MOSFET device models and conventions

Symbols: V_t = threshold voltage; thermal voltage: $V_T=kT/q$,

$\beta_n = \mu_n C_{OX} \dfrac{W_{eff}}{L_{eff}}$. $W_{eff}=W-2W_D$, $L_{eff}=L-2L_D$

μ_n=electron mobility in the MOSFET channel (can be much lower than the mobility in bulk silicon).

1. DRAIN CURRENT

Strong Inversion: $V_{GS}-V_t>4V_T$

$$I_{DS} = \beta_n \frac{(V_{GS}-V_t)^2}{2}(1+\lambda V_{DS}) \quad \text{(saturation region: } V_{DS} > V_{GS}-V_t\text{)} \tag{1}$$

$$I_{DS} = \beta_n \left(V_{GS}-V_t-\frac{V_{DS}}{2}\right)V_{DS} \quad \text{(triode region: } V_{DS} < V_{GS}-V_t\text{)} \tag{2}$$

The threshold voltage depends on the body-source voltage (V_{BS}) according to the law:

$$V_T = V_{T0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right) \quad \text{(body effect).} \tag{3}$$

where gamma (γ) is the body effect coefficient.

Parameter lambda (λ) determines the small signal output resistance of the MOSFET according to:

$$r_d = \frac{1}{\lambda I_D} = \frac{\lambda^{-1}}{I_D} \tag{4}$$

Note that the r_d expression is similar to the r_o expression for BJTs, identifying λ^{-1} with the BJT early voltage (V_A).

Lambda depends on the MOSFET dimensions and also on the operating point (V_{GS}, V_{DS}, V_{BS}). Traditional SPICE models (Level 1,2,3) do not provide a precise representation of λ, and, consequently, of the MOSFET output resistance. More accurate models (EKV, BSIM3, Philps 9) produce improved representations of the I_D vs V_{DS} dependence, resulting in more precise simulations of the effective MOSFET output resistance in most operating conditions.
For hand calculations and design purposes, it is possible to neglect the lambda dependence on the operating point (provided that the MOSFET is in saturation region) and consider only the dependence on the channel length. A simplified linear dependence of λ^{-1} on the channel length can be used only for first order estimations of output resistances, is given by:

$$\lambda^{-1} = k_\lambda L_{eff} \tag{5}$$

where k_λ is a constant. It is important to point out that expression (5) can be used only to have a rough idea of the impact of changing the device lengths on the circuit dc performance. For example, it suggests to the designer that the output resistance of a current mirror can be approximately doubled by doubling the output MOSFET length. The actual gain in circuit performance should be mandatorily checked using an accurate electrical simulator. Equation (5) can be used only for MOSFETS in strong inversion and when L is much larger than the minimum channel length.

Weak inversion: V_{GS}-V_t<<4V_T

Weak inversion is also indicated as Sub-threshold region. Strictly speaking, subthreshold region would require that $V_{GS}-V_t<0$. In practice, the terms weak inversion and sub-threshold are both used to indicate a region where the I_{DS} dependence on both V_{GS} and V_{DS} becomes exponential, according to the equation:

$$I_{DS} = I_{SM} e^{\frac{V_{GS}-V_t}{\varsigma V_T}} \left(1 - e^{-\frac{V_{DS}}{V_T}}\right) \quad (6)$$

where: V_t= threshold voltage, $V_T=kT/q$ and:

$$\varsigma = 1 + \frac{C_D}{C_{OX}} \qquad C_D = \sqrt{\frac{q\varepsilon_{Si} N_A}{2(\phi_{Si} - V_{BS})}} \qquad I_{SM} = \mu_n C_D \frac{W_{eff}}{L_{eff}} V_T^2 \quad (7)$$

Expression (6) includes a dependence on both V_{GS} and V_{DS}. The dependence on V_{DS} vanishes when V_{DS} >> V_T. In practice, due to the exponential dependence, is sufficient that $V_{DS} > 4V_T$ to neglect the dependence on V_{DS}. This condition is similar to saturation region for strong inversion. As for strong inversion, the dependence on V_{DS} is not completely cancelled, but a residual sensitivity of the current to V_{DS} is present. This phenomenon can be modeled with the parameter lambda. Therefore:

$$I_{DS} = I_{SM} e^{\frac{V_{GS}-V_t}{\varsigma V_T}} (1 + \lambda V_{DS}) \quad \text{for } V_{DS} > 100 \text{ mV} \quad \text{(weak inversion + saturation region)} \quad (8)$$

Short channel effects, such as drain induced barrier lowering (DIBL), contribute to the effective lambda value.

Moderate inversion.

Moderate inversion is the transition region between strong inversion and weak inversion. In this region the equations progressively change from exponential (weak inversion) to parabolic.

2. DRAIN-SOURCE SATURATION VOLTAGE

As a result of previous considerations, the saturation voltage V_{DSAT} assumes the following expressions:

$$V_{DSAT} \cong \begin{cases} (V_{GS} - V_t) & \text{in strong inversion} \\ 4V_T \ \ (100 \text{ mV}) & \text{in weak inversion} \end{cases} \quad (9)$$

3. TRANSCONDUCTANCE

Tranconductance (g_m) is the most important small signal parameter of electronic devices. Using the square law drain current formulas (strong inversion) it is possible to derive the following equivalent expressions:

$$g_m = \beta_n (V_{GS} - V_t) \quad \text{(n-MOSFET, saturation +strong inversion)} \tag{10}$$

$$g_m = \sqrt{2\beta_n I_D} \quad \text{(n-MOSFET, saturation +strong inversion)} \tag{11}$$

$$g_m = \frac{2I_D}{(V_{GS} - V_t)} \quad \text{(n-MOSFET, saturation +strong inversion)} \tag{12}$$

In weak inversion, considering equation (8) the g_m becomes:

$$g_m = \frac{I_D}{\varsigma V_T} \quad \text{(n-MOSFET, weak inversion+saturation)} \tag{13}$$

The g_m dependence on the MOSFET current in weak inversion is then similar to that of bipolar transistors, for which:

$$g_m = \frac{I_C}{V_T} \quad \text{(BJT, active region)} \tag{14}$$

It is possible to use a single formula, representative of MOSFETs in strong and weak inversion and BJTs:

$$g_m = \frac{I_C}{V_{TE}} \quad \text{with } V_{TE} = \begin{cases} (V_{GS} - V_t)/2 & \text{MOSFET in strong inversion} \\ \varsigma V_T & \text{MOSFET in weak inversion} \\ V_T & \text{BJT} \end{cases} \tag{15}$$

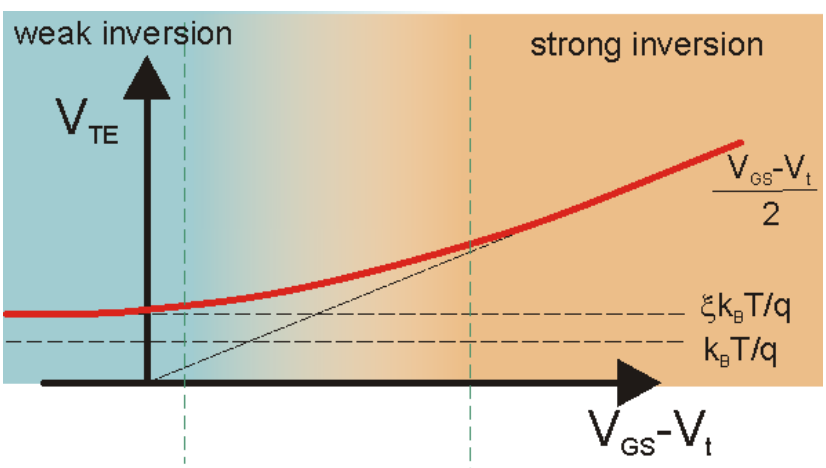

Fig.1: Behavior of V_{TE} as a function of V_{GS}-V_T.

4. PARAMETER MATCHING

$$\sigma_{\frac{\Delta\beta}{\beta}} = \frac{C_\beta}{\sqrt{WL}}; \qquad \sigma_{Vt} = \frac{C_{Vt}}{\sqrt{WL}}$$

5. PARASITIC CAPACITANCES (Meyer model)

$$C_{gs} = \frac{2}{3} C_{ox} WL$$
$$C_{gd} = C_{gdo} W$$
$$C_{bs} = C_{bd} = C_j L_C W$$