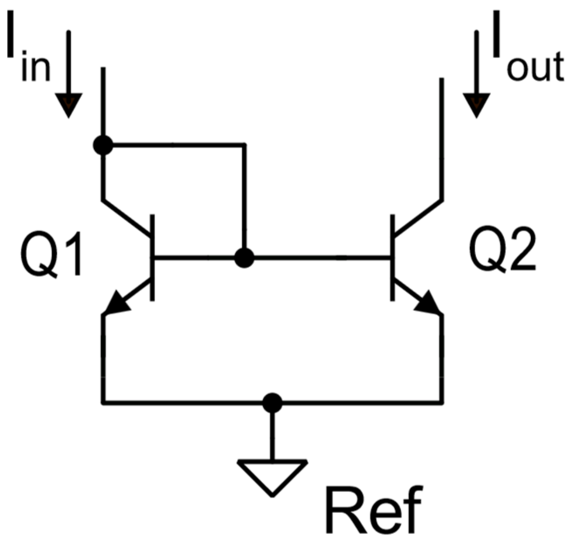
BJT current mirrors

BJT simple current mirror



$$I_{C2} = I_{S2} e^{\frac{V_{BE}}{V_T}} \left(1 + \frac{V_{CB2}}{V_A}\right) = I_{S2} e^{\frac{V_{BE}}{V_T}} \left(1 + \frac{V_{CE2}}{V_A} - \frac{V_{BE}}{V_A}\right)$$

$$I_{C1} = I_{S1} e^{\frac{V_{BE}}{V_T}} \left(1 + \frac{V_{CB1}}{V_A}\right) = I_{S1} e^{\frac{V_{BE}}{V_T}} \left(1 + \frac{V_{CE1}}{V_A} - \frac{V_{BE}}{V_A}\right)$$

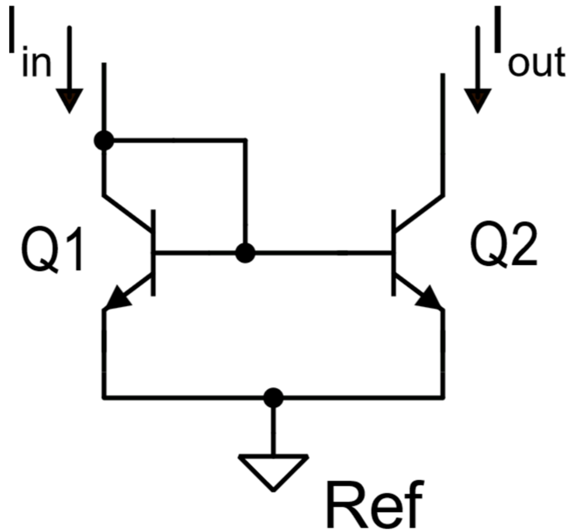
$$k_M \triangleq \frac{I_{S2}}{I_{S1}} = \frac{A_{E2}}{A_{E1}} = \frac{area_2}{area_1}$$

$$V_{CE1} = V_{IN}$$

$$V_{CE2} = V_{OUT}$$

$V_{out}=V_{in}$ ⇒ $$\frac{I_{C2}}{I_{C1}} = k_M$$

BJT simple current mirror: parameters



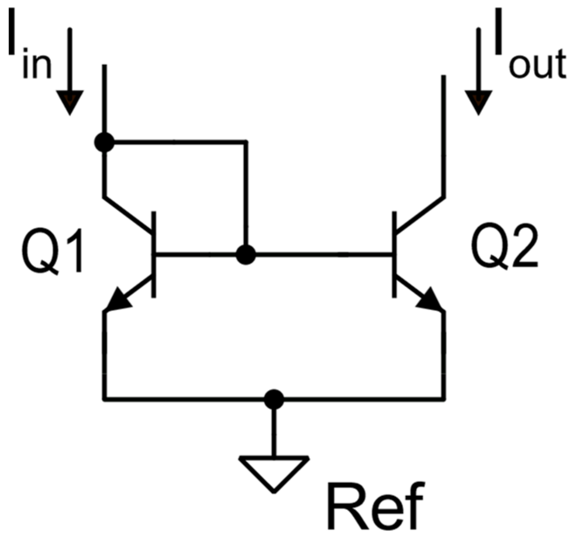
$$V_{IN} = V_{CE1} = V_{BE1} \approx V_{\gamma}$$

$$V_{MIN} = V_{CESAT2} \approx 0.2 \text{ V}$$

$$R_{out} = r_{o2} = \frac{V_A}{I_{C2}} = \frac{V_A}{I_{out}}$$

$$V_{th} = R_{out} I_{out} = V_A$$

BJT simple mirror: impact of base currents



$$\frac{I_{C2}}{I_{C1}} = k_M \qquad I_{C2} = I_{out}$$

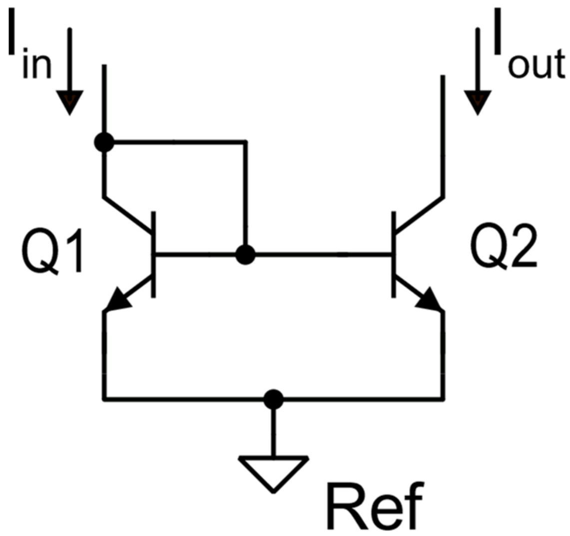
$$I_{C1} \neq I_{in}$$

$$\frac{I_{out}}{I_{in}} = \frac{I_{C2}}{I_{C1} + I_{B1} + I_{B2}} \qquad I_{B1} = \frac{I_{C1}}{\beta_1} \qquad I_{B2} = \frac{I_{C2}}{\beta_2}$$

$$\beta_1 = \beta_2 \triangleq \beta \qquad \frac{I_{out}}{I_{in}} = \frac{I_{C2}}{I_{C1} + \frac{I_{C1}}{\beta} + \frac{I_{C2}}{\beta}}$$

$$\frac{I_{out}}{I_{in}} = \frac{I_{C2}}{I_{C1}} \frac{1}{1 + \frac{1}{\beta} + \frac{k_M}{\beta}} = k_M \frac{1}{1 + \frac{1 + k_M}{\beta}}$$

BJT simple mirror: impact of base currents



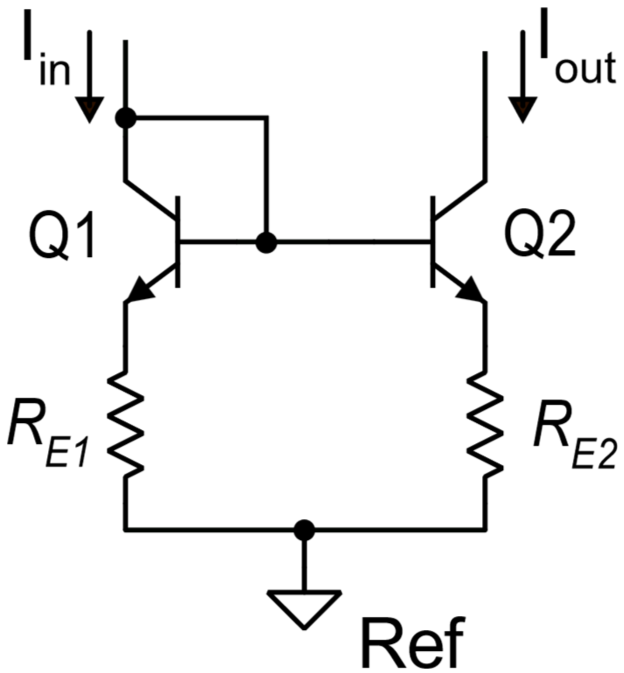
$$\frac{I_{C2}}{I_{C1}} = k_M \qquad \frac{I_{out}}{I_{in}} = k_M \frac{1}{1 + \frac{1 + k_M}{\beta}} < k_M$$

$$\frac{1}{1+x} \cong 1 - x \quad \text{for: } x << 1$$

$$\frac{I_{out}}{I_{in}} \cong k_M \left(1 - \frac{k_M + 1}{\beta} \right) \quad \text{for: } \frac{k_M + 1}{\beta} << 1$$

$$\varepsilon_R \cong \frac{k_M + 1}{\beta}$$

Simple mirror with emitter degeneration to increase the output resistance



$$k_M = \frac{I_{S2}}{I_{S1}} = \frac{A_{E2}}{A_{E1}} = \frac{area_2}{area_1} = \frac{I_{C2}}{I_{C1}}$$

for $V_{in}=V_{out}$, only if: $V_{BE1} = V_{BE2} \Leftarrow I_{E1}R_{E1} = I_{E2}R_{E2}$

$$\frac{R_{E2}}{R_{E1}} = \frac{I_{E1}}{I_{E2}} = \frac{I_{CE1}}{I_{CE2}} = \frac{1}{k_M}$$

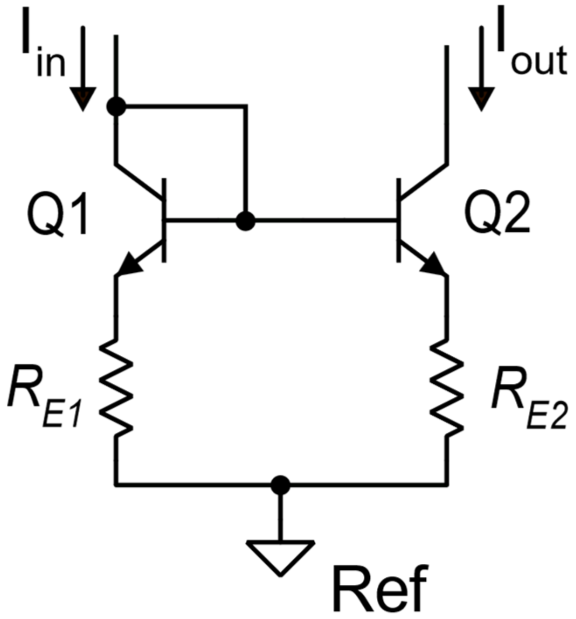
$$V_{IN} = V_{BE1} + R_{E1}I_{E1}$$

$$V_{MIN} = V_{CESAT2} + R_{E2}I_{E2}$$

$$R_{out} \cong r_{o2}\left(1 + g_{m2}R_{E2}\right) \cong r_{o2}\left(1 + \frac{I_{C2}R_{E2}}{V_T}\right)$$

The voltage drop across R_{E1} (R_{E2}) makes V_{IN} and V_{MIN} worse than in the simple mirror

BJT current mirror with emitter degeneration: parameters



$$V_{IN} = V_{BE1} + R_{E1} I_{E1}$$

$$V_{MIN} = V_{CESAT2} + R_{E2} I_{E2}$$

$$R_{out} \cong r_{o2}\left(1 + g_{m2} R_{E2}\right) \cong r_{o2}\left(1 + \frac{I_{C2} R_{E2}}{V_T}\right) = \frac{V_A}{I_{out}}\left(1 + \frac{I_{C2} R_{E2}}{V_T}\right)$$

$$V_{th} \cong V_A \left(1 + \frac{I_{C2} R_{E2}}{V_T}\right)$$

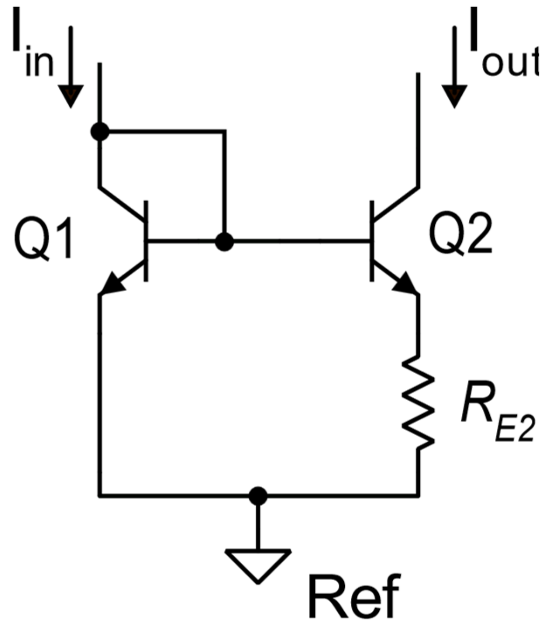
Example: with $I_{C2}R_{E2}$ =225 mV, and V_T=25 mV, R_{out}=10r_{o2}

$$\frac{I_{out}}{I_{in}} = \frac{I_{C2}}{I_{C1} + I_{B1} + I_{B2}}$$

The error due to the base currents is the same as in the simple mirror

$$\varepsilon_R \cong \frac{k_M + 1}{\beta}$$

The Widlar current source



$$V_{BE1} = V_{BE2} + R_{E2} I_{E2}$$

$$\Delta V_{BE} \triangleq V_{BE1} - V_{BE2} = R_{E2} I_{E2} \cong R_{E2} I_{C2}$$

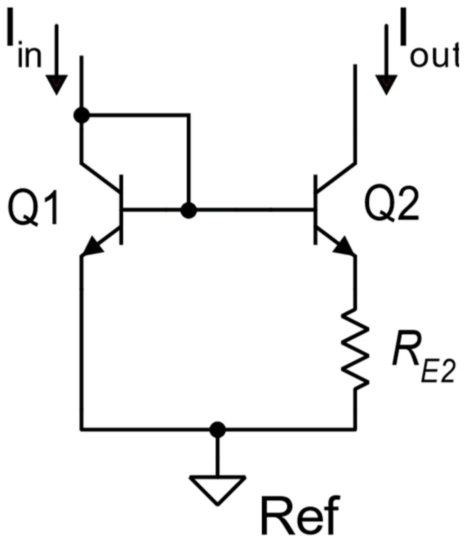
$$I_C = I_S e^{\frac{V_{BE}}{V_T}} \left(1 + \frac{V_{CB2}}{V_A}\right) \cong I_S e^{\frac{V_{BE}}{V_T}} \qquad V_{BE} \cong V_T \ln\left(\frac{I_C}{I_S}\right)$$

$$\Delta V_{BE} \cong V_T \ln\left(\frac{I_{C1}}{I_{S1}}\right) - V_T \ln\left(\frac{I_{C2}}{I_{S2}}\right) = V_T \ln\left(\frac{I_{C1}}{I_{S1}} \frac{I_{S2}}{I_{C2}}\right)$$

If Q1=Q2 (I_{S1}=I_{S2})

$$\Delta V_{BE} \cong V_T \ln\left(\frac{I_{C1}}{I_{C2}}\right) \cong R_{E2} I_{C2} \qquad \frac{I_{C2}}{I_{C1}} \cong e^{-\frac{R_{E2} I_{C2}}{V_T}}$$

The Widlar current source: A **non-linear** current mirror

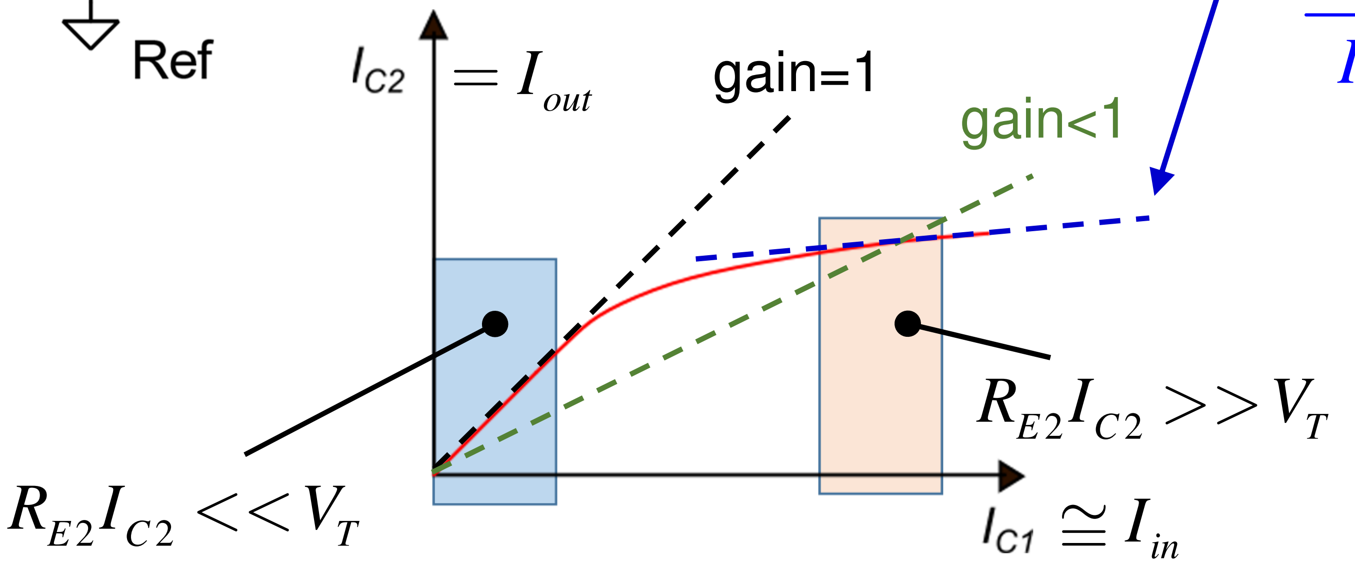


$$\frac{I_{C2}}{I_{C1}} \cong e^{-\frac{R_{E2} I_{C2}}{V_T}}$$

Since the I_{C2}/I_{C1} ratio is not constant, but depends on I_{C2} (and then on the input current), the behavior is non-linear

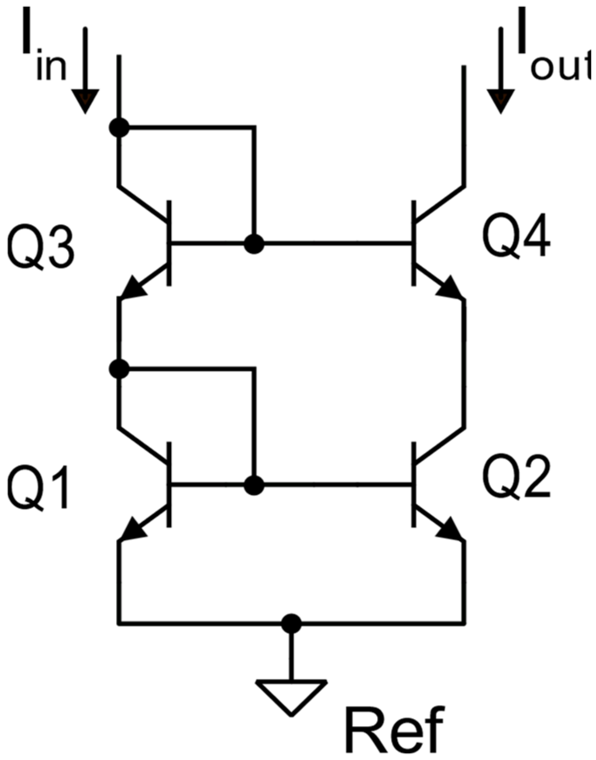
Slope < d.c gain

$$\frac{\Delta I_{out}}{I_{out}} << \frac{\Delta I_{in}}{I_{in}}$$



The Widlar source was used to produce an almost constant current (I_{out}) from a variable current I_{in}

BJT cascode current mirror



$$V_{CE2} = V_{CE1} + V_{BE3} - V_{BE4}$$

If we make: $V_{BE3} = V_{BE4}$ then: $V_{CE2} = V_{CE1}$

$$\frac{I_{C2}}{I_{C1}} = k_M$$ almost independently of V_{out}

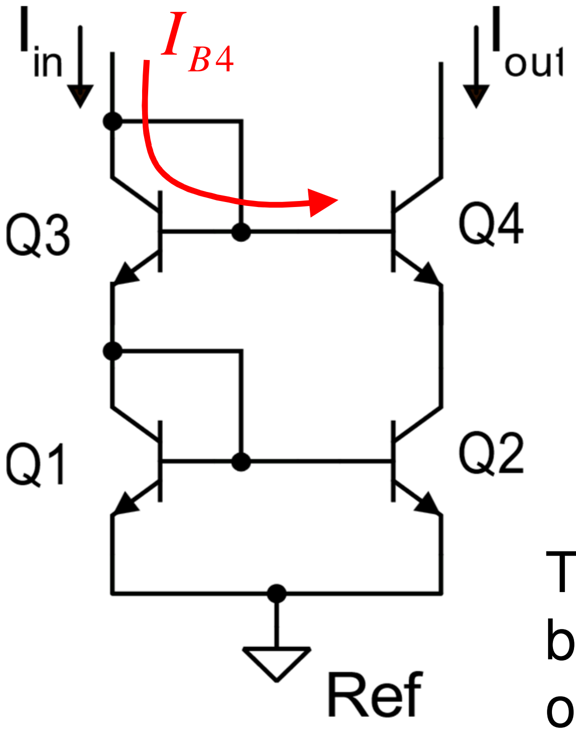
$$\left.\begin{array}{l} V_{BE3} = V_{BE4} \\ V_{BE} \cong V_T \ln\left(\frac{I_C}{I_S}\right) \end{array}\right\} \quad \frac{I_{C4}}{I_{S4}} = \frac{I_{C3}}{I_{S3}}$$

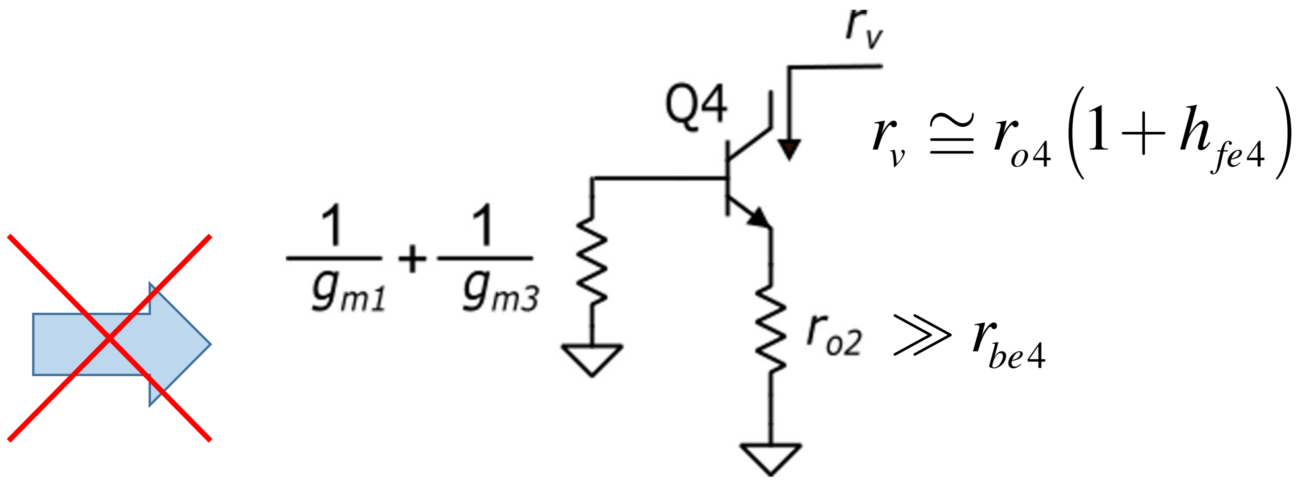
$$\frac{I_{S4}}{I_{S3}} = \frac{I_{C4}}{I_{C3}} \cong \frac{I_{C2}}{I_{C1}} = \frac{I_{S2}}{I_{S1}} = k_M$$

Design rule

$$\frac{I_{S4}}{I_{S3}} = \frac{I_{S2}}{I_{S1}} = k_M$$

BJT cascode current mirror: R_{out}



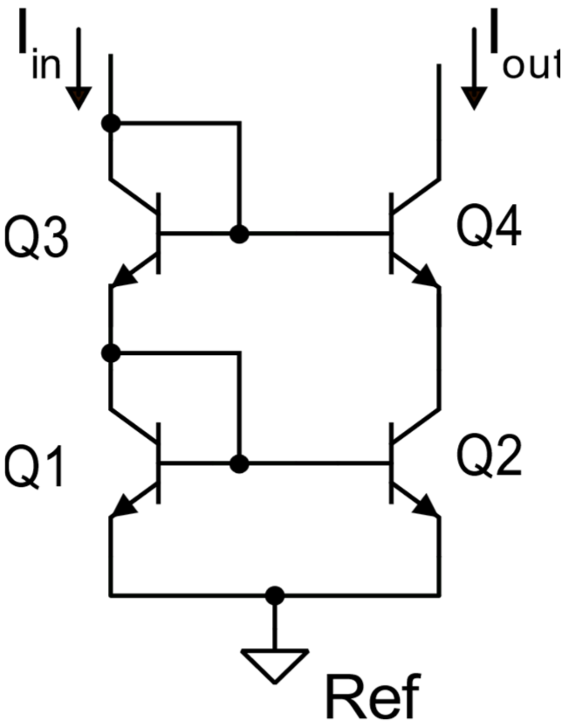


There is interaction of the output branch (Q2,Q4) with the input one (Q3,Q4) caused by the base currents. This interaction is not present in the cascode MOSFET

It can be shown that:

$$R_{out} \cong r_{o4}\left(1+\frac{h_{fe4}}{2}\right)$$

BJT cascode current mirror: effect of base currents



$$\begin{cases} I_{out} = I_{C4} = I_{E4} - I_{B4} = I_{C2} - I_{B4} \\ I_{in} = I_{C1} + I_{B1} + I_{B2} + I_{B4} \end{cases}$$

$$\frac{I_{C4} \cong I_{E4} = I_{C2}}{}$$

$$\begin{cases} I_{B2} = \dfrac{I_{C2}}{\beta} \cong I_{B4} \\ I_{B1} = \dfrac{I_{C1}}{\beta} \end{cases}$$

$$\frac{I_{out}}{I_{in}} = \frac{I_{C2} - I_{B4}}{I_{C1} + I_{B1} + I_{B2} + I_{B4}} \cong \frac{I_{C2} - \dfrac{I_{C2}}{\beta}}{I_{C1} + \dfrac{I_{C1}}{\beta} + 2\dfrac{I_{C2}}{\beta}}$$

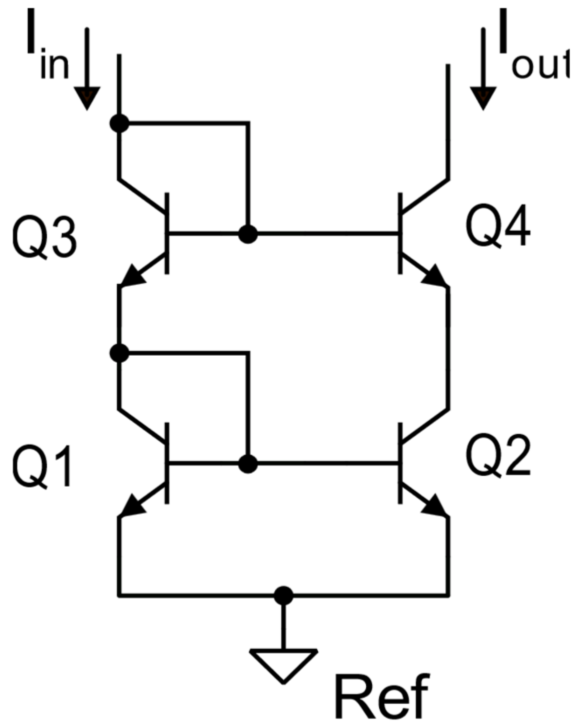
$$\frac{I_{out}}{I_{in}} \cong \frac{I_{C2}}{I_{C1}} \cdot \frac{1 - \dfrac{1}{\beta}}{1 + \dfrac{1}{\beta} + 2\dfrac{k_M}{\beta}}$$

using the approximation:

$$\frac{1-y}{1+x} \cong (1-y)(1-x) = 1 - (x+y) + xy$$

$$\frac{I_{out}}{I_{in}} \cong k_M \cdot \left(1 - 2\frac{1+k_M}{\beta}\right) \longrightarrow \varepsilon_R = -2\frac{1+k_M}{\beta}$$

BJT cascode current mirror: summary of parameters



$$V_{IN} = V_{BE1} + V_{BE3} \approx 2V_\gamma$$
$$V_{MIN} = V_{BE1} + V_{CESAT4} \approx V_\gamma + V_{CESAT}$$

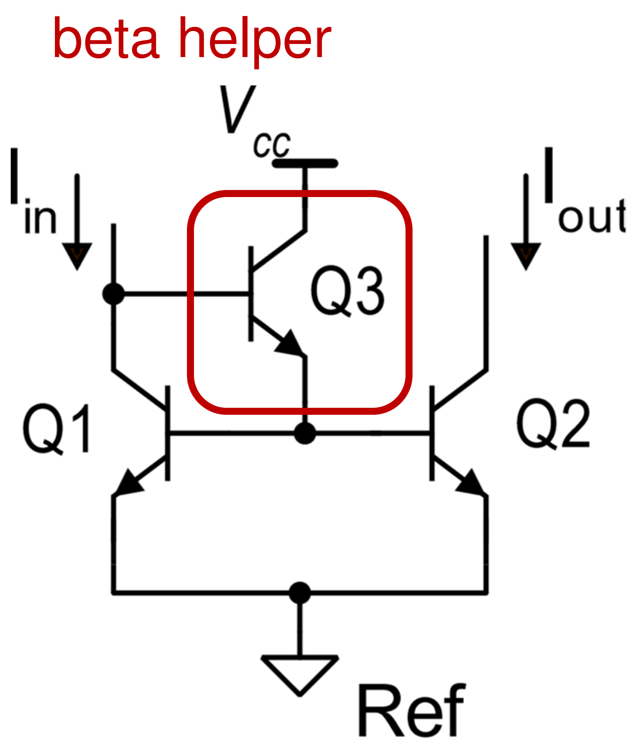
$$R_{out} \cong r_{o4}\left(1 + \frac{h_{fe4}}{2}\right) = \frac{V_A}{I_{out}}\left(1 + \frac{h_{fe4}}{2}\right)$$

$$\varepsilon_R = -2\frac{1 + k_M}{\beta}$$

The output resistance is increased by a factor around $h_{fe}/2$, which is paid with an increase of both V_{IN} and V_{MIN}. The relative error due to the base current is **doubled** with respect of the simple mirror.

Due to the mentioned drawbacks of the BJT cascode current mirror, other solutions, such as the **emitter degenerated mirror** and **Wilson mirror** are often preferred.

Methods to reduce the impact of the base currents: the "beta helper"



$$I_{in} = I_{C1} + I_{B3} \quad = I_{C1} + \frac{I_{E3}}{\beta_3 + 1}$$

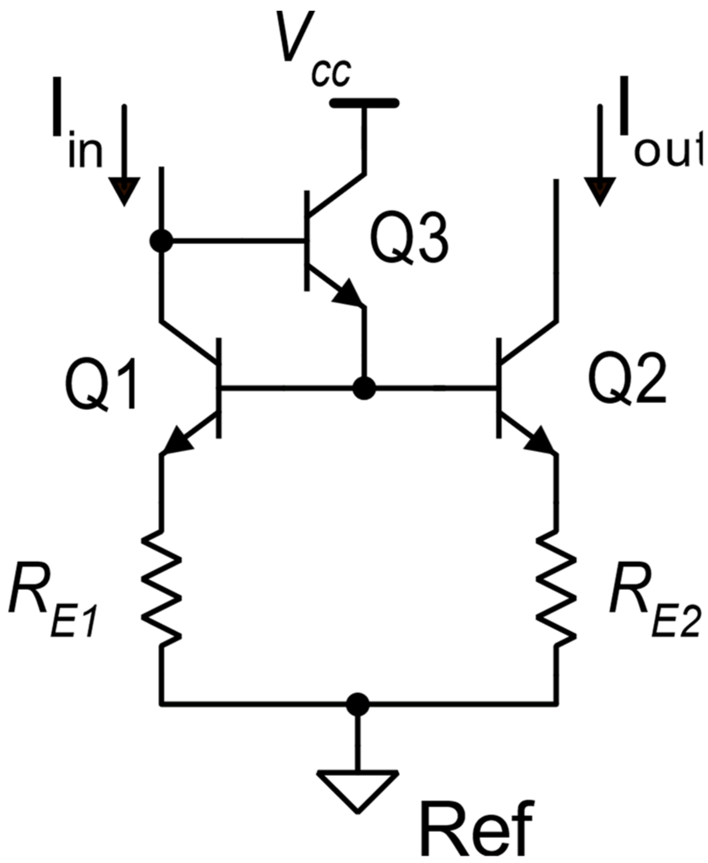
$$I_{E3} = I_{B1} + I_{B2}$$

$$I_{in} = I_{C1} + \frac{I_{B1} + I_{B2}}{\beta_3 + 1} = I_{C1} + \frac{1}{\beta_3 + 1}\left(\frac{I_{C1}}{\beta} + \frac{k_M I_{C1}}{\beta}\right)$$

$$\frac{I_{out}}{I_{in}} = \frac{I_{C2}}{I_{C1} + I_{C1}\dfrac{1+k_M}{(\beta_3+1)\beta}} = k_M \frac{1}{1 + \dfrac{1+k_M}{(\beta_3+1)\beta}}$$

$$\varepsilon_R = -\frac{1+k_M}{(\beta_3+1)\beta} \approx \frac{1}{\beta^2} \qquad V_{IN} = 2V_{\gamma}$$

Simple mirror with emitter degeneration and beta helper



$$R_{out} \cong r_{o2}\left(1+g_{m2}R_{E2}\right)$$

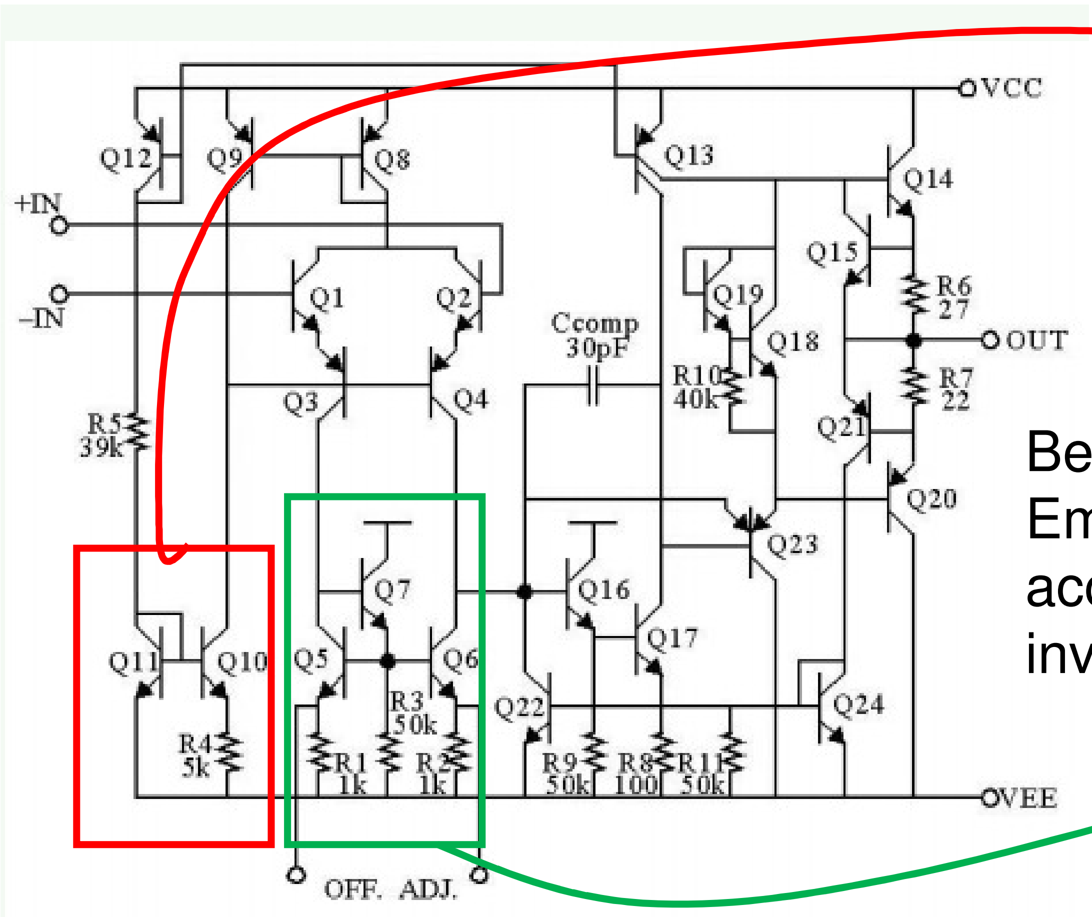
$$V_{IN} = 2V_{\gamma} + R_{E1}I_{E1}$$

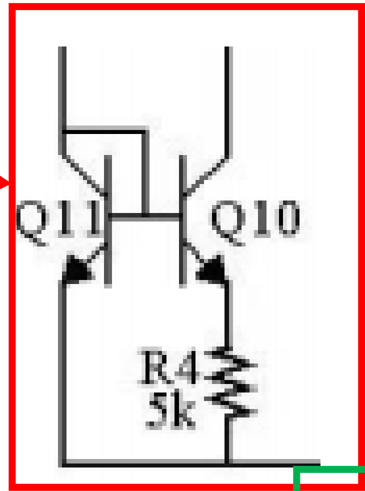
$$V_{MIN} = V_{CESAT} + R_{E2}I_{E2}$$

$$\varepsilon_R = -\frac{1+k_M}{\left(\beta_3+1\right)\beta} \approx \frac{1}{\beta^2}$$

This mirror offers a good output resistance and small base-current related error.
This made it a popular choice for the design of BJT op-amps.
Its large V_{IN} prevents its use in low supply voltage circuits

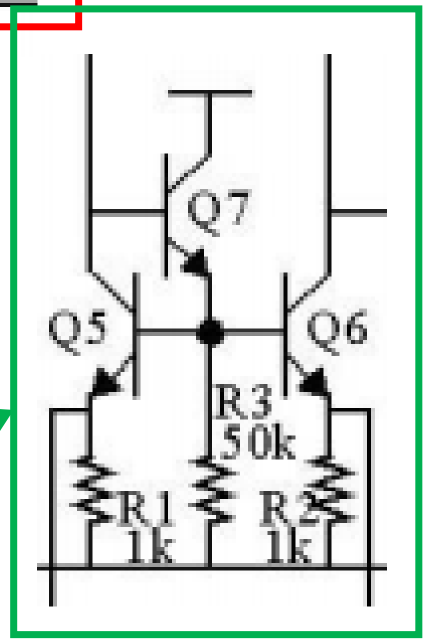
Mirrors in the 741





Widlar
Current source:
Bias current

Beta helper +
Emitter deg:
accurate current
inverter

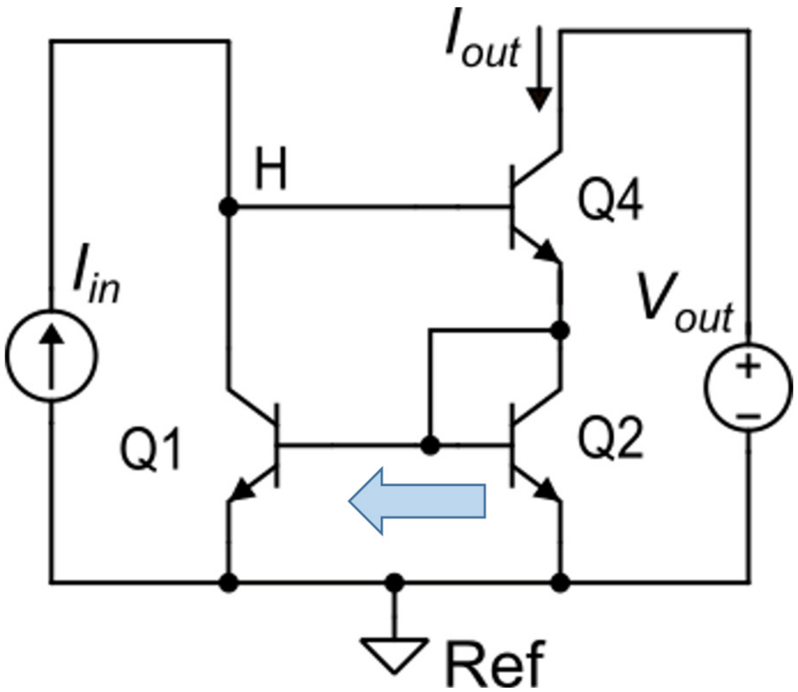


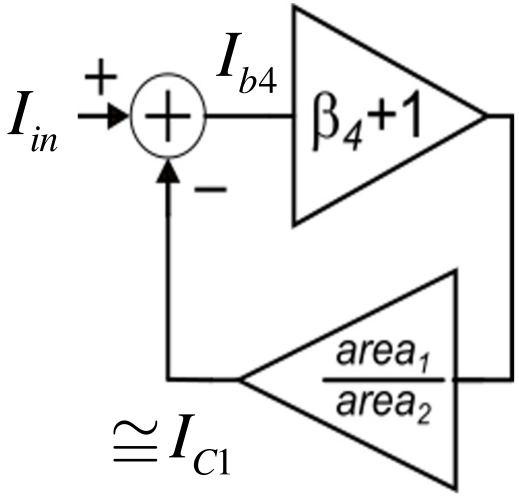
Wilson current mirror

Target: to obtain the same output resistance of a cascode mirror, but with smaller impact of base currents on the current gain accuracy.

Operating principle

Note: the Q2-Q1 mirror transmits the output current (I_{out}) back to the input branch





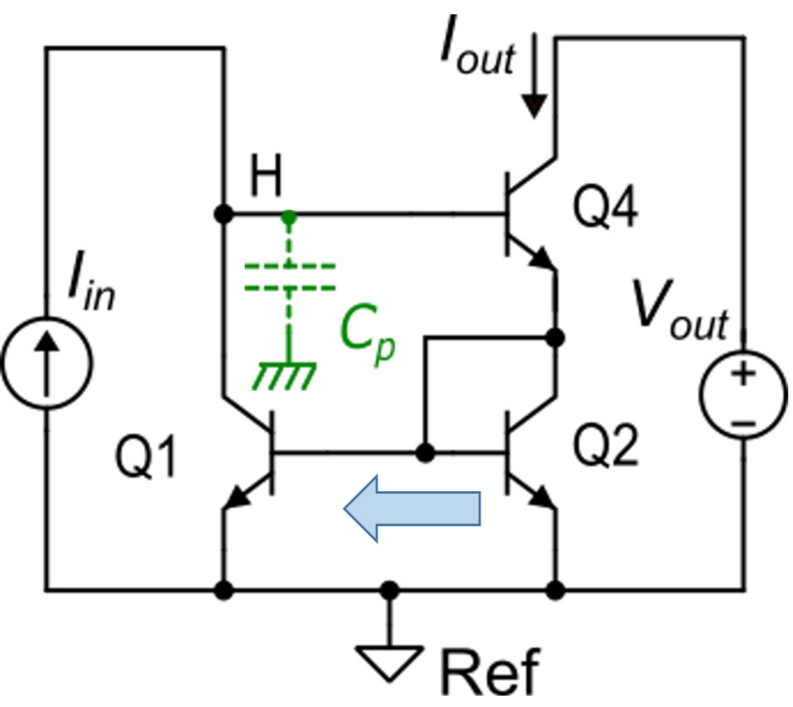
$$I_{E4} \cong I_{out}$$

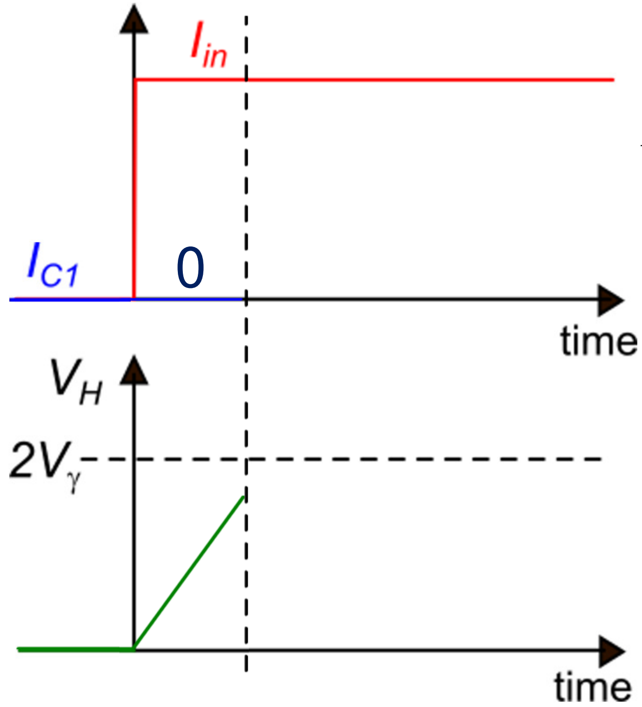
$$I_{out} \cong \left(I_{in} - I_{out} \frac{area_1}{area_2} \right) (\beta_4 + 1)$$

$$I_{out} \cong I_{in} \frac{(\beta_4 + 1)}{\frac{area_1}{area_2}(\beta_4 + 1) + 1} \cong \frac{area_2}{area_1} I_{in}$$

Wilson current mirror: start-up transient

$$I_{Cp} = I_{in} - I_{C1} - I_{B4}$$





until:

$V_H << 2V_\gamma \rightarrow$ Q2, Q4: off

$$I_{B4} = 0, I_{C4} = 0,\ I_{C2} = 0$$

$$I_{Cp} = I_{in} \leftarrow I_{C1} = 0$$

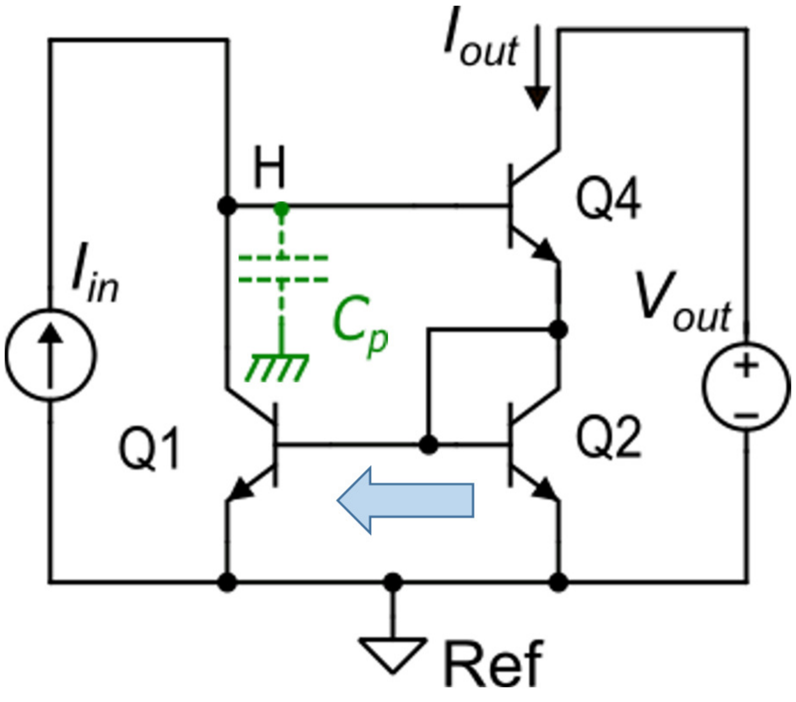
$$\frac{dV_H}{dt} = \frac{I_{Cp}}{C_p} = constant$$

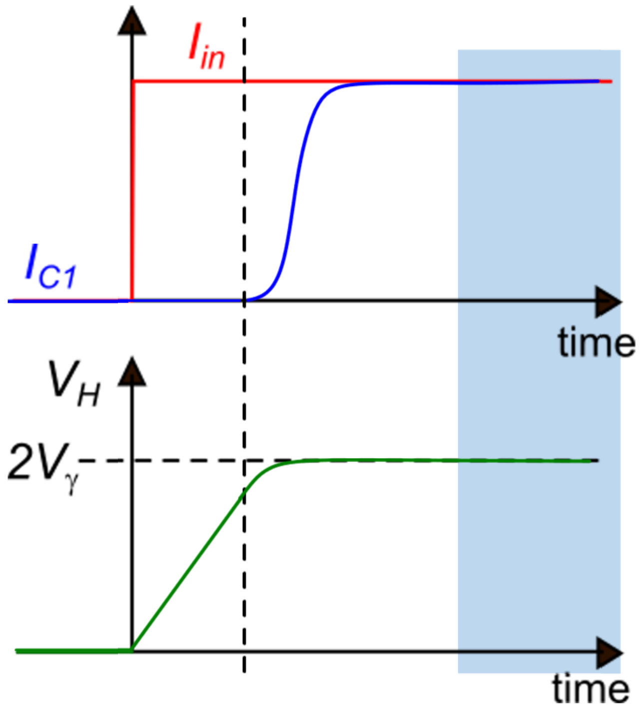
As V_H approaches $2V\gamma$, Q4 and Q2 turn on and the Q2 current is mirrored to M1

Wilson current mirror

I_{C1} starts increasing and taking part of I_{in}, away from C_p ⇨ $I_{Cp} = I_{in} - I_{C1} - I_{B4}$: decreases

$$\frac{dV_H}{dt} = \frac{I_{Cp}}{C_p} \quad \text{: decreases}$$





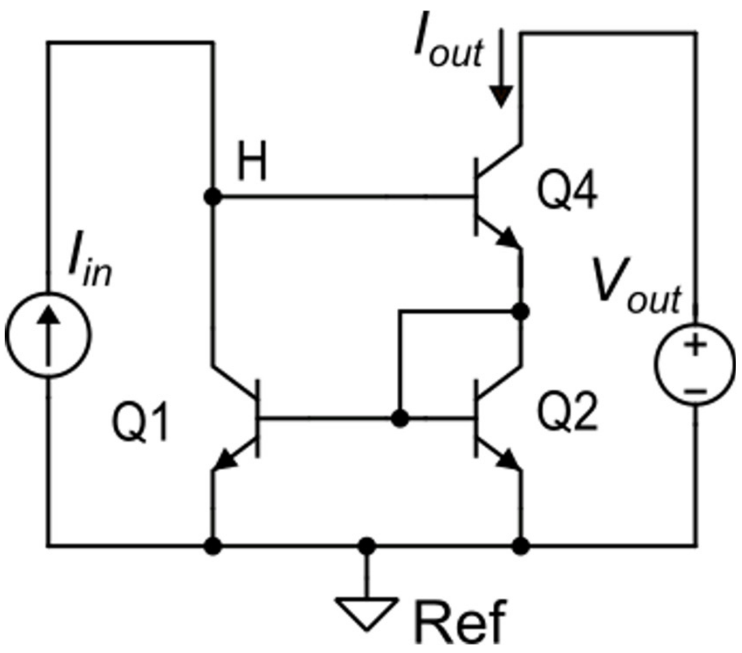
Due to the exponential law, small V_H increments produce large I_{C2} (and then I_{C1}) increases

Equilibrium is reached when:

$$\frac{dV_H}{dt} = 0 \Rightarrow I_{Cp} = 0$$

$$I_{in} = I_{C1} + I_{B4}$$

Wilson Current Mirror: effect of base currents



$$I_{in} = I_{C1} + I_{B4}$$

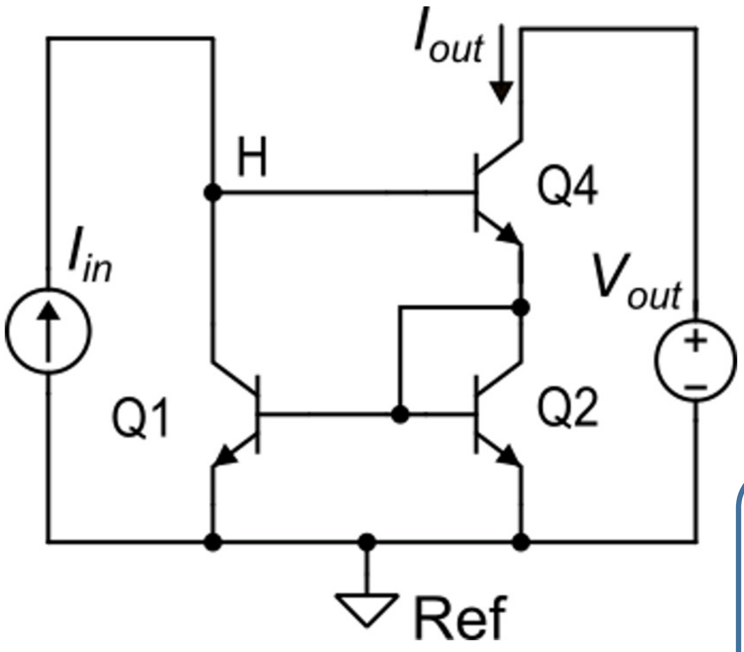
$$I_{out} = I_{C4} = I_{E4} - I_{B4} = \underbrace{I_{C2} + I_{B1} + I_{B2}}_{I_{E4}} - I_{B4}$$

$$\frac{I_{out}}{I_{in}} = \frac{I_{C2} + I_{B1} + I_{B2} - I_{B4}}{I_{C1} + I_{B4}} \cong \frac{I_{C2}}{I_{C1}} \cong \frac{area_2}{area_1} \triangleq k_M$$

$$I_{C2} \cong I_{C4}$$

$$\frac{I_{out}}{I_{in}} = \frac{I_{C2} + \frac{I_{C2}}{k_M \beta_1} + \frac{I_{C2}}{\beta_2} - \frac{I_{C2}}{\beta_4}}{I_{C1} + \frac{k_M I_{C1}}{\beta_4}} = \frac{I_{C2}}{I_{C1}} \frac{1 + \frac{1}{k_M \beta_1} + \frac{1}{\beta_2} - \frac{1}{\beta_4}}{1 + \frac{k_M}{\beta_4}}$$

Wilson current mirror



$$\frac{I_{out}}{I_{in}} = k_M \frac{1+\frac{1}{k_M\beta_1}+\frac{1}{\beta_2}-\frac{1}{\beta_4}}{1+\frac{k_M}{\beta_4}}$$

$$\frac{I_{out}}{I_{in}} \cong k_M\left(1+\frac{1}{k_M\beta_1}-\frac{k_M}{\beta_4}+\frac{1}{\beta_2}-\frac{1}{\beta_4}\right)$$

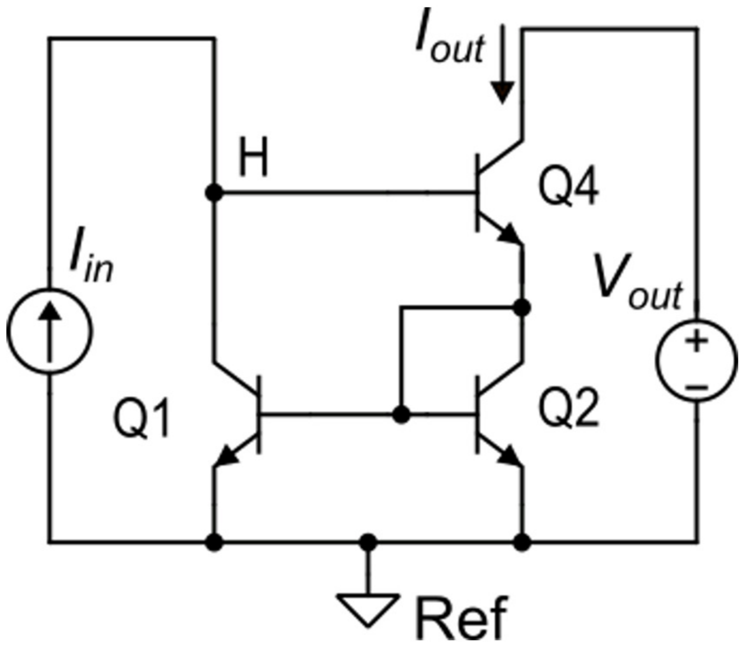
Only if: k_M=1

$$=\frac{1}{\beta_1}-\frac{1}{\beta_4}\approx\frac{1}{\beta^2}$$

$$\frac{1}{\beta_2}-\frac{1}{\beta_4}=\frac{\beta_4-\beta_2}{\beta_2\beta_4}\approx\frac{1}{\beta^2}$$

for: $k_M = 1$ $\qquad \frac{I_{out}}{I_{in}} = k_M\left(1+\varepsilon_R\right), \qquad \varepsilon_R \approx \frac{1}{\beta^2}$

Wilson current mirror



In the Wilson current mirror the relative error due to the base current is of the order of $1/\beta^2$ only when it is designed for $k_M=1$. For current gains different from one, the relative error becomes of the order of $1/\beta$.

Other characteristics:

$$V_{IN} = V_H = V_{BE2} + V_{BE4} \cong 2V_\gamma$$

$$V_{MIN} = V_{BE2} + V_{CESAT4}$$

$$R_{out} \cong r_{o4}\left(1 + \frac{h_{fe4}}{2}\right)$$

due to feedback from I_{out} to node H

(same as the cascode mirror)

$$V_{CE1} = 2V_{BE} > V_{CE2} = V_{BE}$$

There is a large systematic error between I_{C2} and I_{C1}

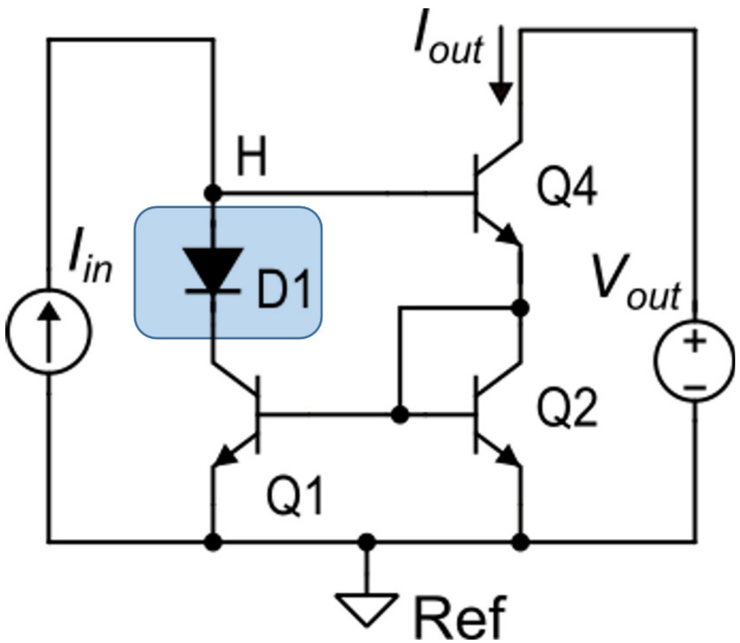
4-transistor Wilson current mirror

$$V_H = V_{BE2} + V_{BE4} = 2V_{BE}$$

$$V_{CE1} = V_H - V_\gamma = V_\gamma$$

$$V_{CE2} = V_\gamma$$

The 4-transistor Wilson Mirror is the optimal choice when an accurate mirror with k_M=1 is required.



Implementing D1 with a diode-connected BJT (Q3)