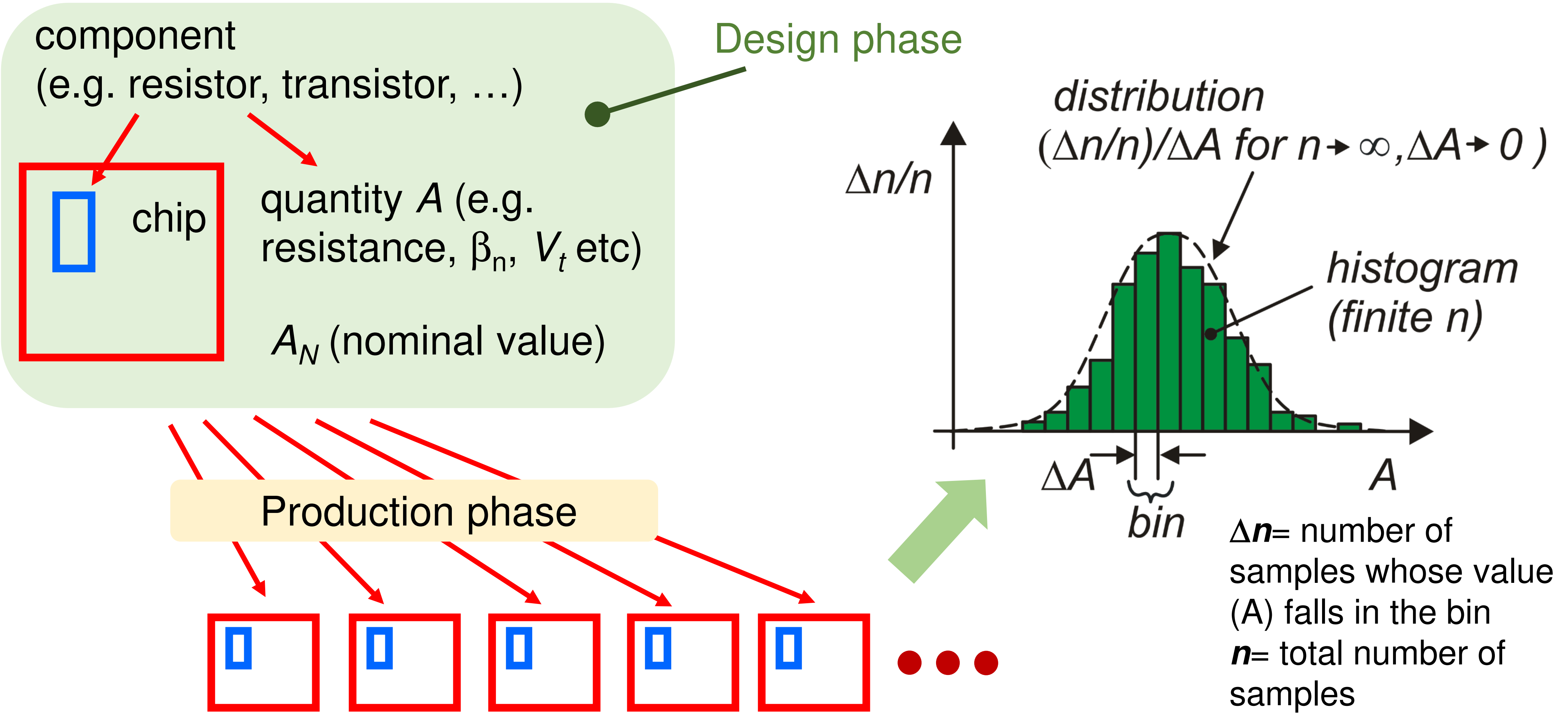
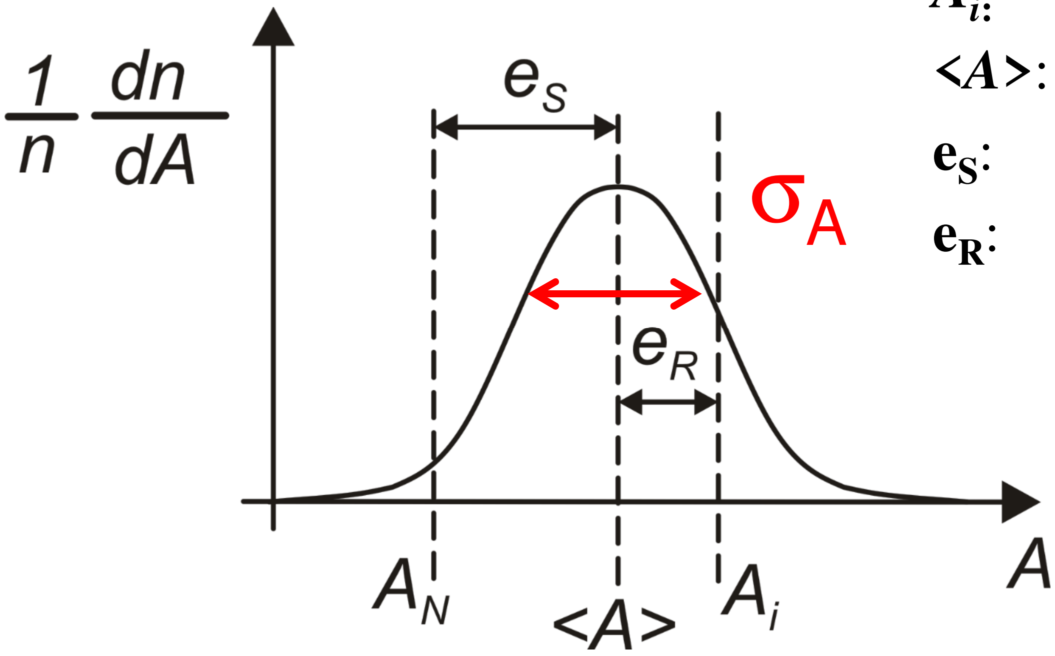
Process Errors in Integrated Circuits



Δn= number of samples whose value (A) falls in the bin
n= total number of samples

Components of the error and statistical representation



A_N: nominal value

A_i: A for a generic i-th component.

$<A>$: the mean of the distribution.

e_S: Systematic error = $<A>-A_N$

e_R: Random error for the i-th component $e_R = A_i - <A>$.

Random error: standard deviation

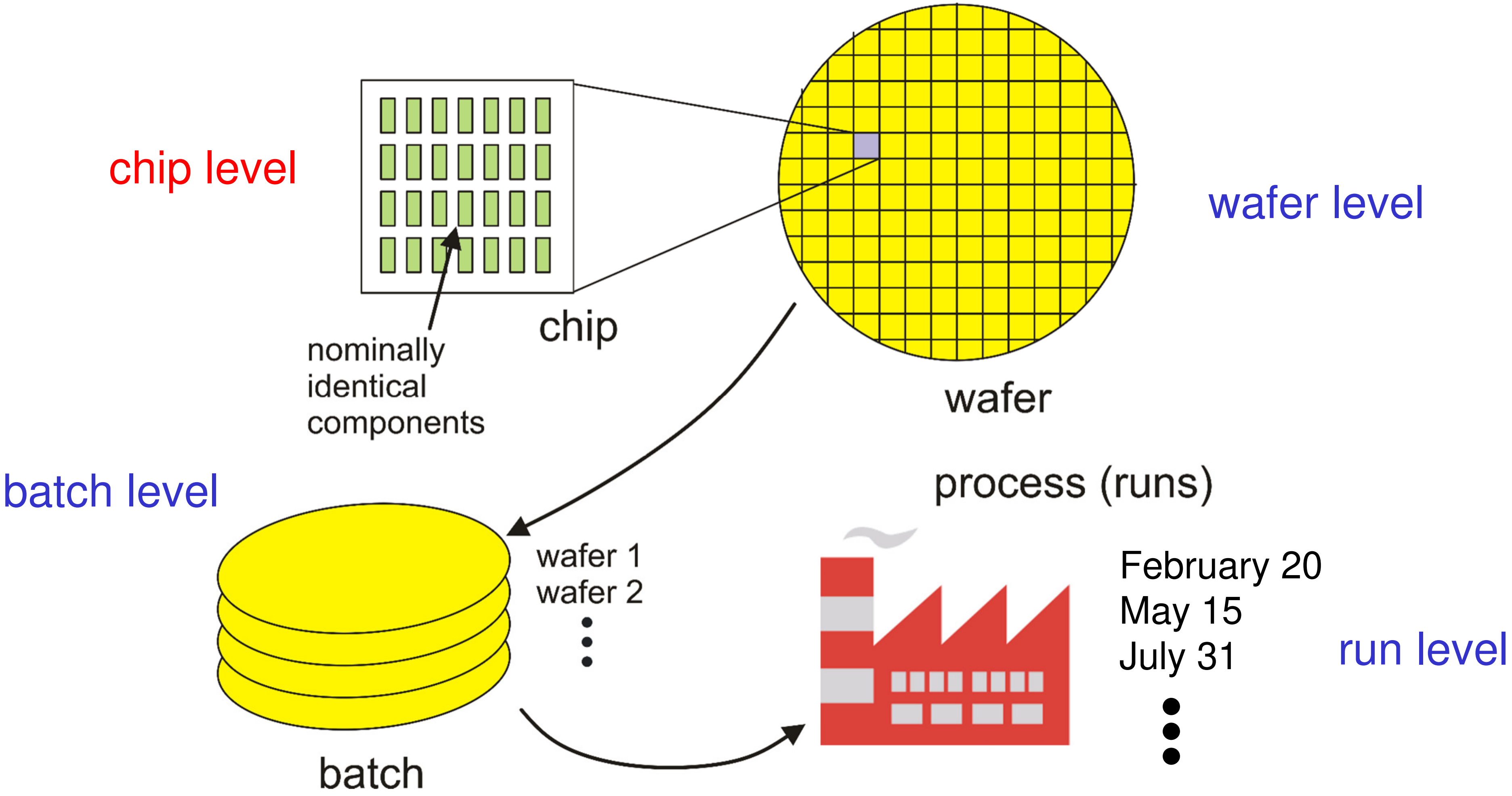
$$\sigma_A = \sqrt{\left\langle \left(A - <A>\right)^2 \right\rangle}$$

Confidence intervals

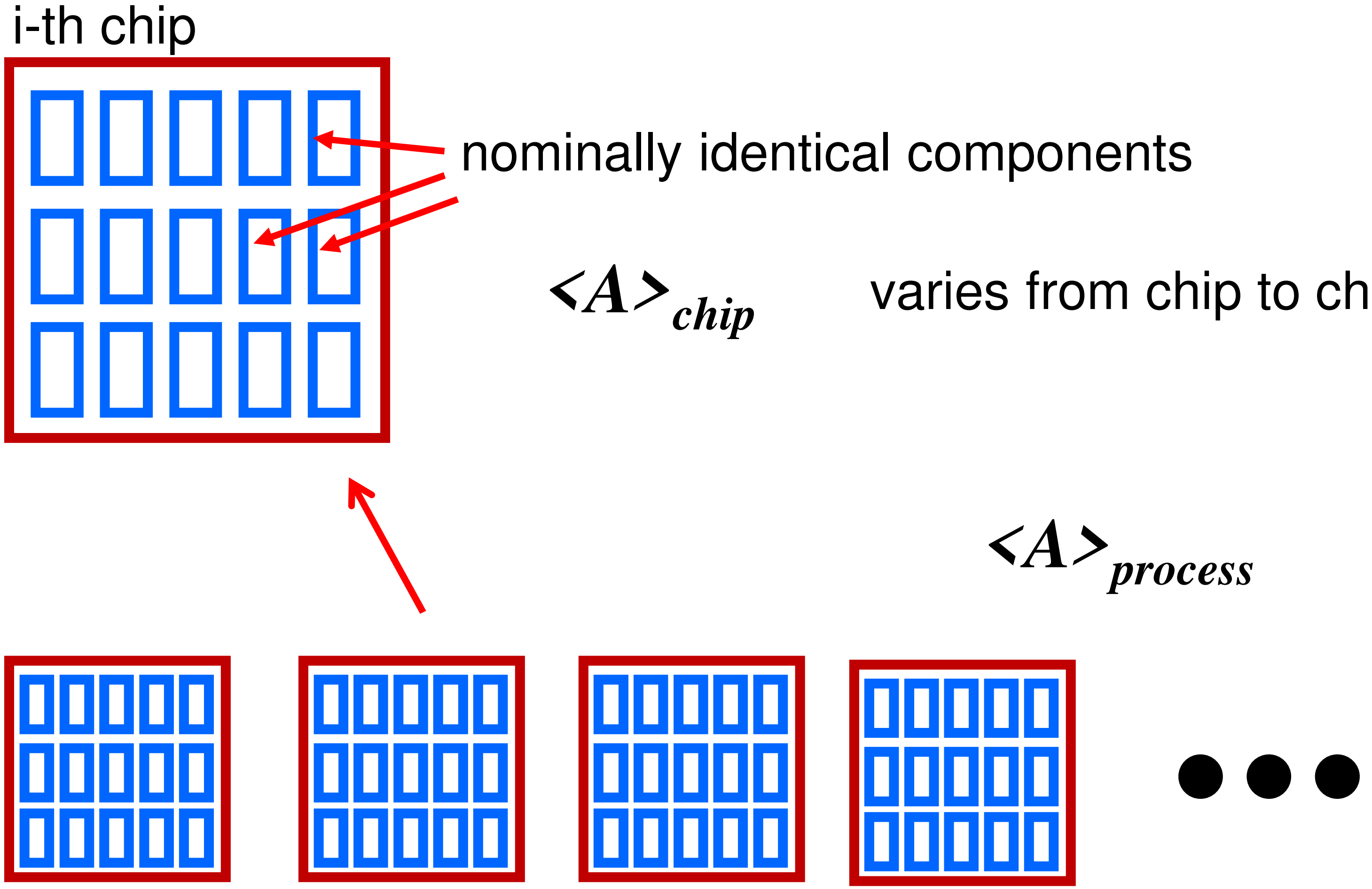
Max deviation from the mean	$\pm\sigma$	$\pm 2\sigma$	$\pm 3\sigma$	$\pm 4\sigma$
Fraction of data within the interval	68.3 %	95.4 %	99.7 %	99.994 %
Fraction of data outside the interval	31.7 %	4.6 %	0.3 %	0.006 %

In 99.7 % of cases, A is in the interval: $[<A> -3\sigma_A \, , <A> +3\sigma_A]$

Errors in Integrated Circuits: Levels



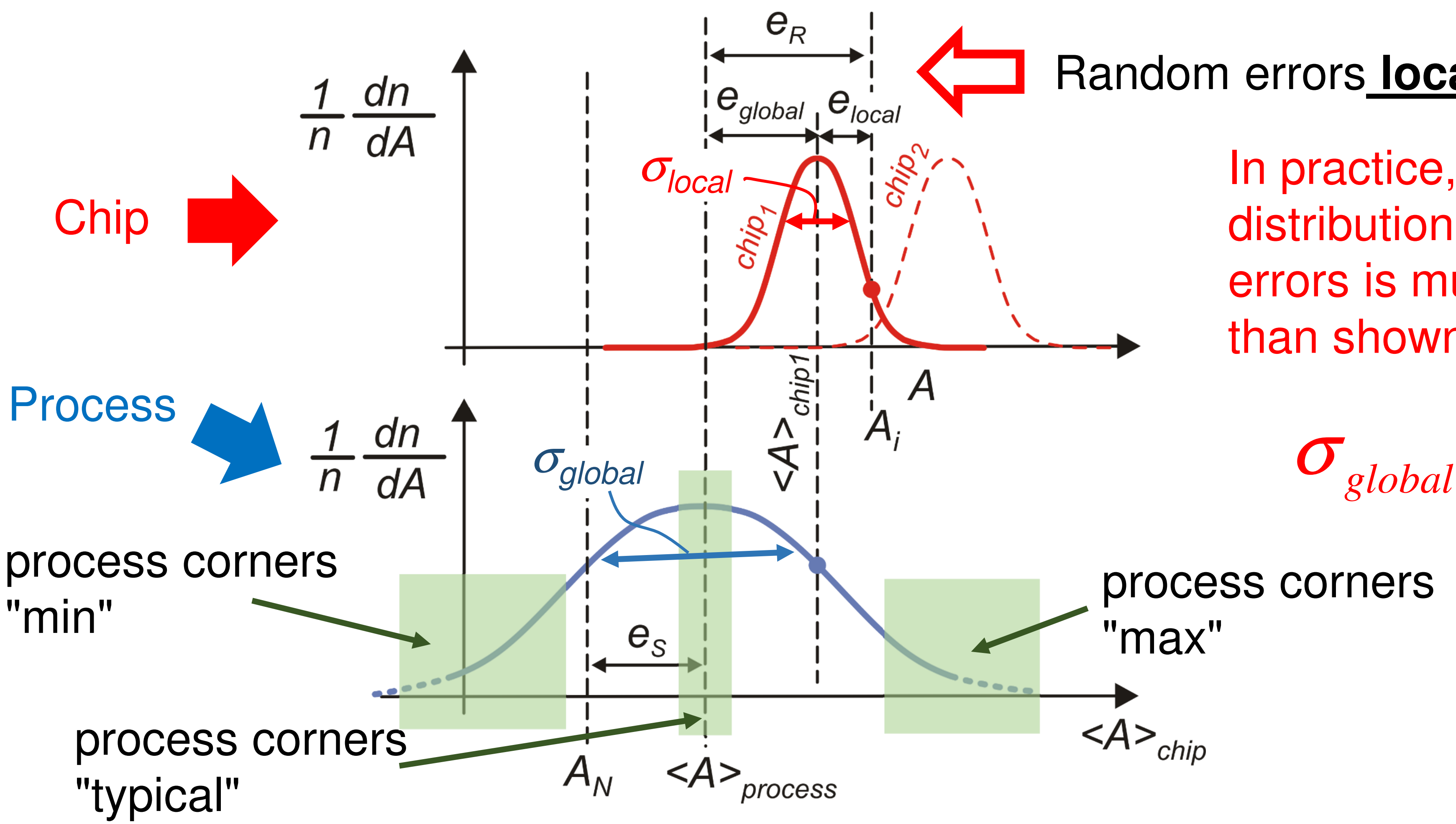
Local and global errors: means



$\langle A \rangle_{chip}$ varies from chip to chip (local mean)

$\langle A \rangle_{process}$ property of the whole process (global mean)

Local and global errors



In practice, the distribution of local errors is much narrower than shown here

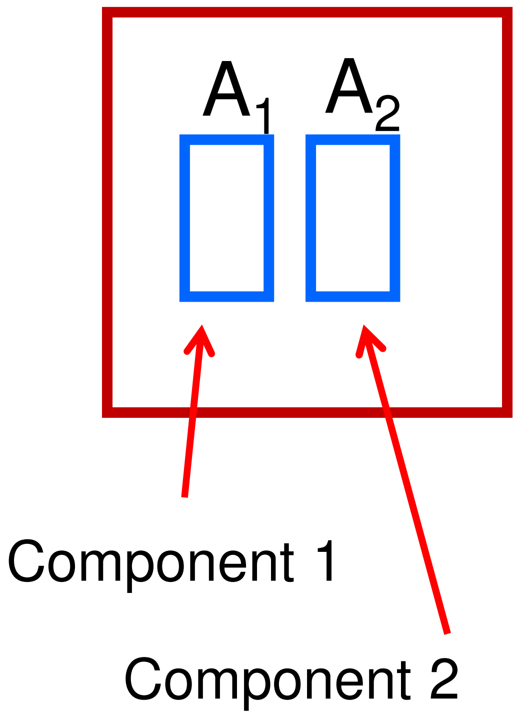
$$\sigma_{global} >> \sigma_{local}$$

Matching errors: definition

Matching error (or mismatch)

$$\begin{cases} \Delta A = A_1 - A_2 \\ \bar{A} = \dfrac{A_1 + A_2}{2} \end{cases} \Leftrightarrow \begin{cases} A_1 = \bar{A} + \dfrac{\Delta A}{2} \\ A_2 = \bar{A} - \dfrac{\Delta A}{2} \end{cases}$$

Mean value



Nominally identical and placed on the same chip

Random Matching errors: Main causes

- Microscopic irregularities (local granularity)
- Parameter gradients

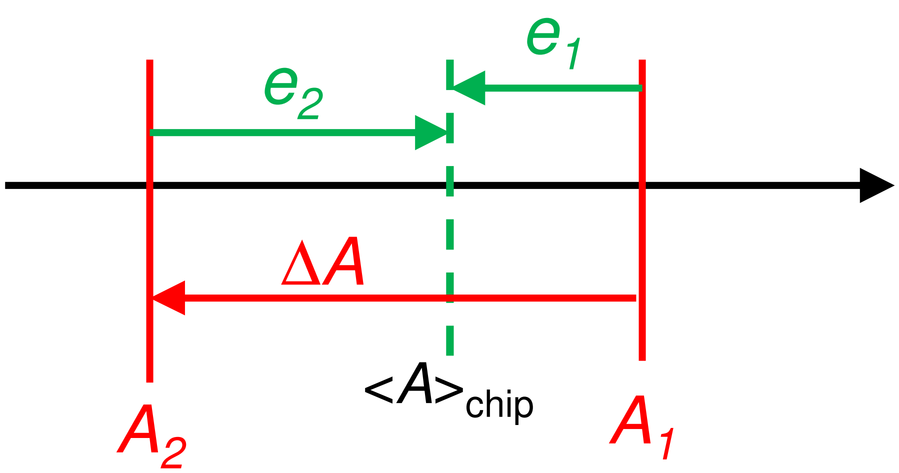
Relationship between matching error and local error

$$A_1 = \langle A \rangle_{chip} + e_1$$

$$A_2 = \langle A \rangle_{chip} + e_2$$

local errors

$$\Delta A = A_1 - A_2 = e_1 - e_2$$



Matching errors: causes

- Matching errors are the consequence of the local errors, which, in turn, derive from non uniformity of physical and chemical parameters over the chip area.
- Matching errors may have a systematic component. This means that one of the two components will be, on average, greater or lesser than the other. A systematic matching error is generally the consequence of a design error.

From here on, we will consider only random matching errors. The causes of random matching error are mainly of two types:

- Microscopic irregularities (local granularity)
- Parameter gradients

Matching errors: Microscopic irregularities

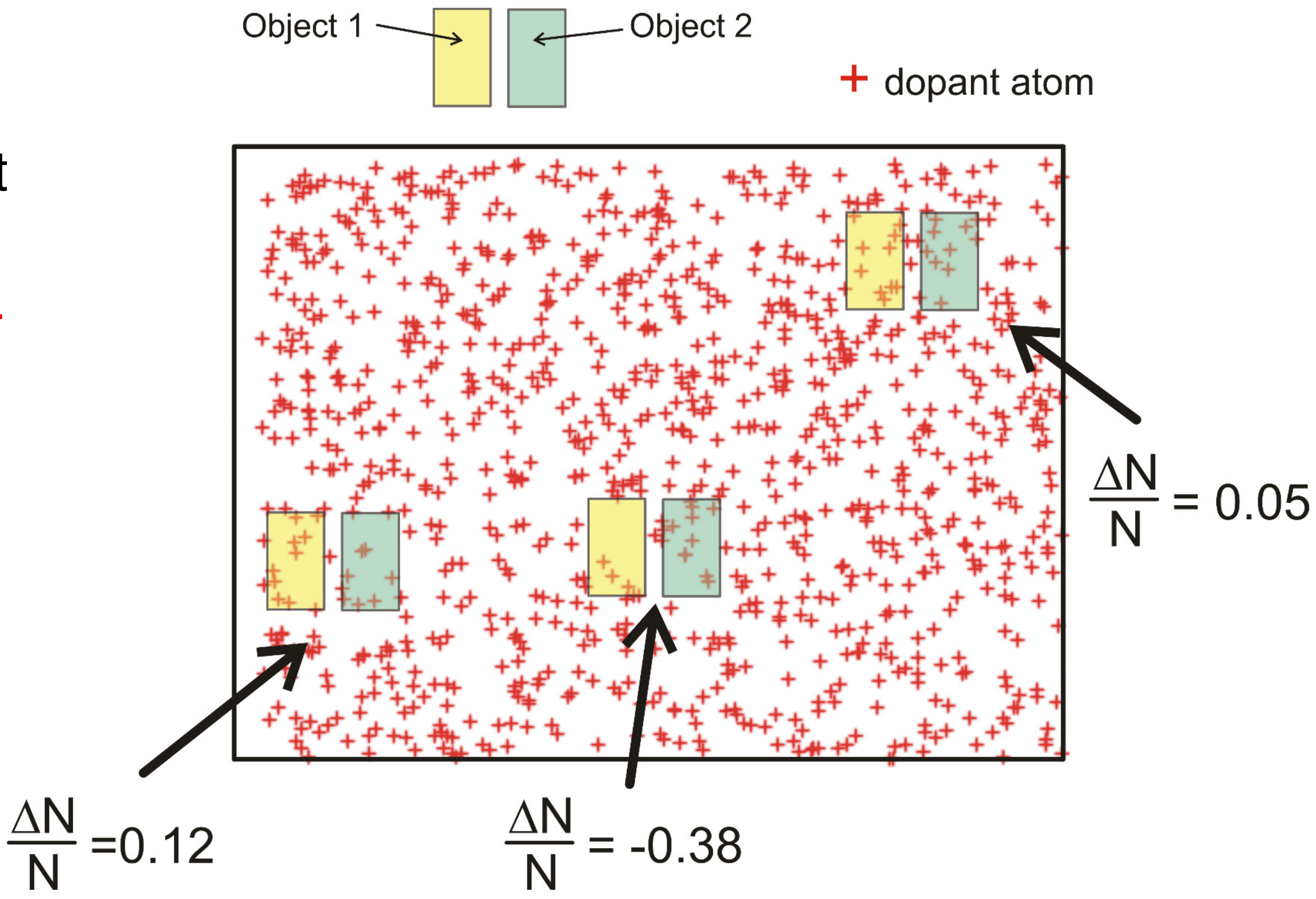
Simple case: A depends on the number of dopants atoms within the component area (N)

$\Delta N = N_1 - N_2$ matching error

Depending on the position ΔN is subjected to large variations.

The important parameter is:
Relative matching error:

$$\frac{\Delta N}{N} \leftarrow \frac{N_1 + N_2}{2}$$

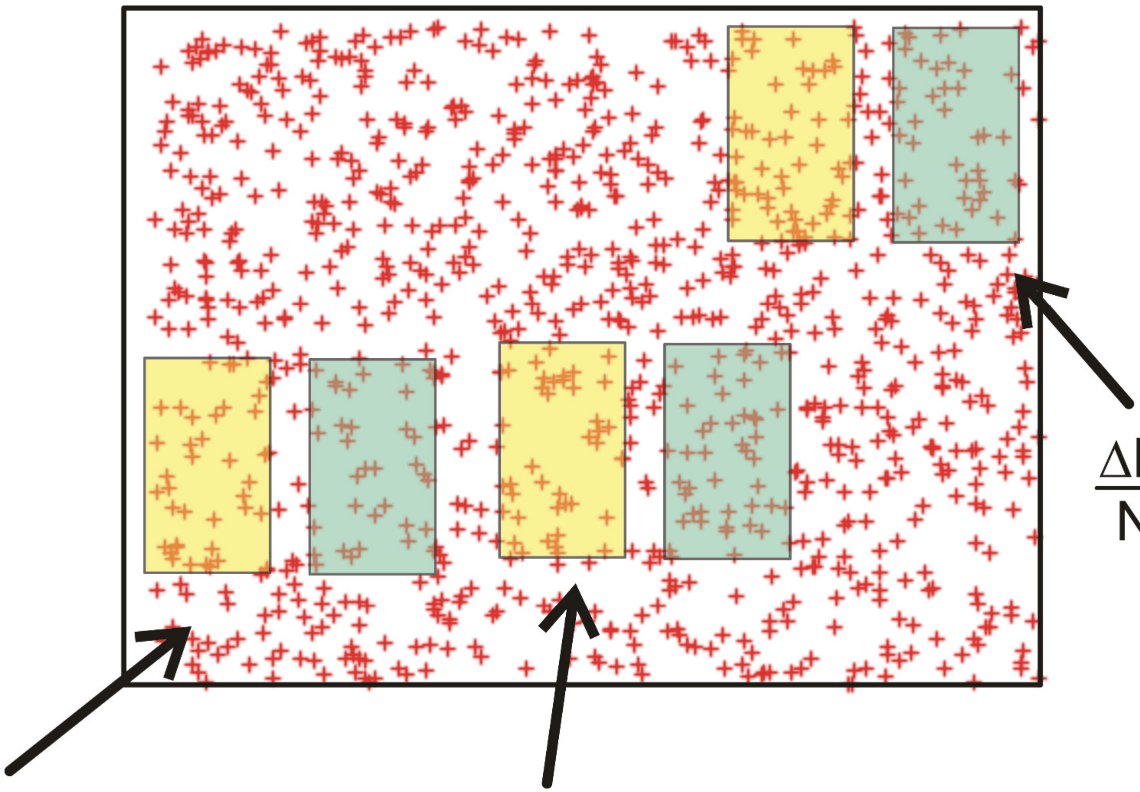


Matching errors: Microscopic irregularities

- The number of dopants atoms affects several properties such as effective sheet resistance and MOSFET threshold voltage
- The example can be replicated for other quantities, such as oxide thickness, where the crosses in the figure may represent local maxima or minima
- The large fluctuation of $\Delta N/N$ can be ascribed to the small area of the devices shown in the example. For even smaller devices its is likely that one of the two devices does not include any dopant atom : $\Delta N/N$ may exceed unity (100 % error).

Microscopic irregularities: effect of device area

Larger device areas result in smaller **relative** matching errors



$$\frac{\Delta N}{N} = 0.01$$

$$\frac{\Delta N}{N} = 0.04$$

$$\frac{\Delta N}{N} = -0.05$$

Microscopic irregularities: the Pelgrom model

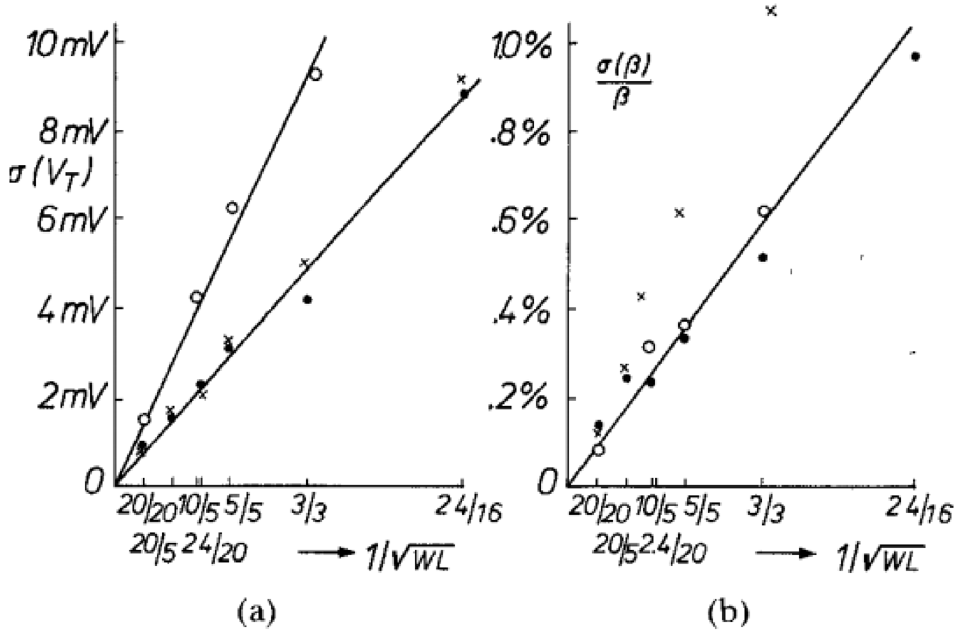
$$\begin{cases} \sigma_{\Delta V_t} = \dfrac{C_{Vt}}{\sqrt{WL}} \\ \sigma_{\frac{\Delta\beta}{\beta}} = \dfrac{C_{\beta}}{\sqrt{WL}} \end{cases}$$

Mosfet

$$\sigma_{\frac{\Delta R}{R}} = \frac{C_R}{\sqrt{WL}}$$

Resistor

C_{vt}, C_{β} and C_R are constant parameters of the process Their values are given in the Design Rule Manual, with names that depend on the foundry (there is no general convention).
C_{vt} units are generally V·µm, while C_{β} and C_R ones are µm.



From: Pelgrom et a IEEE J. of Solid State Circuits, 1989l

Matching Errors: Gradients

- Gradients indicate that important quantities that affect the device properties are not uniformly distributed on a macroscopic scale. This means that these quantities gradually varies across the chip area.

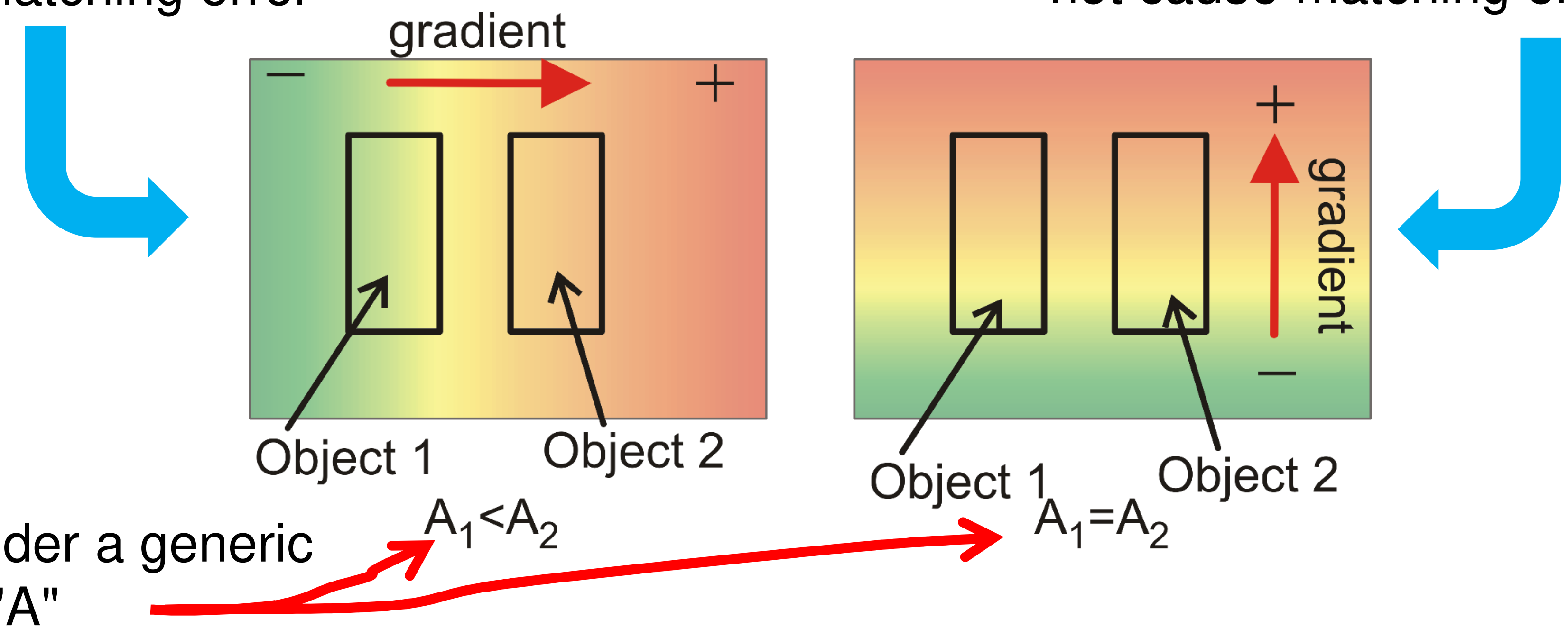
Quantities of interest can be, for example:

- ❑ Dopant density
- ❑ Oxide thickness
- ❑ Geometrical process biases (e.g. etching undercut)
- ❑ Temperature (e.g. due to power devices present on the chip)
- ❑ Mechanical stress (mainly due to the packaging process

Effect of gradients on device matching

In this example radient causes matching error

For this case, gradient does not cause matching error



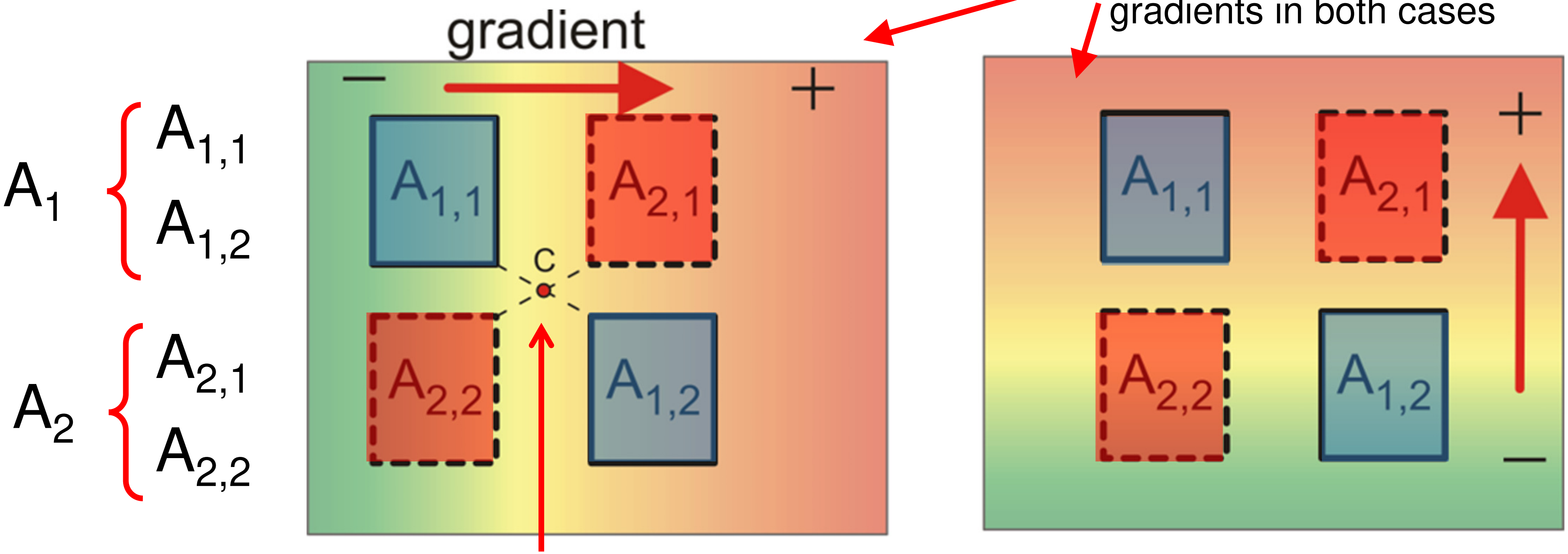
We consider a generic quantity "A"

Unfortunately, in most cases, the direction of gradients is not predictable!

Layout rules that prevent device mismatch caused by gradients

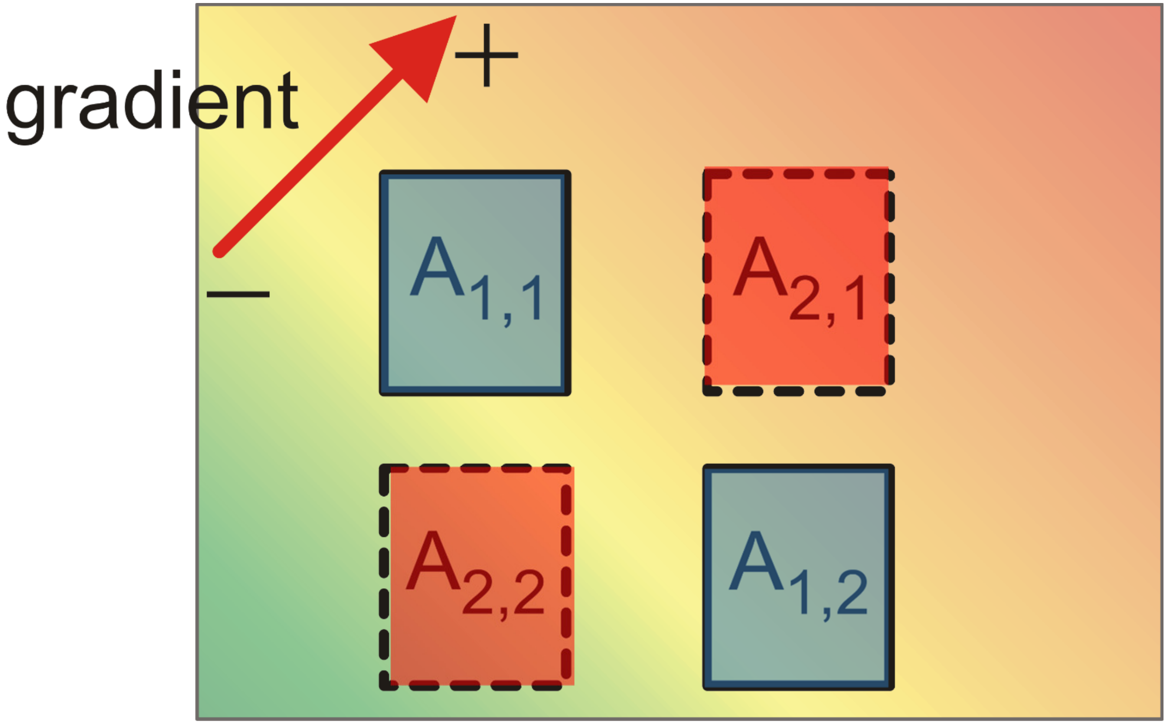
- Rule 1 (obvious): Take the distance between objects as close as possible. (This rule is less effective for large devices)
- Rule 2: Use common centroid configurations.

Object 1 and 2 are placed symmetrically with respect to the gradients in both cases



Centroid of both A_1 and A_2 objects

Common centroid configuration: Oblique gradients



component	part	level	
A_1	$A_{1,1}$	average	average
	$A_{1,2}$	average	
A_2	$A_{2,1}$	high	average
	$A_{2,2}$	low	

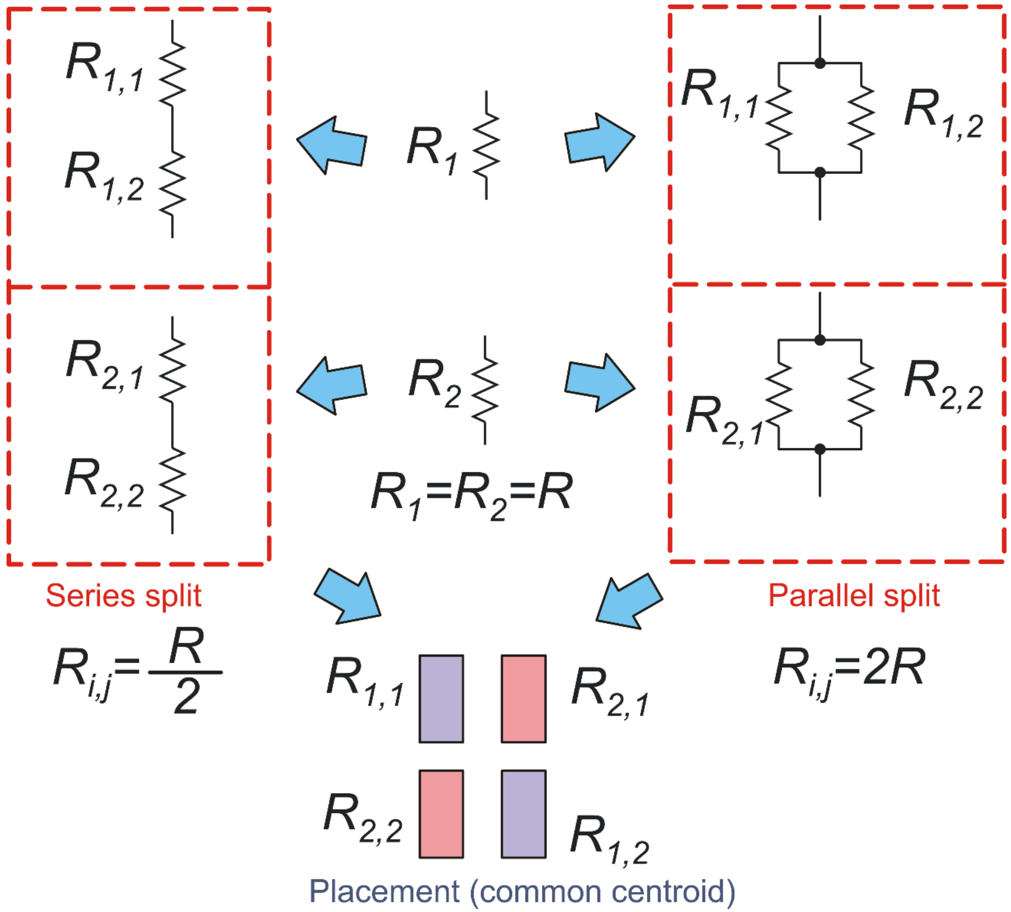
The common centroid configuration is effective also against oblique gradients..

Split and connect components in common centroid configurations

Case 1::Resistors

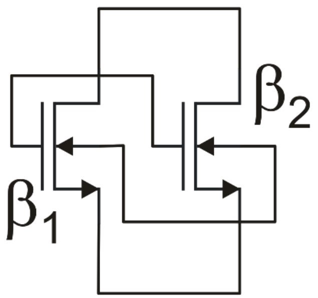
Series are preferred when resistors R_1 and R_2 are large

Parallels have to be preferred for small resistances

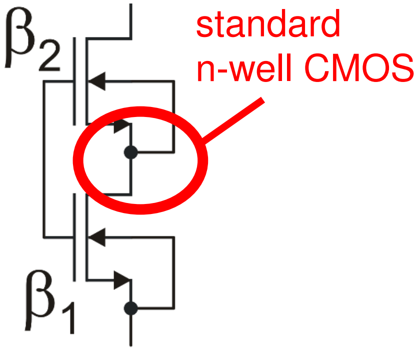


Split and connect components in common centroid configurations

Other devices -Properties of series and parallels

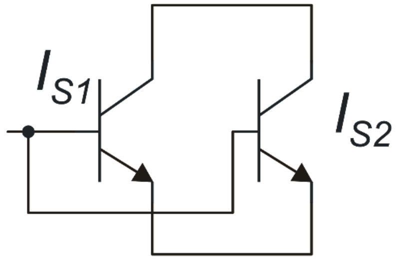


$\beta_{eq}=\beta_1+\beta_2$

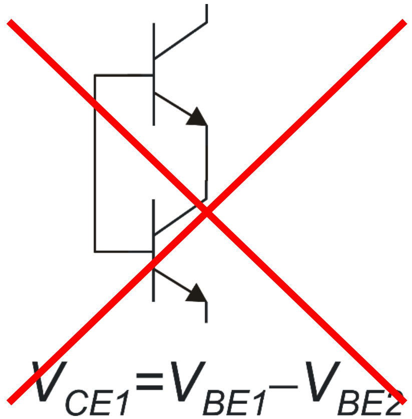


$$\frac{1}{\beta_{eq}} \cong \frac{1}{\beta_1}+\frac{1}{\beta_2}$$

Not accurate



$I_{Seq}=I_{S1}+I_{S2}$



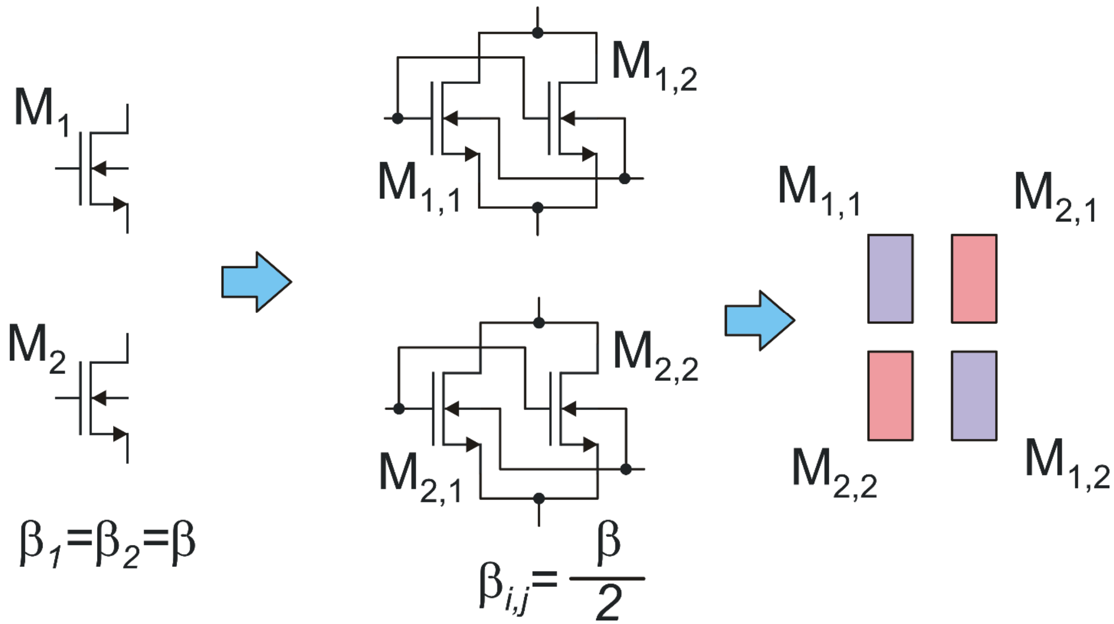
$V_{CE1}=V_{BE1}-V_{BE2} \cong 0$

Don't use!

Accurate: Can be used in common centroid configurations

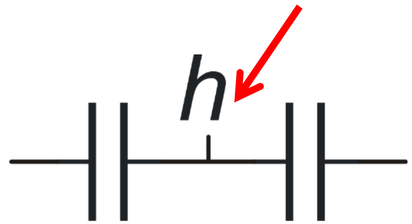
Split and connect components in common centroid configurations

Case 2: MOSFETS



Case 3: CAPACITORS

Floating node (no dc path)



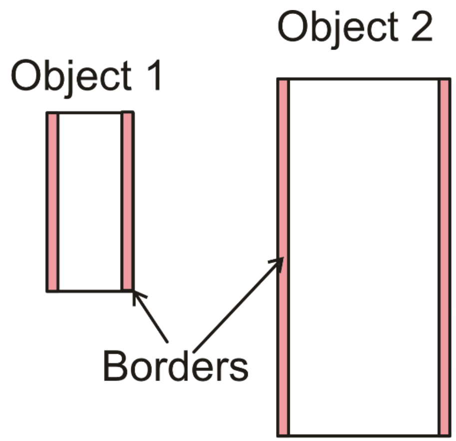
Series connections of capacitors have to be avoided as much as possible unless dc paths are provided for the floating node

BJTs: Same as Mosfets (parallels only)

Capacitors: Parallel connections are preferred (no floating nodes)

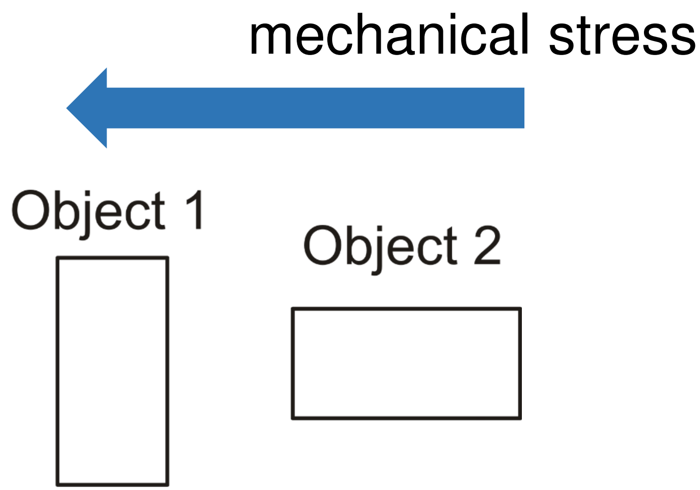
Other Layout rules for improving device matching

Wrong solutions



$$\frac{W_1}{L_1} = \frac{W_2}{L_2} \not\Rightarrow \begin{array}{ll} R_1 = R_2 & \text{resistors} \\ \beta_1 = \beta_2 & \text{mosfets} \end{array}$$

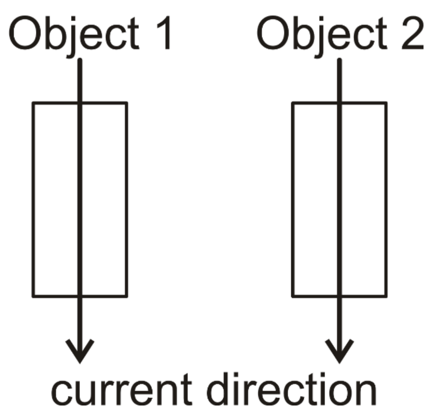
Different size: poor matching



Different orientation: poor matching

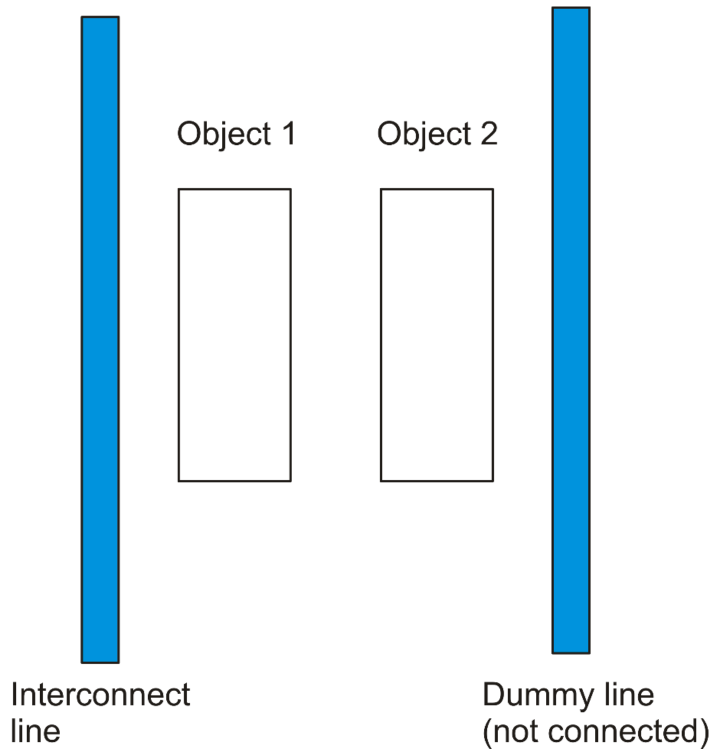
Other Layout rules for improving device matching

Same direction
(to match thermoelectric effects)



Temperature gradients develop extra voltage differences that depend on the current direction (up to several hundred μVs

Same surroundings

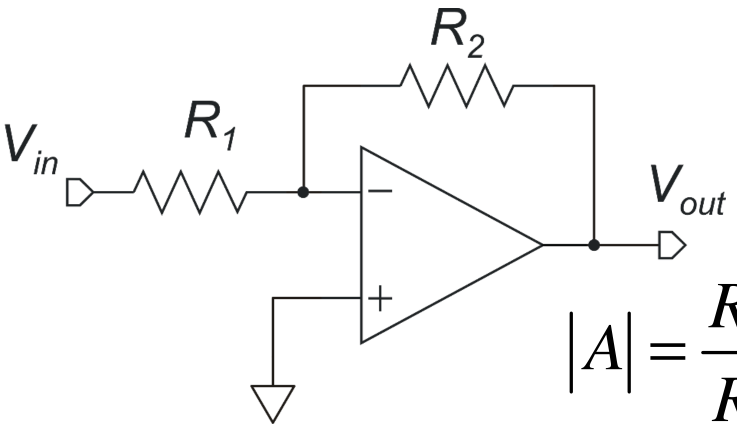


Summary of rules for a good device matching

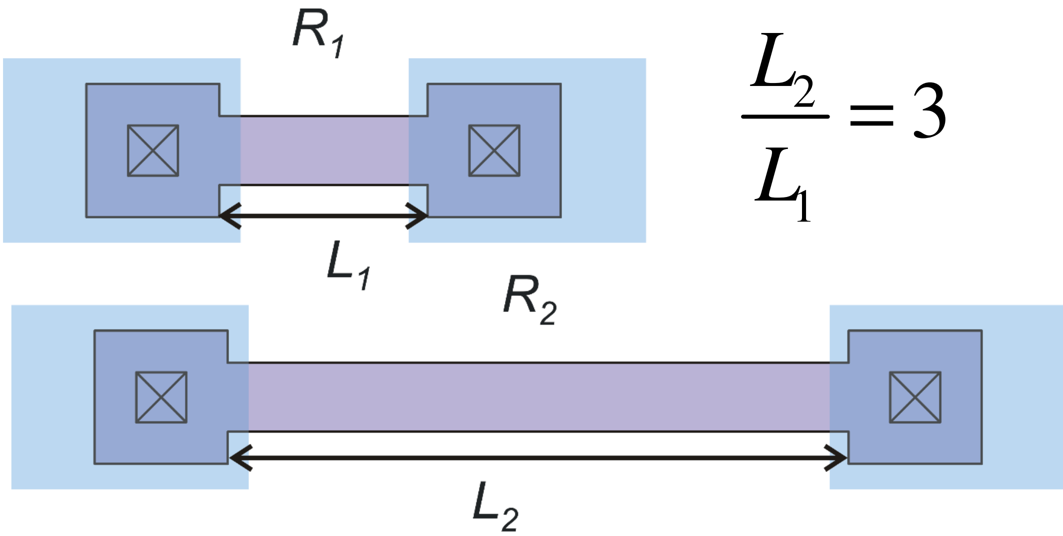
- Devices must be nominally identical (same dimensions, same orientation)
- Device areas should be as large as possible (Pelgrom model)
- Place devices as close as possible
- Use common centroid configurations
- Same current direction for the two devices
- The devices should "see" the same surroundings

Rules to obtain accurate ratios

Example: accurate inverting amplifier with gain magnitude =**3**

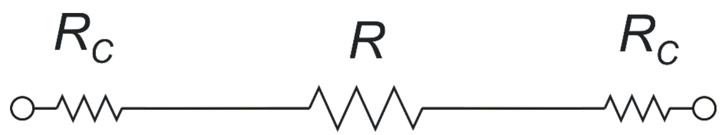


$$|A| = \frac{R_2}{R_1} = 3$$



$$\frac{L_2}{L_1} = 3$$

$$|A| = \frac{3R + 2R_C}{R + 2R_C} = 3\left[\frac{1 + (2/3)x}{1 + 2x}\right] < 3$$

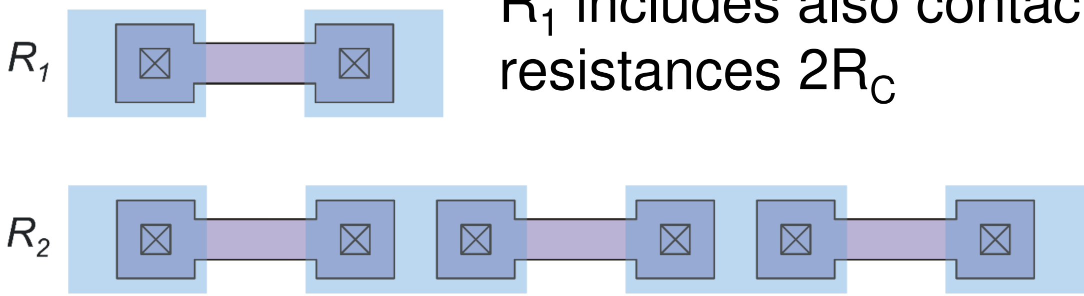


$$x = \frac{R_C}{R}$$

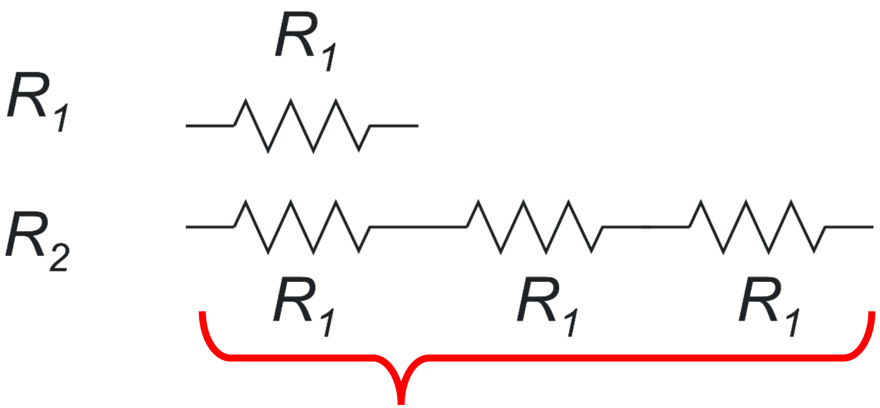
(e.g. with x=0.1, er=**11%**)

(systematic error)

Accurate ratios: modular components

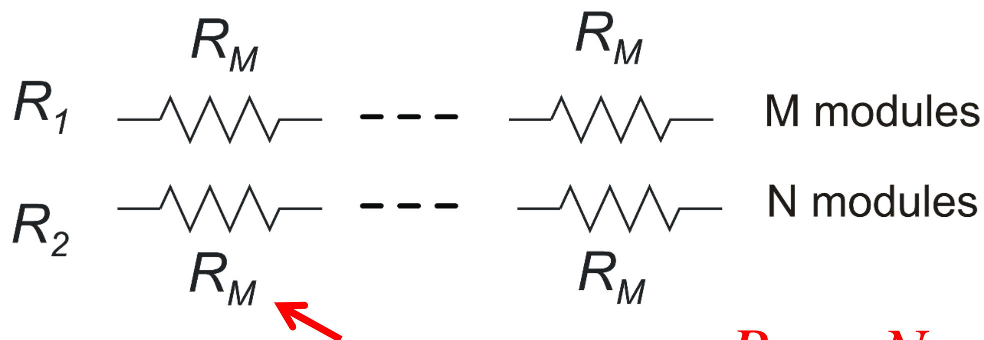


R_1 includes also contact resistances $2R_C$



R_2 is now exactly 3 R_1, independently from contact resistances and other non idealities of R_1

Generalization to Fractional ratios



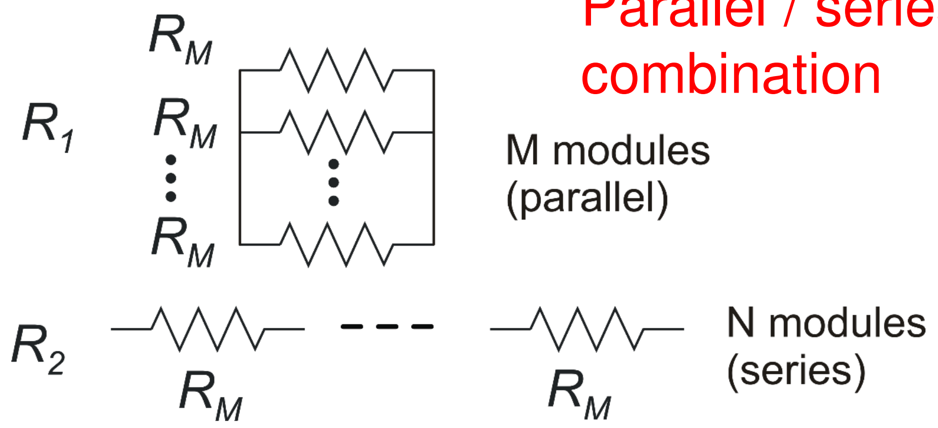
module

$$\frac{R_2}{R_1} = \frac{N}{M}$$

Both R_1 and R_2 are obtained by adding different numbers of a single module R_N.

Accurate ratios: modular components

Parallel / series combination

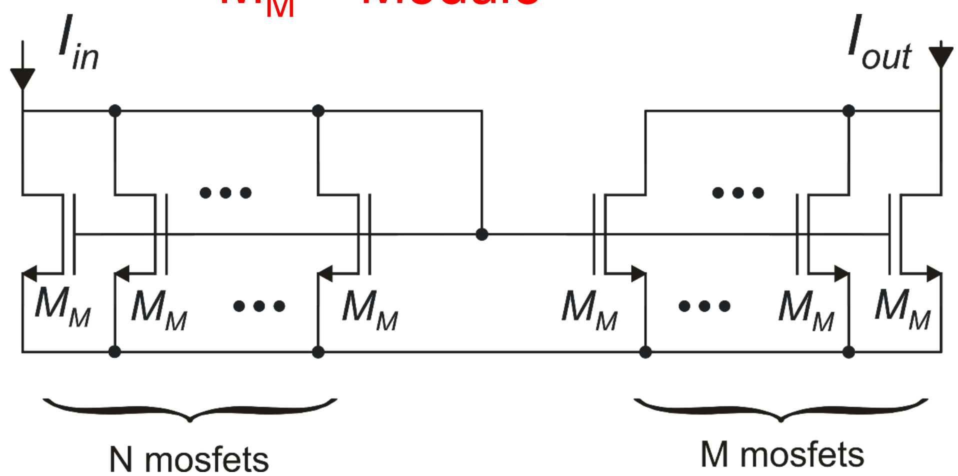


$$\frac{R_2}{R_1} = \frac{NR_M}{\frac{1}{M}R_M} = N \cdot M$$

Large ratios with less components

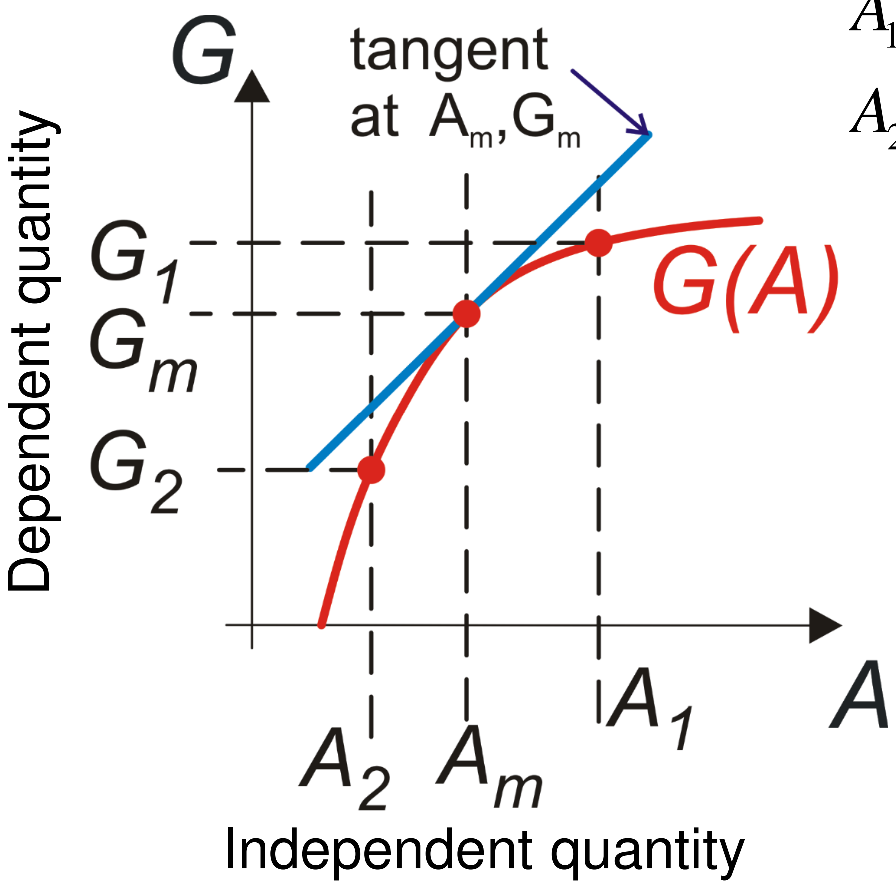
Current mirror with accurate gain

M_M = Module



$$\frac{I_{out}}{I_{in}} = \frac{\beta_2}{\beta_1} = \frac{M\,\beta_M}{N\,\beta_M} = \frac{M}{N}$$

Elements of error propagation theory



$$A_1 = A_m + \Delta A_1$$

$$A_2 = A_m + \Delta A_2$$

$$\Delta A = A_1 - A_2 = \Delta A_1 - \Delta A_2$$

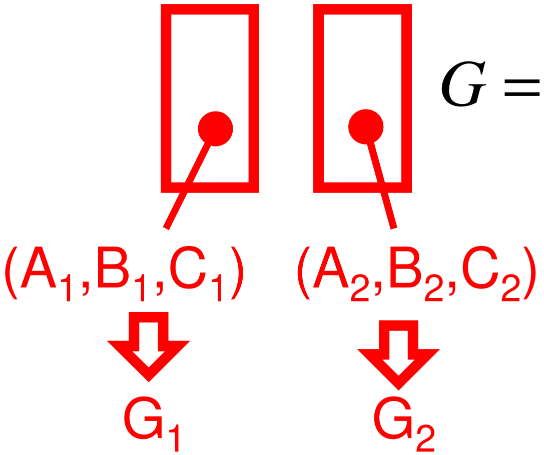
A_m is an arbitrary value between A_1 and A_2

$$G_1 = G\left(A_m + \Delta A_1\right) \cong G\left(A_m\right) + \Delta A_1 \left.\frac{dG}{dA}\right|_{A=A_m}$$

$$G_2 = G\left(A_m + \Delta A_2\right) \cong G\left(A_m\right) + \Delta A_2 \left.\frac{dG}{dA}\right|_{A=A_m}$$

$$G_1 - G_2 \cong \left(\Delta A_1 - \Delta A_2\right) \left.\frac{dG}{dA}\right|_{A=A_m} = \Delta A \left.\frac{dG}{dA}\right|_{A=A_m}$$

For multiple independent variables - general case



$$G = G(A, B, C) \qquad \Delta G = G_1 - G_2 = G(A_1, B_1, C_1) - G(A_2, B_2, C_2)$$

$$P_m = (A_m, B_m, C_m) \quad \begin{cases} G_1 = G(A_m + \Delta A_1, B_m + \Delta B_1, C_m + \Delta C_1) \\ G_2 = G(A_m + \Delta A_2, B_m + \Delta B_2, C_m + \Delta C_2) \end{cases}$$

$$\begin{cases} G_1 = G(A_m, B_m, C_m) + \Delta A_1 \left.\dfrac{\partial G}{\partial A}\right|_{P_m} + \Delta B_1 \left.\dfrac{\partial G}{\partial B}\right|_{P_m} + \Delta C_1 \left.\dfrac{\partial G}{\partial C}\right|_{P_m} \\ G_2 = G(A_m, B_m, C_m) + \Delta A_2 \left.\dfrac{\partial G}{\partial A}\right|_{P_m} + \Delta B_2 \left.\dfrac{\partial G}{\partial B}\right|_{P_m} + \Delta C_2 \left.\dfrac{\partial G}{\partial C}\right|_{P_m} \end{cases}$$

$$\Delta G = G_1 - G_2 = \Delta A \left.\frac{\partial G}{\partial A}\right|_{P_m} + \Delta B \left.\frac{\partial G}{\partial B}\right|_{P_m} + \Delta C \left.\frac{\partial G}{\partial C}\right|_{P_m}$$

Error propagation: particular case 1

case 1:
posynomial expression

$$G(A,B,C) = A^{\alpha} B^{\beta} C^{\gamma}$$

α,β,γ : real exponents
A,B,C real positive variables

$$\left.\frac{\partial G}{\partial A}\right|_{P_m} = \alpha A_m^{\alpha-1} B_m^{\beta} C_m^{\gamma}$$

$$\left.\frac{\partial G}{\partial B}\right|_{P_m} = \beta A_m^{\alpha} B_m^{\beta-1} C_m^{\gamma}$$

$$\left.\frac{\partial G}{\partial C}\right|_{P_m} = \gamma A_m^{\alpha} B_m^{\beta} C_m^{\gamma-1}$$

Using: $\Delta G = \Delta A \left.\frac{\partial G}{\partial A}\right|_{P_m} + \Delta B \left.\frac{\partial G}{\partial B}\right|_{P_m} + \Delta C \left.\frac{\partial G}{\partial C}\right|_{P_m}$

$$\Delta G = \alpha A_m^{\alpha-1} B_m^{\beta} C_m^{\gamma} \Delta A + \beta A_m^{\alpha} B_m^{\beta-1} C_m^{\gamma} \Delta B + \gamma A_m^{\alpha} B_m^{\beta} C_m^{\gamma-1} \Delta C$$

Error propagation: particular case 1

case 1

$$G(A,B,C) = A^{\alpha} B^{\beta} C^{\gamma}$$

Relative variation (or relative error)

$$\frac{\Delta G}{G_m} = \frac{\Delta G}{G\left(P_m\right)} = \frac{\Delta G}{A_m^{\alpha} B_m^{\beta} C_m^{\gamma}}$$

$$\frac{\Delta G}{G_m} = \frac{\alpha A_m^{\alpha-1} B_m^{\beta} C_m^{\gamma} \Delta A + \beta A_m^{\alpha} B_m^{\beta-1} C_m^{\gamma} \Delta B + \gamma A_m^{\alpha} B_m^{\beta} C_m^{\gamma-1} \Delta C}{A_m^{\alpha} B_m^{\beta} C_m^{\gamma}}$$

$$\frac{\Delta G}{G_m} = \alpha \frac{\Delta A}{A_m} + \beta \frac{\Delta B}{B_m} + \gamma \frac{\Delta C}{C_m}$$

Sum of the single relative errors, weighted by the respective exponents

Examples

Power in a resistor $P = \frac{V^2}{R} = V^2 R^{-1}$ ⇨ $\frac{\Delta P}{P_m} = 2\frac{\Delta V}{V_m} - \frac{\Delta R}{R_m}$

If the relative difference in voltage is 1 %, the relative difference in power is 2 %. If the resistance changes by 1 %, we have a power reduction of 1 %.

Resistance of an integrated resistor

$R = R_S \frac{L}{W}$ ⇨ $\frac{\Delta R}{R_m} = \frac{\Delta R_S}{R_{Sm}} + \frac{\Delta L}{L_m} - \frac{\Delta W}{W_m}$

Error propagation: particular case 2

$$G(A,B,C) = \ln\left(A^{\alpha} B^{\beta} C^{\gamma}\right)$$

case 2: logarithm of a posinomial

$$Z = \left(A^{\alpha} B^{\beta} C^{\gamma}\right)$$

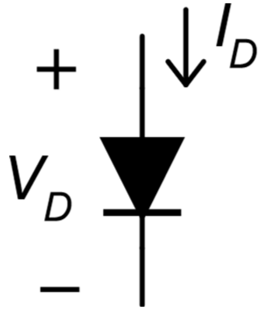
$$Z_m = \left(A_m^{\alpha} B_m^{\beta} C_m^{\gamma}\right)$$

$$\Delta G = \Delta Z \left.\frac{dG}{dZ}\right|_{Z=Z_m} = \Delta Z \left.\frac{d\left[\ln(Z)\right]}{dZ}\right|_{Z=Z_m} = \frac{\Delta Z}{Z_m}$$

$$\Delta G = \frac{\Delta Z}{Z} = \alpha \frac{\Delta A}{A_m} + \beta \frac{\Delta B}{B_m} + \gamma \frac{\Delta C}{C_m}$$

Here we have the absolute error of G, not the relative one.

Example



$$V_D = V_T \ln\left(\frac{I_D}{I_S}\right)$$

where V_T is nearly 25 mV at room temperature

$$\Delta V_D = V_T\left(\frac{\Delta I_D}{I_D} - \frac{\Delta I_S}{I_S}\right)$$

An I_D difference of 10 % produces a V_D difference of 2.5 mV

Application to matching errors

- Generally, when dealing with matching errors: $A_m = \bar{A}$ (mean value) but also the nominal value or other choices are possible

- Matching errors has also the following basic property:

Linearity: *G=A+B* *ΔG=ΔA+ΔB*

G=kA, where k is a constant : *ΔG=kΔA*

- In the case of relative error, if *G=kA:* $\frac{\Delta G}{G} = \frac{\Delta A}{A}$

- Matching errors: statistical independence

- Matching errors of different quantities (e.g., quantities A,B,C) can be often considered independent from each other since they are mostly affected by microscopic irregularities, that do not show significant correlations when pairs of quantities are considered.
- In addition matching errors of different device pairs can be also considered independent, or, at least, uncorrelated.

$$\Delta G = k_1 \Delta A + k_2 \Delta B + k_3 \Delta C \Rightarrow \sigma_{\Delta G} = \sqrt{k_1^2 \sigma_{\Delta A}^2 + k_2^2 \sigma_{\Delta B}^2 + k_3^2 \sigma_{\Delta C}^2}$$