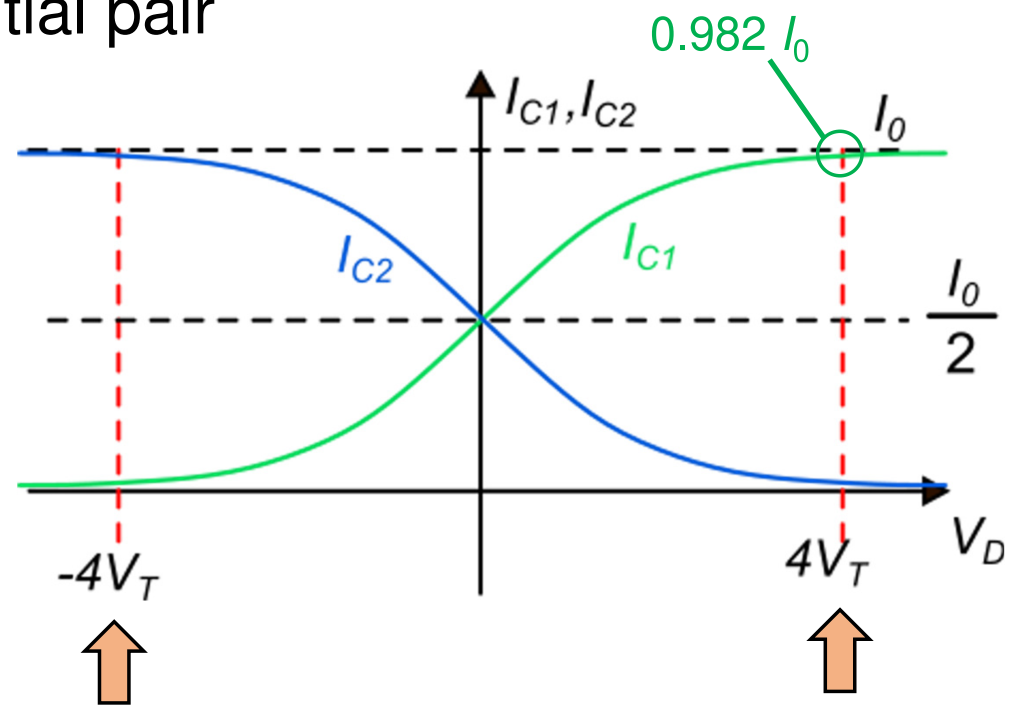
$$I_{C1}\left(1 + e^{\frac{-V_D}{V_T}}\right) = I_0$$

$$\begin{cases} I_{C1} = \dfrac{I_0}{1 + e^{\frac{-V_D}{V_T}}} \\ I_{C2} = \dfrac{I_0 e^{\frac{-V_D}{V_T}}}{1 + e^{\frac{-V_D}{V_T}}} \end{cases}$$

BJT differential pair



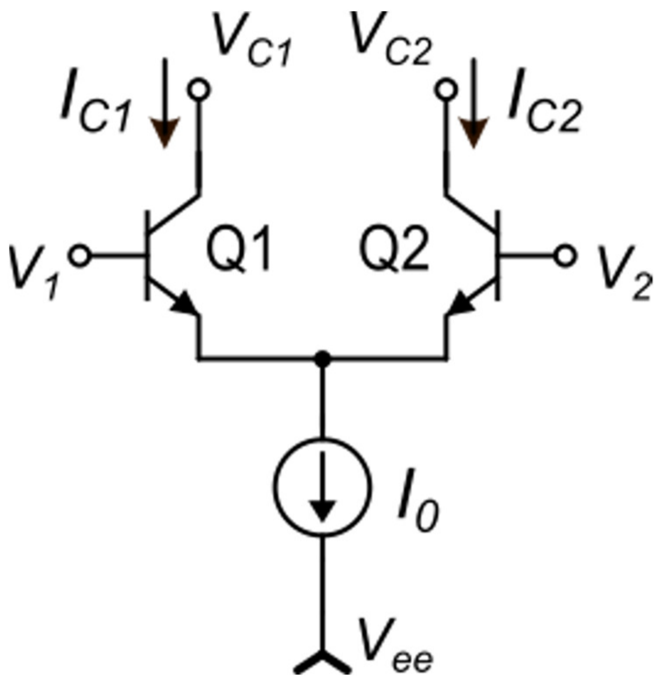
$$\begin{cases} I_{C1} = \dfrac{I_0}{1+e^{\frac{-V_D}{V_T}}} \\ I_{C2} = \dfrac{I_0 e^{\frac{-V_D}{V_T}}}{1+e^{\frac{-V_D}{V_T}}} \end{cases}$$



$$I_{C1} - I_{C2} = I_0 \frac{1-e^{\frac{-V_D}{V_T}}}{1+e^{\frac{-V_D}{V_T}}}$$

$$I_{C1} - I_{C2} = I_0 \frac{e^{\frac{-V_D}{2V_T}}\left(e^{\frac{V_D}{2V_T}} - e^{\frac{-V_D}{2V_T}}\right)}{e^{\frac{-V_D}{2V_T}}\left(e^{\frac{V_D}{2V_T}} + e^{\frac{-V_D}{2V_T}}\right)} = I_0 \tanh\left(\frac{V_D}{2V_T}\right)$$

BJT differential pair - small signal currents



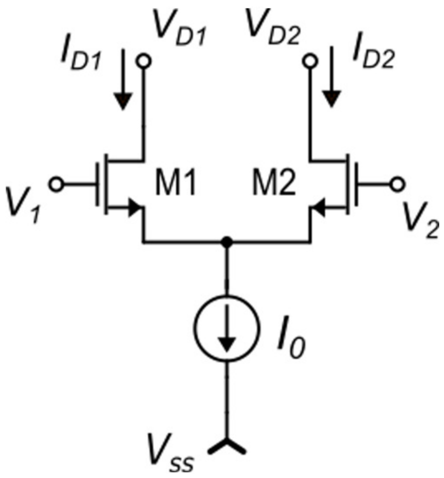
$$I_{C1} = \frac{I_0}{2} + \frac{g_m}{2} v_d$$

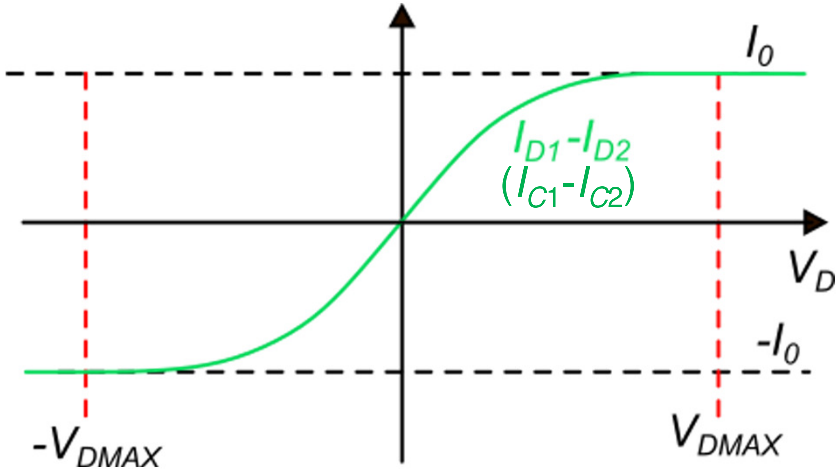
$$I_{C2} = \frac{I_0}{2} - \frac{g_m}{2} v_d$$

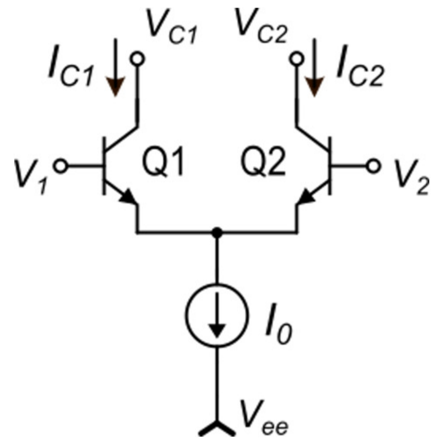
$$I_{C1} - I_{C2} = g_m v_d$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{I_0}{2V_T}$$

MOSFET and BJT differential pairs compared







In the mosfet pair, V_{DMAX} can be varied by modifying β and I_0

Paremeter g_m depends on both I_0 and β:

$$g_m = \sqrt{\beta I_0} \qquad V_{DMAX} = \sqrt{\frac{2I_0}{\beta}}$$

Note: a mosfet pair in subthreshold region behaves like a BJT pair with the substitution:

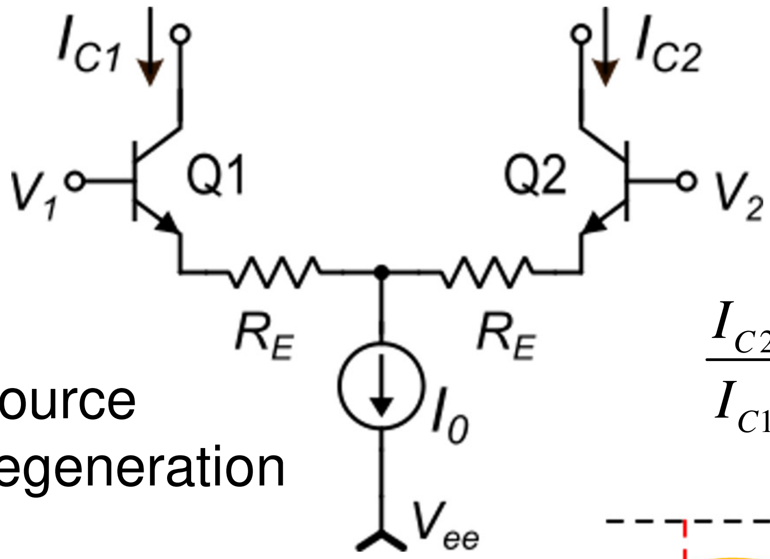
$$mV_T \rightarrow V_T$$

In the BJT pair, V_{DMAX} is fixed to around $4V_T$

Paremeter g_m depends only on I_0

$$g_m = \frac{I_{CQ}}{V_T} = \frac{I_0}{2V_T}$$

How to increase the V_{DMAX} of a BJT pair



Source degeneration

$$V_{DMAX} = V'_{DMAX} + I_0 R_E$$

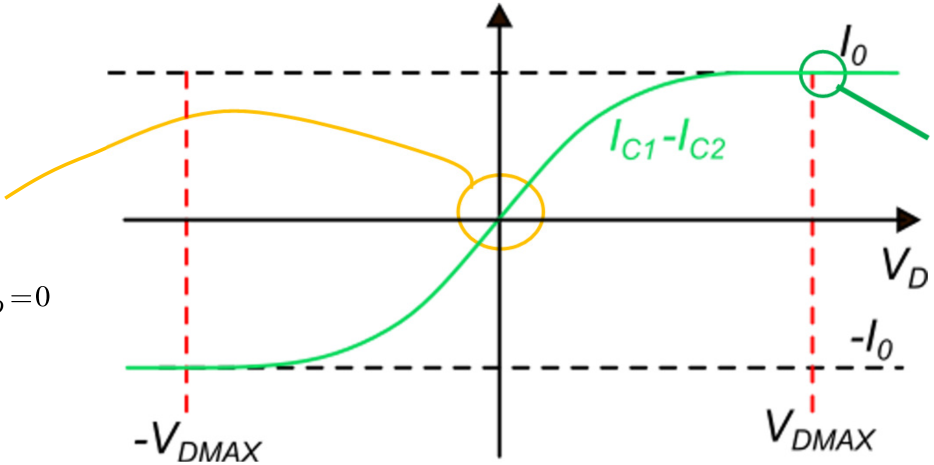
$$\cong 4V_T$$

To demonstrate this, let us write V_D:

$$V_D = V_{BE1} + I_{C1}R_E - \left(V_{BE2} + I_{C2}R_E\right)$$

$$V_D = \underbrace{V_{BE1} - V_{BE2}}_{V_{BE1} - V_{BE2} \cong 4V_T} + R_E \underbrace{\left(I_{C1} - I_{C2}\right)}_{I_{D1} - I_{D2} \cong I_{D1} \cong I_0}$$

$$\frac{I_{C2}}{I_{C1}} = e^{\frac{V_{BE2} - V_{BE1}}{V_T}}$$



$$g_{m-rid} = \left.\frac{d\left(I_{C1} - I_{C2}\right)}{dV_D}\right|_{V_D=0}$$

$$g_{m-rid} = \frac{g_m}{1 + g_m R_E} < g_m$$