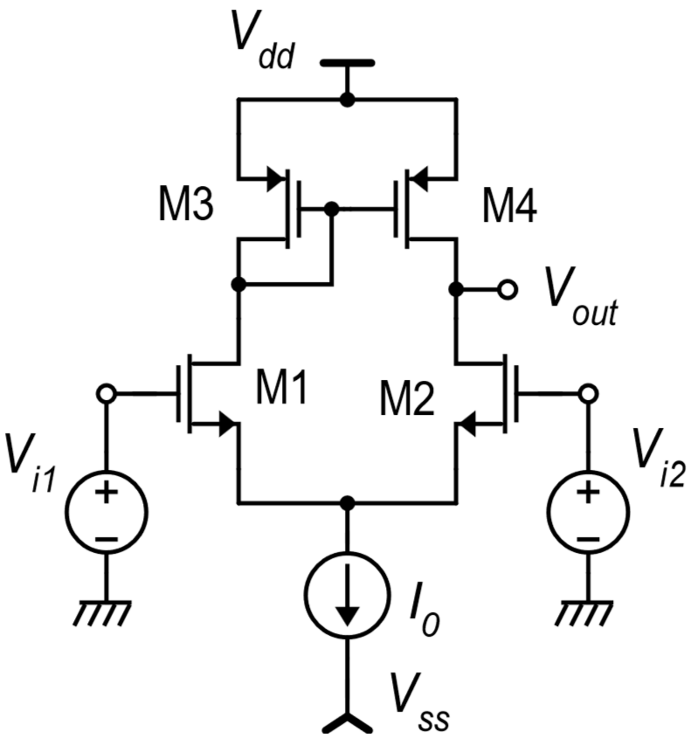
Increasing the gain of the simple differential amplifier with mirror load

For many applications, the gain of the simple CMOS differential is not sufficient

$$A_d = G_m R_{out}$$



$$G_m = g_{m1} = \frac{I_{D1}}{V_{TE}} \qquad R_{out} = r_{d2} \,//\, r_{d4} = \frac{1}{I_{D1}} \frac{1}{\lambda_2 + \lambda_4}$$

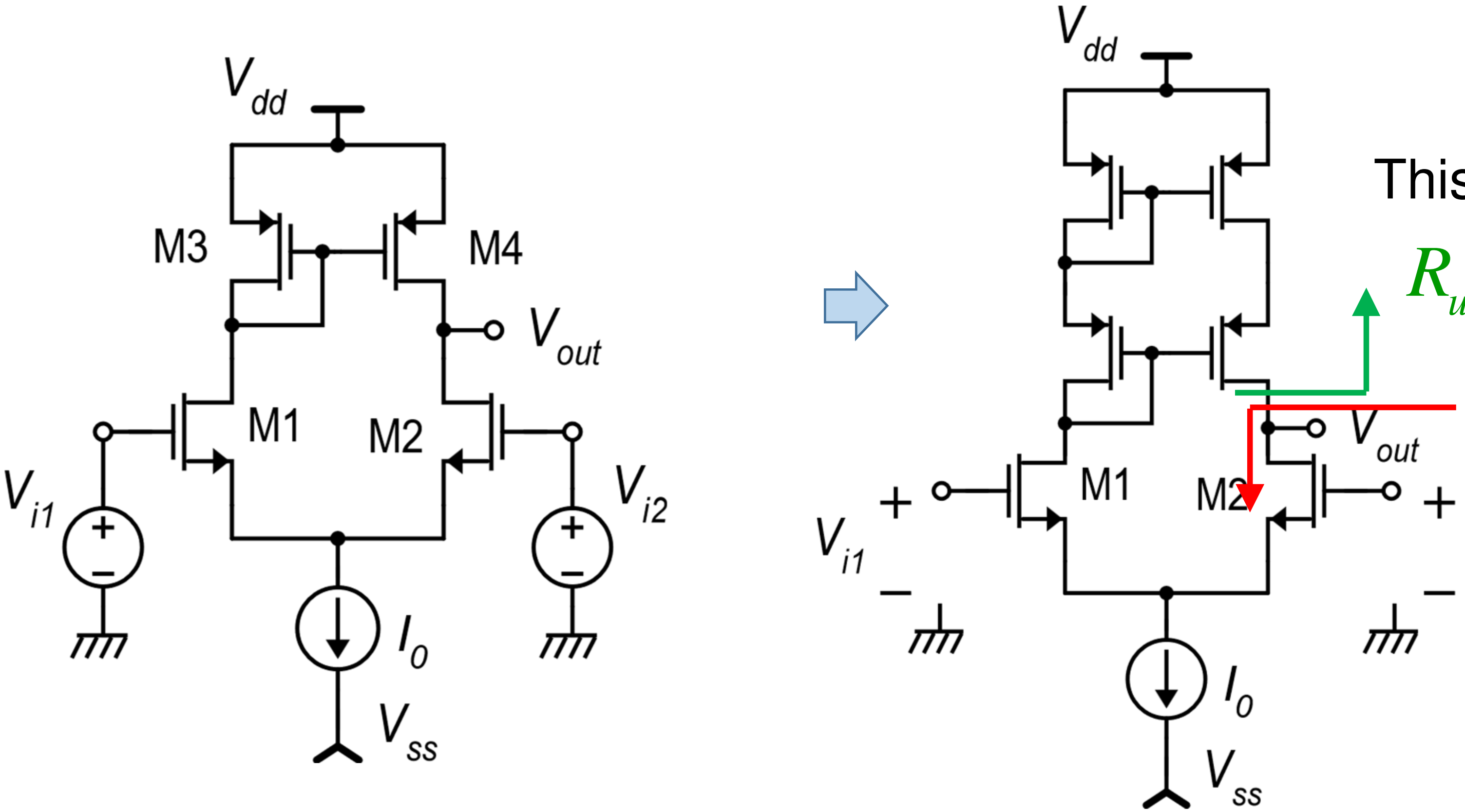
Trying to increase $\boldsymbol{g_{m1}}$ by increasing $\boldsymbol{I_{D1}}$ produces a corresponding decrease of R_{out}.

The gain is not improved

We can increase R_{out} by reducing the lambdas, but this means very long MOSFETs, resulting in large area occupation and parasitic capacitances.

We need to increase R_{out} in a different way!

Fist step: replace the simple current mirror with a cascode mirror



This increases $\boldsymbol{R_{up}}$ …

$$R_{up} \approx r_d \left(g_m r_d \right)$$

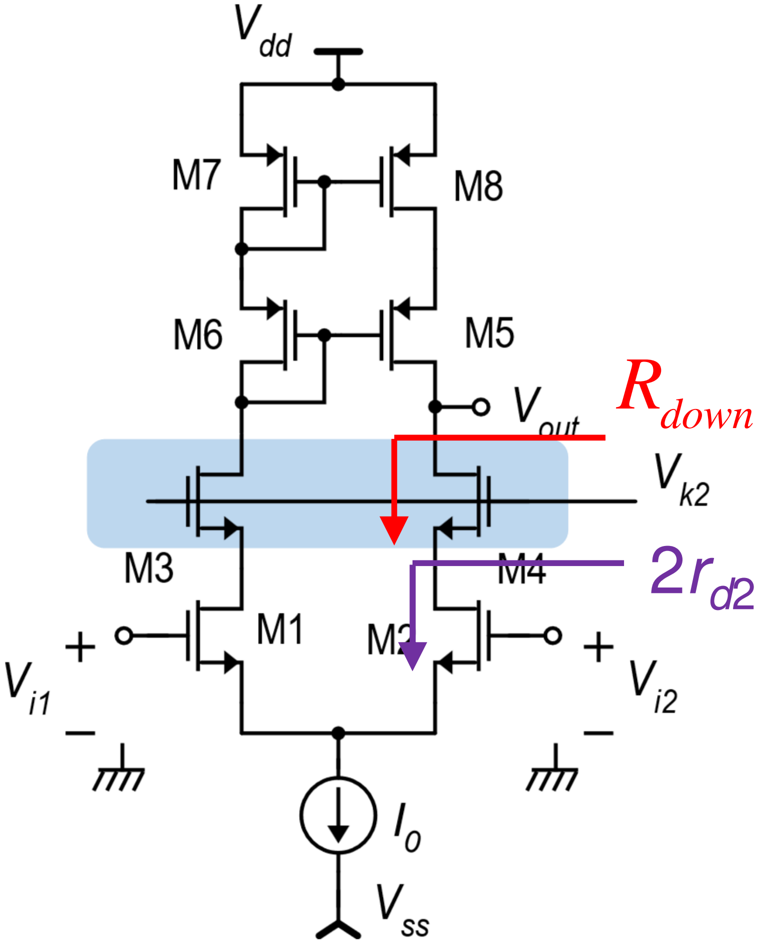
$$R_{down} \approx 2r_d$$

… but $\boldsymbol{R_{down}}$ is not changed

$$R_{out} \cong r_{d2}$$

R_{out} passes from $r_d/2$ to r_d and the gain is only doubled

Second step: add a common-gate also to the differential pair: The cascode differential amplifier

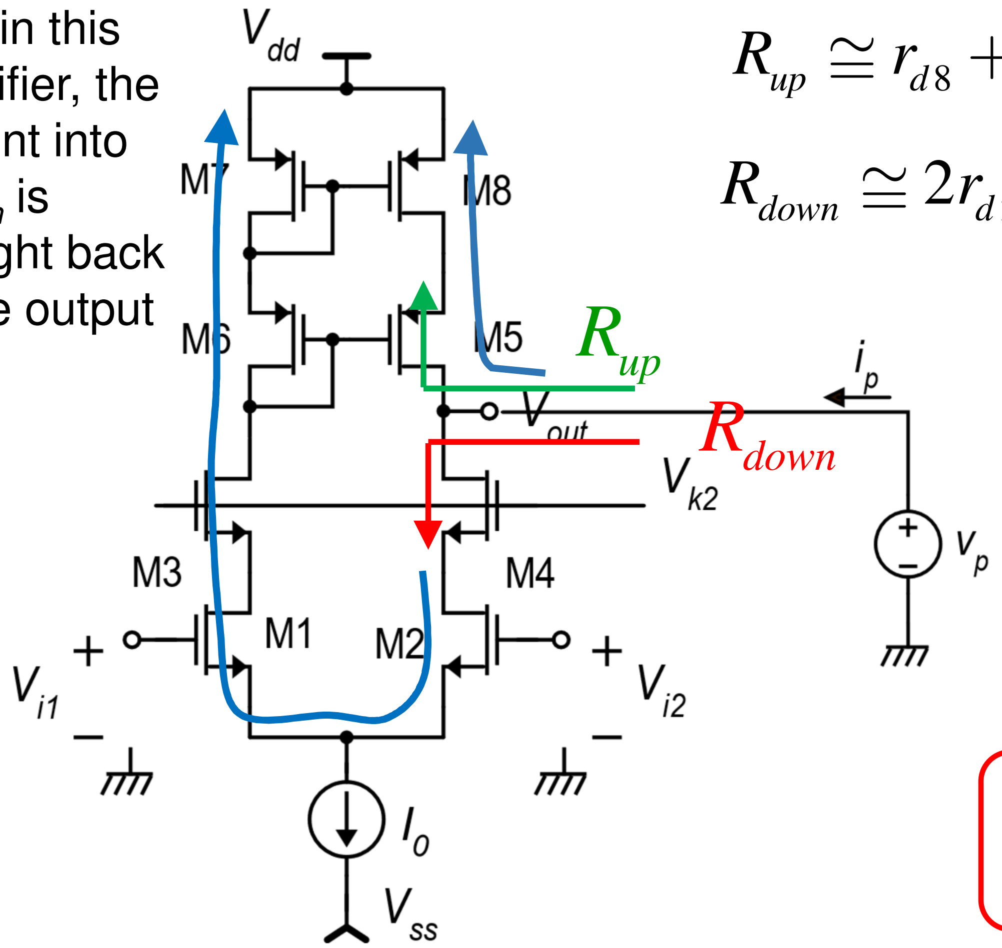


$$R_{down} \cong 2r_{d2} + r_{d4}\left(1 + g_{m4} 2r_{d2}\right)$$

We add a differential common gate stage (M3,M4) to the differential pair: forming a differential cascode stage

Cascode differential amplifier: R_{out}

Also in this amplifier, the current into R_{down} is brought back to the output



$$R_{up} \cong r_{d8} + r_{d5}\left(1 + r_{d8} g_{m5}\right) \cong r_{d5}\left(r_{d8} g_{m5}\right)$$

$$R_{down} \cong 2r_{d2} + r_{d4}\left(1 + g_{m4} \cdot 2r_{d2}\right) \cong 2r_{d4}\left(r_{d2} g_{m4}\right)$$

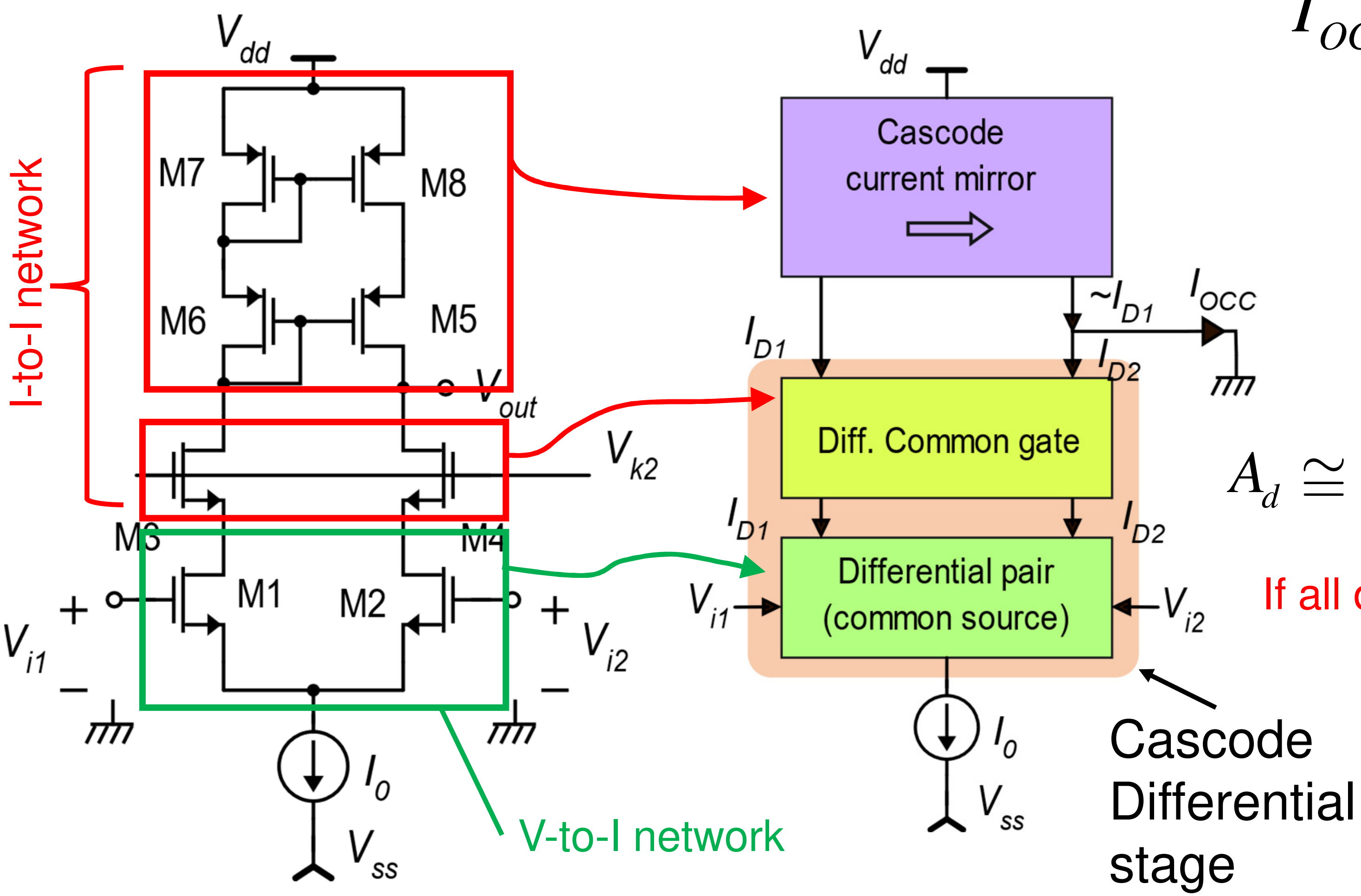
$$i_p = \frac{v_p}{R_{up}} + 2\frac{v_p}{R_{down}}$$

The current into R_{down} is doubled

$$R_{out} = R_{up} // \left(\frac{R_{down}}{2}\right)$$

$$\boxed{R_{out} \cong r_{d5}\left(r_{d8} g_{m5}\right) // r_{d4}\left(r_{d2} g_{m4}\right)}$$

Cascode differential amplifier



$$I_{OCC} \cong I_{D1} - I_{D2} \cong g_{m1} v_d$$

$$I_{OCC} = G_m v_d$$

$$G_m = g_{m1}$$

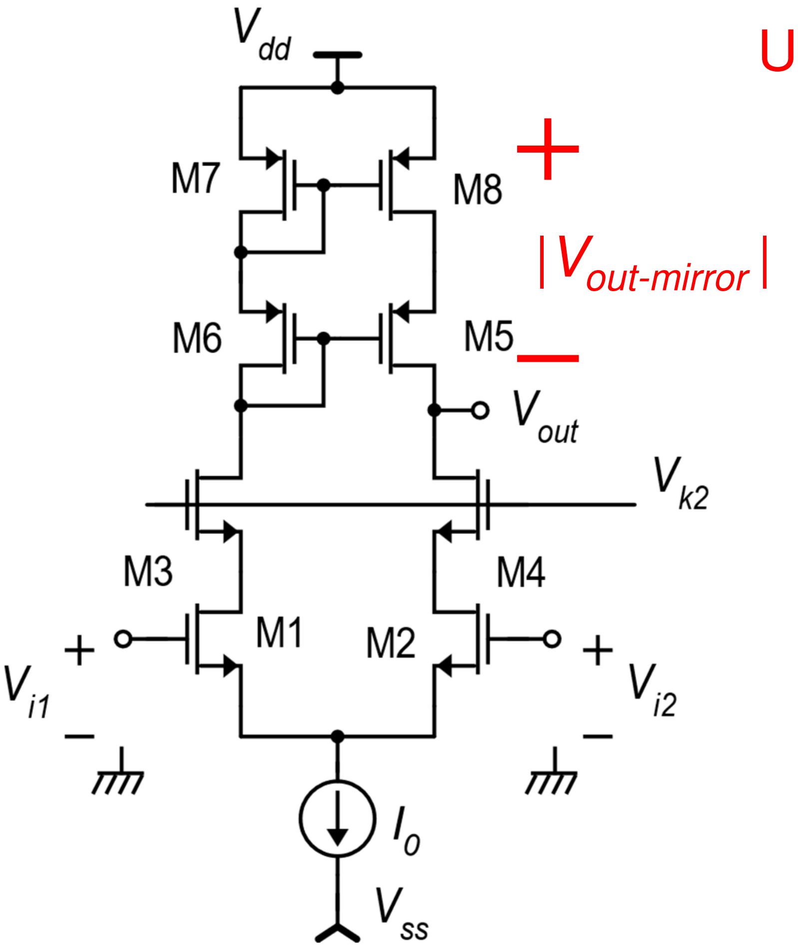
$$A_d = G_m R_{out}$$

$$A_d \cong g_{m1}\left[\left(r_{d2} g_{m4} r_{d4}\right) // \left(r_{d8} g_{m5} r_{d5}\right)\right]$$

If all devices have the same r_d and g_m:

$$\boxed{A_d \cong \frac{\left(g_m r_d\right)^2}{2}}$$

CMOS cascode amplifier: output range



Upper limit

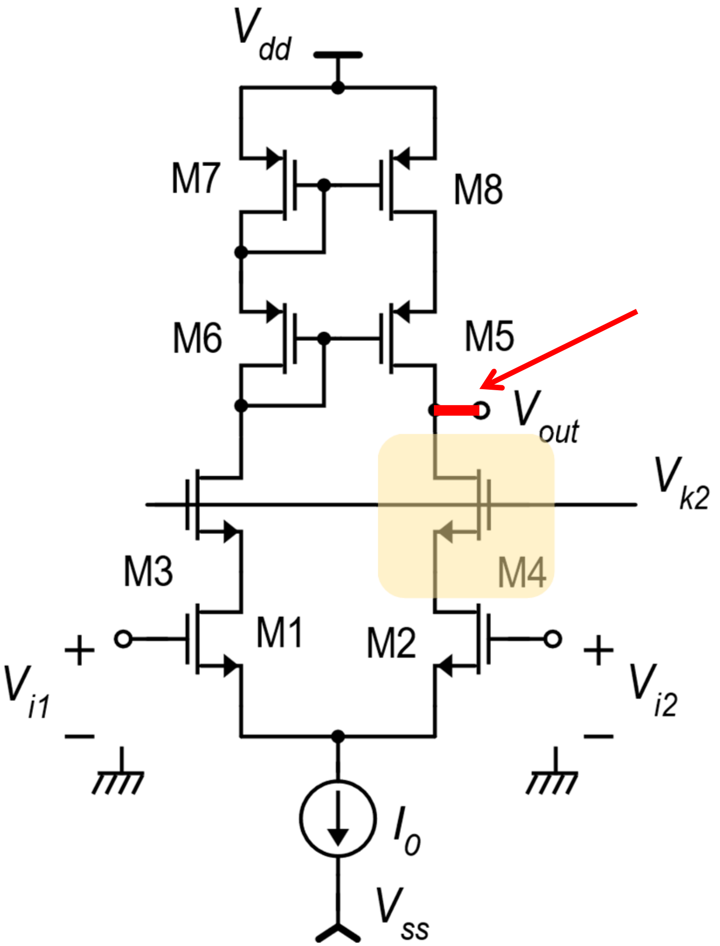
$$\left|V_{out-mirror}\right| = V_{dd} - V_{out} \geq V_{MIN-cascode}$$

$$V_{dd} - V_{MIN-cascode} \geq V_{out}$$

$$\boxed{\max\left(V_{out}\right) = V_{dd} - V_{MIN-cascode}}$$

As V_{out} gets higher than max(V_{out}), R_{up} decreases making the gain decrease

CMOS cascode amplifier: output range



Lower limit

$$V_{DS4} = V_{out} - V_{S4} \geq V_{DSAT4}$$

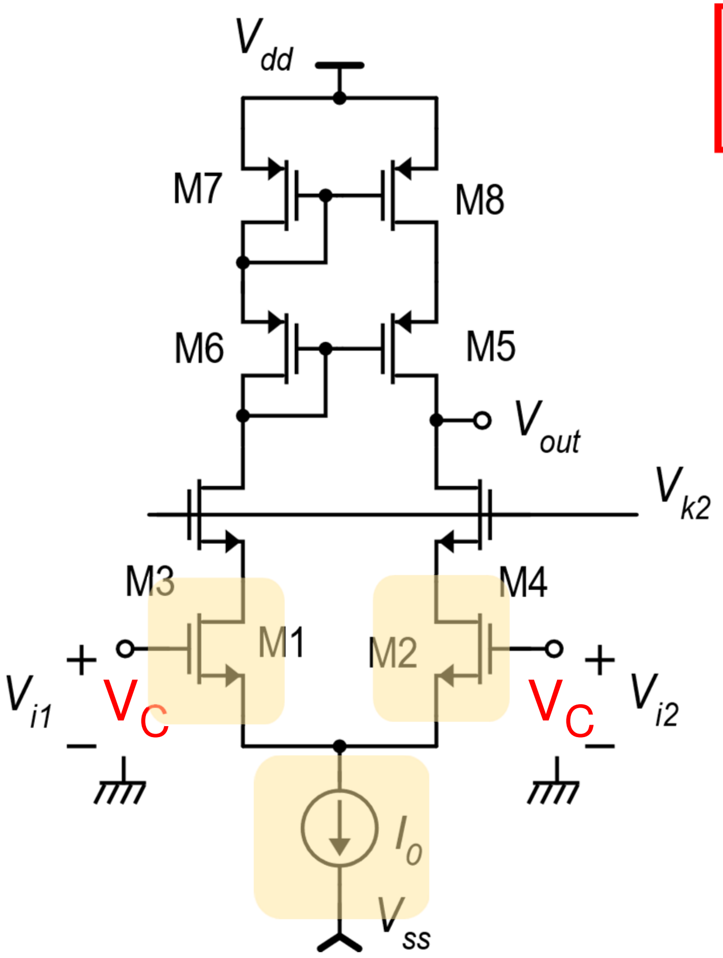
$$V_{S4} = V_{k2} - V_{GS4}$$

$$V_{out} - V_{k2} + V_{GS4} \geq V_{DSAT4}$$

$$\min(V_{out}) = V_{k2} - V_{GS4} + V_{DSAT4}$$

In strong inversion: $\min(V_{out}) = V_{k2} - V_{t4}$

For an approximation of min(V_{out}) in weak inversion just add 100 mV



Input common mode range

$$\boxed{\min(V_{iC}) = V_{SS} + V_{MIN-tail} + V_{GS1}}$$

$$V_{DS1} = V_{D1} - V_{S1} \geq V_{DSAT1}$$

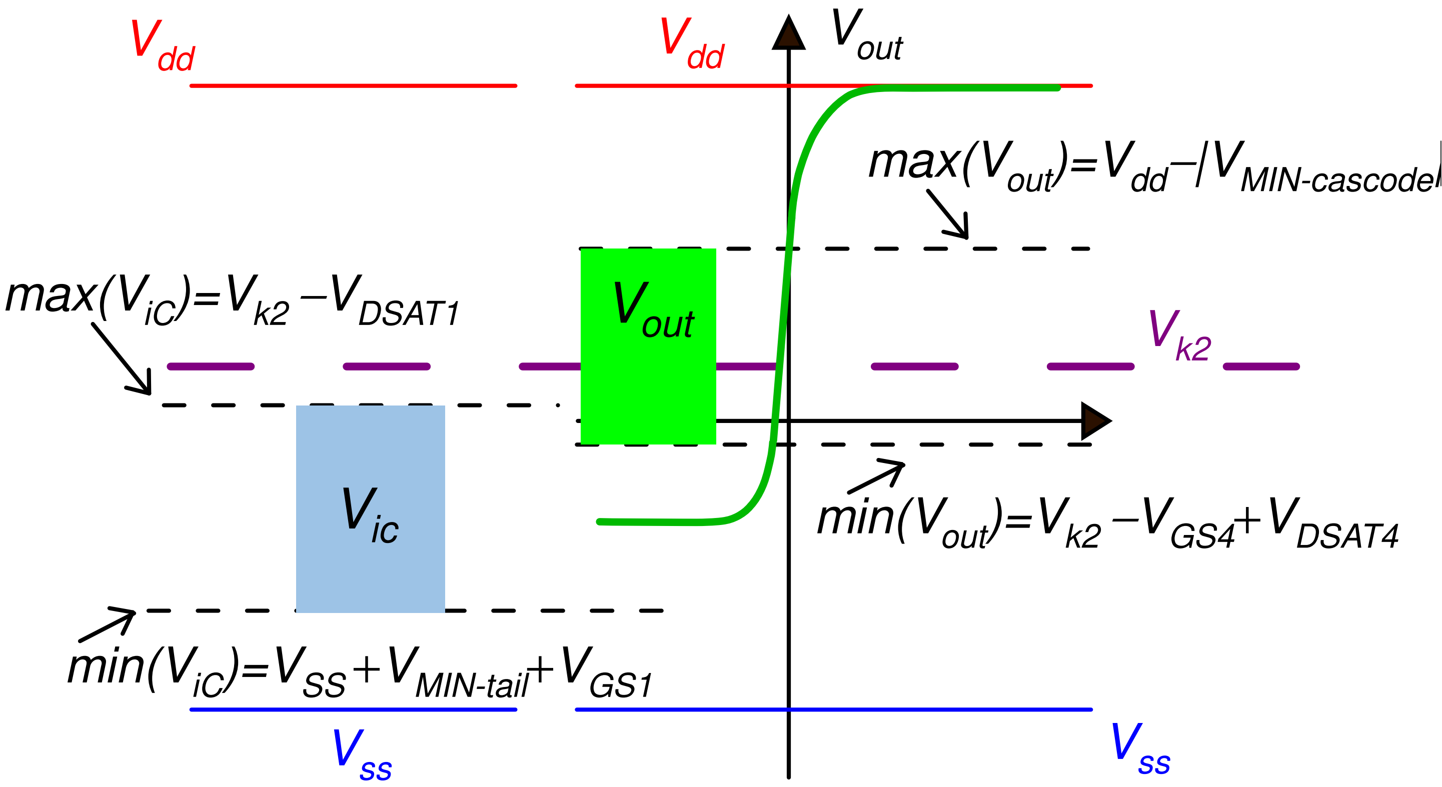
$$\begin{cases} V_{D1} = V_{k2} - V_{GS3} \\ V_{S1} = V_{iC} - V_{GS1} \end{cases}$$

$$V_{k2} - V_{GS3} - V_{iC} + V_{GS1} \geq V_{DSAT1}$$

$$\max(V_{iC}) = V_{k2} \underline{- V_{GS3} + V_{GS1}} - V_{DSAT1}$$

$$\max(V_{iC}) \cong V_{k2} - V_{DSAT1}$$

Cascode amplifier: all ranges

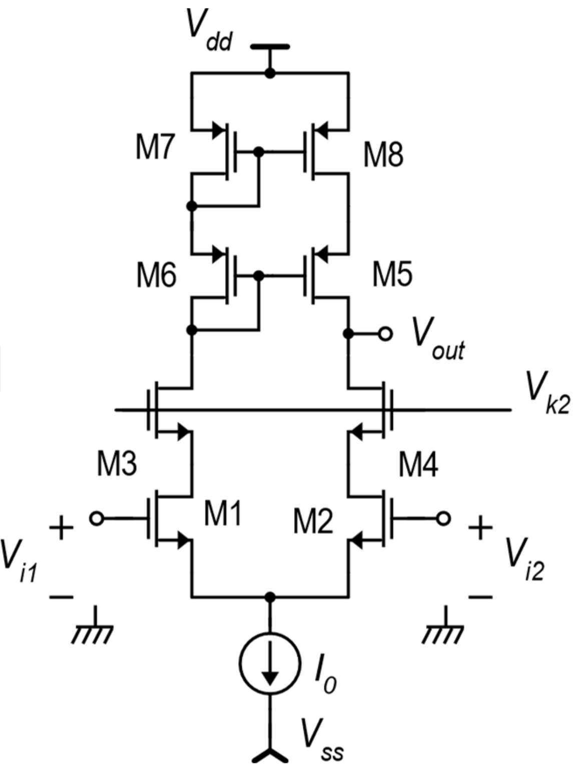


$\max(V_{iC})=V_{k2}-V_{DSAT1}$

$\min(V_{iC})=V_{SS}+V_{MIN\text{-}tail}+V_{GS1}$

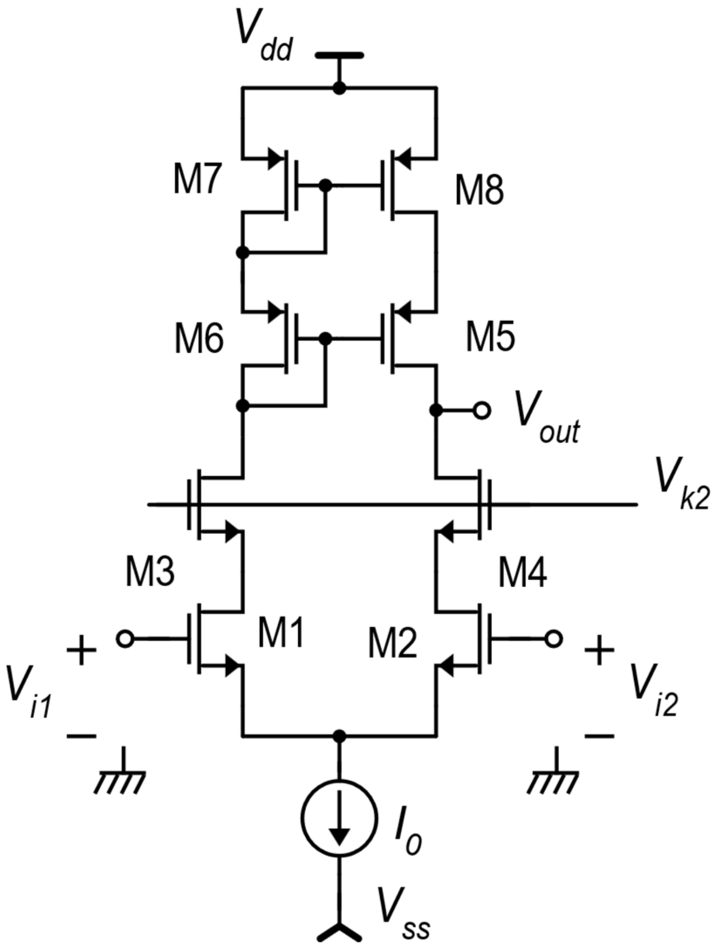
$\max(V_{out})=V_{dd}-|V_{MIN\text{-}cascode}|$

$\min(V_{out})=V_{k2}-V_{GS4}+V_{DSAT4}$



V_{k2} determines a trade-off between the input common mode range and the output swing

Adaptive V_{k2}



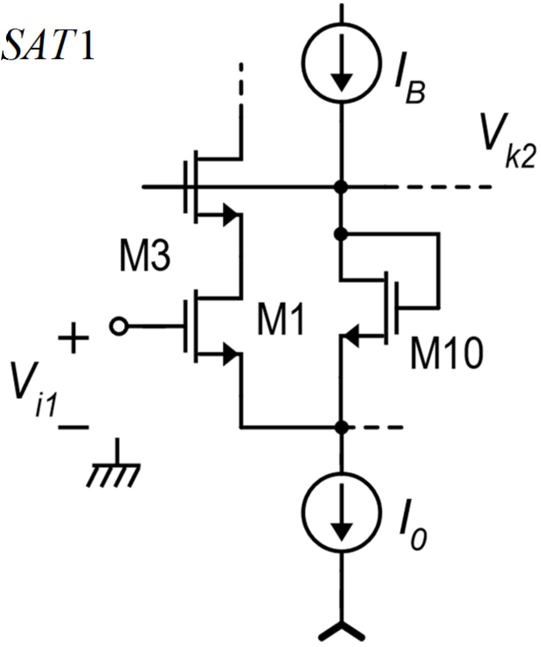
$$V_{k2} - V_{GS3} - V_{iC} + V_{GS1} \geq V_{DSAT1}$$

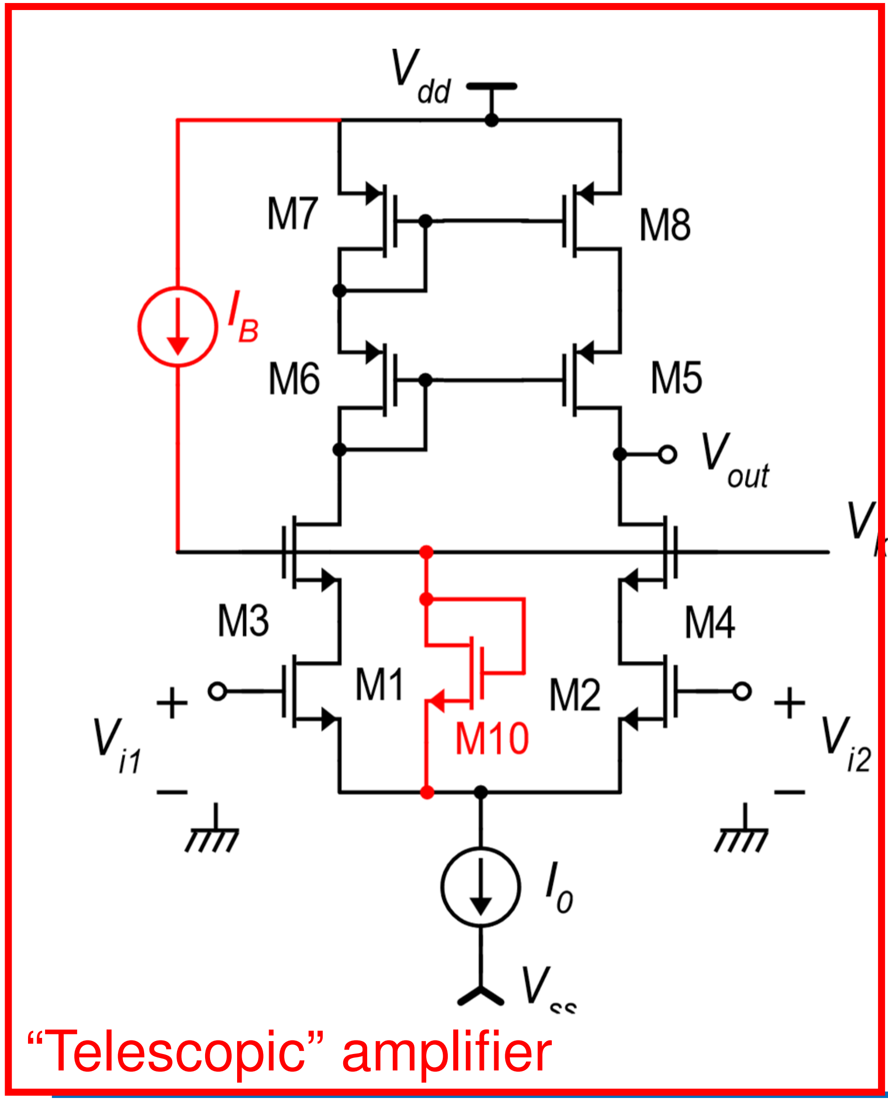
$$V_{k2} \geq \underbrace{V_{iC} - V_{GS1}}_{V_{S1}} + V_{GS3} + V_{DSAT1}$$

$$V_{k2} = V_{S1} + \underbrace{V_{GS3} + V_{DSAT1}}$$

$$V_{k2} = V_{S1} + V_{GS10}$$

$$V_{GS10} = V_{GS3} + V_{DSAT1}$$





"Telescopic" amplifier

Adaptive V_{k2}

Same as in the case of a general cascode structure

$$V_{GS10} = V_{GS3} + V_{DSAT1}$$

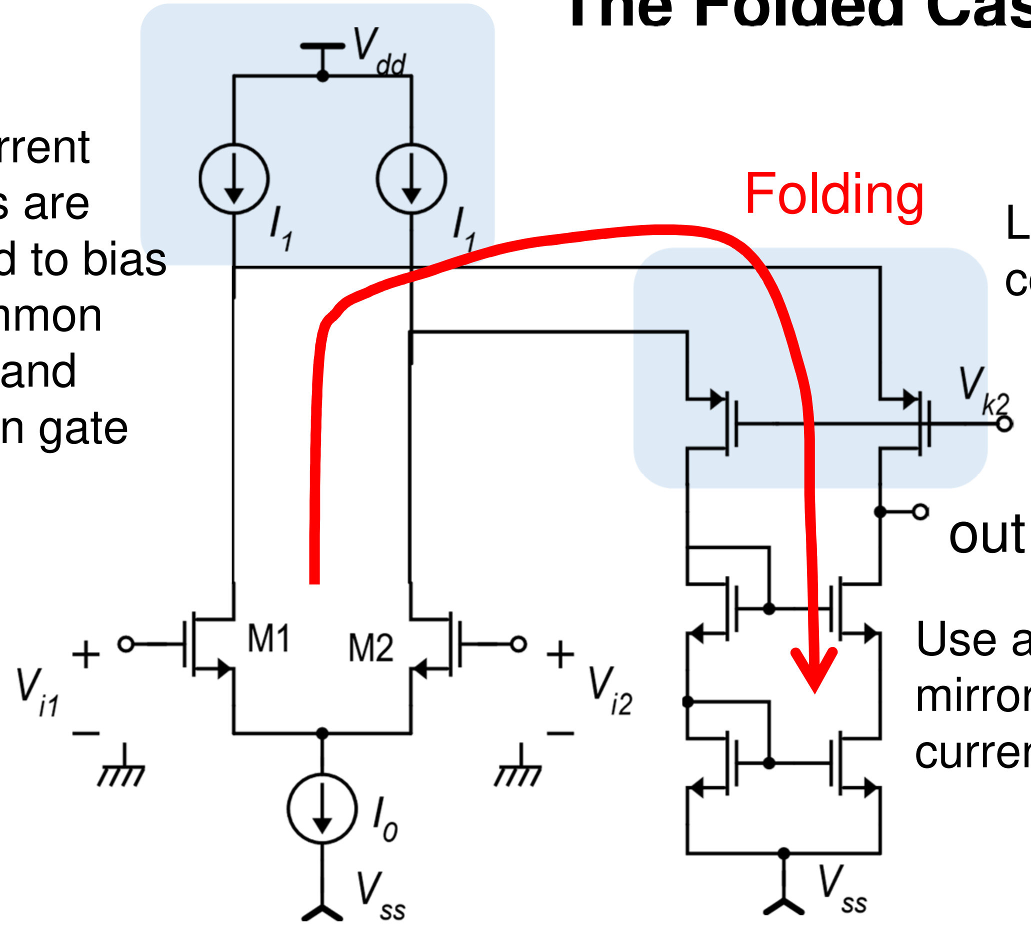
$$\boxed{\left(V_{GS} - V_t\right)_{10} = m_3 V_{DSAT1} + \left(V_{GS} - V_t\right)_3}$$

$$I_{0-eff} = I_0 - I_B$$

The adaptive V_{k2} does not solve the problem that, as V_{iC} increases, the output swing gets smaller. The advantage is that the amplifier is more flexible and no trade-off on V_{K2} should be made in the design phase.

Removing the interaction between input and output range: **The Folded Cascode**

Two current sources are required to bias the common source and common gate

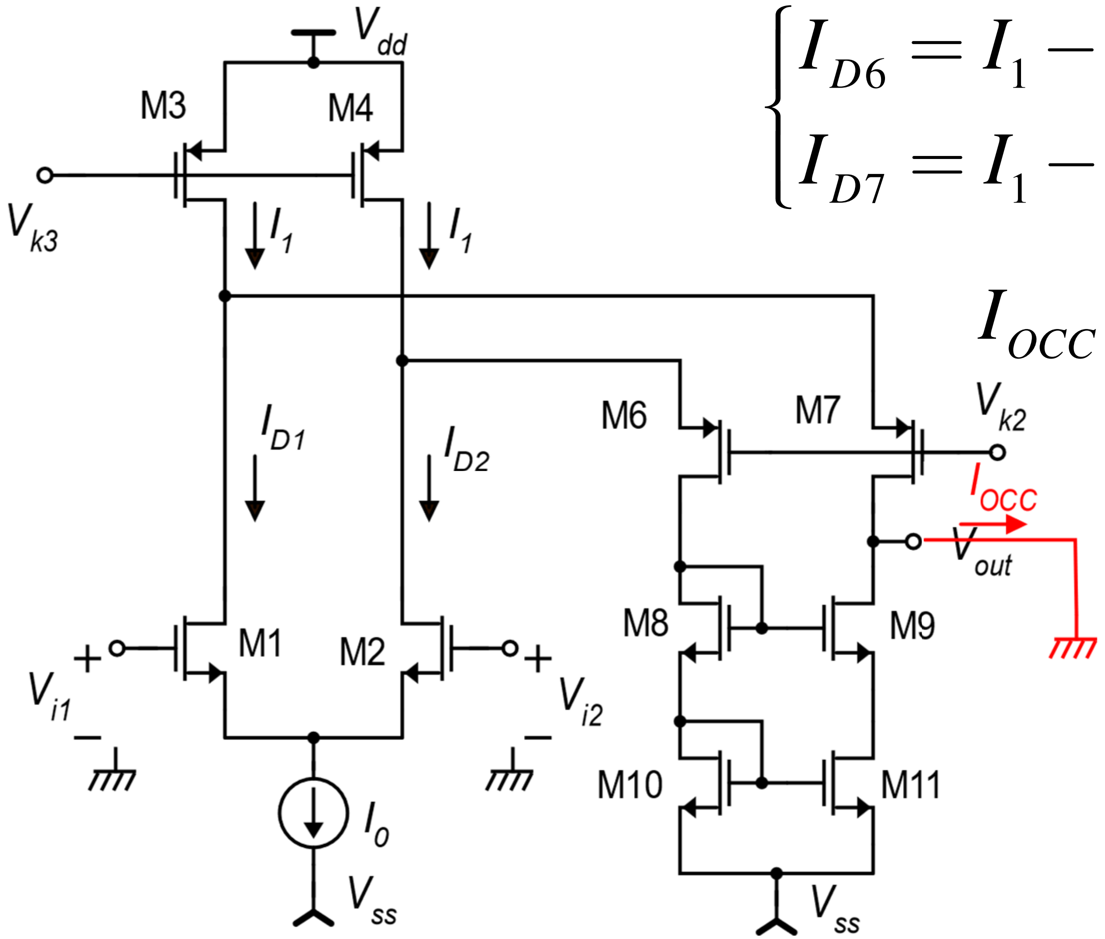


Let us introduce a type-p common gate stage

Use an n-type current mirror to subtract the currents

According to the folding mechanism, the signal (variations) initially travels from one rail to the other and then invert direction and goes back to the initial rail

Folded cascode amplifier: device currents



$$\begin{cases} I_{D6} = I_1 - I_{D2} \\ I_{D7} = I_1 - I_{D1} \end{cases}$$

Condition:

$$\begin{cases} I_{D6} \geq 0 \\ I_{D7} \geq 0 \end{cases}$$

$$I_{OCC} = I_{D7} - I_{D9} \cong I_{D7} - I_{D6}$$

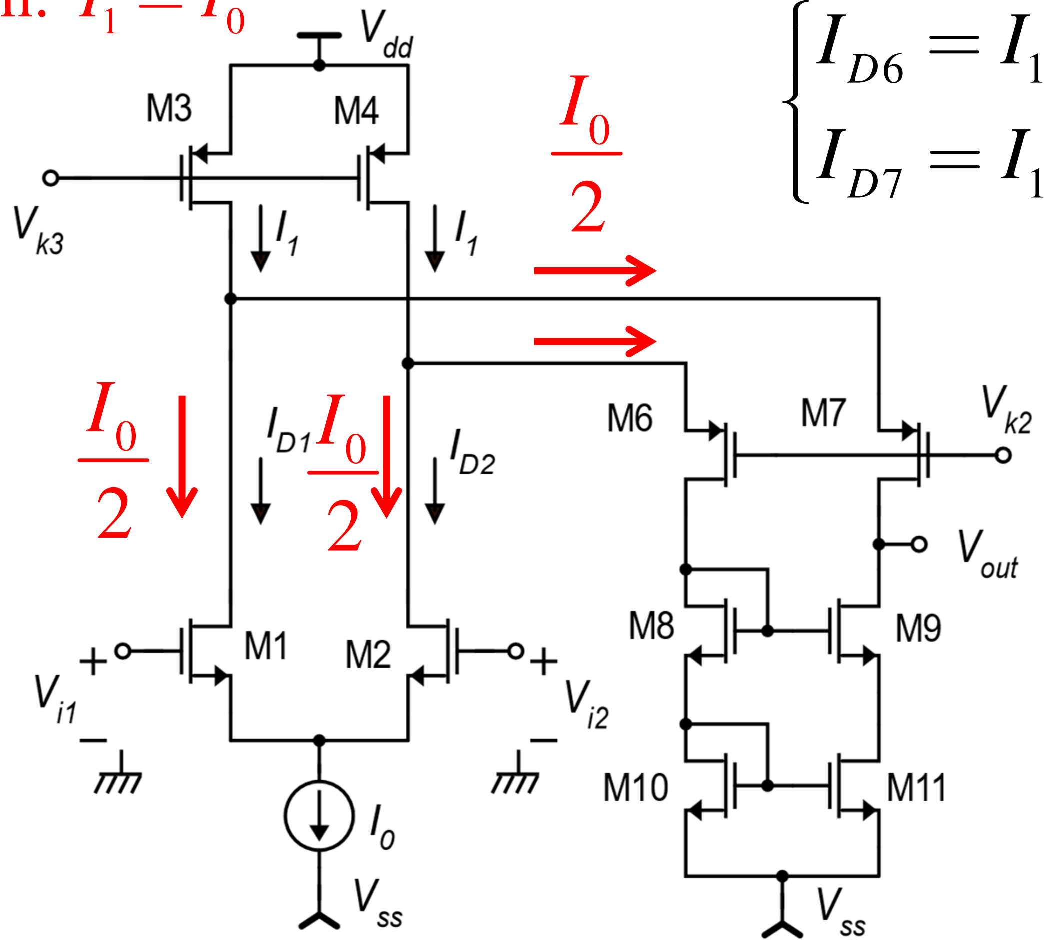
$$I_{OCC} = \left(I_1 - I_{D1}\right) - \left(I_1 - I_{D2}\right)$$

$$V_D = V_{i1} - V_{i2}$$

$$I_{OCC} = -\left(I_{D1} - I_{D2}\right) \cong -g_{m1} v_d$$

Folded cascode: setting the correct I_1 value

with: $I_1 = I_0$



$$\begin{cases} I_{D6} = I_1 - I_{D2} \geq 0 \\ I_{D7} = I_1 - I_{D1} \geq 0 \end{cases}$$

Quiescent point:

$$I_{D1} = I_{D2} = \frac{I_0}{2}$$

$$I_1 > \frac{I_0}{2}$$

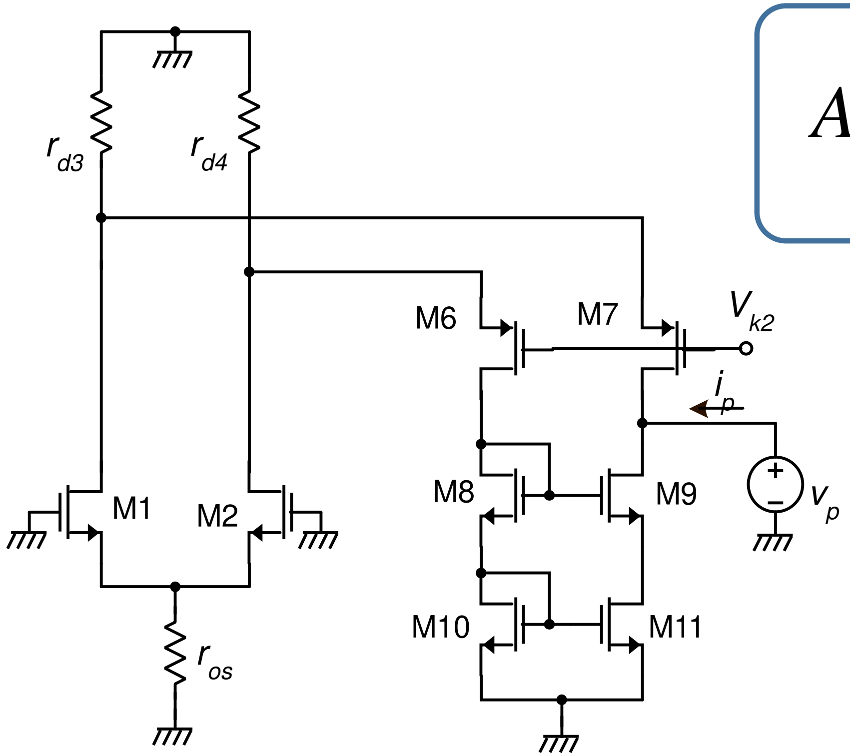
With a large input signal ($|V_{id}| > V_{DMAX}$):

$$V_{id} > V_{DMAX} : I_{D1} = I_0$$

$$V_{id} < -V_{DMAX} : I_{D2} = I_0$$

$$I_1 \geq I_0 \qquad \text{usually} : I_1 = I_0$$

Folded cascode: differential mode gain



Using a Norton equivalent circuit of the output port

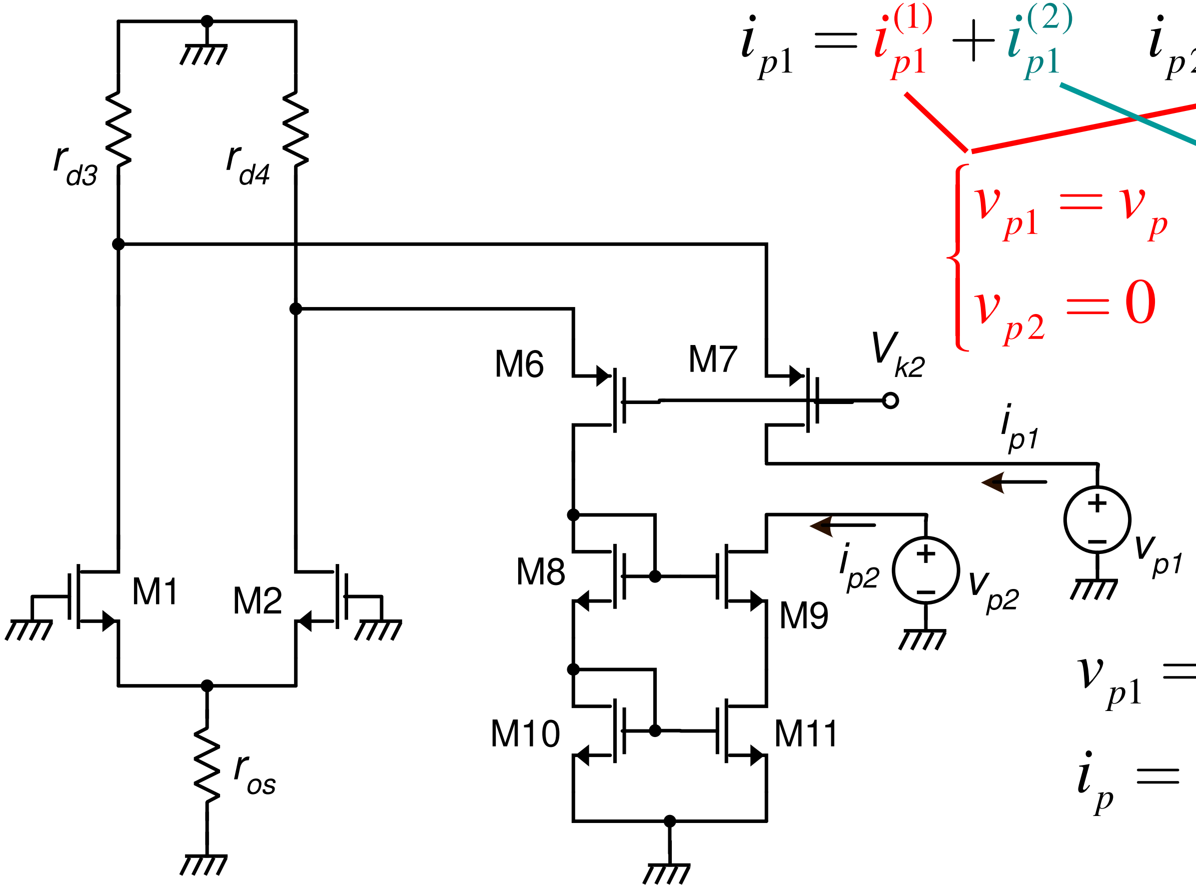
$$A_d = G_m R_{out} \qquad G_m = \frac{i_{occ}}{v_d}$$

$$I_{OCC} \cong -g_{m1} v_d \Rightarrow G_m = -g_{m1}$$

$$A_d = -g_{m1} R_{out}$$

$$R_{out} = \frac{v_p}{i_p}$$

Folded cascode: output resistance (2)



$$i_{p1} = i_{p1}^{(1)} + i_{p1}^{(2)} \qquad i_{p2} = i_{p2}^{(1)} + i_{p2}^{(2)}$$

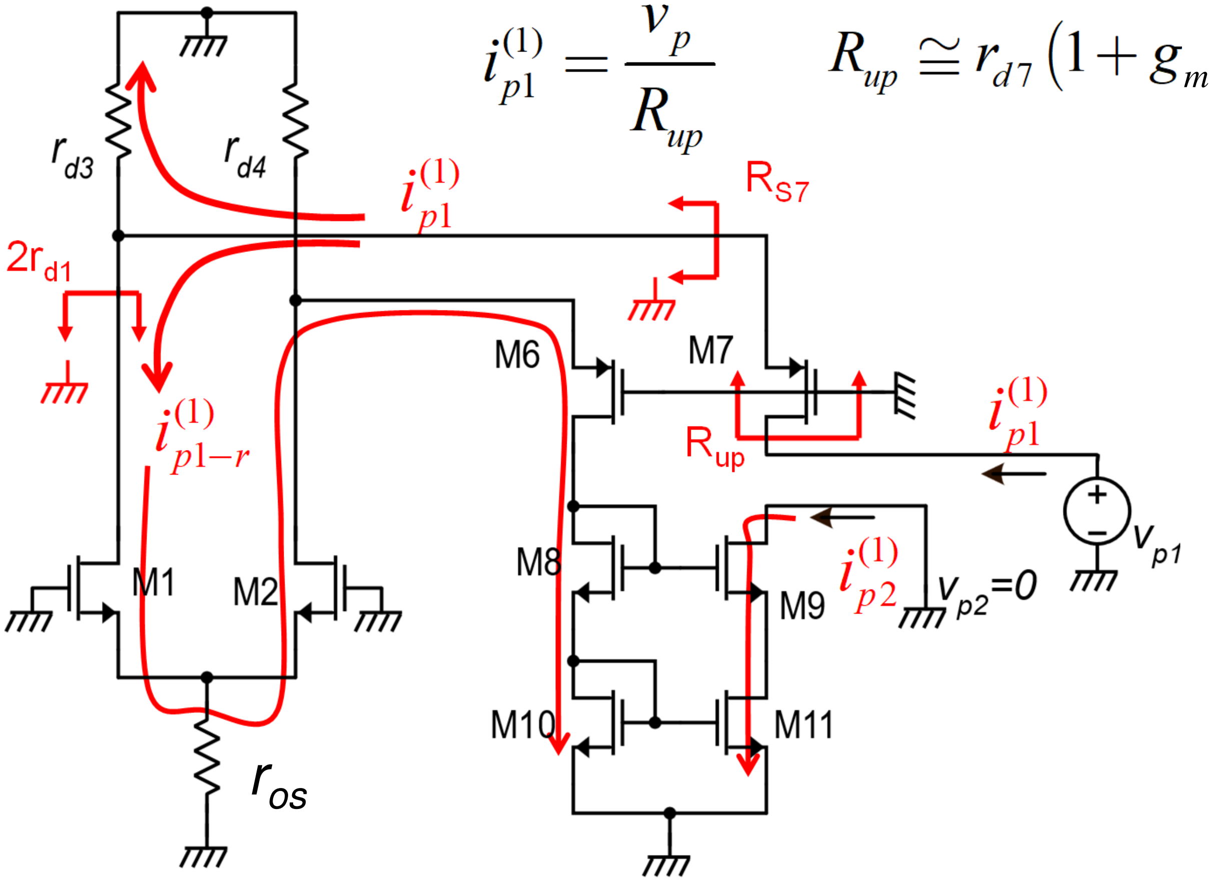
$$\begin{cases} v_{p1} = v_p \\ v_{p2} = 0 \end{cases}$$

$$\begin{cases} v_{p1} = 0 \\ v_{p2} = v_p \end{cases}$$

$$v_{p1} = v_{p2} = v_p$$

$$i_p = i_{p1} + i_{p2}$$

Folded cascode: output resistance (2)



$$i_{p1}^{(1)} = \frac{v_p}{R_{up}}$$

$$R_{up} \cong r_{d7}\left(1+g_{m7}R_{S7}\right)$$

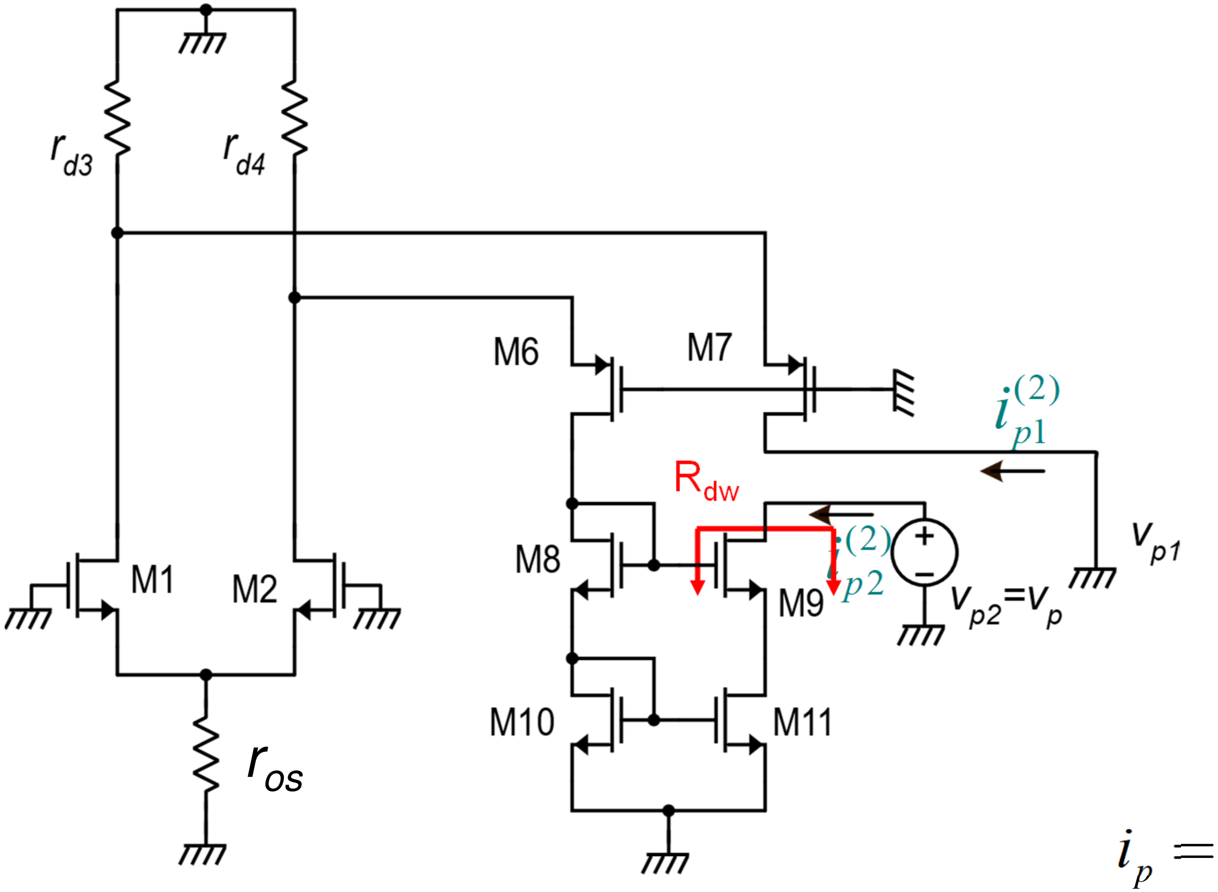
$$R_{S7} = r_{d3}//2r_{d1}$$

$$R_{up} \cong r_{d7}g_{m7}\left(r_{d3}//2r_{d1}\right)$$

$$i_{p1-r}^{(1)} = i_{p1}^{(1)}\frac{r_{d3}}{r_{d3}+2r_{d1}}$$

$$i_{p2}^{(1)} = i_{p1-r}^{(1)} = i_{p1}^{(1)}\frac{r_{d3}}{r_{d3}+2r_{d1}}$$

Folded cascode: output resistance (2)



$$R_{dw} \cong r_{d9} g_{m9} r_{d11}$$

$$i_{p2}^{(2)} = \frac{v_p}{R_{dw}}$$

$$i_{p1}^{(2)} = 0$$

$$i_{p1}^{(1)} = \frac{v_p}{R_{up}}$$

$$i_{p2}^{(1)} = i_{p1}^{(1)} \frac{r_{d3}}{r_{d3} + 2r_{d1}}$$

$$i_p = \frac{v_p}{R_{up}} \left(1 + \frac{r_{d3}}{r_{d3} + 2r_{d1}} \right) + \frac{v_p}{R_{dw}}$$

Folded cascode: output resistance (3)

$$i_p = v_p\left(\frac{1}{R_{up}}\left(1+\frac{r_{d3}}{r_{d3}+2r_{d1}}\right)+\frac{1}{R_{dw}}\right)$$

$$R_{out} = \left(\frac{1}{R_{up}}\left(1+\frac{r_{d3}}{r_{d3}+2r_{d2}}\right)+\frac{1}{R_{dw}}\right)^{-1}$$

$$R_{out} = \left(\frac{1}{\dfrac{R_{up}}{\left(1+\dfrac{r_{d3}}{r_{d3}+2r_{d1}}\right)}}+\frac{1}{R_{dw}}\right)^{-1}$$

$$R_{out} = R_{dw} // R_{up-r}$$

$$R_{dw} = r_{d9}\, g_{m9}\, r_{d11} \qquad R_{up-r} = \frac{r_{d7}\, g_{m7}\left(r_{d3} // 2r_{d1}\right)}{\left(1+\dfrac{r_{d3}}{r_{d3}+2r_{d1}}\right)}$$

If $r_{d2}=r_{d3}=r_{d7}=r_{d9}=r_{d11} = r_d$

$g_{m7}=g_{m9}=g_m$

$$\frac{2}{3}r_d$$

$$\frac{4}{3}$$

$$R_{dw} = r_d\left(g_m r_d\right)$$

$$R_{up-r} = \frac{r_d\left(g_m r_d\right)}{2}$$

$$R_{out} = \frac{r_d\left(g_m r_d\right)}{3}$$

Folded cascode: differential mode gain

$$A_d = -g_{m1} R_{out}$$

If $r_{d2}=r_{d3}=r_{d7}=r_{d9}=r_{d11}=r_d$
$g_{m7}=g_{m9}=g_{m1}=g_m$

$$A_d = -g_m \frac{r_d\left(g_m r_d\right)}{3} = -\frac{\left(g_m r_d\right)^2}{3}$$

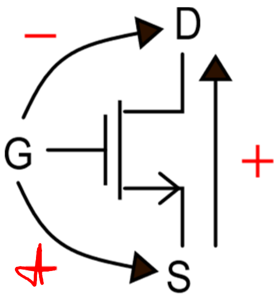
Compare with the "Telescopic" amplifier

$$A_d = \frac{\left(g_m r_d\right)^2}{2}$$

… and with the simple amplifier with mirror load (non-cascode)

$$A_d = \frac{\left(g_m r_d\right)}{2}$$

Simple method to find if a terminal is inverting / non-inverting

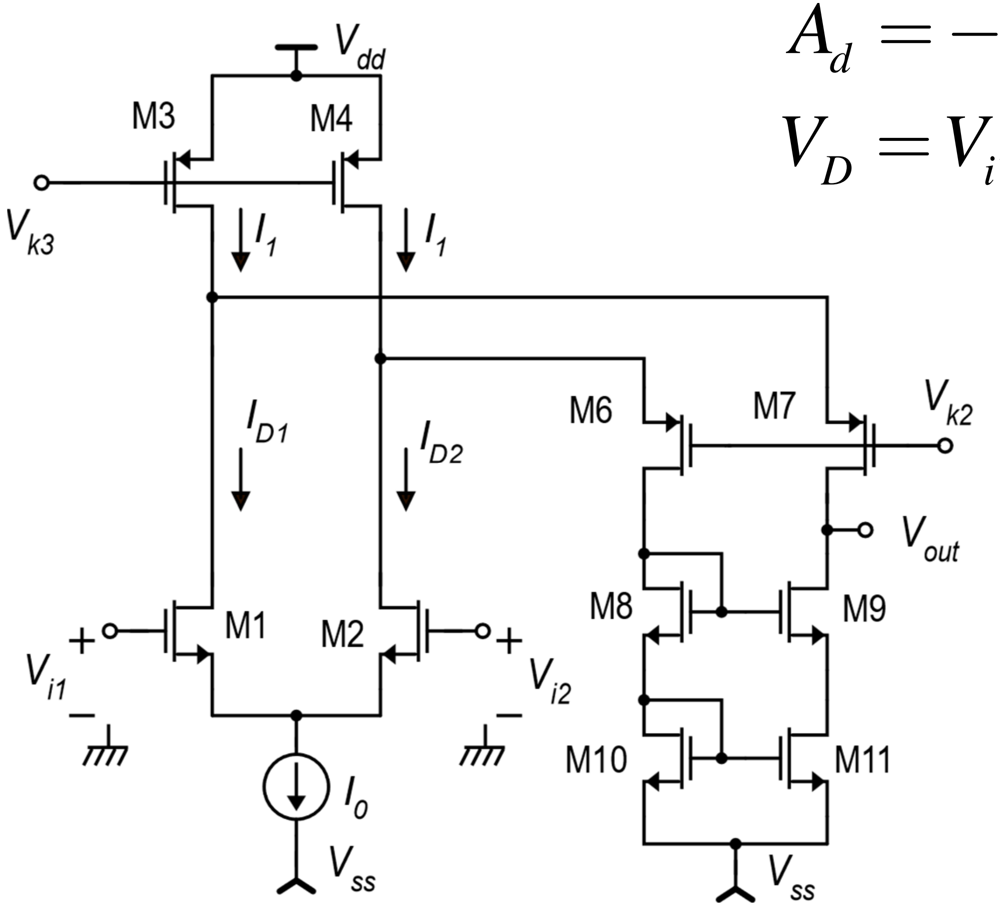


Signal paths:

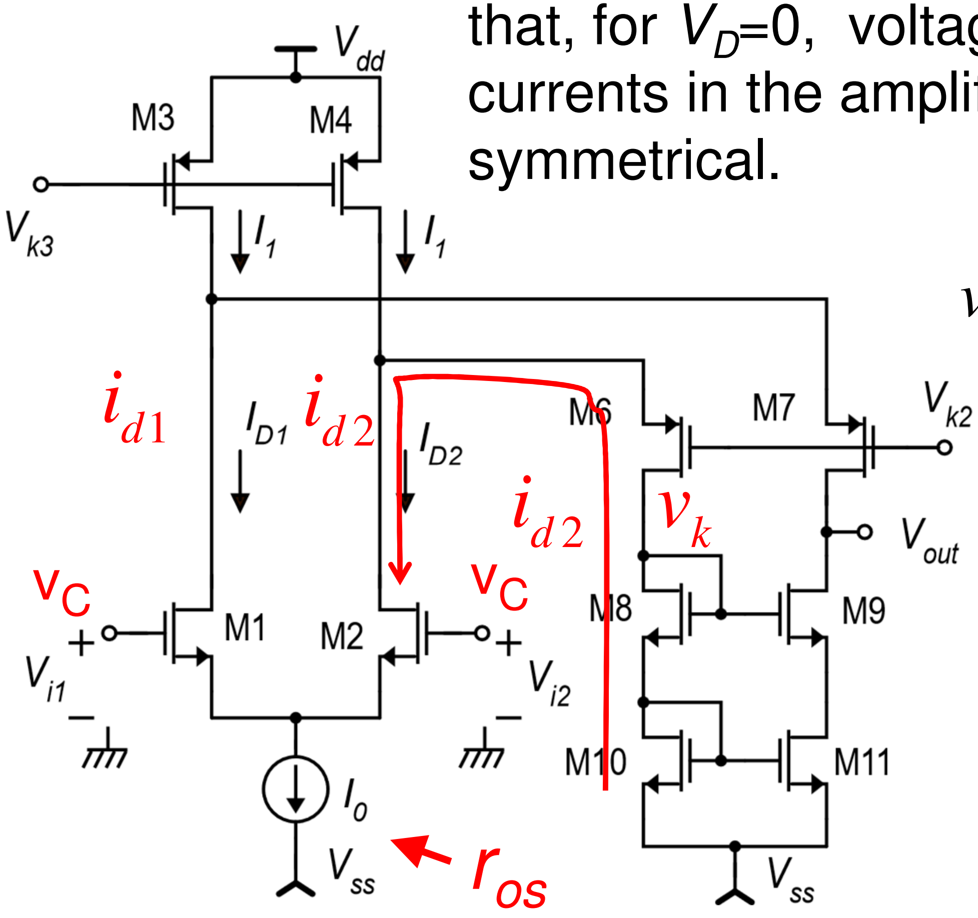
1. From G to D: inversion
2. From G to S: no inversion
3. From S to D: no inversion

$$A_d = -g_{m1} R_{out}$$

$$V_D = V_{i1} - V_{i2}$$



Ac, CMRR



It is possible to demonstrate that, for V_D=0, voltage and currents in the amplifiers are symmetrical.

Then:
$$\begin{cases} i_{d1} = i_{d2} \cong \dfrac{v_c}{2r_{os}} \\ v_{out} = v_k \end{cases}$$

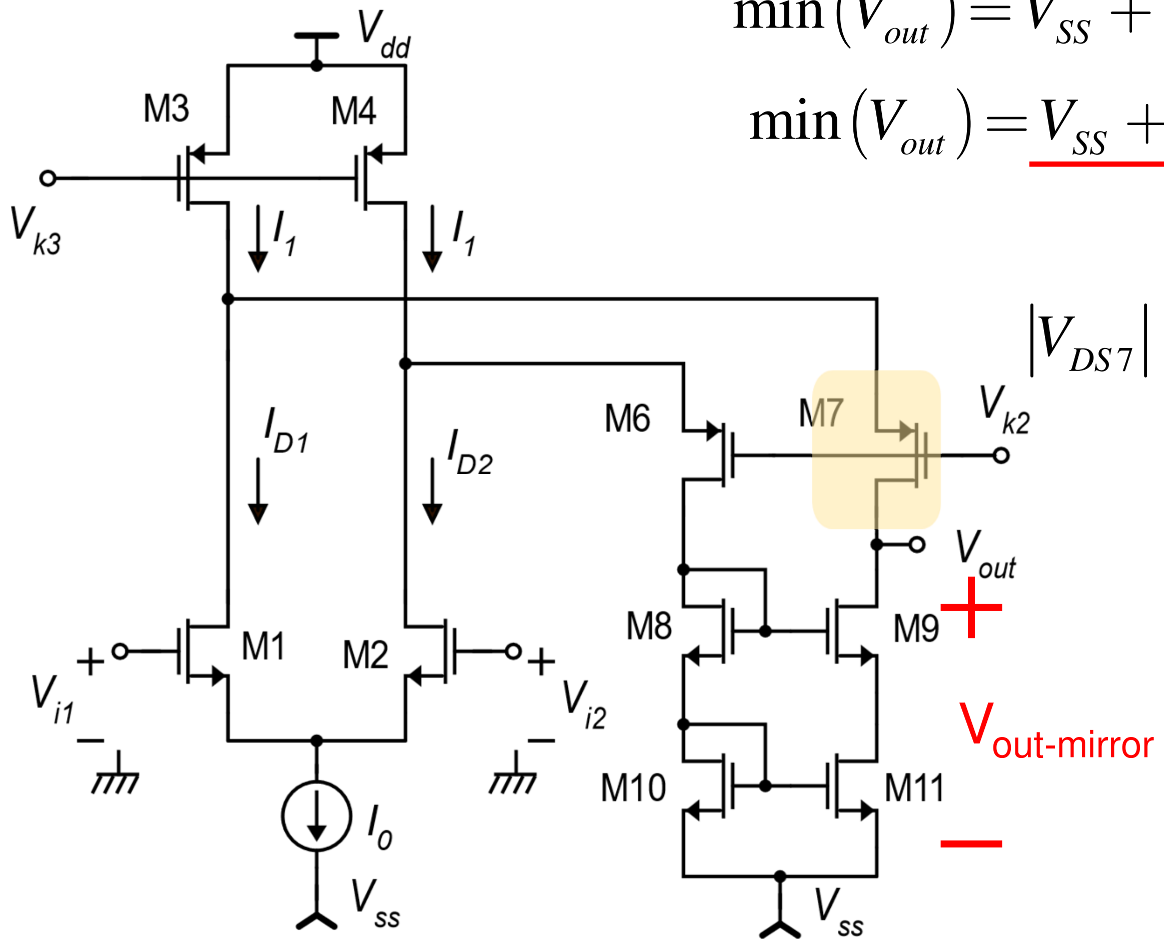
$$v_{out} = v_k = -i_{d2}\left(\frac{1}{g_{m8}} + \frac{1}{g_{m10}}\right)$$

$$v_{out} = -\frac{v_c}{2r_{os}}\left(\frac{1}{g_{m8}} + \frac{1}{g_{m10}}\right) = -\frac{v_c}{g_m r_{os}}$$

for $g_{m8} = g_{m10} = g_m$

$$A_C = -\frac{1}{g_m r_{os}} \qquad CMRR \approx \frac{(g_m r_d)^2}{3} g_m r_{os}$$

Folded cascode: output range



$$\min\left(V_{out}\right)=V_{SS}+V_{MIN-cascode}$$

Lower limit

$$\min\left(V_{out}\right)=\underline{V_{SS}+V_{GS11}+V_{DSAT9}}$$

Upper limit

$$\left|V_{DS7}\right|=V_{S7}-V_{D7}=V_{k2}+\left|V_{GS7}\right|-V_{out}\geq\left|V_{DSAT7}\right|$$

$$V_{k2}+\left|V_{GS7}\right|-\left|V_{DSAT7}\right|\geq V_{out}$$

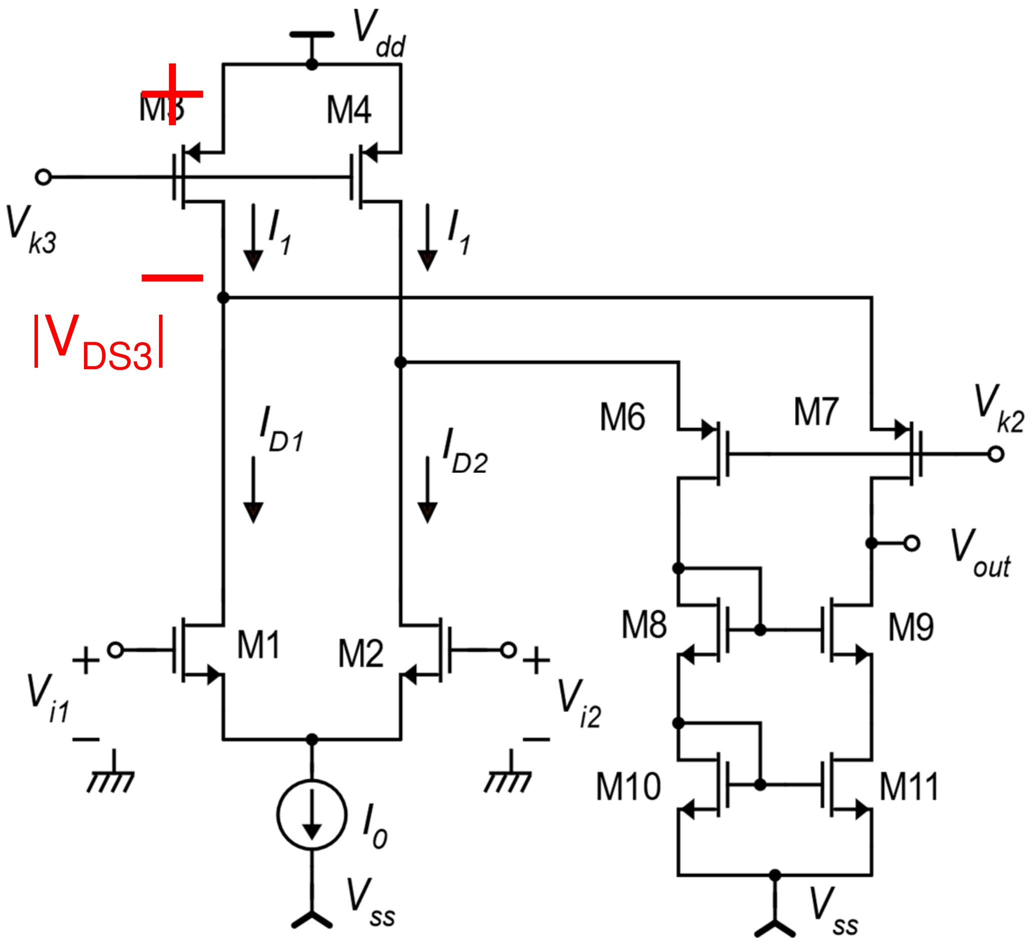
$$\max\left(V_{out}\right)=\underline{V_{k2}+\left|V_{GS7}\right|-\left|V_{DSAT7}\right|}$$

In strong inversion:

$$\left|V_{DSAT7}\right|=\left|V_{GS7}\right|-\left|V_{t7}\right|$$

$$\max\left(V_{out}\right)=V_{k2}+\left|V_{t7}\right|$$

Maximum V_{k2}



$$\max(V_{out}) = V_{k2} + |V_{GS7}| - |V_{DSAT7}|$$

$$V_{D3} = V_{D1} = V_{k2} + |V_{GS7}|$$

V_{k2} sets the voltage of M1-M3 and M2-M4 drains

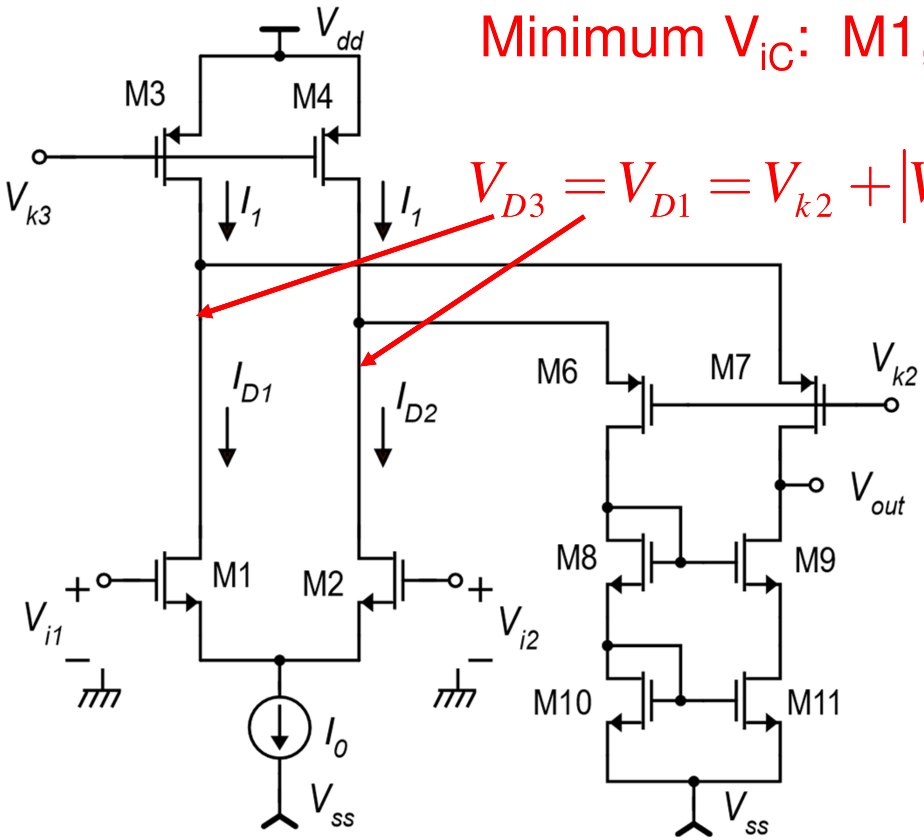
$$|V_{DS3}| = V_{dd} - V_{D3} = V_{dd} - V_{k2} - |V_{GS7}| \geq |V_{DSAT3}|$$

$$\max(V_{k2}) = V_{dd} - |V_{DSAT3}| - |V_{GS7}|$$

$$\max(V_{out}) = V_{dd} - |V_{DSAT3}| - |V_{DSAT7}|$$

Input common mode range

Minimum V_{iC}: M1,M2 pair ⇒ $\min\left(V_{iC}\right)=V_{SS}+V_{MIN-tail}+V_{GS1}$



$$V_{D3}=V_{D1}=V_{k2}+\left|V_{GS7}\right|$$

Maximum V_{iC}:

M1 (M2) has the drain at a fixed voltage. If the gate voltage increases, M1 (M2) will eventually leave saturation

$$V_{DS1}=V_{D1}-V_{S1}=$$

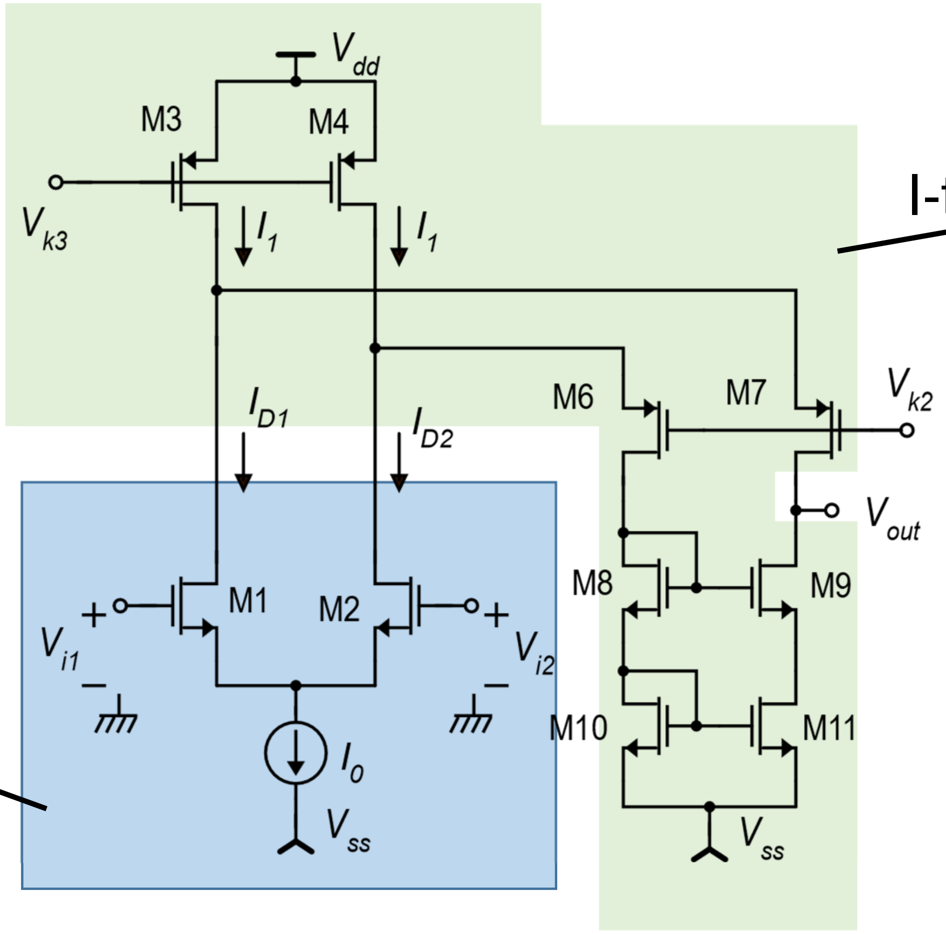
$$=V_{k2}+\left|V_{GS7}\right|-\left(V_C-V_{GS1}\right)\geq V_{DSAT1}$$

$$\max\left(V_{iC}\right)=V_{k2}+\left|V_{GS7}\right|+V_{GS1}-V_{DSAT1}$$

$$\max\left(V_{k2}\right)=V_{dd}-\left|V_{DSAT3}\right|-\left|V_{GS7}\right|$$

$$\max\left(V_{iC}\right)\Big|_{V_{k2-\max}}=V_{dd}-\left|V_{DSAT3}\right|+V_{GS1}-V_{DSAT1}$$

The folded cascode as a single stage amplifier



I-to-I network (K_I=1)

V-to-I network
(G_{m1}=g_{m1})

Folded cascode: summary of properties

DC Gain: Slightly smaller than the telescopic amplifier (non-folded cascode) gain. May reach several thousands or even 10^4 (80 dB) with long mosfets. Larger than the gain of the cascade of two common source stages.

Ranges: In the the Folded cascode, the output range is not affected by the input common mode voltage. As a result, the input CM range and output range are much wider than in the telescopic amplifier.

Output range: (swing)

$$\begin{cases} \max\left(V_{out}\right) = V_{dd} - \left|V_{DSAT3}\right| - \left|V_{DSAT7}\right| \\ \min\left(V_{out}\right) = V_{SS} + V_{GS11} + V_{DSAT9} \end{cases}$$

Approaches the V_{dd} rail

May approach the V_{ss} rail if a wide swing mirror is used

Input CM range:

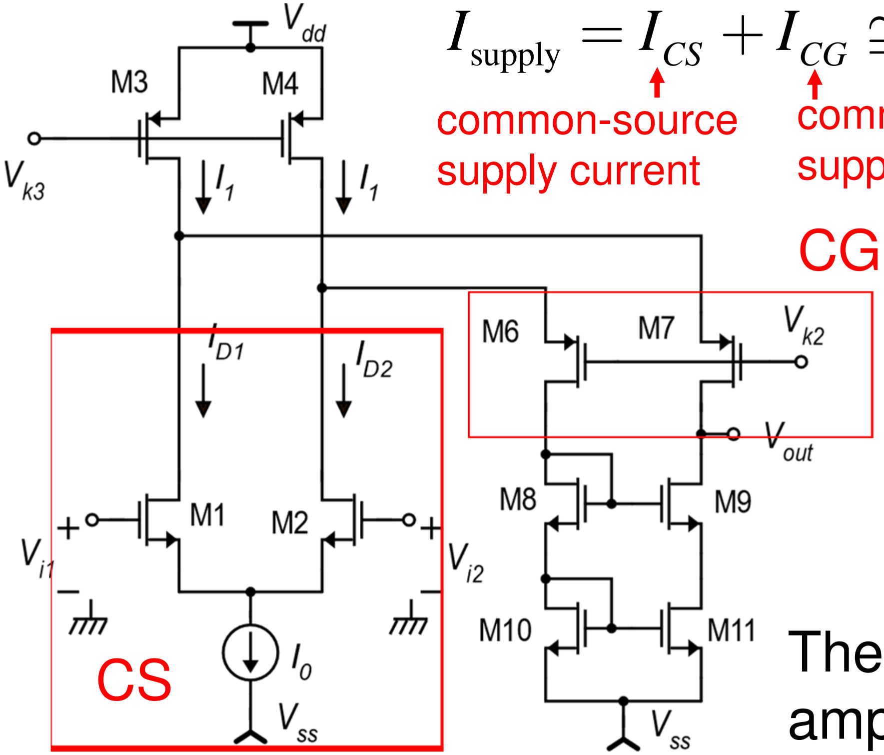
$$\begin{cases} \min\left(V_{iC}\right) = V_{SS} + V_{MIN-tail} + V_{GS1} \\ \max\left(V_{C}\right) = V_{dd} - \left|V_{DSAT3}\right| + V_{GS1} - V_{DSAT1} \end{cases}$$

The only critical limit

Goes over the V_{dd} rail

The only true advantage of the telescopic amplifier

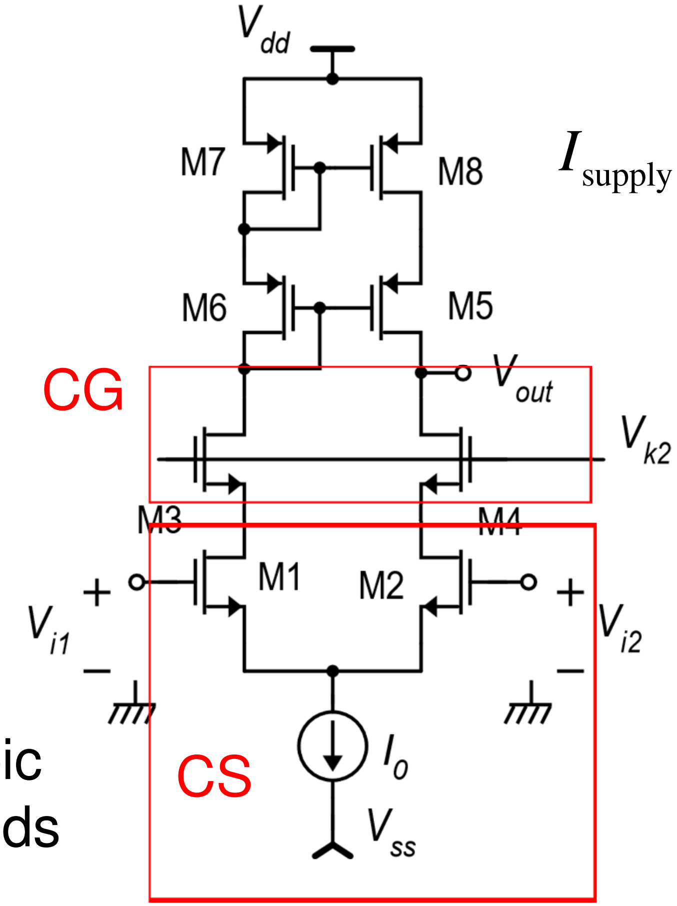
Current consumption (I_{supply})



$$I_{\text{supply}} = \underset{\uparrow}{I_{CS}} + \underset{\uparrow}{I_{CG}} \cong 2I_0$$

common-source supply current

common-gate supply current



$$I_{\text{supply}} = I_{CS} \cong I_0$$

Current re-use

The telescopic amplifier needs less current

Example: Analog Devices ADA4530

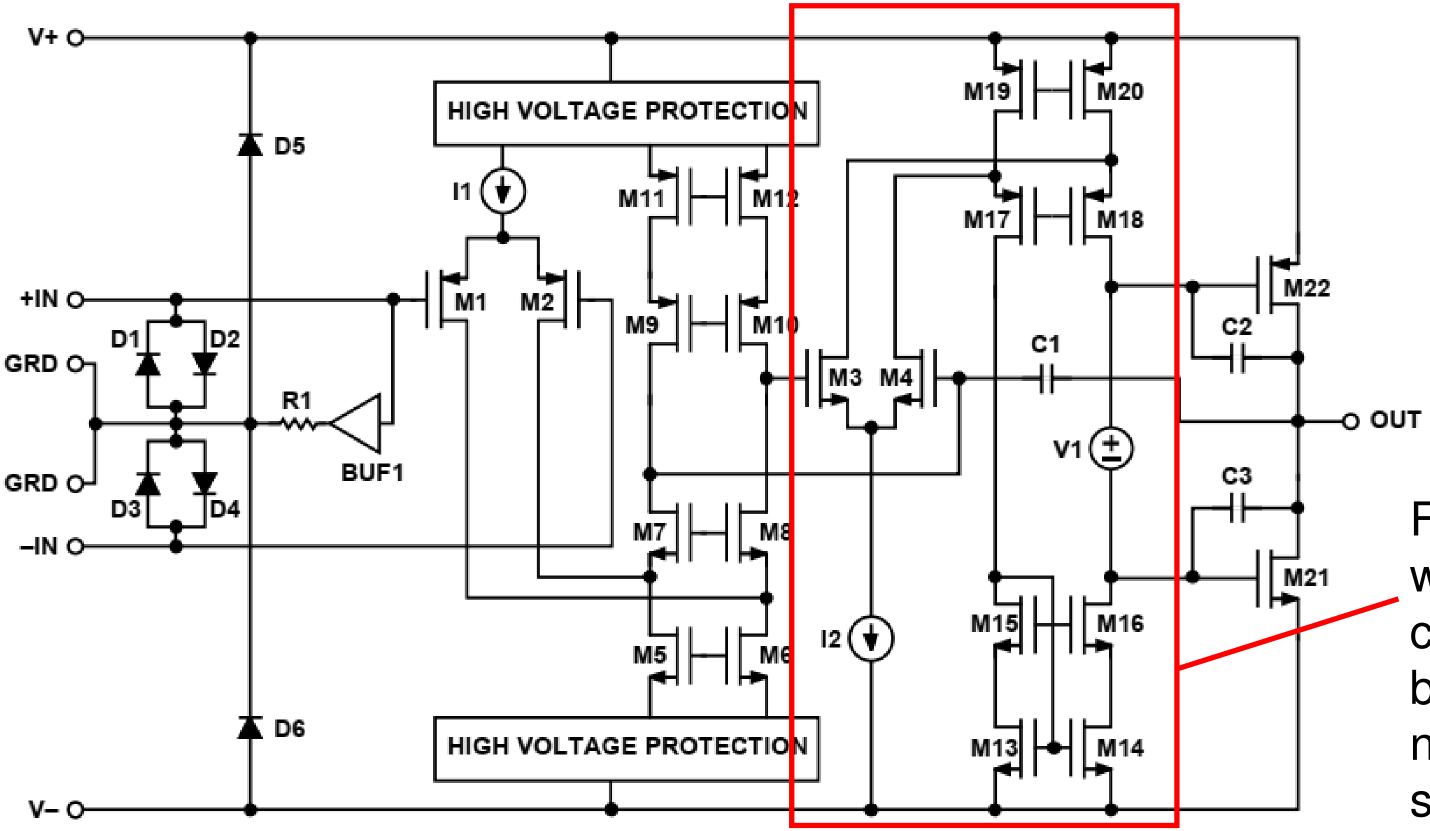


Figure 99. Simplified Schematic

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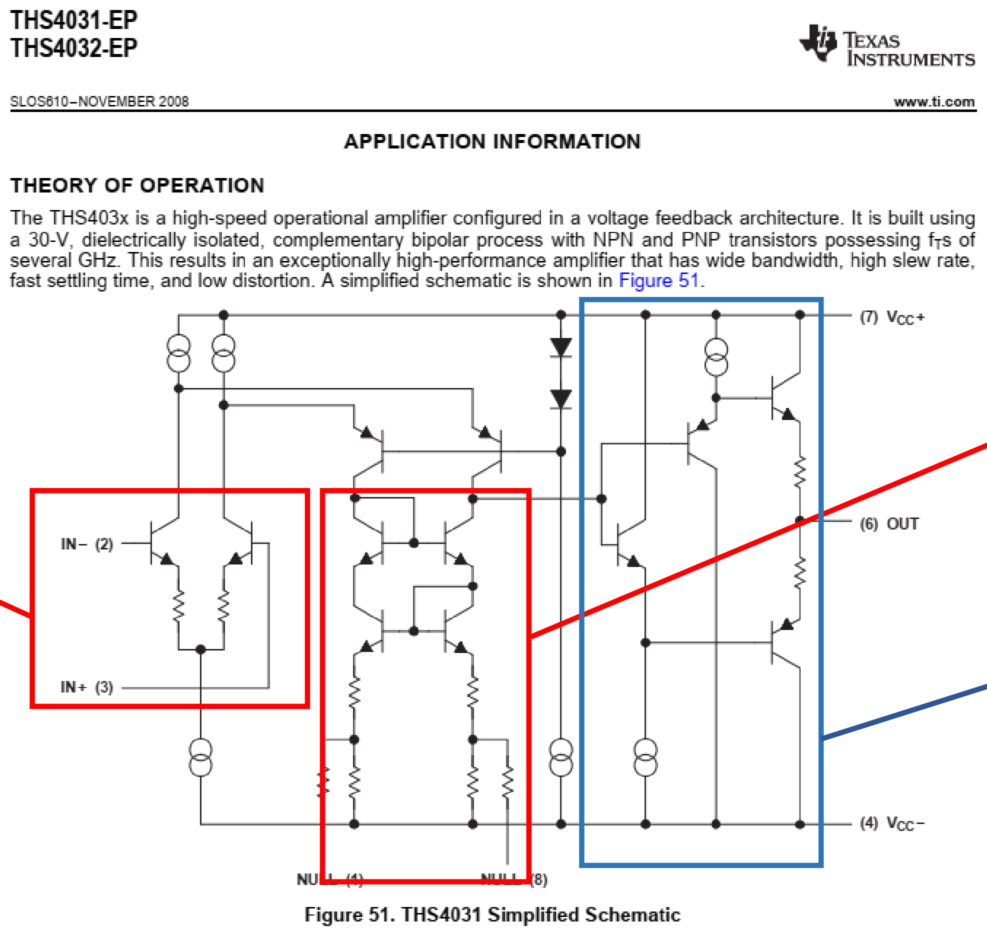
Folded cascode with wide-swing current mirror (gate bias voltages are not indicated for simplicity)

Example of BJT Op-Amp with a single folded cascode gain stage



THS4031-EP
THS4032-EP

TEXAS INSTRUMENTS

SLOS610–NOVEMBER 2008

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APPLICATION INFORMATION

THEORY OF OPERATION

The THS403x is a high-speed operational amplifier configured in a voltage feedback architecture. It is built using a 30-V, dielectrically isolated, complementary bipolar process with NPN and PNP transistors possessing f_Ts of several GHz. This results in an exceptionally high-performance amplifier that has wide bandwidth, high slew rate, fast settling time, and low distortion. A simplified schematic is shown in Figure 51.

Figure 51. THS4031 Simplified Schematic

Emitter-degenerated input pair (improves input differential range, with benefits in terms of Slew-Rate

Wilson current mirror with emitter degeneration.

Class-AB emitter follower (gain ≅ 1)