1 Effect of matching errors on the precision of ratios in integrated circuits.

Let us define the ratio:

$$r = \frac{R_1 + ... + R_M}{R_{M+1} + ... + R_{M+N}} \tag{1.1}$$

where R is a quantity (e.g. the resistance) associated to a type of component (e.g. a resistor) and R_i are the value assumed by R on N+M nominally identical components. Clearly, the nominal value of r is M/N.

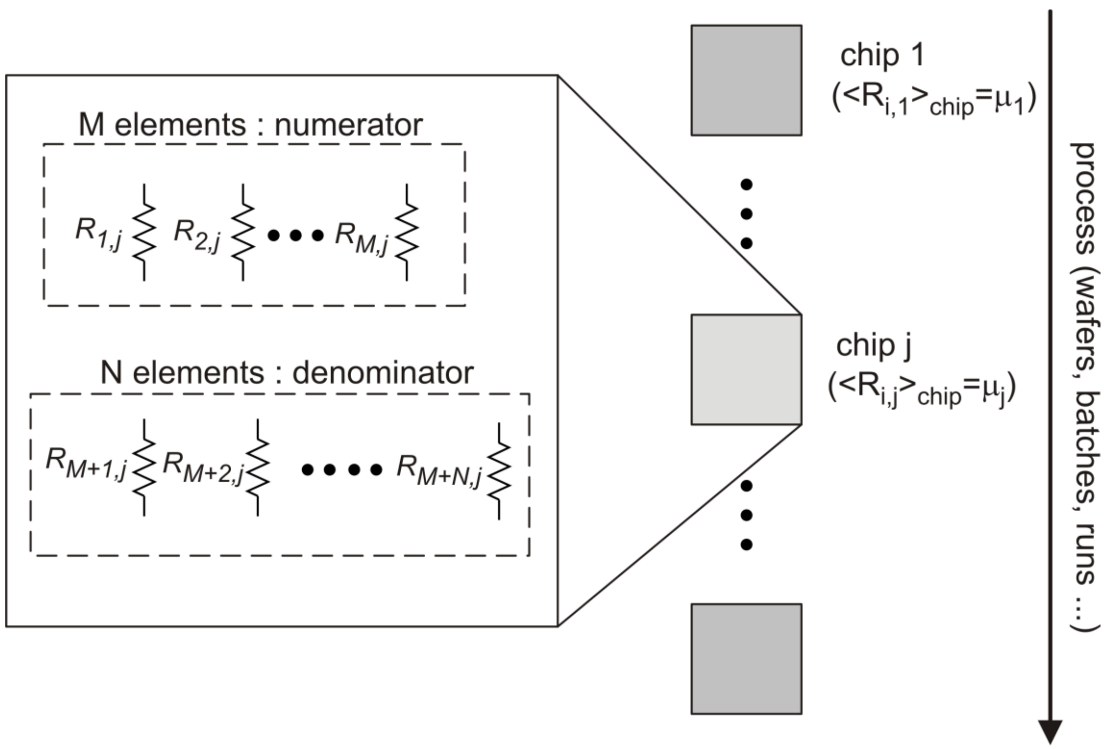

Fig. 1.1. Ratio of *M* over *N* nominally identical resistors.

Let us now consider a series of distinct samples of the same chip, and identify each sample (i.e. each chip) with number j. Figure 1.1 illustrate these concepts. With obvious notation, we will indicate the ratio r_j of chip j as:

$$r_j = \frac{R_{1,j} + ... + R_{M,j}}{R_{M+1,j} + ... + R_{M+N,j}} \tag{1.2}$$

The problem here is to determine the standard deviation of ratio r , which expresses the variability of r over a large number of different samples.

For each chip we can consider that the values $R_{i,j}$ are the result of a stochastic sub-process which is peculiar of that chip. We can define the mean μ_j of that sub-process, which can be regarded as the average of an infinite number of nominally identical components $R_{i,j}$, all produced in the same chip j. This is clearly an ideal operation, since the number of component that can be included into the same single chip is finite and generally small. We also observe that μ_j varies widely from one chip to another owing to process variations (global variations or global errors). These variations are clearly larger if we consider chips that comes from different runs, but this point is not essential here. We can write the $R_{i,j}$ values as:

$$R_{i,j} = \mu_j + \delta R_{i,j} \tag{1.3}$$

It can be easily demonstrated that, if the mean μ_j is really the limit of the average calculated on an ideally infinite number of components, the deviations $\delta R_{i,j}$ (local variations or local errors) of two distinct components are independent of each other, that is:

$$< \delta R_{k,j} \delta R_{h,j} >_{proc} = 0 \quad \text{for } h \neq k \tag{1.4}$$

where operator $< >_{proc}$ indicates the average performed over the whole process, i.e. when j varies from 1 to ∞.

Using (1.3) we can write the ratio (1.2) as:

$$r_j = \frac{\left(\mu_j + \delta R_{1,j}\right) + ... + \left(\mu_j + \delta R_{M,j}\right)}{\left(\mu_j + \delta R_{M+1,j}\right) + ... + \left(\mu_j + \delta R_{M+N,j}\right)} = \frac{M\mu_j + \sum_{i=1}^{M} \delta R_{i,j}}{N\mu_j + \sum_{i=M+1}^{M+N} \delta R_{i,j}} \tag{1.5}$$

which can be rewritten as:

$$r_j = \frac{M}{N} \frac{1 + \frac{1}{M}\sum_{i=1}^{M} \frac{\delta R_{i,j}}{\mu_j}}{1 + \frac{1}{N}\sum_{i=M+1}^{M+N} \frac{\delta R_{i,j}}{\mu_j}} \tag{1.6}$$

If $$\frac{1}{M}\left|\sum_{i=1}^{M} \frac{\delta R_{i,j}}{\mu_j}\right| << 1 \quad \text{and} \quad \frac{1}{N}\left|\sum_{i=M+1}^{M+N} \frac{\delta R_{i,j}}{\mu_j}\right| << 1 \tag{1.7}$$

we can apply first order Taylor approximation and easily find that:

$$r_j \cong \frac{M}{N}\left(1+\sum_{i=1}^{M}\frac{\delta R_{i,j}}{M\mu_j}-\sum_{i=M+1}^{M+N}\frac{\delta R_{i,j}}{N\mu_j}\right) \tag{1.8}$$

$$\frac{\left(r_j-\frac{M}{N}\right)}{\frac{M}{N}}=\frac{\Delta r_j}{r_{nom}}\cong\left(\sum_{i=1}^{M}\frac{\delta R_{i,j}}{M\mu_j}-\sum_{i=M+1}^{M+N}\frac{\delta R_{i,j}}{N\mu_j}\right) \tag{1.9}$$

where r_{nom} is the nominal value of ratio r, i.e. M/N. Now, considering Eq. (1.4), stating that all $\delta R_{i,j}$ are uncorrelated, we find:

$$\frac{\sigma_{\Delta r}^2}{r_{nom}^2}\cong\frac{1}{M^2}\sum_{i=1}^{M}\sigma_{\frac{\delta R_i}{\mu_j}}^2+\frac{1}{N^2}\sum_{i=M+1}^{M+N}\sigma_{\frac{\delta R_i}{\mu_j}}^2 \tag{1.10}$$

Note that averages μ_j is the average of the quantity R of elements in chip j, thus the ratios $\delta R_{i,j}/\mu_j$ are the relative ($\delta R/R$) local errors. Considering that no systematic error is present, the standard deviations of $\delta R_{i,j}/\mu_j$ of all components on the chip (R_1 .... R_{N+M}) are equal, so that, by simple passages, we get:

$$\frac{\sigma_{\Delta r}}{r_{nom}}\cong\sigma_{\frac{\delta R}{R}}\sqrt{\frac{1}{N}+\frac{1}{M}} \tag{1.11}$$

Eq. (1.11) relates the standard deviation of the relative error on the ratio r to the standard deviation of the relative mismatch error of the quantity R with respect of the average over a single chip, μ_j. Since the data that is generally reported is the standard deviation of the relative *matching error* between <u>two elements</u>, ΔR, than we have to find a relationship between $\sigma_{\delta R}$ and $\sigma_{\Delta R}$.

Considering for simplicity two elements R_1 and R_2, we can write:

$$\Delta R = R_{1,j}-R_{2,j}=\delta R_{1,j}-\delta R_{2,j} \tag{1.12}$$

Using again the statistical independence of the terms $\delta R_{i,j}$, we get:

$$\sigma_{\Delta R}^2 = \sigma_{\delta R_1}^2 + \sigma_{\delta R_2}^2 = 2\sigma_{\delta R}^2 \;\Rightarrow\; \sigma_{\delta R} = \frac{1}{\sqrt{2}}\sigma_{\Delta R} \tag{1.13}$$

With simple substitutions, we can rewrite (1.11) as:

$$\frac{\sigma_r}{r_{nom}} \cong \sigma_{\frac{\Delta R}{R}} \left[\frac{1}{\sqrt{2}} \sqrt{\frac{1}{N} + \frac{1}{M}} \right] \tag{1.14}$$