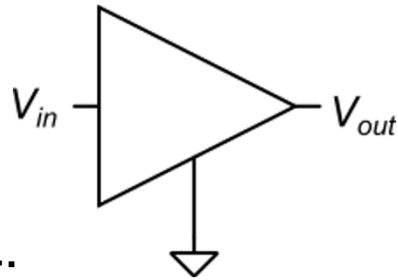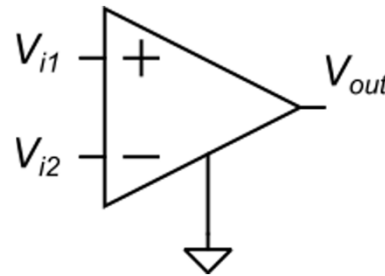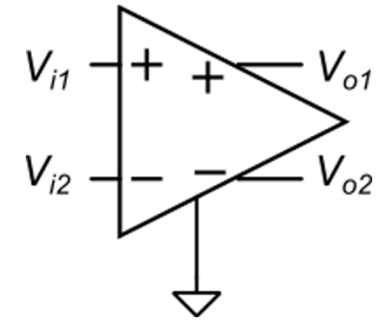# Voltage amplifiers: number of inputs and outputs

Unipolar:
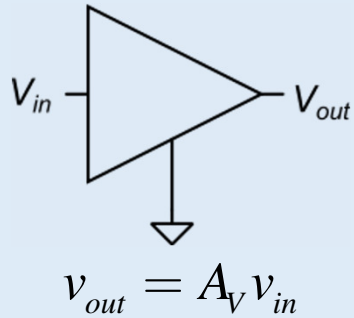single input
single output (single-ended, S/E)

Differential:
differential input,
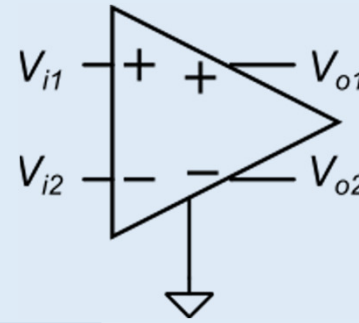single output (single-ended, S/E)

Fully Differential:
differential input,
differential output

Voltage Amplifiers: gains

$$v_{out} = A_V v_{in}$$

$$v_{id} = v_{i1} - v_{i2}$$

$$v_{ic} = \frac{v_{i1} + v_{i2}}{2}$$

$$v_{od} = v_{o1} - v_{o2}$$

$$v_{oc} = \frac{v_{o1} + v_{o2}}{2}$$

$$A_{dd} = \left.\frac{v_{od}}{v_{id}}\right|_{v_{ic}=0}$$

$$A_{cd} = \left.\frac{v_{od}}{v_{ic}}\right|_{v_{id}=0}$$
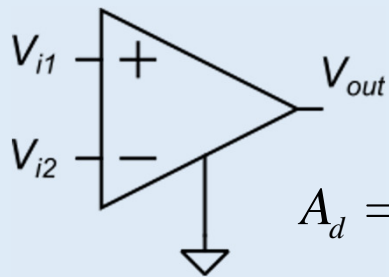
$$A_{dc} = \left.\frac{v_{oc}}{v_{id}}\right|_{v_{ic}=0}$$

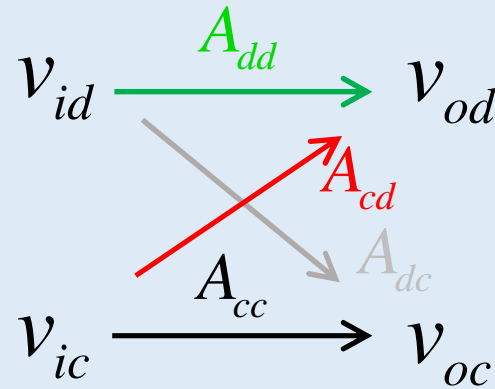$$A_{cc} = \left.\frac{v_{oc}}{v_{ic}}\right|_{v_{id}=0}$$

$$A_d = \left.\frac{v_{out}}{v_d}\right|_{v_c=0} \qquad A_c = \left.\frac{v_{out}}{v_c}\right|_{v_d=0}$$

$$v_d = v_{i1} - v_{i2}$$

$$v_c = \frac{v_{i1} + v_{i2}}{2}$$

$$CMRR = \left|\frac{A_d}{A_c}\right|$$

$$CMRR = \left|\frac{A_{dd}}{A_{cd}}\right|$$

Differential amplifiers: Ideal behavior and ranges
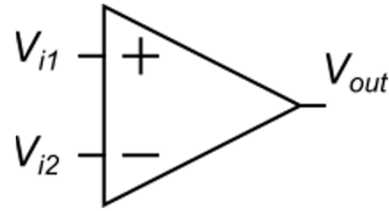
$$V_{out} = A_d \left(V_{id} - V_{io} \right)$$

Linear response
(with input offset voltage)

Input voltage ranges

Input differential range
(maximum V_{ID} (V_D) to maintain an acceptable input-output linearity

$$-V_{DMAX} \leq V_D \leq V_{DMAX}$$

Input common-mode range: Interval of V_{IC} (V_C) values where the amplifier behaves as designed
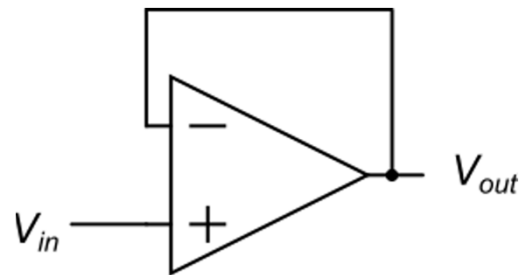
$$V_{CMIN} \leq V_C \leq V_{CMAX}$$

Output voltage range (Output voltage swing)

$$V_{OMIN} \leq V_{out} \leq V_{OMAX}$$

Outside the output range the amplifier stop working correctly (e.g. the output voltage tends to saturate)

Differential amplifiers: role of the input common mode range

Example: voltage follower (buffer amplifier)

In the ideal case:

$$V_{io} = 0, \ A_d \to \infty$$

We have:

$$V_C = V_{in}$$

$$V_D \cong 0$$

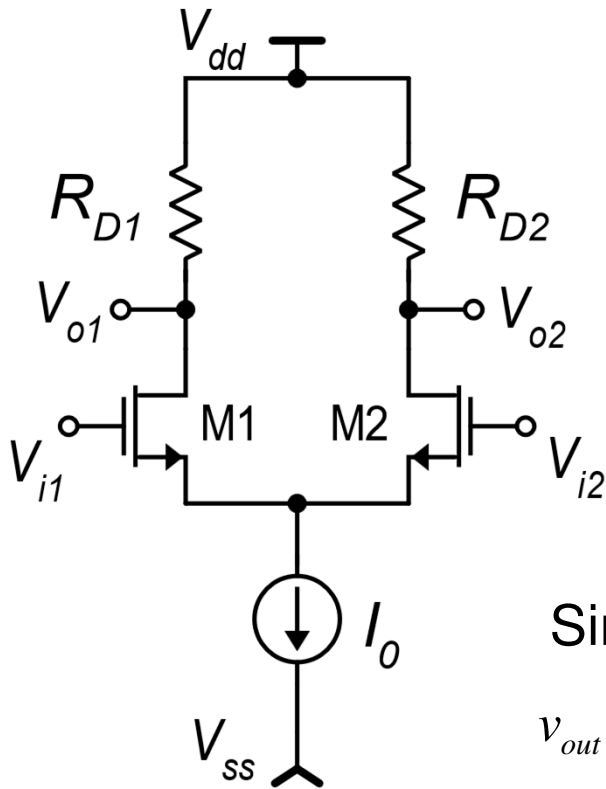The voltage follower operates correctly only for V_{in} within the CM range

Another condition is clearly that V_{out} is within the output range

Then the conditions for correct operation are:

$$V_{out} = V_{in} \ \text{only for} \ \begin{cases} V_{CMIN} \leq V_{in} \leq V_{CMAX} \\ V_{OMIN} \leq V_{out} \leq V_{OMAX} \end{cases}$$

$$V_{in} \in Input \ CM \ range \ \cap \ Output \ range$$

Differential amplifier with resistive loads

$$V_{ID} = V_{i1} - V_{i2} \quad V_{OD} = V_{o1} - V_{o2}$$

$$V_{o1} = V_{DD} - R_{D1}I_{D1} \quad V_{o2} = V_{DD} - R_{D2}I_{D2} \quad V_{OD} = R_{D2}I_{D2} - R_{D1}I_{D1}$$

Small signal analysis

$$v_{o1} = -R_{D1}i_{d1} \quad v_{o2} = -R_{D2}i_{d2} \quad v_{od} = i_{d2}R_{D2} - i_{d1}R_{D1}$$

Differential mode: $i_{d1} = \dfrac{g_m}{2}v_{id} \quad i_{d2} = -\dfrac{g_m}{2}v_{id}$

Single-ended case:
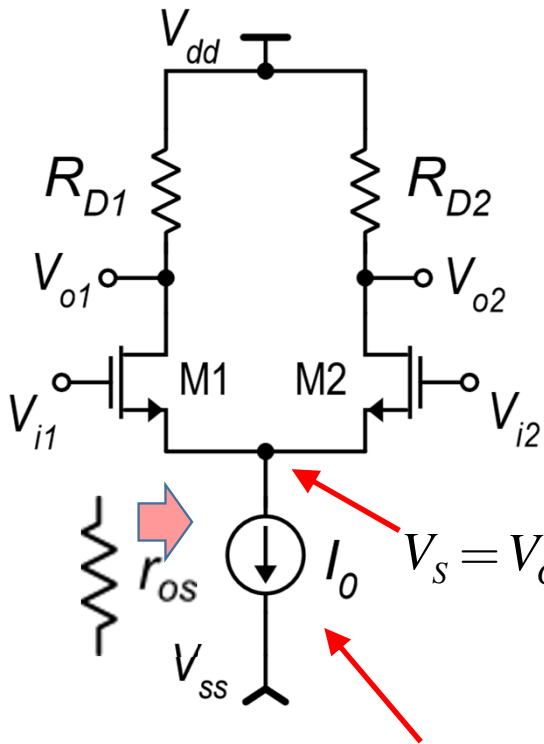
$$v_{out} = v_{o1} = -R_{D1}\frac{g_m}{2}v_{id}$$

$$A_d = -R_{D1}\frac{g_m}{2}$$

Fully-differential case:

$$v_{out} = v_{od} = -\frac{g_m}{2}\left(R_{D1} + R_{D2}\right)v_{id}$$

$$A_{dd} = -g_m\frac{\left(R_{D1} + R_{D2}\right)}{2}$$

Common mode Analysis

$$v_{id} = 0 \implies i_{d1} = i_{d2}$$

$$v_{i1} = v_{i2} = v_c$$

$$i_0 = i_{d1} + i_{d2} = \frac{v_s}{r_{os}}$$

$$v_s = v_{i1} - v_{gs1} = v_c - v_{gs1}$$

$$i_{d1} \cong g_m v_{gs1}$$

$$v_{gs1} = \frac{i_{d1}}{g_m}$$

$$V_S = V_C - V_{GS}$$

$$V_{GS} \cong constant$$

$$v_s = v_c - \frac{i_{d1}}{g_m}$$

$$i_{d1} + i_{d2} = 2i_{d1} = i_0$$

$$i_{d1} = \frac{i_0}{2} = \frac{v_s}{2r_{os}}$$
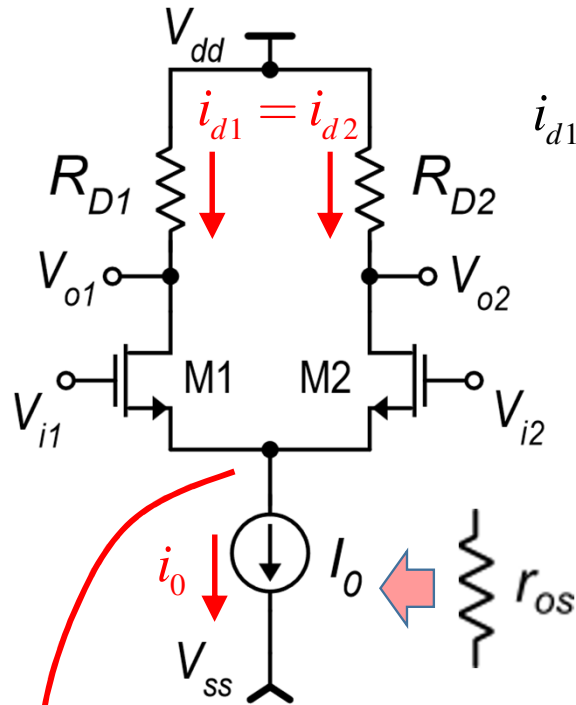
$$v_s = v_c - \frac{v_s}{2r_{os} g_m}$$

$$v_s = v_c \frac{1}{1 + \frac{1}{2g_m r_{os}}} \cong v_c$$

This, actually, is not an ideal current source. It has a finite output resistance = r_{os} If it is implemented by a simple current mirror, r_{os}= r_d otherwise it can be even higher

The source voltage practically follows the common mode variations. V_S is V_C shifted by V_{GS}

Common mode gain and CMRR

output voltages

$$i_{d1} = i_{d2} \cong \frac{i_0}{2} \qquad v_{o1} = -R_{D1}i_{d1} \quad v_{o2} = -R_{D2}i_{d2} \quad v_{od} = i_{d2}R_{D2} - i_{d1}R_{D1}$$

Single-ended case

$$v_{out} = v_{o1} = -R_{D1}i_{d1} \qquad v_{out} - R_{D1}\frac{i_0}{2} \cong -R_{D1}\frac{v_c}{2r_{os}}$$

$$A_c = \frac{v_{out}}{v_c}\bigg|_{v_d=0} \cong -\frac{R_{D1}}{2r_{os}} \qquad CMRR = \frac{A_d}{A_c} = \frac{g_m R_{D1}}{2}\frac{2r_{os}}{R_{D1}}$$

$|A_d|$

$$CMRR = \left|\frac{A_d}{A_c}\right| = g_m r_{os}$$

$v_{i1} = v_{i2} = v_c$

$v_s \cong v_c$

$$i_o = \frac{v_s}{r_{os}} \cong \frac{v_c}{r_{os}}$$

Considering that generally I_0 is produced by a simple current mirror, $r_{os}=r_d$, then CMRR~40 dB,

This CMRR is not sufficient for many applications

Common mode gain and CMRR
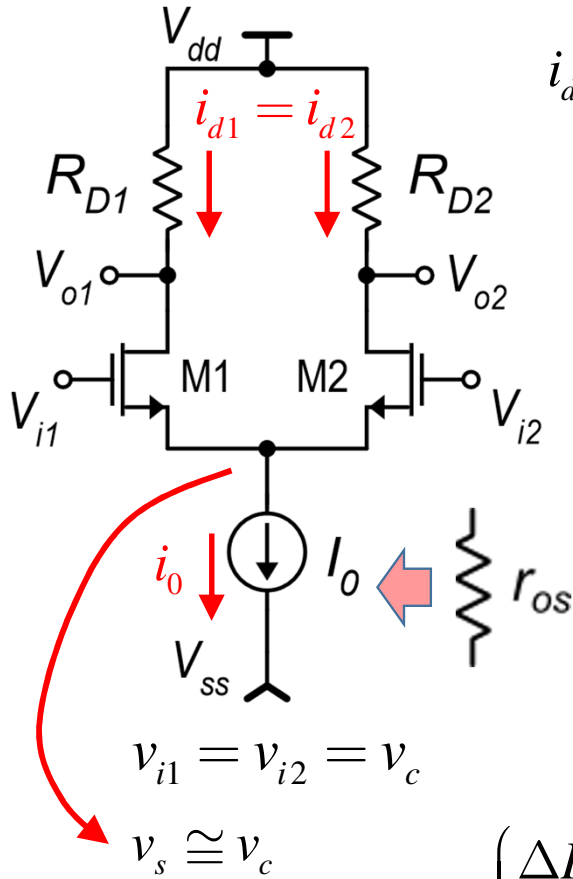
$$i_{d1} = i_{d2} \cong \frac{i_0}{2} \qquad v_{o1} = -R_{D1}i_{d1} \quad v_{o2} = -R_{D2}i_{d2} \quad v_{od} = i_{d2}R_{D2} - i_{d1}R_{D1}$$

Fully differential case

$$v_{out} = v_{od} = \frac{i_0}{2}R_{D2} - \frac{i_0}{2}R_{D1} = \frac{i_0}{2}\left(R_{D2} - R_{D1}\right)$$

$$v_{out} = \frac{v_c}{2r_{os}}\left(R_{D2} - R_{D1}\right) \qquad A_{cd} = \left.\frac{v_{od}}{v_{ic}}\right|_{v_{id}=0} = -\frac{1}{2r_{os}}\left(R_{D1} - R_{D2}\right)$$

$$CMRR = \left|\frac{A_{dd}}{A_{cd}}\right| = \frac{g_m}{2}\left(R_{D1} + R_{D2}\right)\frac{2r_{os}}{\left(R_{D1} - R_{D2}\right)} = 2g_m r_{os}\left(\frac{\Delta R_D}{R_D}\right)^{-1}$$

$\overline{R}_D$ ΔR_D

$\left(\dfrac{\Delta R_D}{R_D}\right)$ is a relative matching error ~ 0.01 $\implies$ $\left(\dfrac{\Delta R_D}{R_D}\right)^{-1} \approx 100$

$CMRR \approx 10^4 = 80\,\text{dB}$

Adequate for most applications

BJT case

In the case of a BJT differential amplifier, the expressions are exactly the same as for the MOSFET case.

The only important difference is the presence of a base current that is drawn from the voltage sources that provide V_{i1} and V_{i2}. In terms of small signal analysis, this base current is the cause of a finite input resistance that may result in an input attenuation if the signal voltage sources (V_{i1}, V_{i2}) have a significant internal resistance.
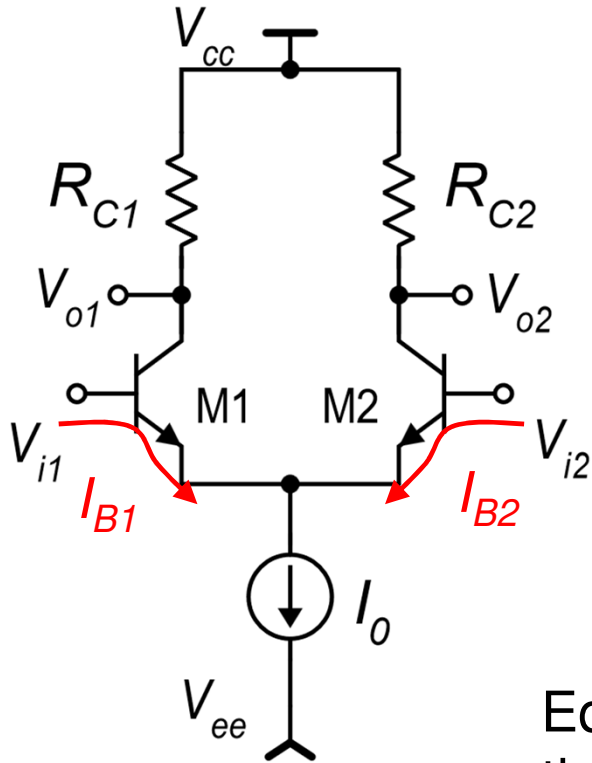
Equivalent circuit of the input terminals
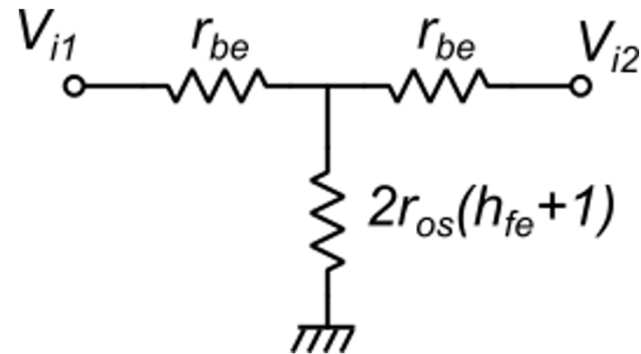
Maximum gain for a given supply voltage
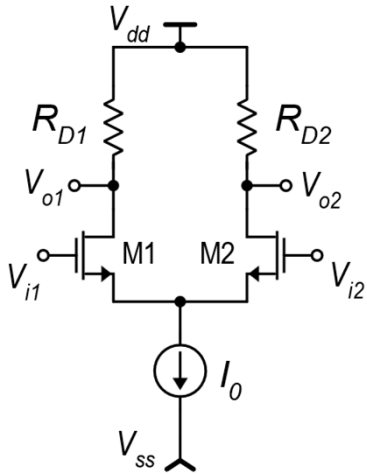
single ended $\quad A_d = -R_{D1} \dfrac{g_m}{2}$ $\qquad\qquad |A_d| = \dfrac{g_m R_D}{2}$ $\left.\rule{0pt}{60pt}\right\}$ nominal case: $R_{D1}=R_{D2}=R_D$

fully-differential $\quad A_{dd} = -g_m \dfrac{(R_{D1}+R_{D2})}{2}$ $\quad |A_{dd}| = g_m R_D$

using: $g_m = \dfrac{I_D}{V_{TE}}$ $\quad |A_{dd}| = \dfrac{R_D I_D}{V_{TE}}$

$R_D I_D$ is the static voltage drop across resistors R_D. It cannot exceed the total supply voltage **V_{DD}–V_{SS}**

$$R_D I_D < (V_{DD} - V_{SS})$$

$$|A_{dd}| < \dfrac{(V_{DD} - V_{SS})}{V_{TE}}$$

Example: V_{TE}=50 mV, V_{DD}-V_{SS}=3 V: $|A_{dd}| < 60$

With a BJT: $V_{TE}=V_T$=25 mV: $|A_{dd}| < 120$

To get the highest gain for a given supply voltage it is convenient to bias the MOSFETS with the smallest V_{TE}.
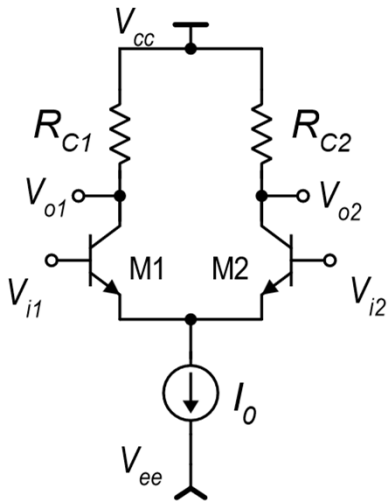
Unipolar case
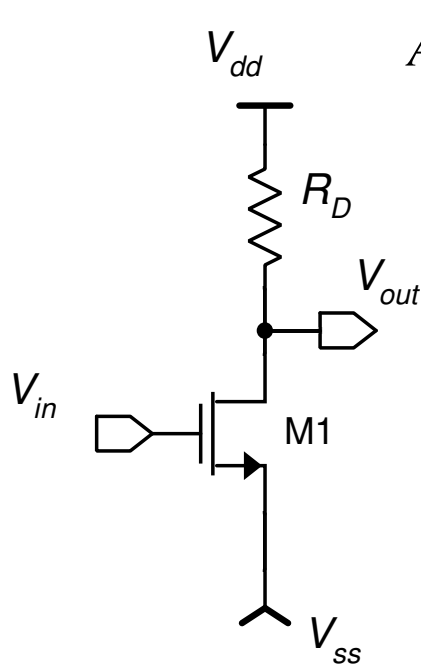
$$A = \frac{v_{out}}{v_{in}} \cong -g_m \left(R_D \; / \! / \; r_d \right)$$

$$\left| A \right| \leq g_m R_D$$

This is the same espression as in the case of differential amplifier with resistive load.
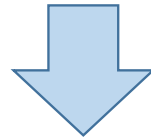
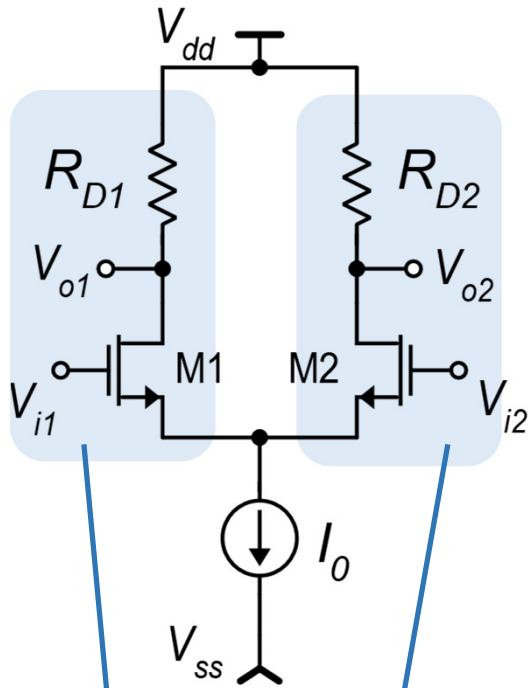$$\left| A \right| < \frac{\left(V_{DD} - V_{SS} \right)}{V_{TE}}$$

In the case of resistive load, the maximum voltage gain of a unipolar amplifier is subjected to the same limitation as the fully differential amplifier

Offset voltage of differential amplifiers with resistive load

MOSFET case - fully-differential - Strong inversion

$$v_{io} = V_D \big|_{V_{out}=0}$$

$$V_{out} = V_{o1} - V_{o2}$$

$$V_D = V_{GS1} - V_{GS2} \triangleq \Delta V_{GS}$$

We can consider the difference of V_{GS} as the matching error between two nominally identical circuits: M1,R_{D1} and M2,R_{D2}

$$V_{GS} = V_t + \sqrt{\frac{2I_D}{\beta}} \qquad A = V_t \quad B = \sqrt{\frac{2I_D}{\beta}} = \sqrt{2} \cdot I_D^{\frac{1}{2}} \beta^{-\frac{1}{2}}$$

posynomial form

$$V_{GS} = A + B \qquad \Delta V_{GS} = \Delta A + \Delta B$$

Nominally identical circuits

$$\Delta A = \Delta V_t \qquad \Delta B = B \cdot \frac{\Delta B}{B} = B \cdot \left(\frac{1}{2} \frac{\Delta I_D}{I_D} - \frac{1}{2} \frac{\Delta \beta}{\beta} \right)$$

Offset voltage of differential amplifiers with resistive load

MOSFET case - fully-differential

$$V_D = \Delta V_{GS} = \Delta A + \Delta B$$

$$A$$

$$B$$
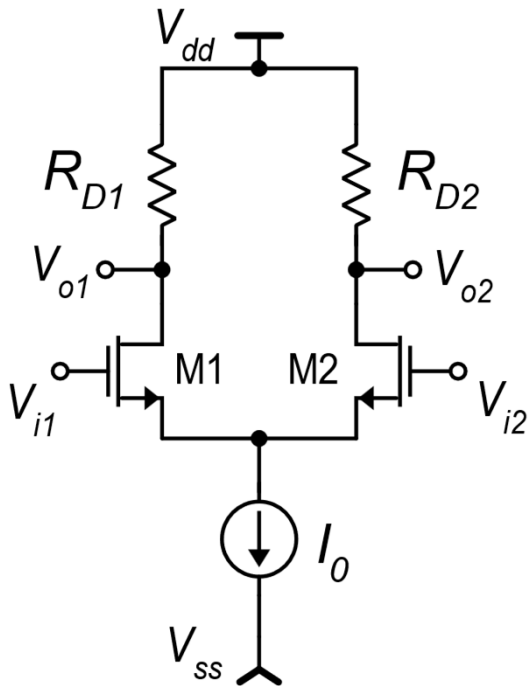
$$V_{GS} = V_t + \sqrt{\frac{2I_D}{\beta}}$$

$$\Delta A = \Delta V_t \qquad \Delta B = B \cdot \left(\frac{1}{2} \frac{\Delta I_D}{I_D} - \frac{1}{2} \frac{\Delta \beta}{\beta} \right)$$

$$V_D = \Delta V_{GS} = \Delta V_t + \sqrt{\frac{2I_D}{\beta}} \cdot \left(\frac{1}{2} \frac{\Delta I_D}{I_D} - \frac{1}{2} \frac{\Delta \beta}{\beta} \right)$$

This is a generic expression of V_D, corresponding to a generic value of $\dfrac{\Delta I_D}{I_D}$

It gives V_{io} only for the particular $\dfrac{\Delta I_D}{I_D}$ value that results in $V_{out}=0$

Offset voltage of differential amplifiers with resistive load

MOSFET case - fully-differential

$$V_{out} = V_{OD} = \underbrace{R_{D2}I_{D2} - R_{D1}I_{D1}}$$

defining: $Z = R_D I_D \qquad \Delta Z = V_{out}$

Imposing V_{out}=0 is the same as imposing ΔZ=0

$$\Delta Z = \frac{\Delta Z}{Z} Z = \underbrace{\left(\frac{\Delta R_D}{R_D} + \frac{\Delta I_D}{I_D} \right)} R_D I_D = 0$$

$R_D I_D$ is the nominal value and cannot be zero

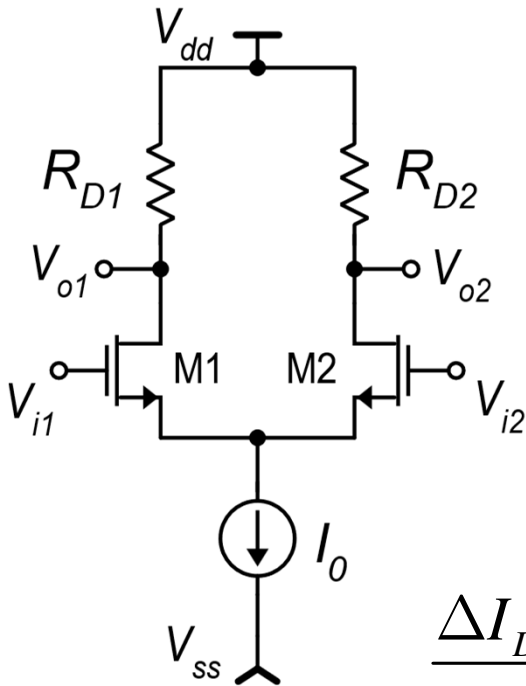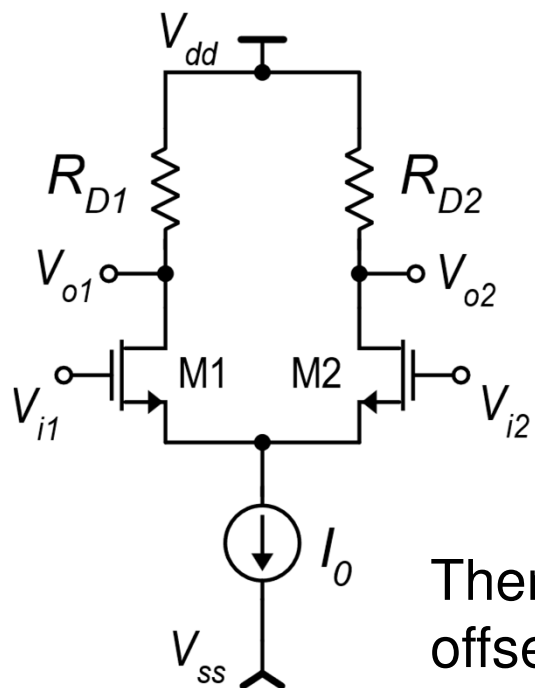$$\frac{\Delta I_D}{I_D} = -\frac{\Delta R_D}{R_D} \qquad \Longleftarrow \qquad = 0 \qquad V_D = \Delta V_{GS} = \Delta V_t + \sqrt{\frac{2I_D}{\beta}} \cdot \left(\frac{1}{2} \frac{\Delta I_D}{I_D} - \frac{1}{2} \frac{\Delta \beta}{\beta} \right)$$

$$V_{io} = \Delta V_t + \sqrt{\frac{2I_D}{\beta}} \cdot \left(-\frac{1}{2} \frac{\Delta R_D}{R_D} - \frac{1}{2} \frac{\Delta \beta}{\beta} \right) \qquad -\frac{\Delta R_D}{R_D}$$

Offset voltage of differential amplifiers with resistive load

MOSFET case - fully-differential

$$V_{io} = \Delta V_t + \sqrt{\frac{2I_D}{\beta} \cdot \left(-\frac{1}{2}\frac{\Delta R_D}{R_D} - \frac{1}{2}\frac{\Delta \beta}{\beta} \right)}$$
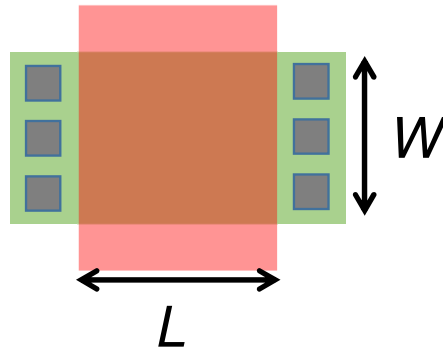
$$\sqrt{\frac{2I_D}{\beta}} = \left(V_{GS} - V_t \right)$$

$$V_{io} = \Delta V_t - \frac{\left(V_{GS} - V_t \right)}{2}\left(\frac{\Delta R_D}{R_D} + \frac{\Delta \beta}{\beta} \right)$$

Then, we can calculate the standard deviation of the offsest voltage:

$$\sigma_{Vio} = \sqrt{\sigma_{Vt}^2 + \left[\sigma_{\frac{\Delta \beta}{\beta}} \frac{\left(V_{GS} - V_t \right)}{2} \right]^2 + \left[\sigma_{\frac{\Delta R_D}{R_D}} \frac{\left(V_{GS} - V_t \right)}{2} \right]^2}$$

Role of the design parameters
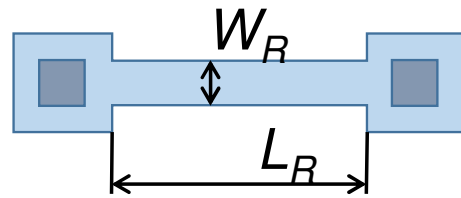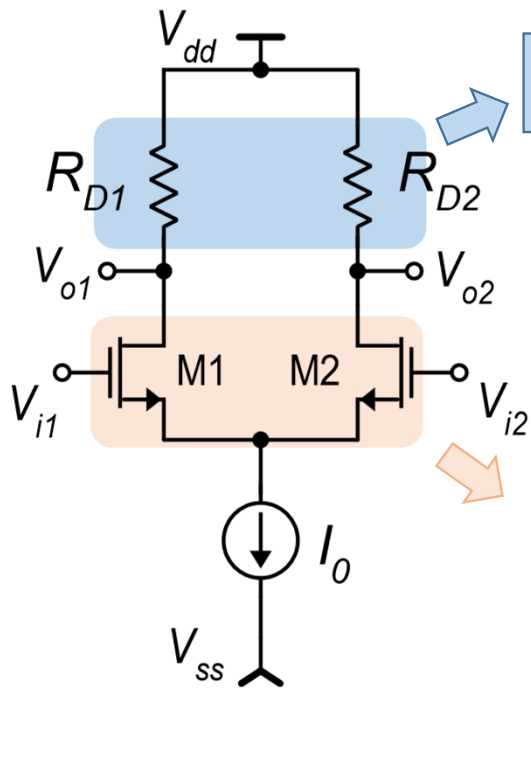
$$\sigma_{Vio} = \sqrt{\sigma_{Vt}^2 + \left[\sigma_{\frac{\Delta\beta}{\beta}} \frac{(V_{GS} - V_t)}{2}\right]^2 + \left[\sigma_{\frac{\Delta R_D}{R_D}} \frac{(V_{GS} - V_t)}{2}\right]^2}$$

$$\frac{L_R}{W_R} \rightarrow R_D \qquad W_R L_R \rightarrow \sigma_{\frac{\Delta R_D}{R_D}} = \frac{C_R}{\sqrt{W_R L_R}}$$

To be made as small as possible, to reduce σ_{Vio}

$$I_0 \rightarrow (V_{GS} - V_t)$$

$$\frac{W}{L} \rightarrow \beta$$

$$WL \rightarrow \begin{cases} \sigma_{Vt} = \dfrac{C_{vt}}{\sqrt{WL}} \\[2em] \sigma_{\frac{\Delta\beta}{\beta}} = \dfrac{C_\beta}{\sqrt{WL}} \end{cases}$$

Small offset means large silicon area

Offset voltage of a BJT differential amplifier with resistor load
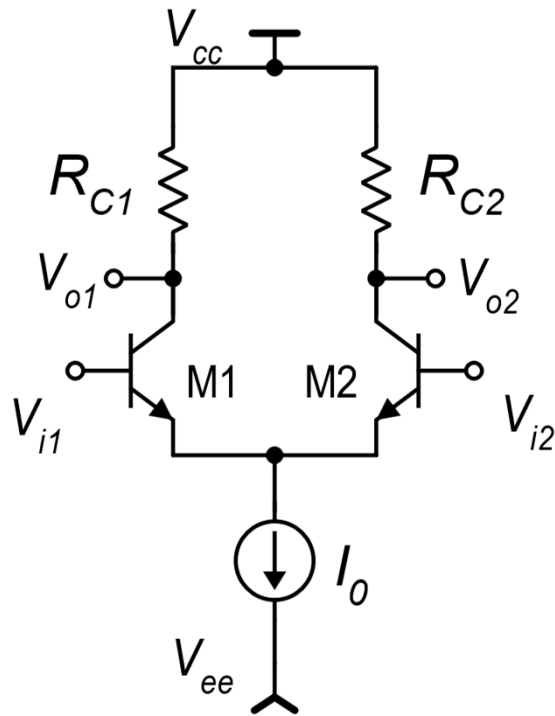
$$v_{io} = V_{id}\big|_{V_{od}=0} \qquad V_{od} = V_{o1} - V_{o2} = R_{C2}I_{C2} - R_{C2}I_{C2}$$
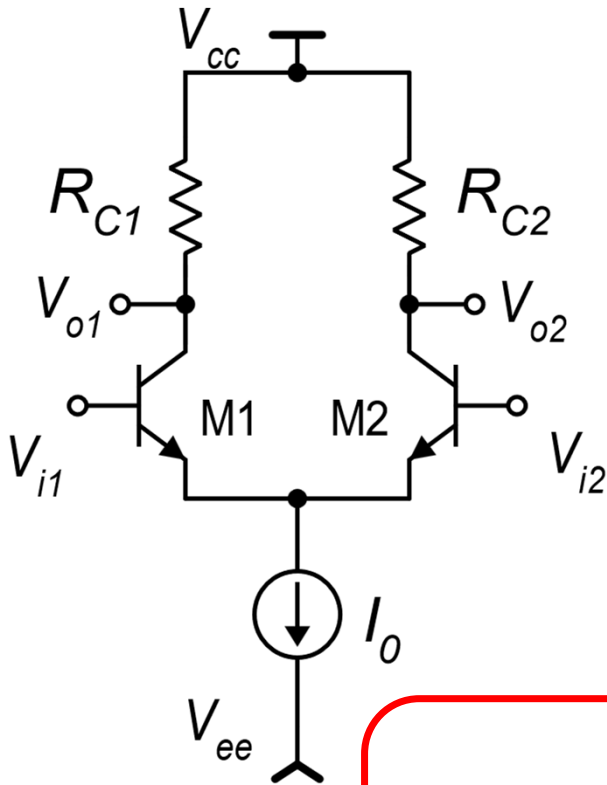
$$V_{id} = V_{i1} - V_{i2} = V_{BE1} - V_{BE2} = \Delta V_{BE}$$

$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) \qquad V_{id} = \Delta V_{BE} = V_T\left(\frac{\Delta I_C}{I_C} - \frac{\Delta I_S}{I_S}\right)$$

$$V_{od} = 0 \implies \Delta(R_C I_C) = 0 \implies \frac{\Delta I_C}{I_C} = -\frac{\Delta R_C}{R_C}$$

$$v_{io} = V_T\left(-\frac{\Delta R_C}{R_C} - \frac{\Delta I_S}{I_S}\right)$$

Offset voltage of a BJT differential amplifier with resistor load

$$v_{io} = V_T \left(-\frac{\Delta R_C}{R_C} - \frac{\Delta I_S}{I_S} \right)$$

$$\sigma_{vio} = \sqrt{ \left(V_T \sigma_{\frac{\Delta R_C}{R_C}} \right)^2 + \left(V_T \sigma_{\frac{\Delta I_S}{I_S}} \right)^2 }$$

$$V_T \sigma_{\frac{\Delta I_S}{I_S}} = \sigma_{\Delta V_{BE}^*}$$

$$\sigma_{\frac{\Delta R_C}{R_C}} = \frac{C_R}{\sqrt{W_R L_R}}$$

Minimum size BJT (elemental BJT) of the process

$$\sigma_{\Delta V_{BE-e}^*}$$
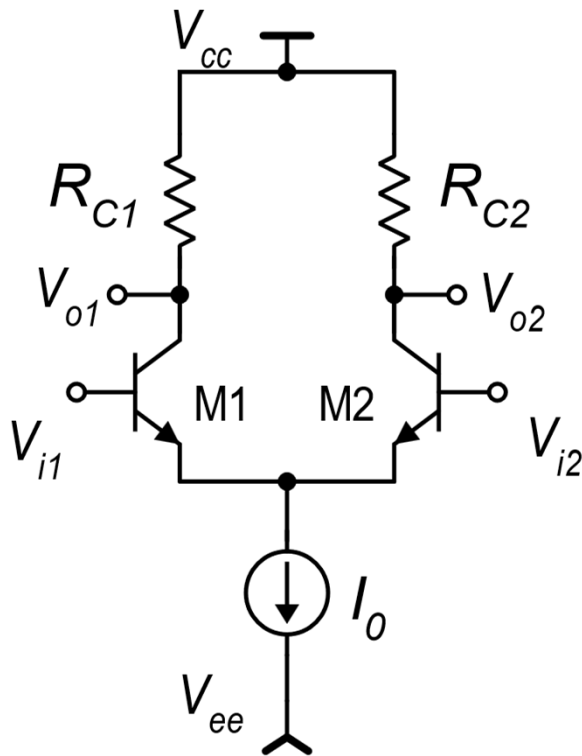
$$\boxed{\sigma_{vio} = \sqrt{ \left(V_T \sigma_{\frac{\Delta R_C}{R_C}} \right)^2 + \sigma_{\Delta V_{BE}^*}^2 }}$$

Small offset means large silicon area

$$\sigma_{\Delta V_{BE}^*} = \frac{\sigma_{\Delta V_{BE-e}^*}}{\sqrt{area}}$$

Temperature drift of the input offset voltage: BJT case

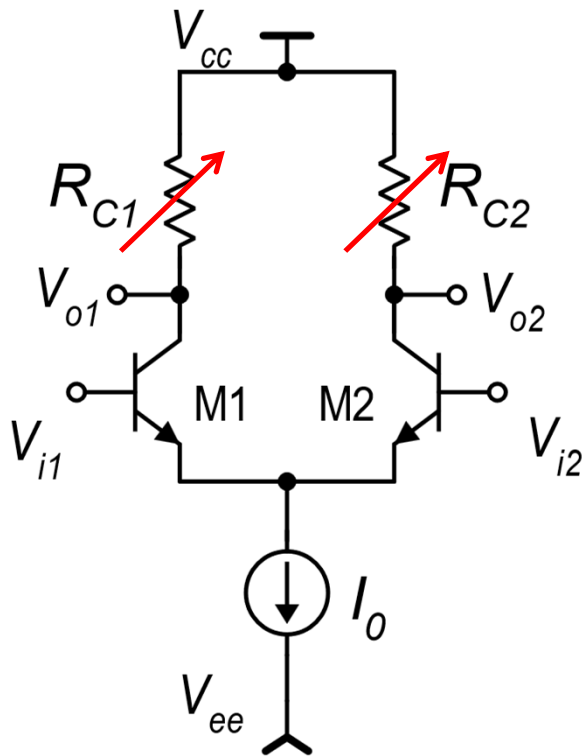$$v_{io} = \frac{kT}{q}\left(-\frac{\Delta R_C}{R_C} - \frac{\Delta I_S}{I_S}\right)$$

$$\frac{dv_{io}}{dT} = \frac{k}{q}\left(-\frac{\Delta R_C}{R_C} - \frac{\Delta I_S}{I_S}\right)$$

These terms are practically temperature independent (ratios of homogeneous quantities)

$$\frac{dv_{io}}{dT} = \frac{kT}{q}\left(-\frac{\Delta R_C}{R_C} - \frac{\Delta I_S}{I_S}\right)\frac{1}{T} = \frac{v_{io}}{T}$$

A method to reduce the offset voltage and its drift

$$\frac{dv_{io}}{dT} = \frac{v_{io}}{T}$$

Example: v_{io}=1 mV
T=300 K

$$\frac{dv_{io}}{dT} = \frac{1.0 \times 10^{-3}}{300} = 3.3 \ \mu V/K$$

By trimming the resistors, I can null the offset. In this way, also the temperature drift is nulled.

This is an advantage of the BJT amplifier. Trimming the offset of a MOSFET differential amplifier does not null the drift.

Example: the OP07

1. Resistors are varied by laser trimming to null the offset
2. In this way, also the offset drift is strongly reduced
3. The base currents of the input devices (Q1,Q2) are reproduced by Q3 and Q4 and than fed back to the input terminals by mirrors Q8,Q6 and Q7,Q5. This approach allows to reduce the input bias currents.
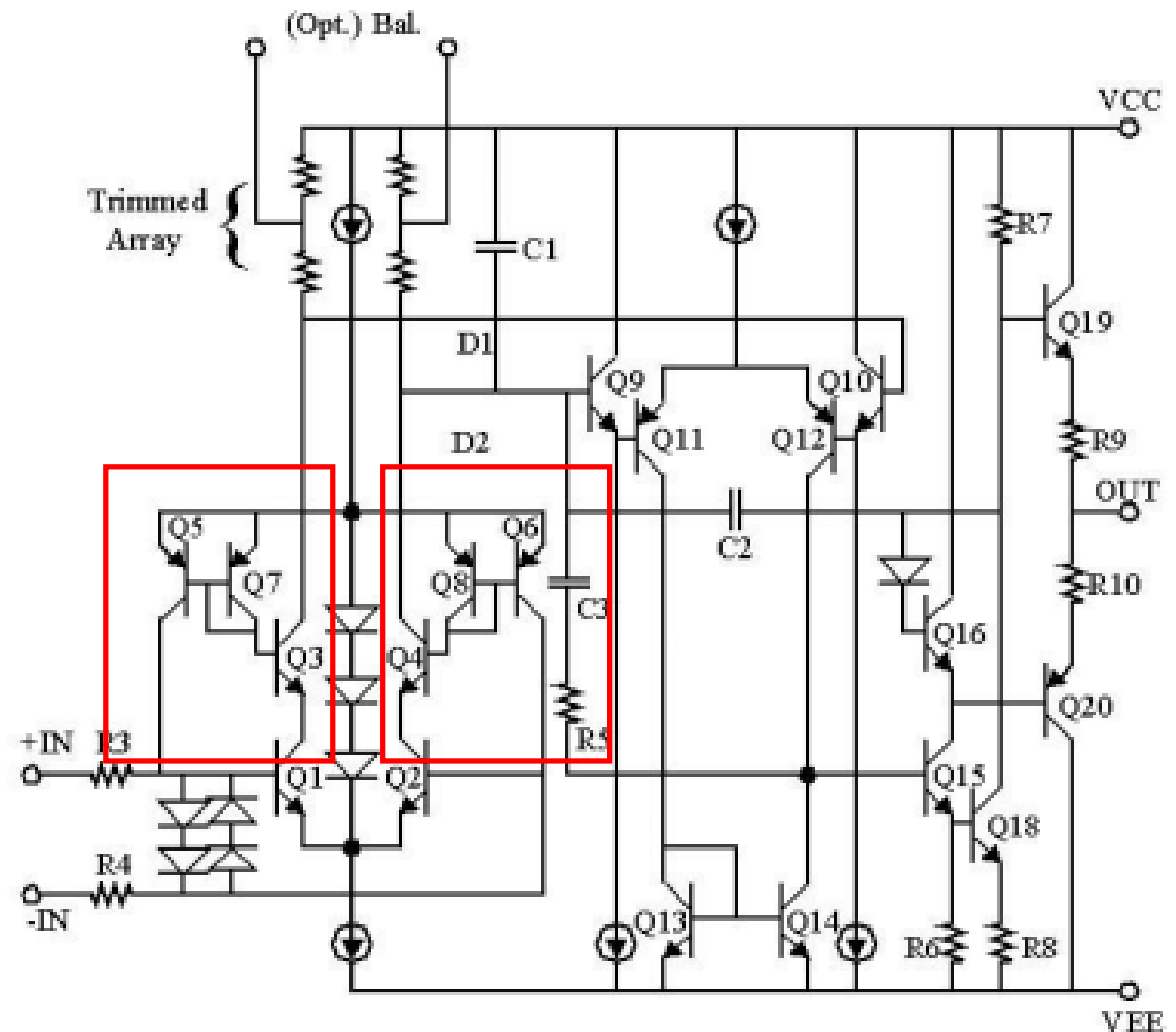
OP07 vs µA741

OP07

Table 1.

Parameter	Symbol	Conditions	Min	Typ	Max	Unit
INPUT CHARACTERISTICS						
$T_A = 25°C$						
Input Offset Voltage[1]	V_{OS}			30	75	µV
Long-Term V_{OS} Stability[2]	$V_{OS}/Time$			0.3	1.5	µV/Month
Input Offset Current	I_{OS}			0.5	3.8	nA
Input Bias Current	I_B			±1.2	±4.0	nA
$0°C \leq T_A \leq 70°C$						
Input Offset Voltage[1]	V_{OS}			45	130	µV
Voltage Drift Without External Trim[4]	TCV_{OS}			0.3	1.3	µV/°C
Voltage Drift with External Trim[3]	TCV_{OSN}	$R_P = 20\ k\Omega$		0.3	1.3	µV/°C

The OP07 has a much better offset than the mA 741, whose first stage has not restive loads

µA741

at specified virtual junction temperature, $V_{CC\pm} = \pm15\ V$ (unless otherwise noted)

PARAMETER		TEST CONDITIONS[1]		MIN	TYP	MAX	UNIT
V_{IO}	Input offset voltage	$V_O = 0$	25°C		1	6	mV
			Full range			7.5	
$\Delta V_{IO(adj)}$	Offset voltage adjust range	$V_O = 0$	25°C		±15		mV
I_{IO}	Input offset current	$V_O = 0$	25°C		20	200	nA
			Full range			300	
I_{IB}	Input bias current	$V_O = 0$	25°C		80	500	nA
			Full range			800	

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How is the amplifier offset when the single ended option is chosen?
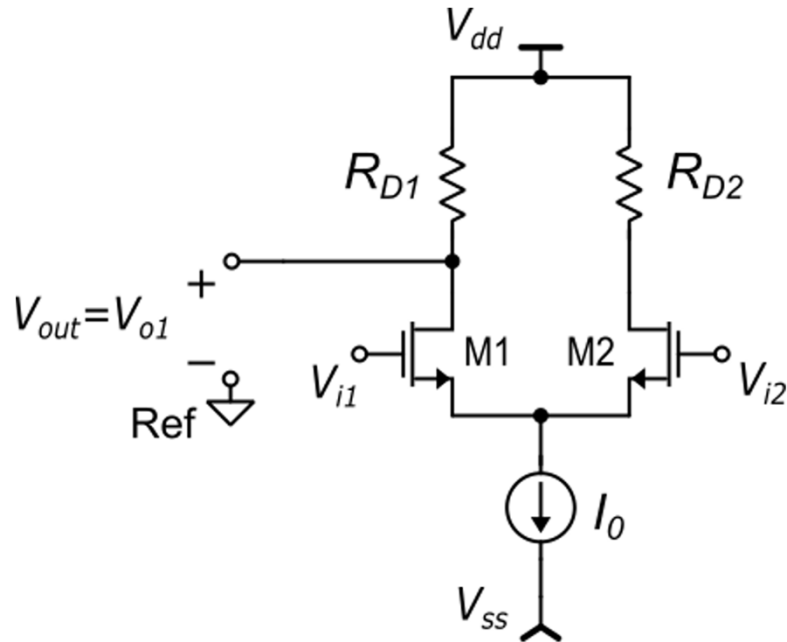
$$V_{out} = V_{O1} = V_{dd} - R_1 I_{D1}$$

Single ended (S/E): device parameters do not appear as differences: the output voltage is affected mainly by global errors

The single ended case is subjected to a much larger offset!

Compare with the fully-differential case: the output voltage depends only on matching differences

$$V_{od} = R_{D2} I_{D2} - R_{D1} I_{D1}$$