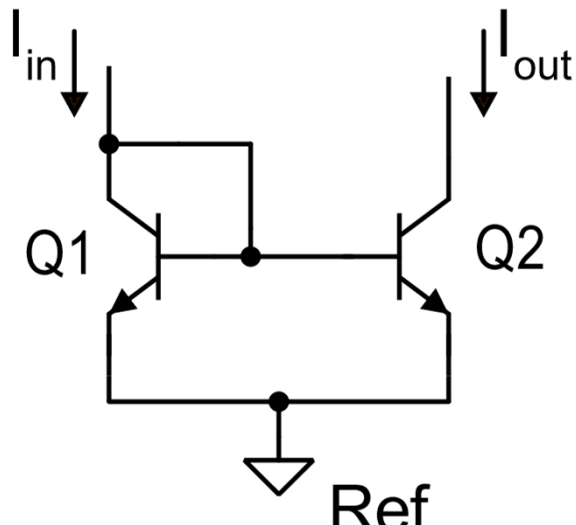


BJT current mirrors

BJT simple current mirror



$$I_{C2} = I_{S2} e^{\frac{V_{BE}}{V_T}} \left(1 + \frac{V_{CB2}}{V_A} \right) = I_{S2} e^{\frac{V_{BE}}{V_T}} \left(1 + \frac{V_{CE2}}{V_A} - \frac{V_{BE}}{V_A} \right)$$

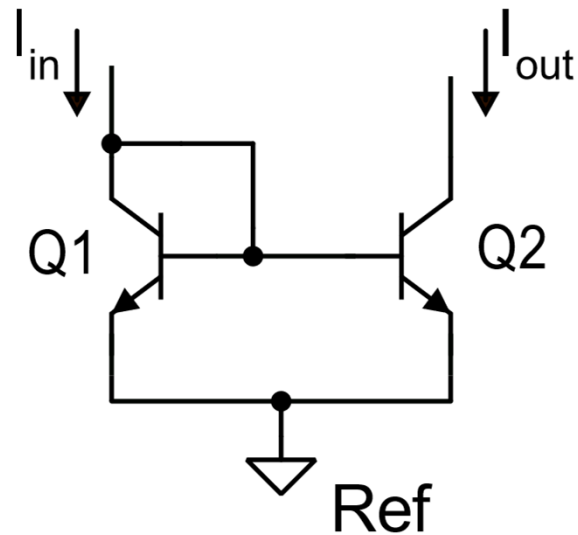
$$I_{C1} = I_{S1} e^{\frac{V_{BE}}{V_T}} \left(1 + \frac{V_{CB1}}{V_A} \right) = I_{S1} e^{\frac{V_{BE}}{V_T}} \left(1 + \frac{V_{CE1}}{V_A} - \frac{V_{BE}}{V_A} \right)$$

$$k_M \triangleq \frac{I_{S2}}{I_{S1}} = \frac{A_{E2}}{A_{E1}} = \frac{area_2}{area_1}$$

$V_{CE1} = V_{IN}$
 $V_{CE2} = V_{OUT}$

$V_{out} = V_{in}$ \longrightarrow $\frac{I_{C2}}{I_{C1}} = k_M$

BJT simple current mirror: parameters



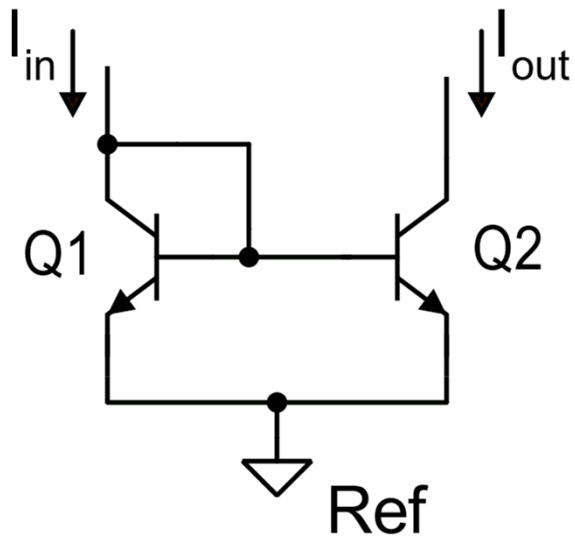
$$V_{IN} = V_{CE1} = V_{BE1} \approx V_{\gamma}$$

$$V_{MIN} = V_{CESAT2} \approx 0.2 \text{ V}$$

$$R_{out} = r_{o2} = \frac{V_A}{I_{C2}} = \frac{V_A}{I_{out}}$$

$$V_{th} = R_{out} I_{out} = V_A$$

BJT simple mirror: impact of base currents



$$\frac{I_{C2}}{I_{C1}} = k_M \quad I_{C2} = I_{out}$$

$$I_{C1} \neq I_{in}$$

$$\frac{I_{out}}{I_{in}} = \frac{I_{C2}}{I_{C1} + I_{B1} + I_{B2}}$$

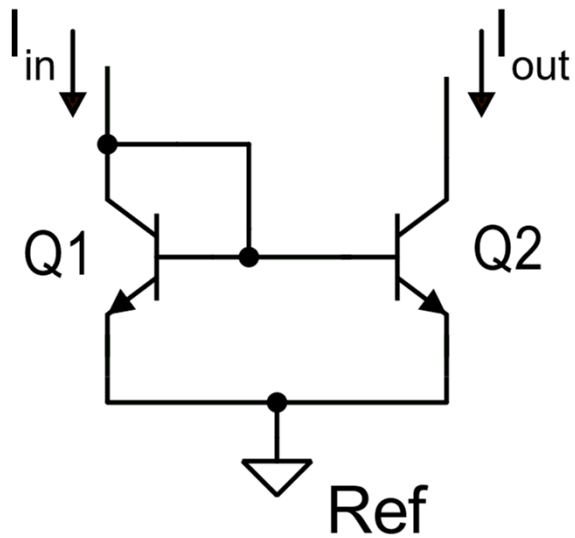
$$I_{B1} = \frac{I_{C1}}{\beta_1} \quad I_{B2} = \frac{I_{C2}}{\beta_2}$$

$$\beta_1 = \beta_2 \triangleq \beta$$

$$\frac{I_{out}}{I_{in}} = \frac{I_{C2}}{I_{C1} + \frac{I_{C1}}{\beta} + \frac{I_{C2}}{\beta}}$$

$$\frac{I_{out}}{I_{in}} = \frac{I_{C2}}{I_{C1}} \frac{1}{1 + \frac{1}{\beta} + \frac{k_M}{\beta}} = k_M \frac{1}{1 + \frac{1 + k_M}{\beta}}$$

BJT simple mirror: impact of base currents



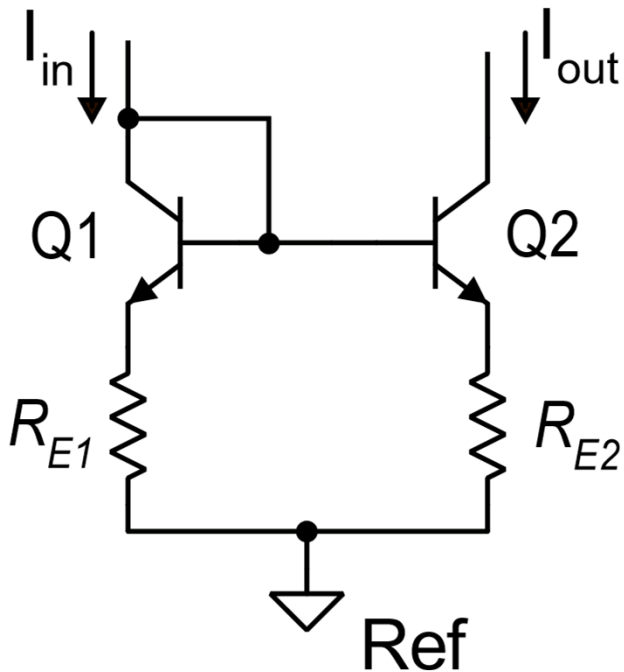
$$\frac{I_{C2}}{I_{C1}} = k_M \quad \frac{I_{out}}{I_{in}} = k_M \frac{1}{1 + \frac{1+k_M}{\beta}} < k_M$$

$$\frac{1}{1+x} \cong 1-x \quad \text{for: } x \ll 1$$

$$\frac{I_{out}}{I_{in}} \cong k_M \left(1 - \frac{k_M + 1}{\beta} \right) \quad \text{for: } \frac{k_M + 1}{\beta} \ll 1$$

$$\epsilon_R \cong \frac{k_M + 1}{\beta}$$

Simple mirror with emitter degeneration to increase the output resistance



$$k_M = \frac{I_{S2}}{I_{S1}} = \frac{A_{E2}}{A_{E1}} = \frac{area_2}{area_1} = \frac{I_{C2}}{I_{C1}}$$

for $V_{in} = V_{out}$, only if: $V_{BE1} = V_{BE2} \Leftrightarrow I_{E1}R_{E1} = I_{E2}R_{E2}$

$$\frac{R_{E2}}{R_{E1}} = \frac{I_{E1}}{I_{E2}} = \frac{I_{CE1}}{I_{CE2}} = \frac{1}{k_M}$$

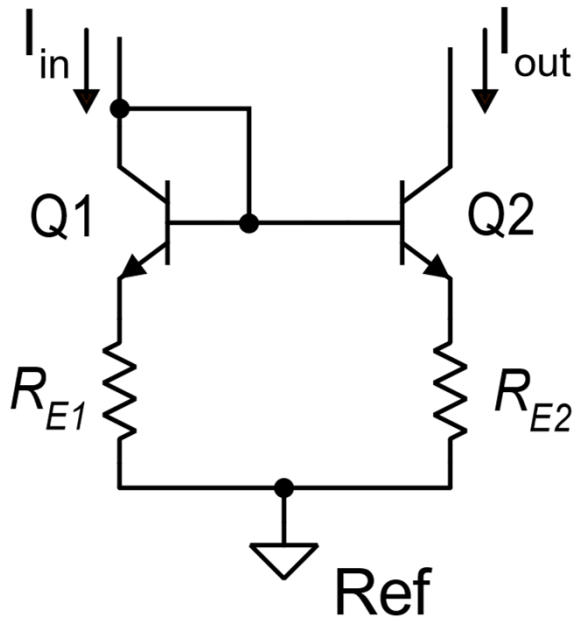
$$V_{IN} = V_{BE1} + R_{E1}I_{E1}$$

$$V_{MIN} = V_{CESAT2} + R_{E2}I_{E2}$$

$$R_{out} \cong r_{o2} (1 + g_{m2}R_{E2}) \cong r_{o2} \left(1 + \frac{I_{C2}R_{E2}}{V_T} \right)$$

The voltage drop across R_{E1} (R_{E2}) makes V_{IN} and V_{MIN} worse than in the simple mirror

BJT current mirror with emitter degeneration: parameters



$$V_{IN} = V_{BE1} + R_{E1} I_{E1}$$

$$V_{MIN} = V_{CESAT2} + R_{E2} I_{E2}$$

The voltage drop across R_{E1} (R_{E2}) makes V_{IN} and V_{MIN} worse than in the simple mirror

$$R_{out} \cong r_{o2} (1 + g_{m2} R_{E2}) \cong r_{o2} \left(1 + \frac{I_{C2} R_{E2}}{V_T} \right) = \frac{V_A}{I_{out}} \left(1 + \frac{I_{C2} R_{E2}}{V_T} \right)$$

$$V_{th} \cong V_A \left(1 + \frac{I_{C2} R_{E2}}{V_T} \right)$$

Example: with $I_{C2} R_{E2} = 225$ mV, and $V_T = 25$ mV, $R_{out} = 10 r_{o2}$

$$\frac{I_{out}}{I_{in}} = \frac{I_{C2}}{I_{C1} + I_{B1} + I_{B2}}$$

The error due to the base currents is the same as in the simple mirror

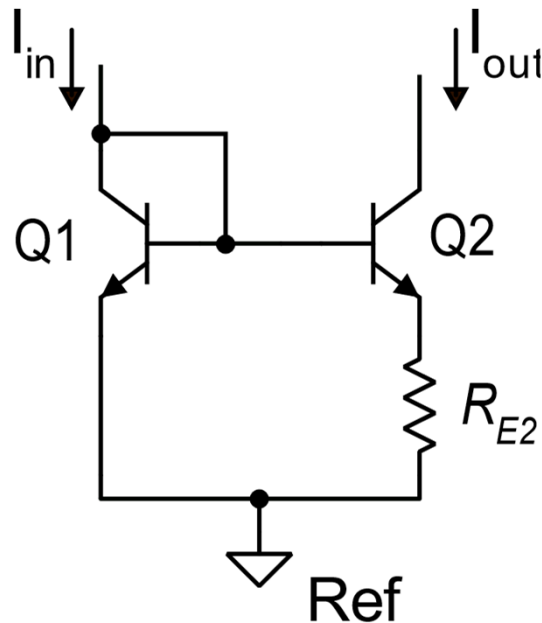
$$\varepsilon_R \cong \frac{k_M + 1}{\beta}$$

The Widlar current source

A non-linear current mirror

$$V_{BE1} = V_{BE2} + R_{E2} I_{E2}$$

$$\Delta V_{BE} \triangleq V_{BE1} - V_{BE2} = R_{E2} I_{E2} \cong R_{E2} I_{C2}$$



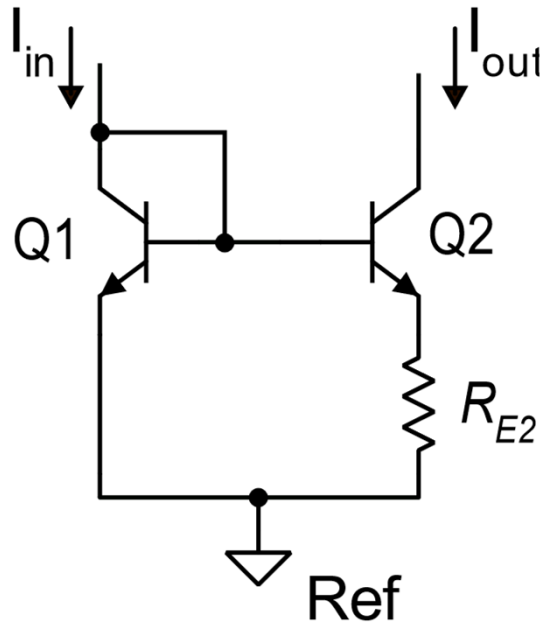
$$I_C = I_S e^{\frac{V_{BE}}{V_T}} \left(1 + \frac{V_{CB2}}{V_A} \right) \cong I_S e^{\frac{V_{BE}}{V_T}} \quad V_{BE} \cong V_T \ln \left(\frac{I_C}{I_S} \right)$$

$$\Delta V_{BE} \cong V_T \ln \left(\frac{I_{C1}}{I_{S1}} \right) - V_T \ln \left(\frac{I_{C2}}{I_{S2}} \right) = V_T \ln \left(\frac{I_{C1}}{I_{S1}} \frac{I_{S2}}{I_{C2}} \right)$$

If $Q1=Q2$ ($I_{S1}=I_{S2}$)

$$\Delta V_{BE} \cong V_T \ln \left(\frac{I_{C1}}{I_{C2}} \right) \cong R_{E2} I_{C2} \quad \frac{I_{C2}}{I_{C1}} \cong e^{-\frac{R_{E2} I_{C2}}{V_T}}$$

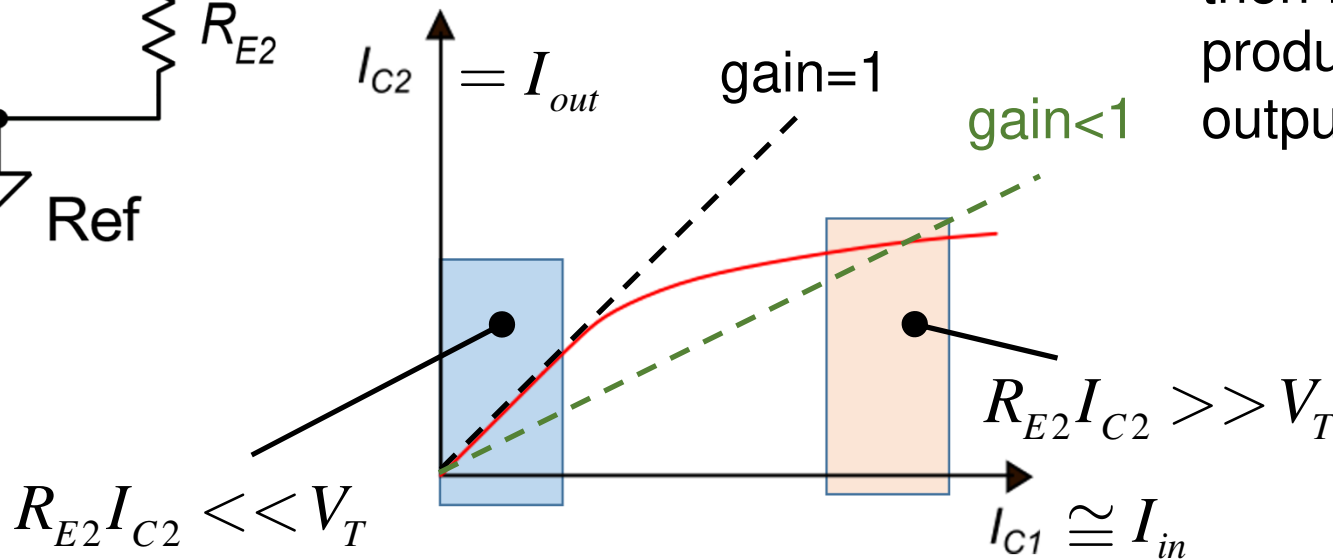
The Widlar current source



$$\frac{I_{C2}}{I_{C1}} \cong e^{-\frac{R_{E2}I_{C2}}{V_T}}$$

Since the I_{C2}/I_{C1} ratio is not constant, but depends on I_{C2} (and then on the input current), the behavior is non-linear

I_{out} is much less sensitive to I_{in} variations than for a linear mirror producing the same output current.



It was used to produce an almost constant current (I_{out}) from a variable current I_{in}

BJT cascode current mirror

$$V_{CE2} = V_{CE1} + V_{BE3} - V_{BE4}$$

If we make: $V_{BE3} = V_{BE4}$ then: $V_{CE2} = V_{CE1}$

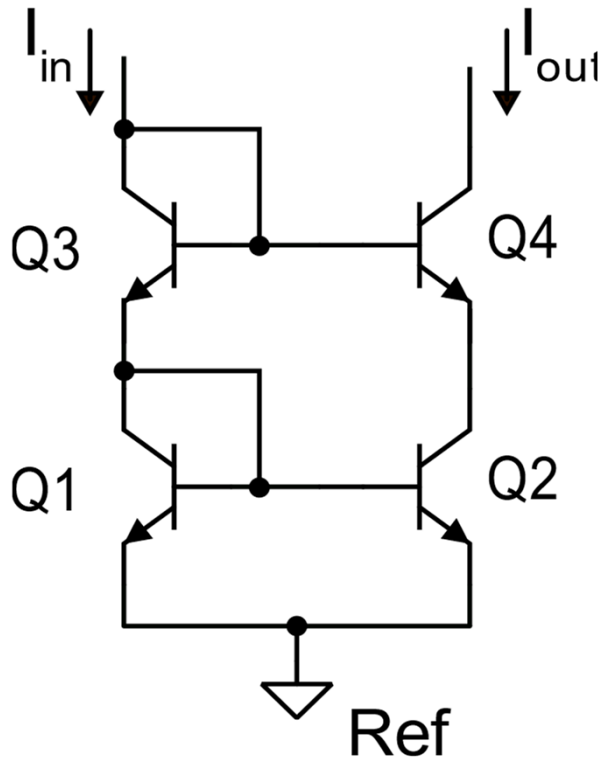
$$\frac{I_{C2}}{I_{C1}} = k_M \quad \text{almost independently of } V_{out}$$

$$\left. \begin{aligned} V_{BE3} &= V_{BE4} \\ V_{BE} &\cong V_T \ln\left(\frac{I_C}{I_S}\right) \end{aligned} \right\} \frac{I_{C4}}{I_{S4}} = \frac{I_{C3}}{I_{S3}}$$

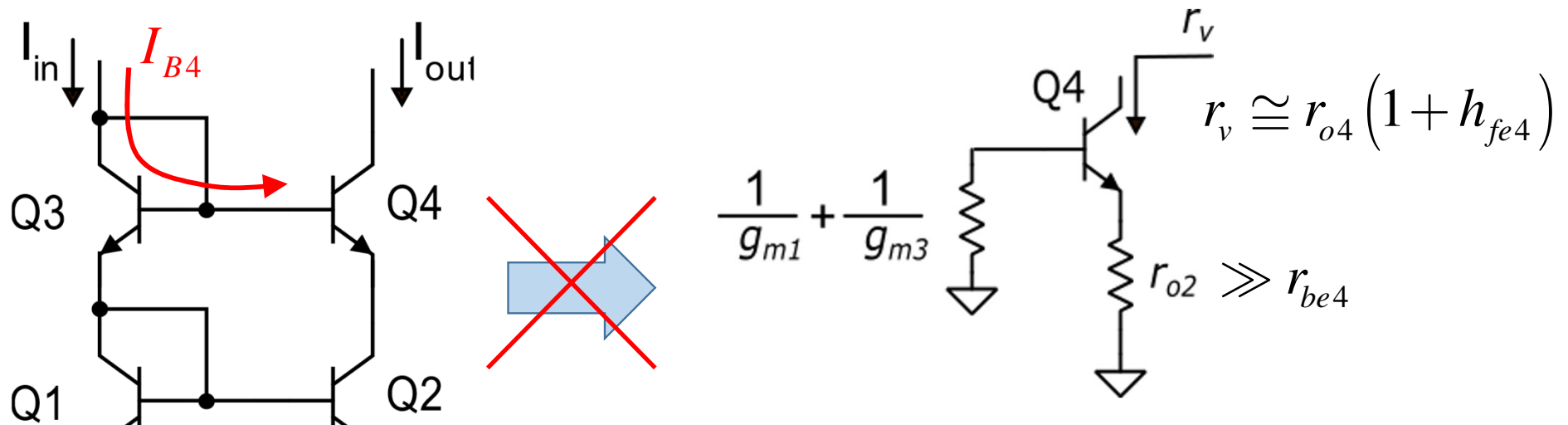
$$\frac{I_{S4}}{I_{S3}} = \frac{I_{C4}}{I_{C3}} \cong \frac{I_{C2}}{I_{C1}} = \frac{I_{S2}}{I_{S1}} = k_M$$

Design rule

$$\frac{I_{S4}}{I_{S3}} = \frac{I_{S2}}{I_{S1}} = k_M$$



BJT cascode current mirror: R_{out}

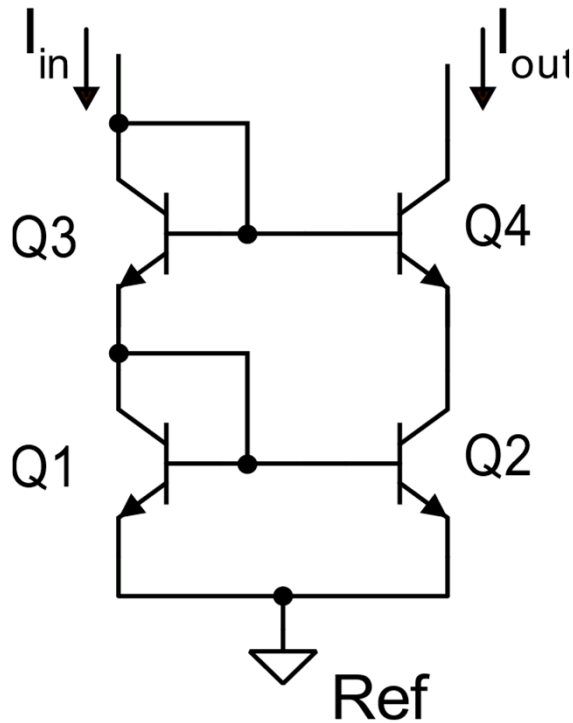


There is interaction of the output branch (Q2,Q4) with the input one (Q3,Q4) caused by the base currents. This interaction is not present in the cascode MOSFET

It can be shown that:

$$R_{out} \cong r_{o4} \left(1 + \frac{h_{fe4}}{2} \right)$$

BJT cascode current mirror: effect of base currents



$$\begin{cases} I_{out} = I_{C4} = I_{E4} - I_{B4} = I_{C2} - I_{B4} \\ I_{in} = I_{C1} + I_{B1} + I_{B2} + I_{B4} \end{cases}$$

$$\frac{I_{out}}{I_{in}} = \frac{I_{C2} - I_{B4}}{I_{C1} + I_{B1} + I_{B2} + I_{B4}} \approx \frac{I_{C2} - \frac{I_{C2}}{\beta}}{I_{C1} + \frac{I_{C1}}{\beta} + 2\frac{I_{C2}}{\beta}}$$

$$\begin{cases} I_{B2} = \frac{I_{C2}}{\beta} \approx I_{B4} \\ I_{B1} = \frac{I_{C1}}{\beta} \end{cases}$$

$$\frac{I_{C4} \approx I_{E4} = I_{C2}}{\downarrow}$$

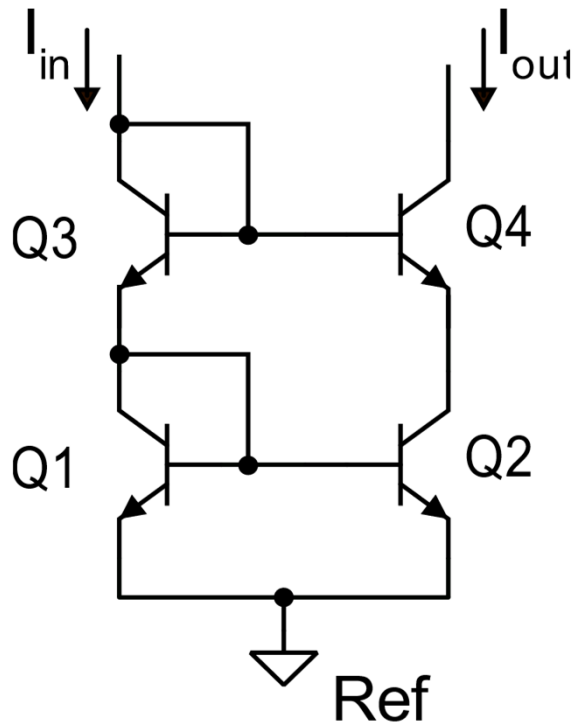
using the approximation:

$$\frac{1-y}{1+x} \approx (1-y)(1-x) = 1 - (x+y) + xy$$

$$\frac{I_{out}}{I_{in}} \approx \frac{I_{C2}}{I_{C1}} \cdot \frac{1 - \frac{1}{\beta}}{1 + \frac{1}{\beta} + 2\frac{k_M}{\beta}}$$

$$\frac{I_{out}}{I_{in}} \approx k_M \cdot \left(1 - 2\frac{1+k_M}{\beta} \right) \quad \longrightarrow \quad \varepsilon_R = -2\frac{1+k_M}{\beta}$$

BJT cascode current mirror: summary of parameters



$$V_{IN} = V_{BE1} + V_{BE3} \approx 2V_{\gamma}$$

$$V_{MIN} = V_{BE1} + V_{CESAT4} \approx V_{\gamma} + V_{CESAT}$$

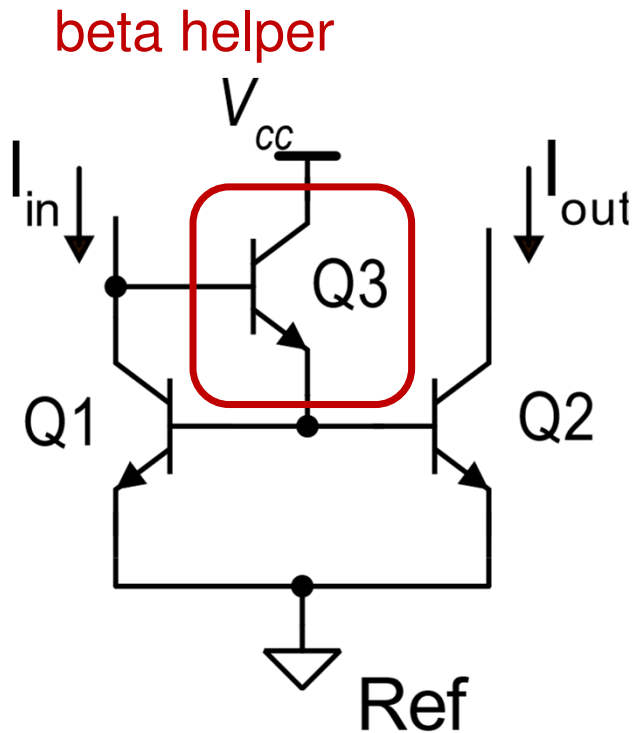
$$R_{out} \cong r_{o4} \left(1 + \frac{h_{fe4}}{2} \right) = \frac{V_A}{I_{out}} \left(1 + \frac{h_{fe4}}{2} \right)$$

$$\varepsilon_R = -2 \frac{1 + k_M}{\beta}$$

The output resistance is increased by a factor around $h_{fe}/2$, which is paid with an increase of both V_{IN} and V_{MIN} . The relative error due to the base current is **doubled** with respect of the simple mirror.

Due to the mentioned drawbacks of the BJT cascode current mirror, other solutions, such as the **emitter degenerated mirror** and **Wilson mirror** are often preferred.

Methods to reduce the impact of the base currents: the "beta helper"



$$I_{in} = I_{C1} + I_{B3} = I_{C1} + \frac{I_{E3}}{\beta_3 + 1}$$

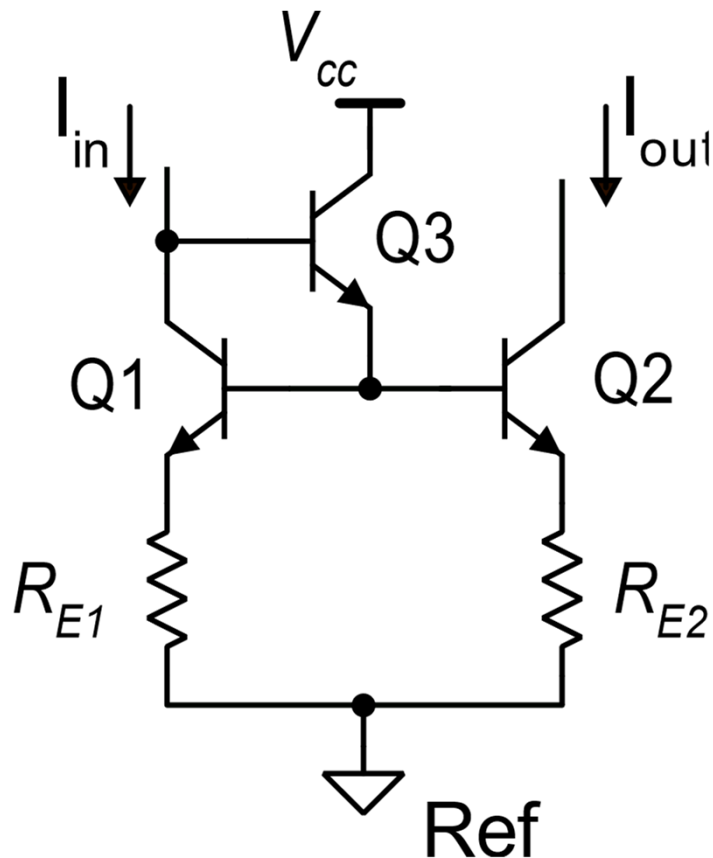
$$I_{E3} = I_{B1} + I_{B2}$$

$$I_{in} = I_{C1} + \frac{I_{B1} + I_{B2}}{\beta_3 + 1} = I_{C1} + \frac{1}{\beta_3 + 1} \left(\frac{I_{C1}}{\beta} + \frac{k_M I_{C1}}{\beta} \right)$$

$$\frac{I_{out}}{I_{in}} = \frac{I_{C2}}{I_{C1} + I_{C1} \frac{1 + k_M}{(\beta_3 + 1)\beta}} = k_M \frac{1}{1 + \frac{1 + k_M}{(\beta_3 + 1)\beta}}$$

$$\varepsilon_R = -\frac{1 + k_M}{(\beta_3 + 1)\beta} \approx \frac{1}{\beta^2} \quad V_{IN} = 2V_\gamma$$

Simple mirror with emitter degeneration and beta helper



$$R_{out} \cong r_{o2} (1 + g_{m2} R_{E2})$$

$$V_{IN} = 2V_{\gamma} + R_{E1} I_{E1}$$

$$V_{MIN} = V_{CESAT} + R_{E2} I_{E2}$$

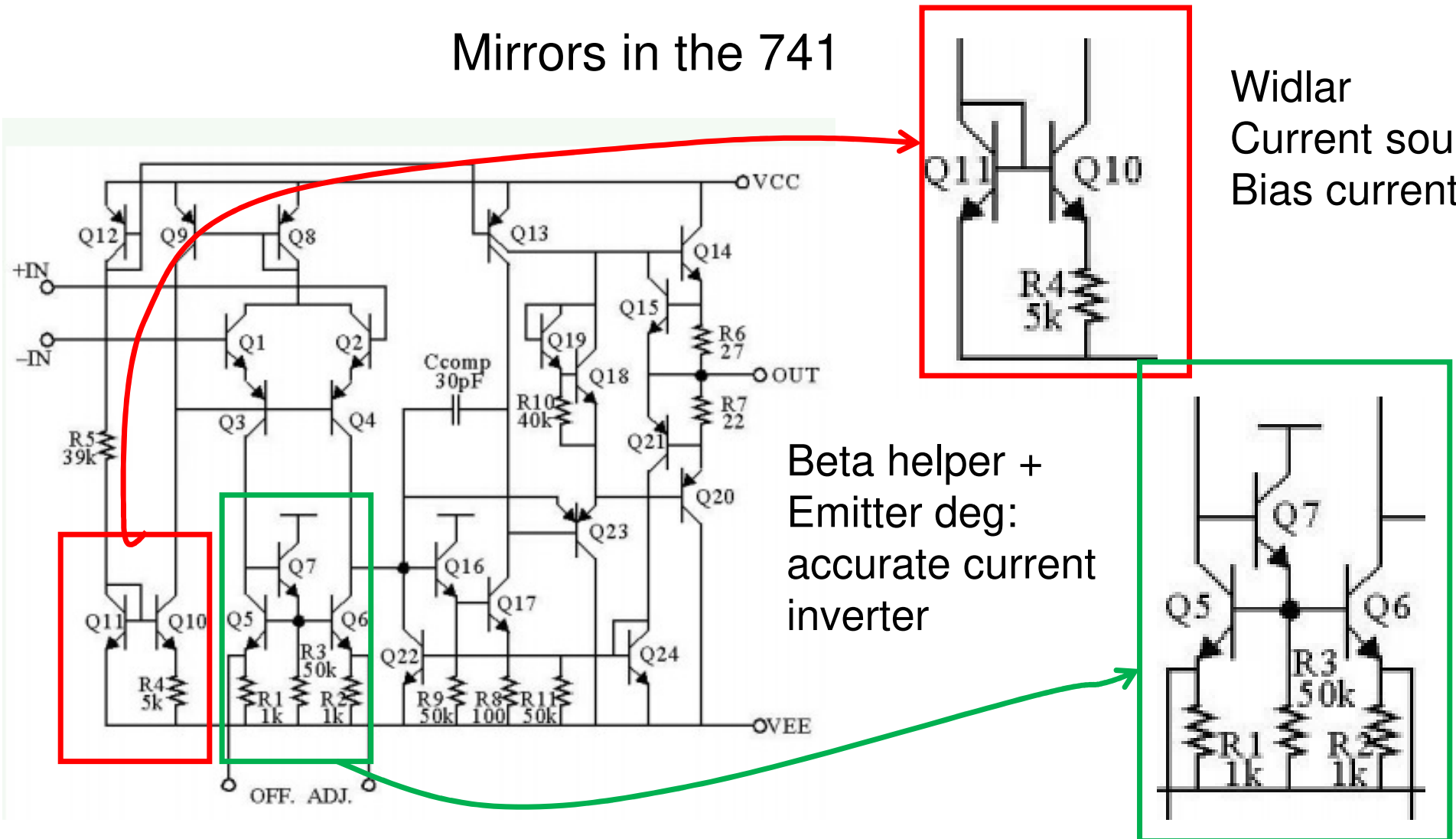
$$\varepsilon_R = -\frac{1 + k_M}{(\beta_3 + 1)\beta} \approx \frac{1}{\beta^2}$$

This mirror offers a good output resistance and small base-current related error.

This made it a popular choice for the design of BJT op-amps.

Its large V_{IN} prevents its use in low supply voltage circuits

Mirrors in the 741



Widlar
Current source:
Bias current

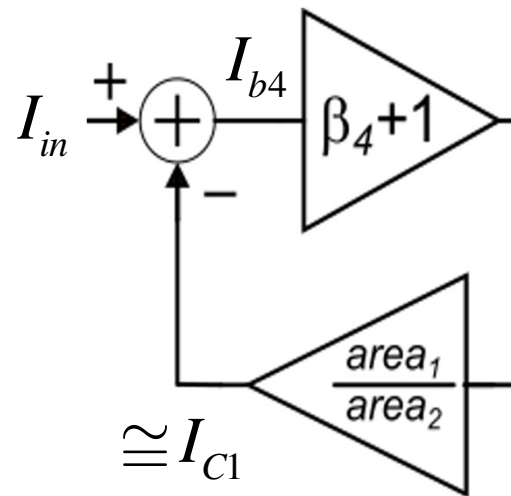
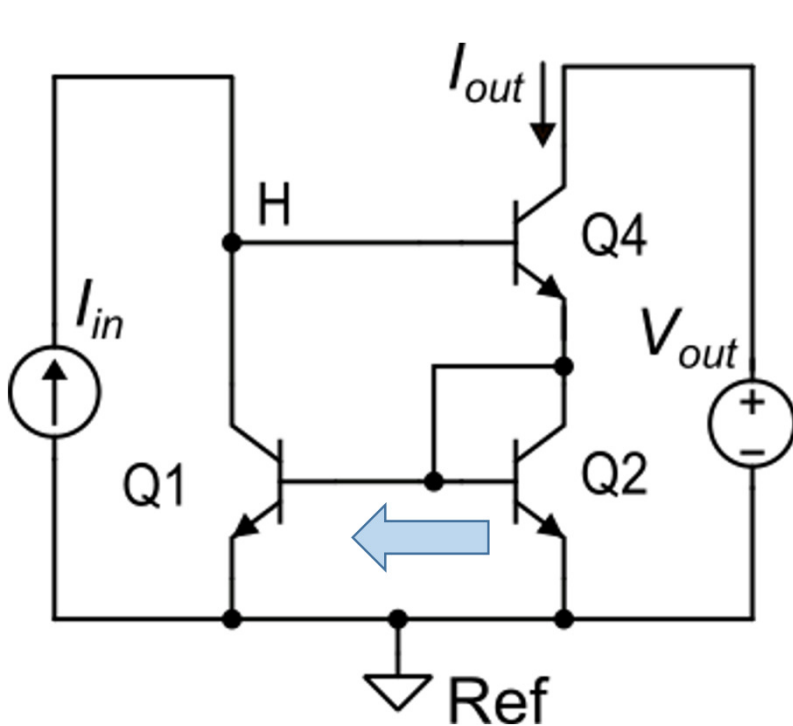
Beta helper +
Emitter deg:
accurate current
inverter

Wilson current mirror

Target: to obtain the same output resistance of a cascode mirror, but with smaller impact of base currents on the current gain accuracy.

Operating principle

Note: the Q2-Q1 mirror transmits the output current (I_{out}) back to the input branch



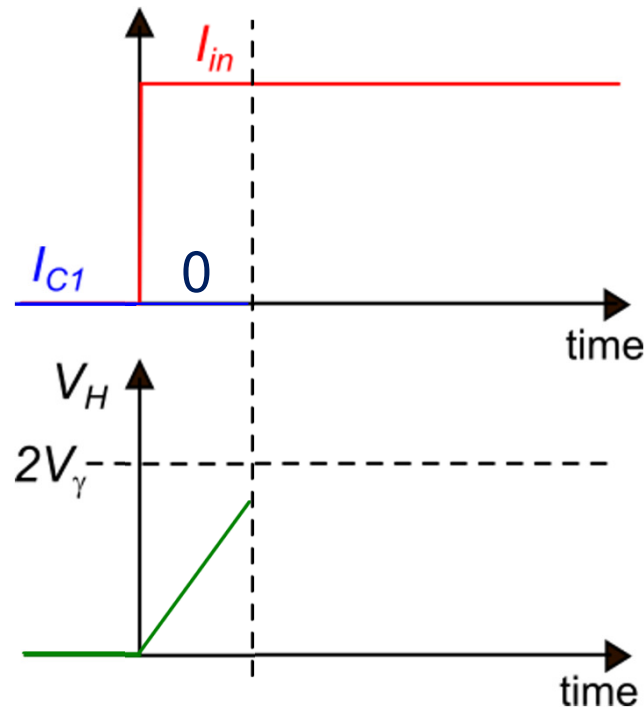
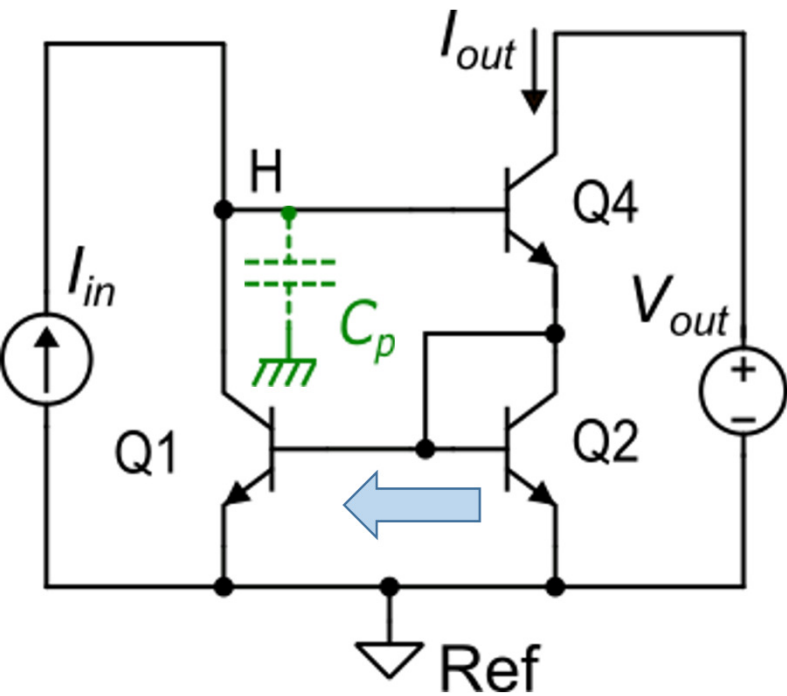
$$I_{E4} \cong I_{out}$$

$$I_{out} \cong I_{E4} \cong \left(I_{in} - I_{E4} \frac{area_1}{area_2} \right) (\beta_4 + 1)$$

$$I_{out} \cong I_{in} \frac{(\beta_4 + 1)}{\frac{area_1}{area_2} (\beta_4 + 1) + 1} \cong \frac{area_2}{area_1} I_{in}$$

Wilson current mirror: start-up transient

$$I_{Cp} = I_{in} - I_{C1} - I_{B4}$$

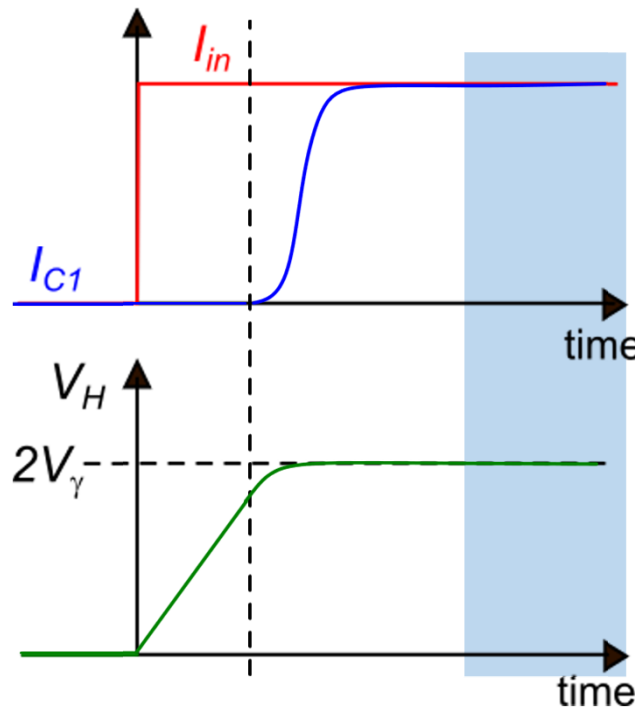
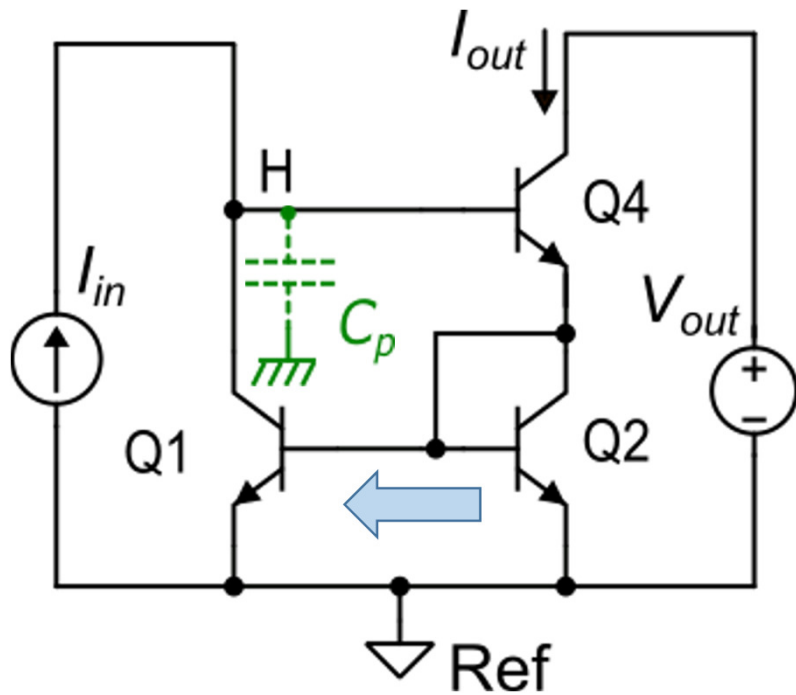


until:
 $V_H \ll 2V_\gamma \rightarrow$ Q2, Q4: off
 $I_{B4} = 0, I_{C4} = 0, I_{C2} = 0$
 $I_{Cp} = I_{in} \leftarrow I_{C1} = 0$
 $\frac{dV_H}{dt} = \frac{I_{Cp}}{C_p} = \text{constant}$
 As V_H approaches $2V_\gamma$, Q4 and Q2 turn on and the Q2 current is mirrored to M1

Wilson current mirror

I_{C1} starts increasing and taking part of I_{in} , away from $C_p \Rightarrow I_{Cp} = I_{in} - I_{C1} - I_{B4}$: decreases

$$\frac{dV_H}{dt} = \frac{I_{Cp}}{C_p} \quad : \text{decreases}$$



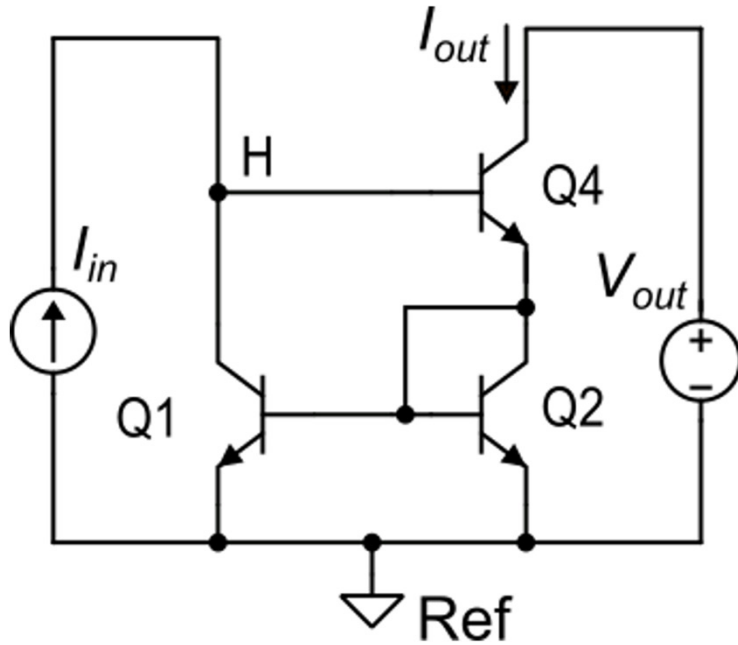
Due to the exponential law, small V_H increments produce large I_{C2} (and then I_{C1}) increases

Equilibrium is reached when:

$$\frac{dV_H}{dt} = 0 \Rightarrow I_{Cp} = 0$$

$$I_{in} = I_{C1} + I_{B4}$$

Wilson Current Mirror: effect of base currents



$$I_{in} = I_{C1} + I_{B4}$$

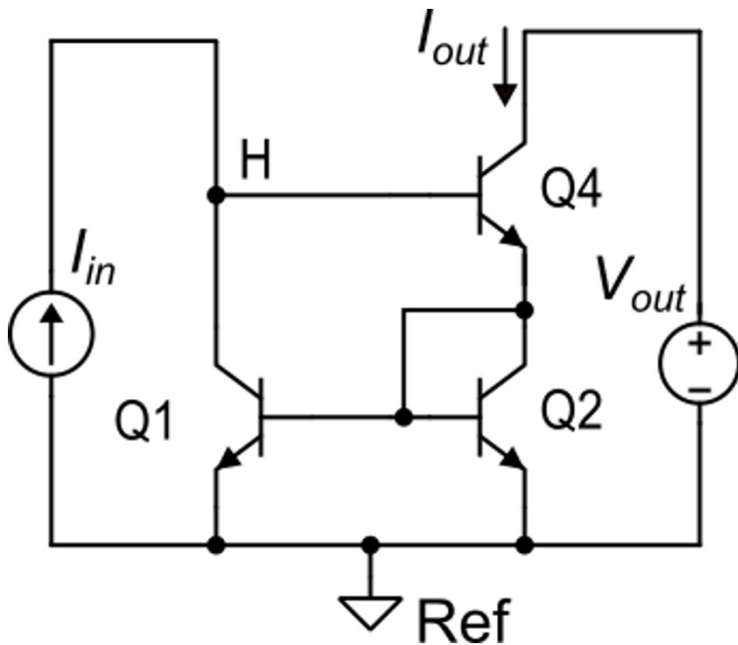
$$I_{out} = I_{C4} = I_{E4} - I_{B4} = \overbrace{I_{C2} + I_{B1} + I_{B2}}^{I_{E4}} - I_{B4}$$

$$\frac{I_{out}}{I_{in}} = \frac{I_{C2} + I_{B1} + I_{B2} - I_{B4}}{I_{C1} + I_{B4}} \cong \frac{I_{C2}}{I_{C1}} \cong \frac{area_2}{area_1} \triangleq k_M$$

$$I_{C2} \cong I_{C4}$$

$$\frac{I_{out}}{I_{in}} = \frac{I_{C2} + \frac{I_{C2}}{k_M \beta_1} + \frac{I_{C2}}{\beta_2} - \frac{I_{C2}}{\beta_4}}{I_{C1} + \frac{k_M I_{C1}}{\beta_4}} = \frac{I_{C2}}{I_{C1}} \frac{1 + \frac{1}{k_M \beta_1} + \frac{1}{\beta_2} - \frac{1}{\beta_4}}{1 + \frac{k_M}{\beta_4}}$$

Wilson current mirror



$$\frac{I_{out}}{I_{in}} = k_M \frac{1 + \frac{1}{k_M \beta_1} + \frac{1}{\beta_2} - \frac{1}{\beta_4}}{1 + \frac{k_M}{\beta_4}}$$

$$\frac{I_{out}}{I_{in}} \approx k_M \left(1 + \frac{1}{k_M \beta_1} - \frac{k_M}{\beta_4} + \frac{1}{\beta_2} - \frac{1}{\beta_4} \right)$$

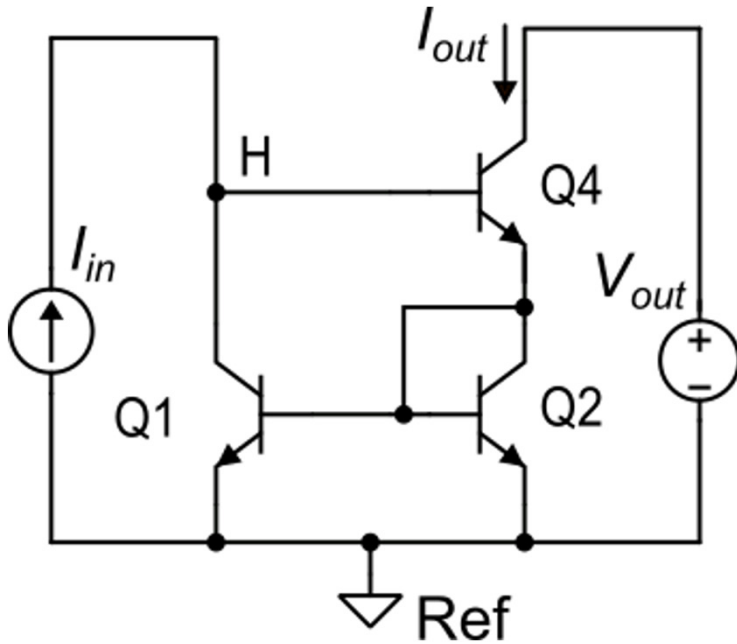
Only if: $k_M=1$

$$= \frac{1}{\beta_1} - \frac{1}{\beta_4} \approx \frac{1}{\beta^2}$$

$$\frac{1}{\beta_2} - \frac{1}{\beta_4} = \frac{\beta_4 - \beta_2}{\beta_2 \beta_4} \approx \frac{1}{\beta^2}$$

for: $k_M = 1$ $\frac{I_{out}}{I_{in}} = k_M (1 + \varepsilon_R), \quad \varepsilon_R \approx \frac{1}{\beta^2}$

Wilson current mirror



In the Wilson current mirror the relative error due to the base current is of the order of $1/\beta^2$ only when it is designed for $k_M=1$. For current gains different from one, the relative error becomes of the order of $1/\beta$.

Other characteristics:

$$V_{IN} = V_H = V_{BE2} + V_{BE4} \cong 2V_\gamma$$

$$V_{MIN} = V_{BE2} + V_{CESAT4}$$

$$R_{out} \cong r_{o4} \left(1 + \frac{h_{fe4}}{2} \right) \quad \text{due to feedback from } I_{out} \text{ to node H}$$

(same as the cascode mirror)

$$V_{CE1} = 2V_{BE} > V_{CE2} = V_{BE}$$

There is a large systematic error between I_{C2} and I_{C1}

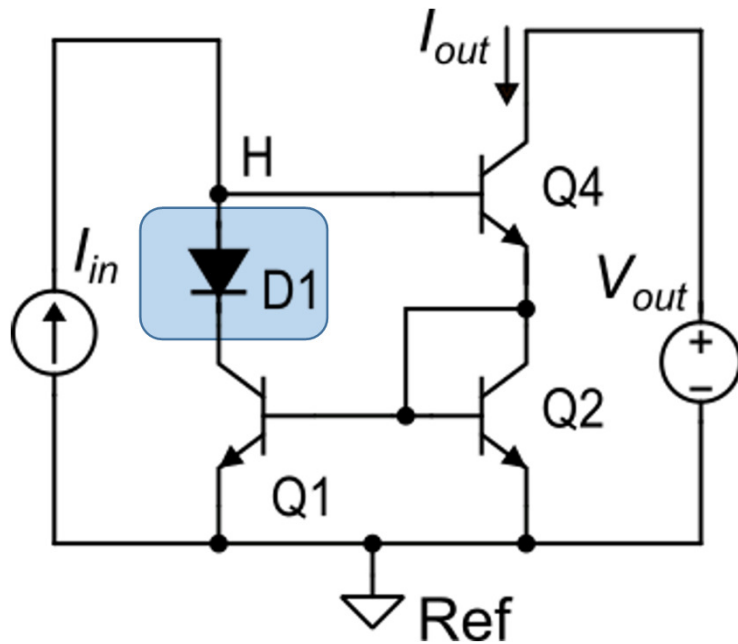
4-transistor Wilson current mirror

$$V_H = V_{BE2} + V_{BE4} = 2V_{BE}$$

$$V_{CE1} = V_H - V_\gamma = V_\gamma$$

$$V_{CE2} = V_\gamma$$

The 4-transistor Wilson Mirror is the optimal choice when an accurate mirror with $k_M=1$ is required.



Implementing D1 with a diode-connected BJT (Q3)

