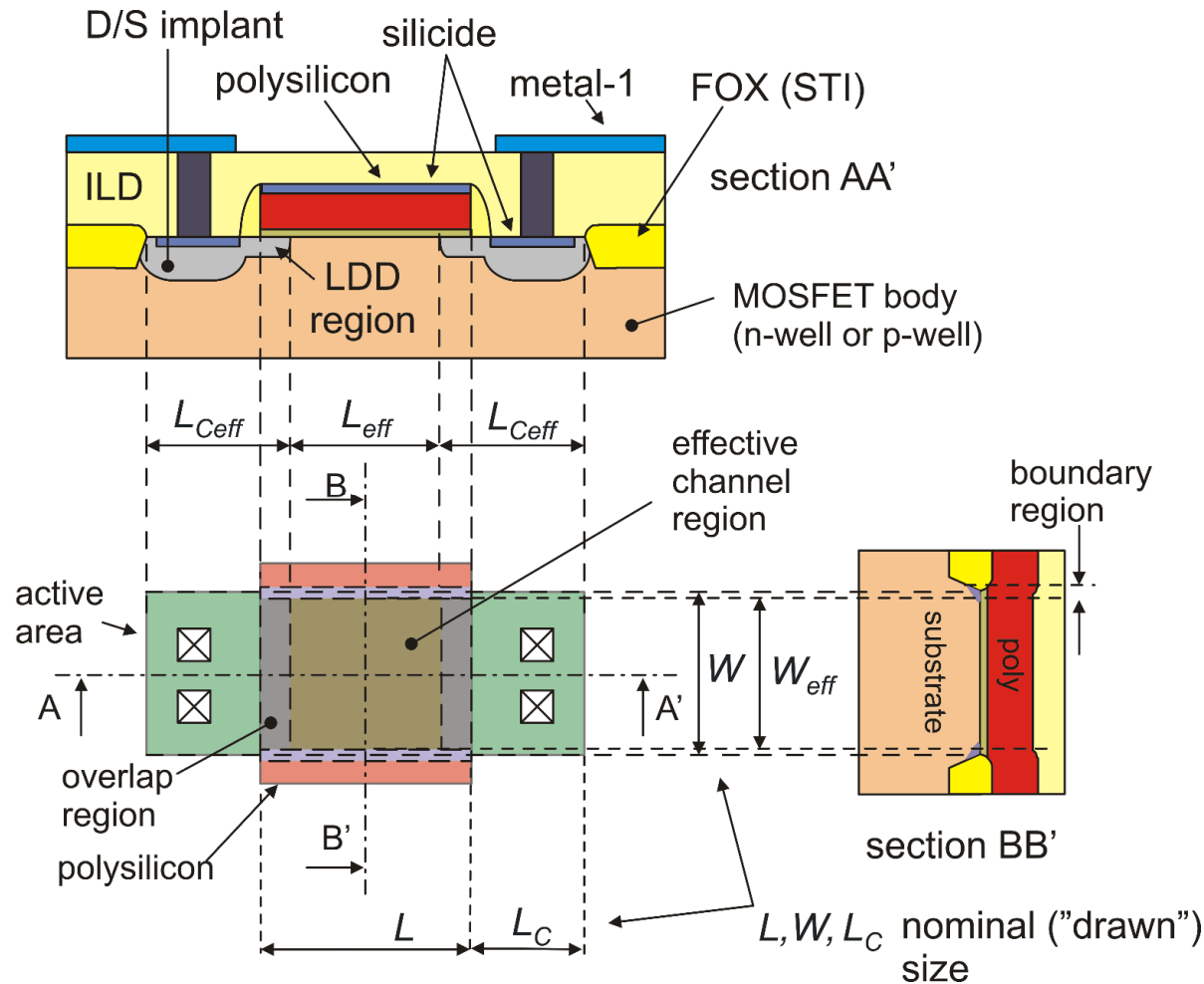


Planar n-MOSFET cross-section and layout

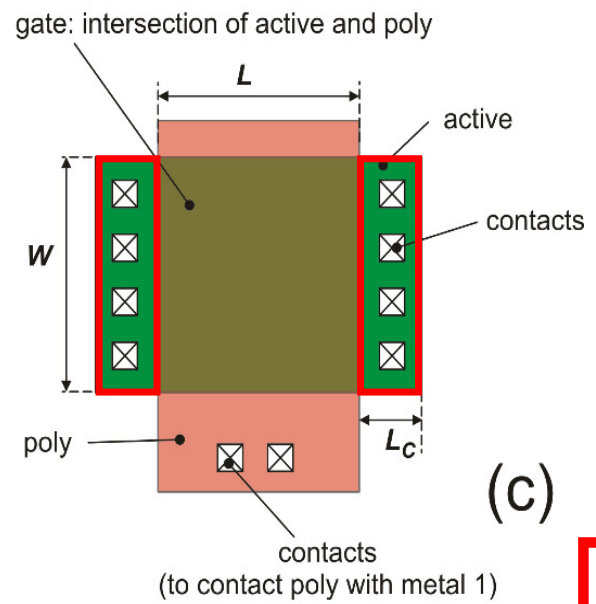
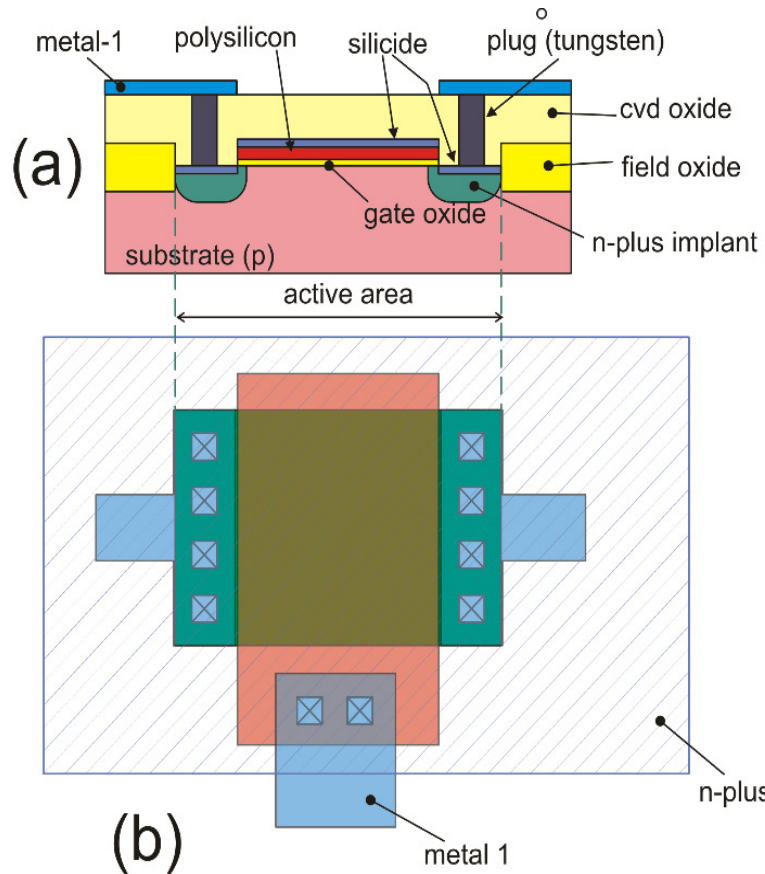


The designer introduces ideal geometrical values ($L, W ..$), while the electrical properties are determined by "effective" values:

$$L_{eff} = L - 2L_D$$

$$W_{eff} = W - 2W_D$$

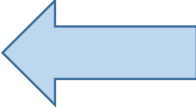
Simplified layout and cross-section ("designer view")



$$A_D = A_S = WL_C$$

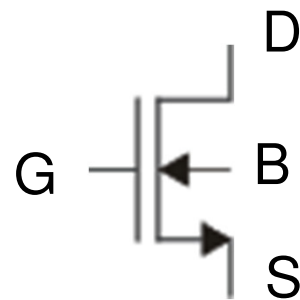
$$P_D = P_S = 2L_C + 2W$$

MOSFET models

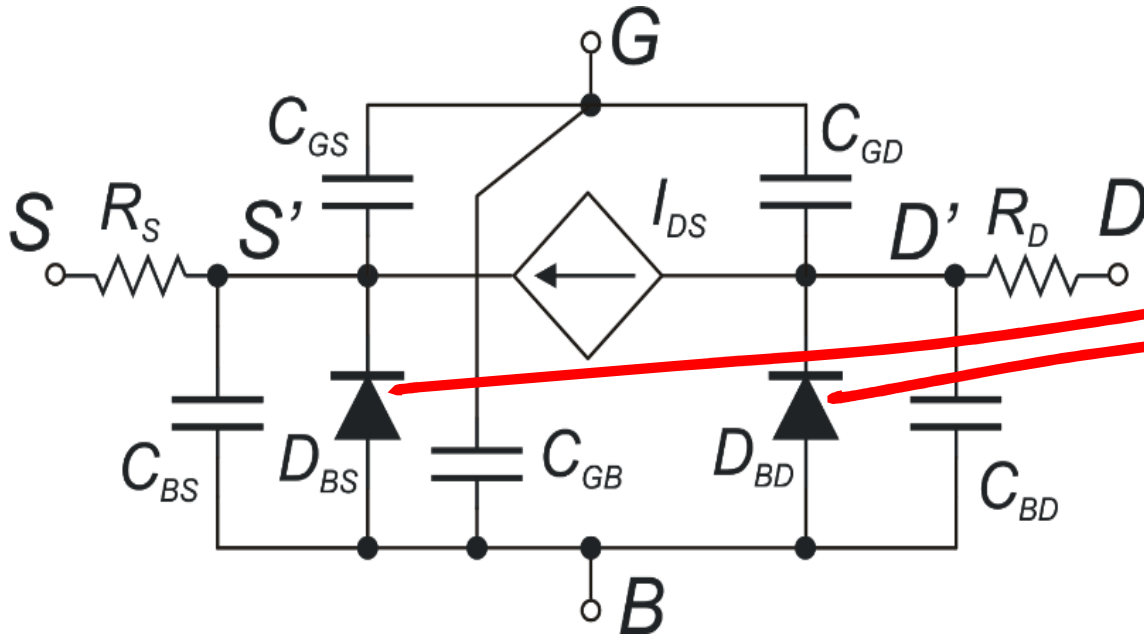
- Models for accurate electrical simulations: BSIM models (Berkeley Short-channel IGFET Model), EKV (Enz, Kruppenacher, Vittoz) ...
- Models for "hand calculations": square law (strong inversion)
exponential laws (weak inversion) 
- It is of primary importance to be able to manually perform first order device sizing and first order performance estimation.
- Only very simple and intuitive model enable the designer to create cells that need only a final refinement and verification in the simulation phase
- The simulator is useless if we do not know how to produce a circuit on scrap-paper. The simulator obeys to the law:
garbage in – garbage out

MOSFET models: The n-MOSFET

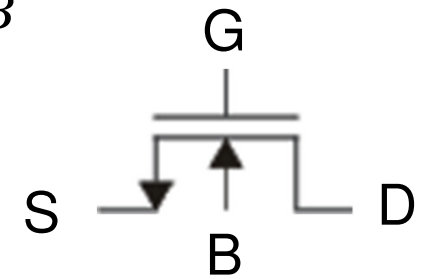
- From this point on, we will consider the behavior of the n-MOSFET, unless otherwise specified. In the end, we will suggest a simple way to transfer all the considerations made for the n-MOSFET to the p-MOSFET
- In integrated circuits, the MOSFET is a four terminal devices: Drain, Source, Gate and Body. In discrete MOSFETs, the body is generally connected to the source internally.



Large signal MOSFET model (n-MOSFET)



$$V_D, V_S \geq V_B$$

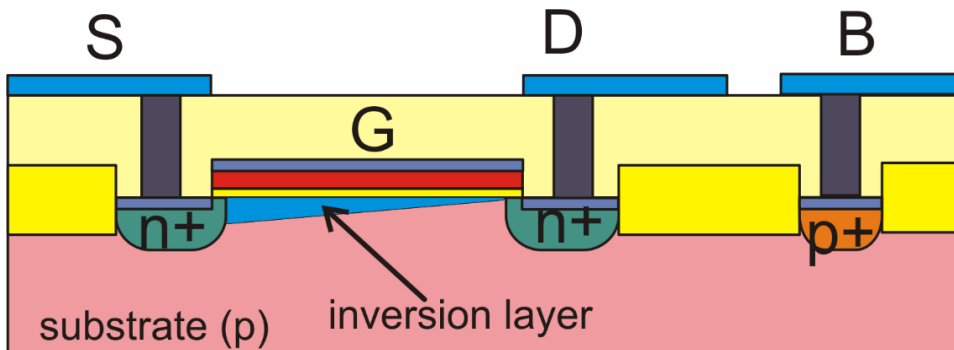


$$I_D \neq I_S \neq I_{DS}$$

In DC, we will always assume:

$$I_D \cong I_S \cong I_{DS}$$

$$R_S = R_D \cong 0$$

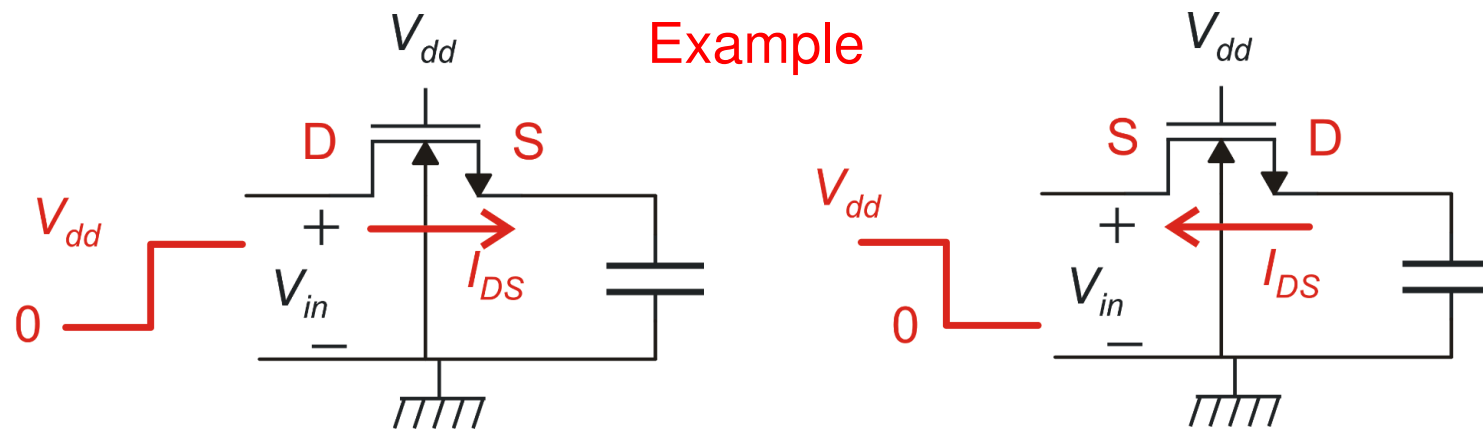


Source and Drain Symmetry (1)

- The planar MOSFET is symmetric so that drain and source can be swapped with no consequences in the electrical characteristics.
- Equations that use the source as a reference terminal for all relevant voltages can be applied only after finding which terminal is actually playing the role of the source.
- In an **n-MOSFET**, the effective source is the terminal that has the **lower** voltage; the other one of the two, is the actual drain
- In a **p-MOSFET**, the effective source is the terminal that has the **higher** voltage; the other one of the two, is the actual drain

Source and Drain Symmetry (2)

- With this definition, it is clear that in transient situations, the effective drain and source can swap, depending on the voltage assumed by the terminals.



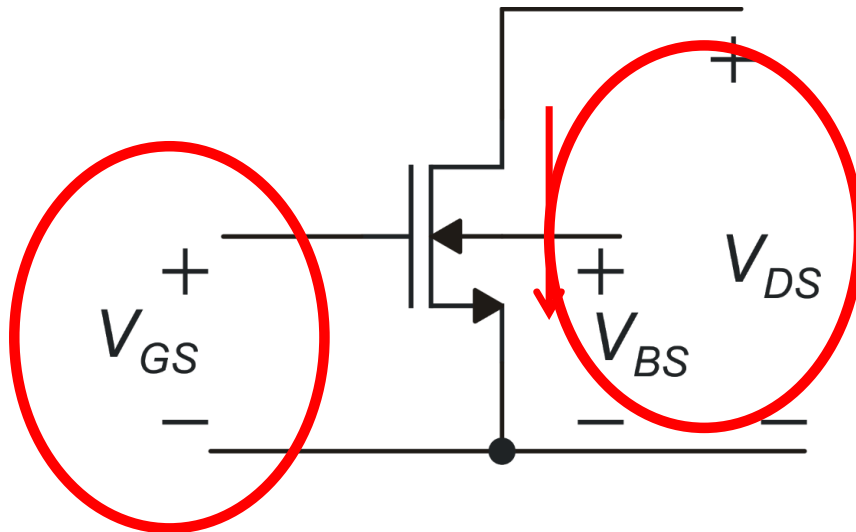
- In a schematic editor it is necessary to indicate which terminal is the drain and the source. These "conventional" terminals are used to mark all voltages for printing and plotting purposes. This choice does not affect the circuit behavior during the simulations.

Source and Drain Symmetry (3)

- If the circuit has a clear static operating point (like most analog circuits), it is convenient to mark as source the terminal that in the operating point is actually working as the source. This will facilitate reading device voltages produced as textual or graphical outputs by the simulator.
- Models like the EKV use the body as the reference for all voltages. In this way drain and sources are perfectly symmetrical also in the equations and there is no need to decide which one is actually working as the source.
- Maintaining the distinction between source and drain is more intuitive and most models oriented to hand calculations are actually based on this choice.

The I_{DS} model: control voltages

$$I_{DS}(V_{GS}, V_{BS}, V_{DS})$$



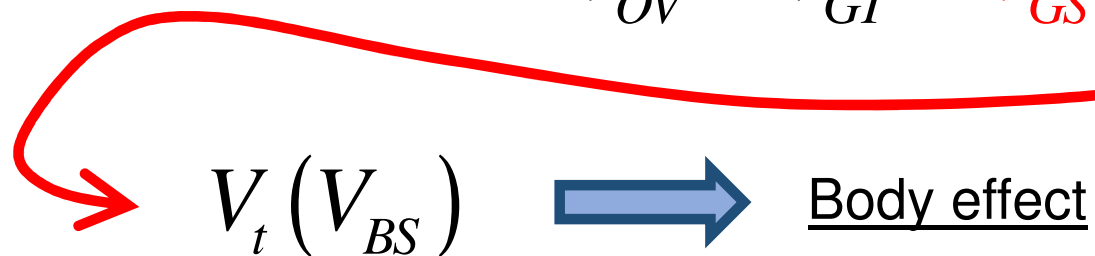
secondary effects:
generally they are **unwanted**

primary effect
it is the **wanted** current control

V_{GS} , V_{BS} and "overdrive voltage"

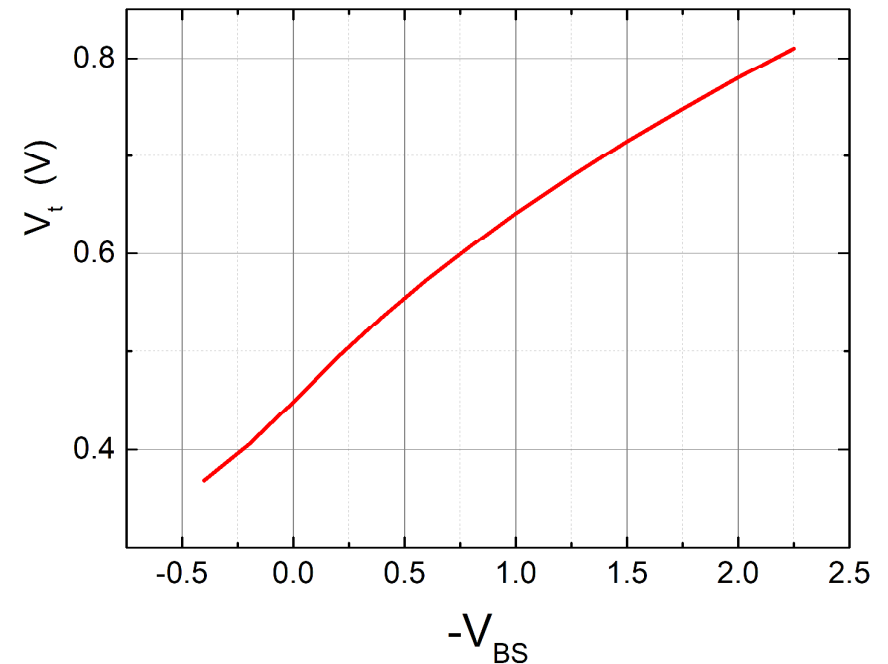
The voltage that really affects the current is the "useful" part of the V_{GS} , often called "overdrive voltage".

$$V_{OV} = V_{GT} = V_{GS} - V_t$$

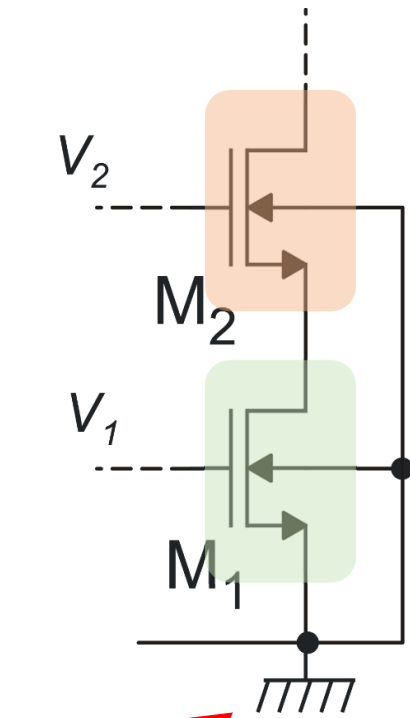


$$V_t = V_{t0} + \gamma \left(\sqrt{\phi_s - V_{BS}} - \sqrt{\phi_s} \right)$$

$$V_{t0} = V_t (V_{BS} = 0) \quad \gamma: \text{body effect coefficient} \quad \phi_s: \text{surface potential}$$



More on body effect: example



$V_{SS} = gnd$

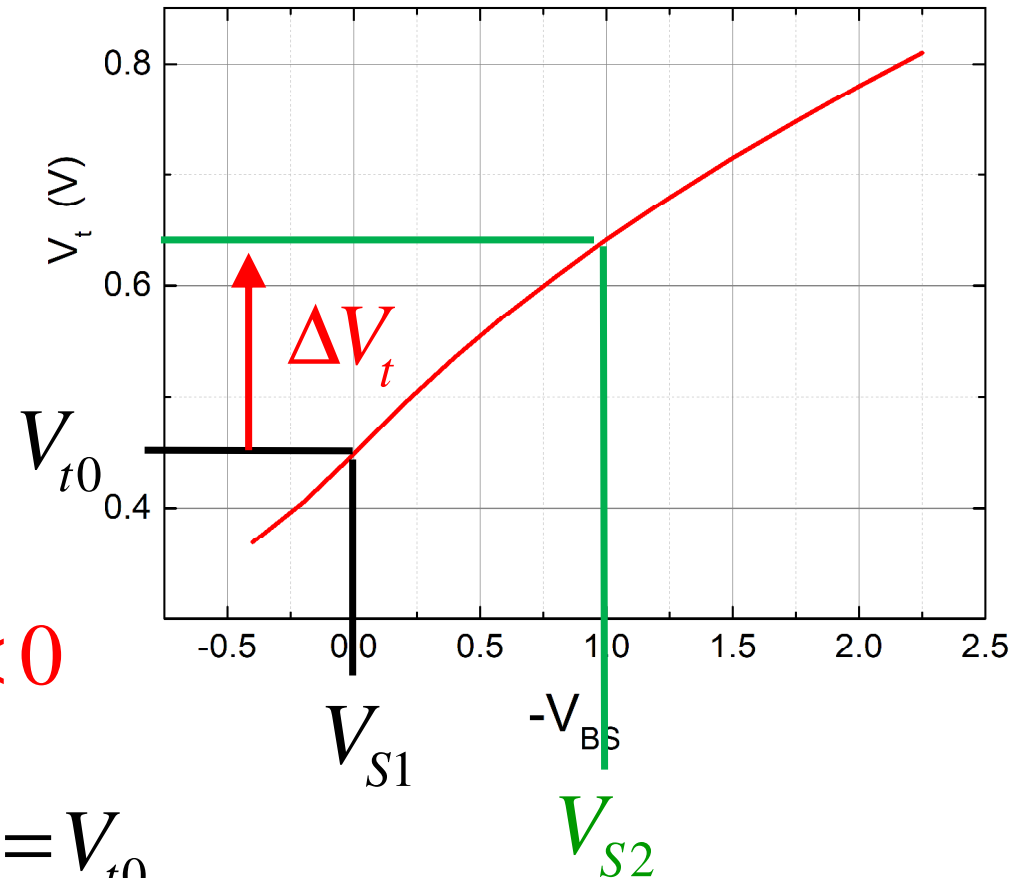
$$V_{B2} = V_{B1} = 0$$

$$V_{S1} = 0$$

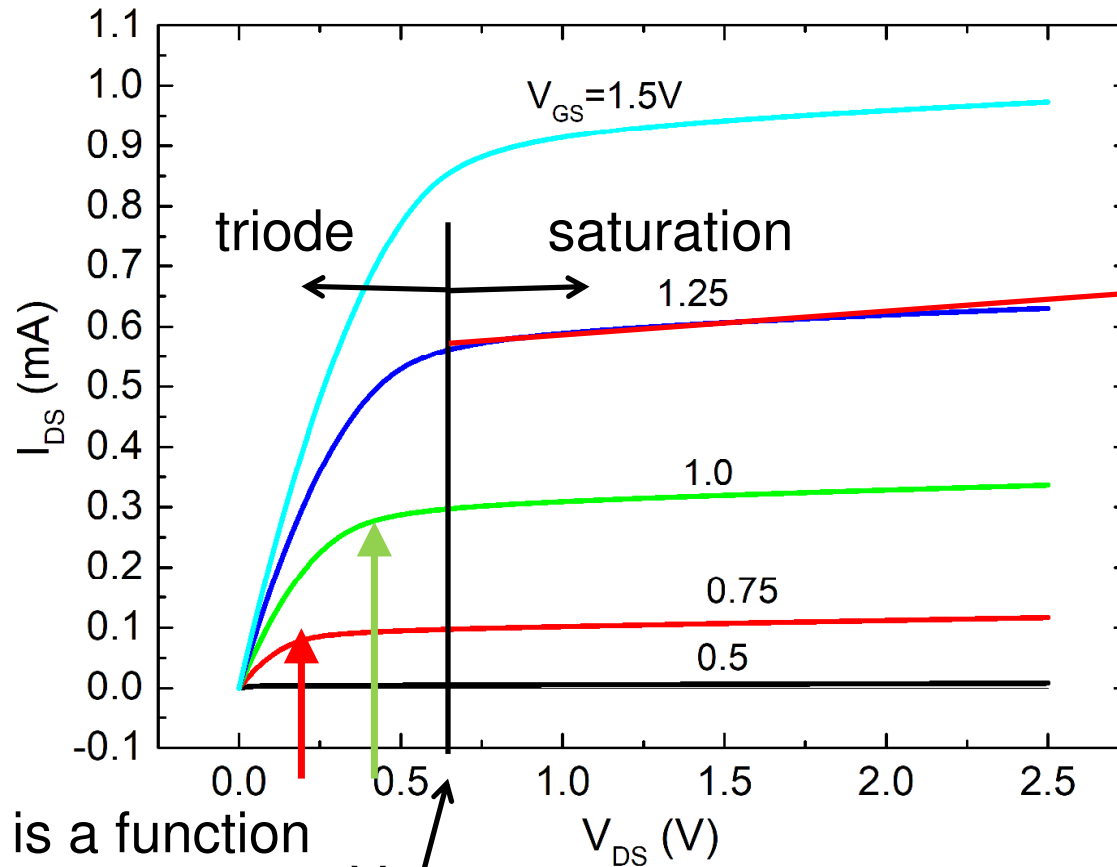
$$V_{S2} = V_{D1} > 0$$

$$V_{BS2} = V_{B2} - V_{S2} < 0$$

$$\begin{cases} V_{BS1} = 0 \Rightarrow V_{t1} = V_{t0} \\ -V_{BS2} = V_{S2} - V_{B2} = V_{DS1} > 0 \Rightarrow V_{t2} = V_{t0} + \Delta V_t \end{cases}$$



I_{DS} : operating zones on the basis of V_{DS}



Triode: I_{DS} is strongly dependent on V_{DS}

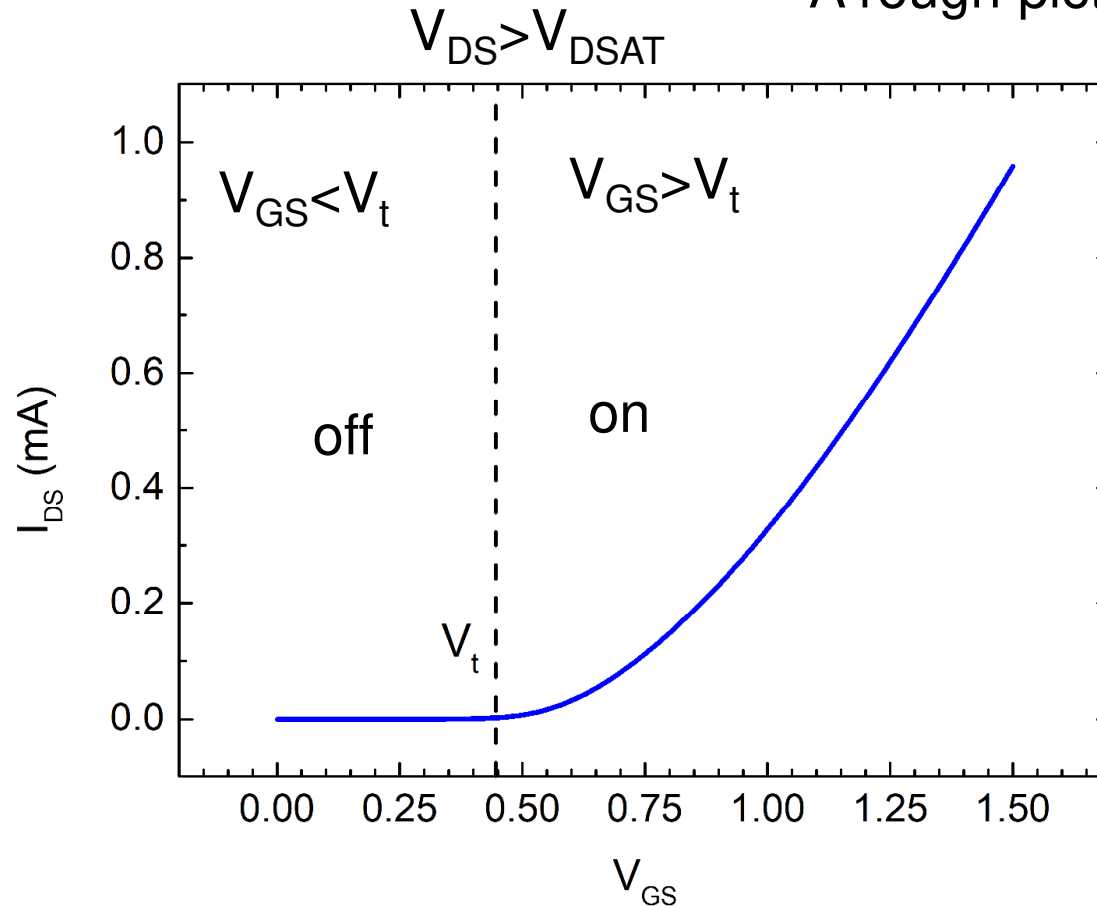
Saturation: I_{DS} shows a weak and almost linear dependence on V_{DS}

V_{DSAT} is a function of $V_{GS} - V_t$

V_{DSAT}

Operating zones on the basis of $V_{GS} - V_t$

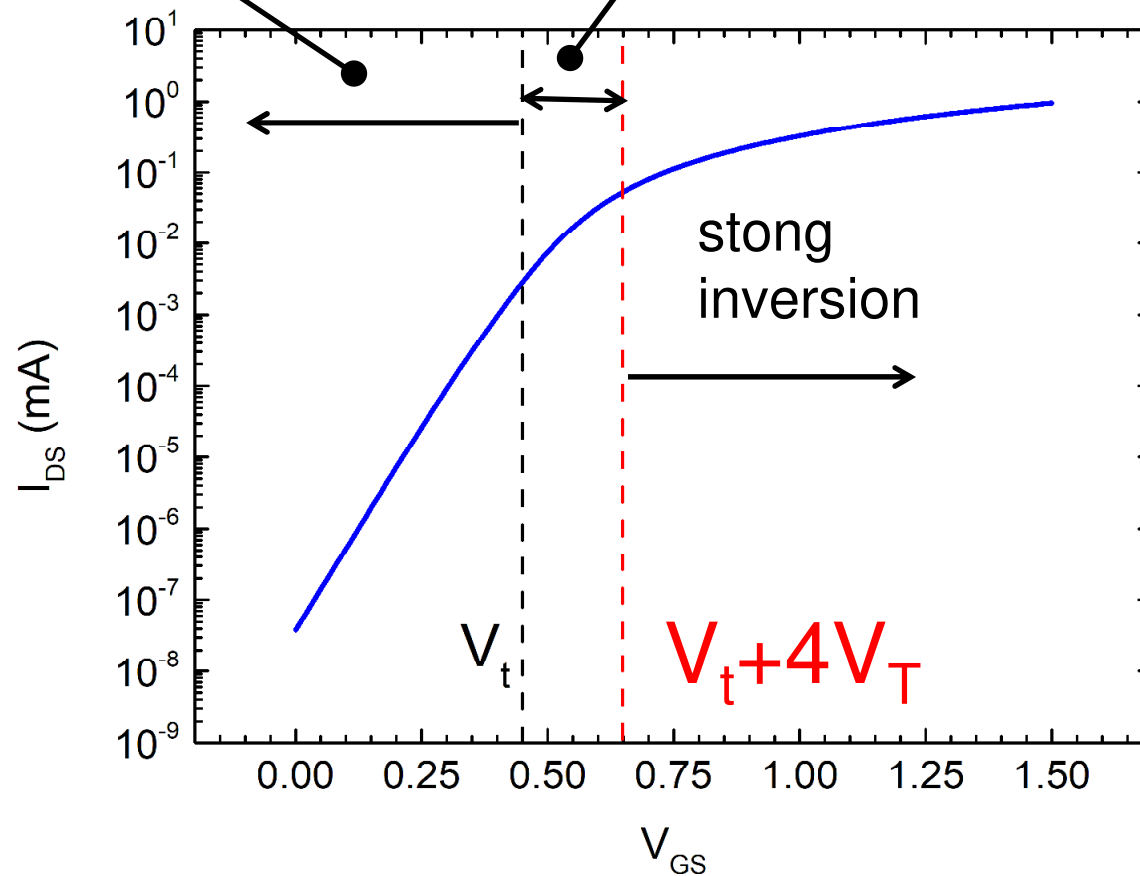
A rough picture:



A more gradual picture: same characteristic with logarithmic y-axis

weak inversion
(sub-threshold)

moderate inversion



$$V_T = kT/q$$

I_{DS} : operating zones

	$V_{GS} - V_t \leq 0$	$0 \leq V_{GS} - V_t \leq 4V_T$	$V_{GS} - V_t \geq 4V_T$
$V_{DS} \leq V_{DSAT}$	Triode – Weak Inversion	Triode – Moderate Inversion	Triode – Strong Inversion
$V_{DS} \geq V_{DSAT}$	Saturation – Weak Inversion	Saturation – Moderate Inversion	Saturation – Strong Inversion

$$V_{DSAT} \cong \begin{cases} (V_{GS} - V_t) & \text{in strong inversion} \\ 4V_T \text{ (100 mV)} & \text{in moderate and weak inversion} \end{cases}$$

$V_{GS} - V_t > 4V_T$ Strong inversion: I_{DS} equations

$$V_{DS} \leq V_{DSAT} \text{ (Triode)} \quad I_{DS} = \beta_n \left(V_{GS} - V_t - \frac{V_{DS}}{2} \right) V_{DS}$$

$$V_{DS} \geq V_{DSAT} \text{ (Saturation)} \quad I_{DS} = \beta_n \frac{(V_{GS} - V_t)^2}{2} \left[1 + \lambda (V_{DS} - V_{DSAT}) \right]$$

$$\beta_n = \mu_n C_{OX} \frac{W_{eff}}{L_{eff}} \quad V_{DSAT} = V_{GS} - V_t$$

$$\lambda^{-1} = k_\lambda L_{eff}$$

In some textbooks this term is omitted (V_{DSAT}) for simplicity, but this cause a discontinuity between the triode and saturation region

I_{DS} simplified model in weak inversion

$$I_{DS} = I_{SM} e^{\frac{V_{GS} - V_t}{mV_T}} \left(1 - e^{\frac{-V_{DS}}{V_T}} \right) \left[1 + \lambda (V_{DS} - V_{DSAT}) \right]$$

m : subthreshold slope factor

$$I_{SM} = \mu_n C_{dm} \frac{W_{eff}}{L_{eff}} V_T^2 = \mu_n C_{ox} (m-1) V_T^2 \frac{W_{eff}}{L_{eff}}$$

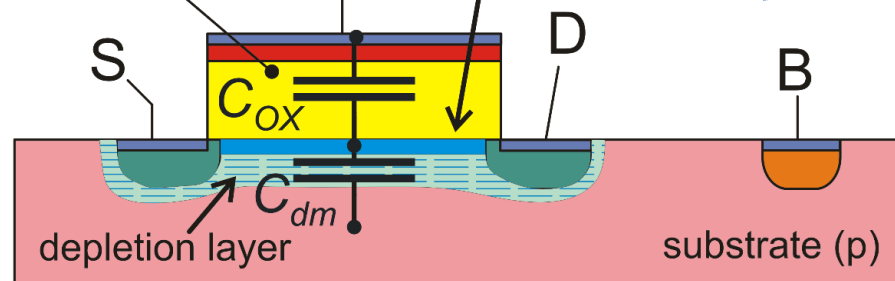
$$m = 1 + \frac{C_{dm}}{C_{ox}}$$

β_n

gate oxide
(not to scale)

inversion layer

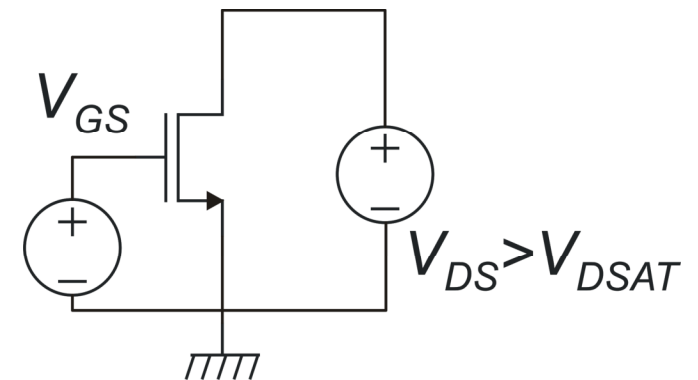
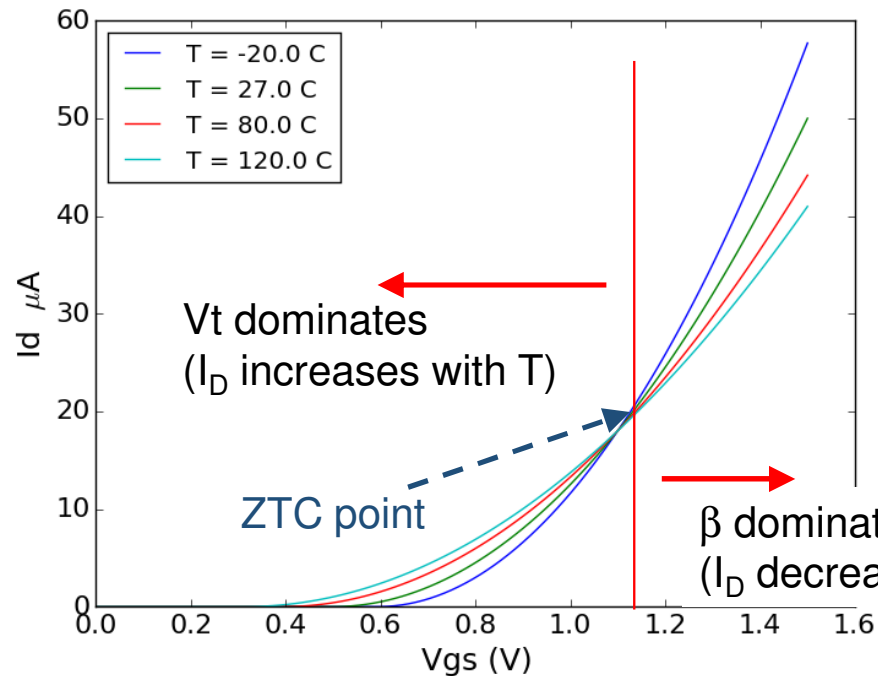
$m \approx 1.2 - 1.3$



Temperature effects on MOSFET characteristics

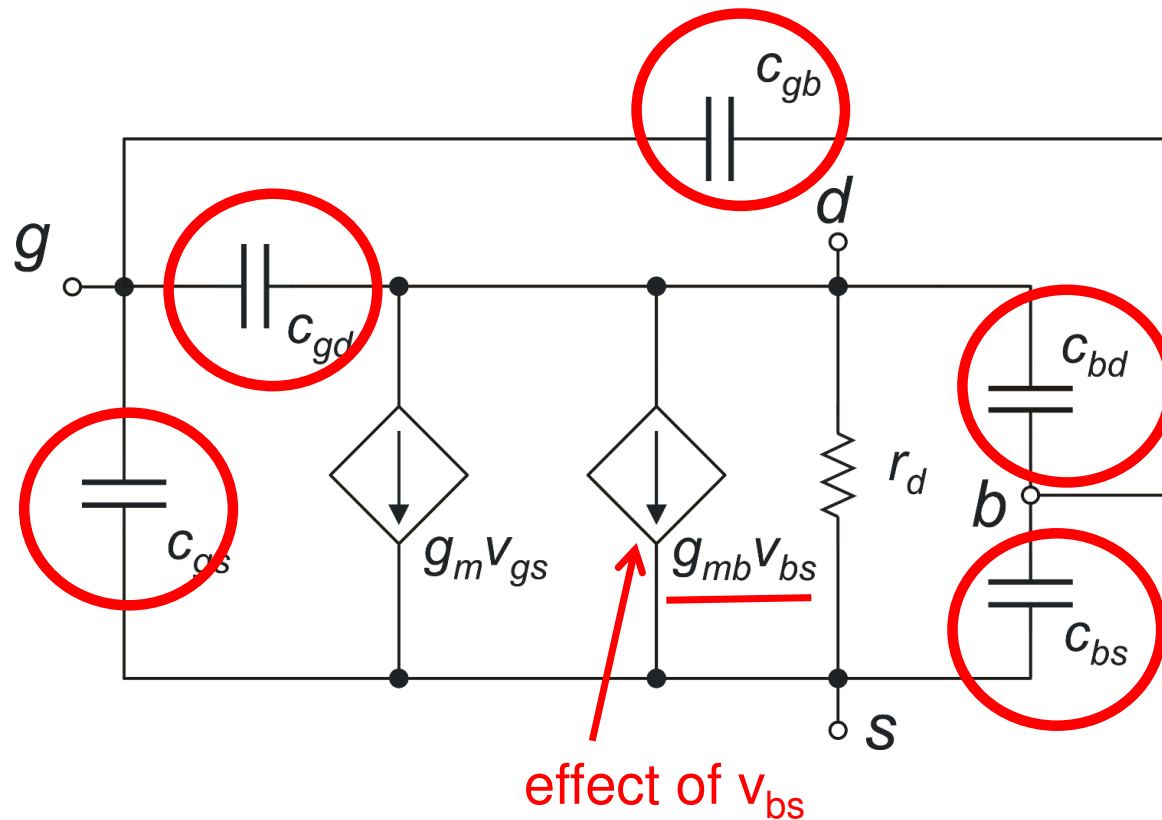
$$\beta_n(T) = \beta_n(T_0) \left(\frac{T}{T_0} \right)^{-\alpha_\mu} \quad \alpha_\mu = 1.2 - 2.4 \text{ (typical 1.5)}$$

$$V_t(T) = V_t(T_0) - \alpha_{VT} (T - T_0) \quad 1 \text{ mV/K} \leq \alpha_{VT} \leq 4 \text{ mV/K}$$

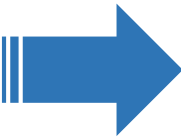


MOSFET Small Signal model

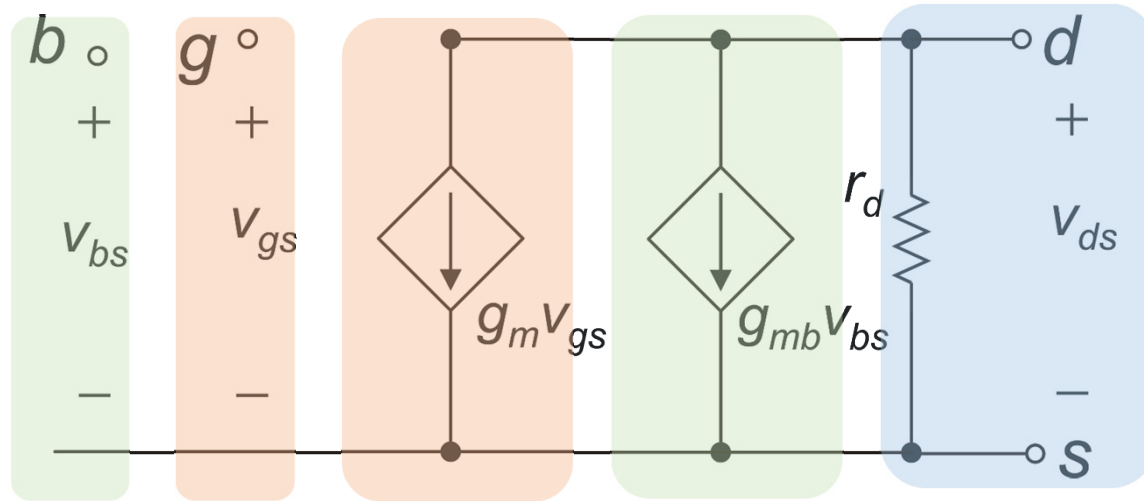
C_{gs} , C_{gd} , C_{gb} , C_{bd} , C_{bs} : *small signal capacitances*



Let's start from
the dc model
(capacitances are
removed)



MOSFET small signal model: dc limit



$$I_D(V_{GS}, V_{BS}, V_{DS})$$



small signal

$$i_d = g_m v_{gs} + g_{mb} v_{bs} + g_d v_{ds}$$

$$g_m = \left. \frac{i_d}{v_{gs}} \right|_{v_{ds}, v_{bs}=0} = \left(\frac{\partial I_D}{\partial V_{GS}} \right)_{V_{DS}, V_{BS}}$$

$$g_{mb} = \left. \frac{i_d}{v_{bs}} \right|_{v_{ds}, v_{gs}=0} = \left(\frac{\partial I_D}{\partial V_{BS}} \right)_{V_{DS}, V_{GS}}$$

$$\frac{1}{r_d} = g_d = \left. \frac{i_d}{v_{ds}} \right|_{v_{gs}, v_{bs}=0} = \left(\frac{\partial I_D}{\partial V_{DS}} \right)_{V_{GS}, V_{BS}}$$

Body transconductance: g_{mb}

$$g_{mb} = \left(\frac{\partial I_D}{\partial V_{BS}} \right)_{V_{DS}=const; V_{GS}=const} \quad I_D(V_{GS}, V_{BS}, V_{DS}) \cong I_D \left[(V_{GS} - V_t), V_{DS} \right]$$

effect of V_{BS}

let's recall the g_m definition

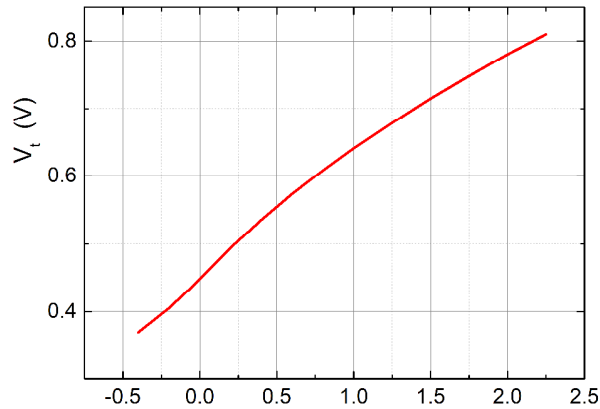
1

$$g_m = \left(\frac{\partial I_D}{\partial V_{GS}} \right)_{V_{DS}, V_{BS}} = \left(\frac{\partial I_D}{\partial (V_{GS} - V_t)} \right)_{V_{DS}} \left(\frac{\partial (V_{GS} - V_t)}{\partial V_{GS}} \right)_{V_{BS}} = \left(\frac{\partial I_D}{\partial (V_{GS} - V_t)} \right)_{V_{DS}} \quad g_m$$

$$g_{mb} = \left(\frac{\partial I_D}{\partial V_{BS}} \right)_{V_{GS}, V_{DS}} = \left(\frac{\partial I_D}{\partial (V_{GS} - V_t)} \right)_{V_{DS}} \left(\frac{\partial (V_{GS} - V_t)}{\partial V_{BS}} \right)_{V_{GS}} = g_m \left(-\frac{\partial V_t}{\partial V_{BS}} \right)_{V_{DS}} \quad g_{mb}$$

≡≡≡

Body transconductance: g_{mb}

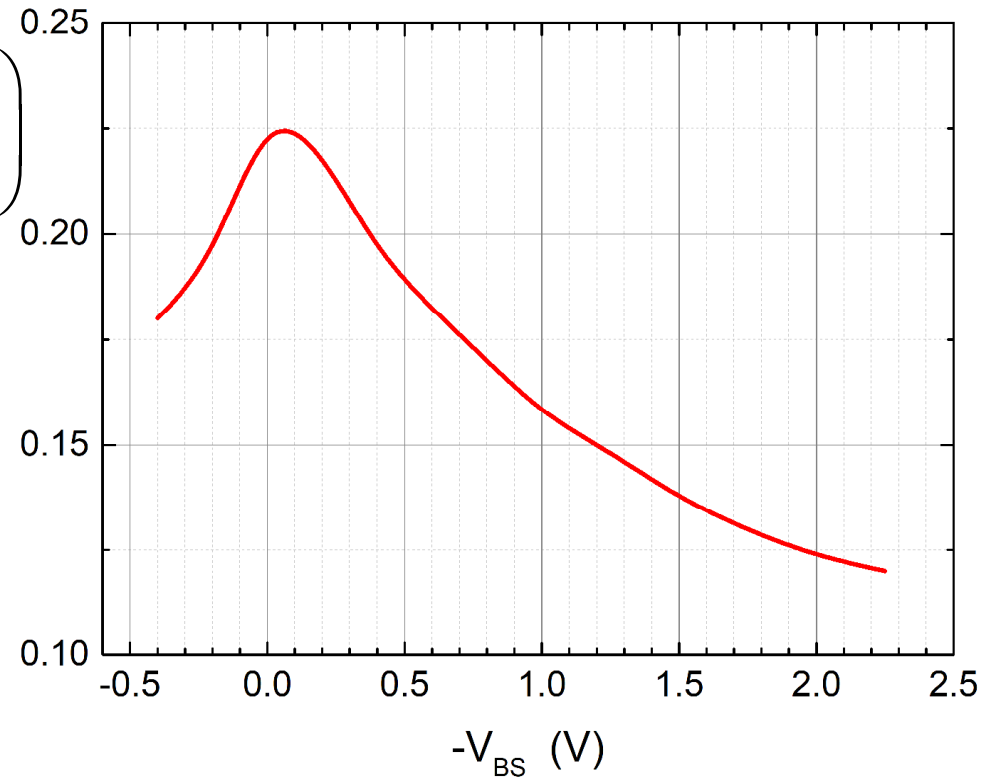


$$\left(-\frac{\partial V_t}{\partial V_{BS}} \right)$$

$$g_{mb} = g_m \left(-\frac{\partial V_t}{\partial V_{BS}} \right) = g_m (m-1)$$

$$m \sim 1.2 \quad \rightarrow \quad g_{mb} \sim 0.2g_m$$

Example from simulation



g_m , g_d in strong inversion

Triode region $I_{DS} = \beta_n \left(V_{GS} - V_t - \frac{V_{DS}}{2} \right) V_{DS}$

g_m $g_m = \left(\frac{\partial I_D}{\partial V_{GS}} \right)_{V_{DS}, V_{BS}} = \underline{\beta_n V_{DS}}$

g_d $\frac{1}{r_d} = g_d = \left(\frac{\partial I_D}{\partial V_{DS}} \right)_{V_{GS}, V_{BS}} = \beta_n \left[(V_{GS} - V_t) - \frac{V_{DS}}{2} \right] - \beta_n \frac{V_{DS}}{2} = \underline{\beta_n [(V_{GS} - V_t) - V_{DS}]}$

g_m , g_d in strong inversion

Saturation region

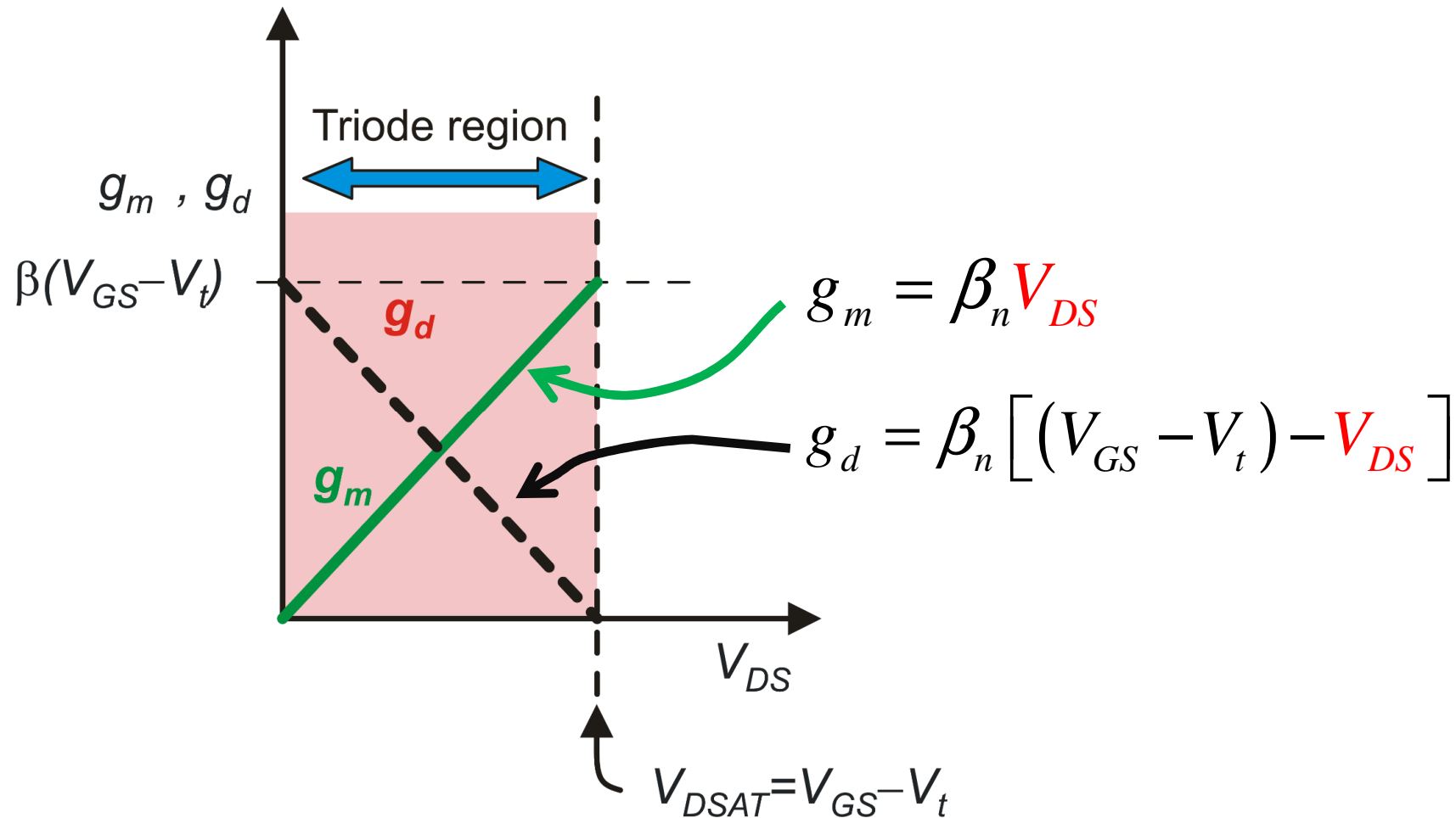
$$I_{DS} = \beta_n \frac{(V_{GS} - V_t)^2}{2} [1 + \lambda(V_{DS} - V_{DSAT})]$$

neglecting the dependence of V_{DSAT} on V_{GS}

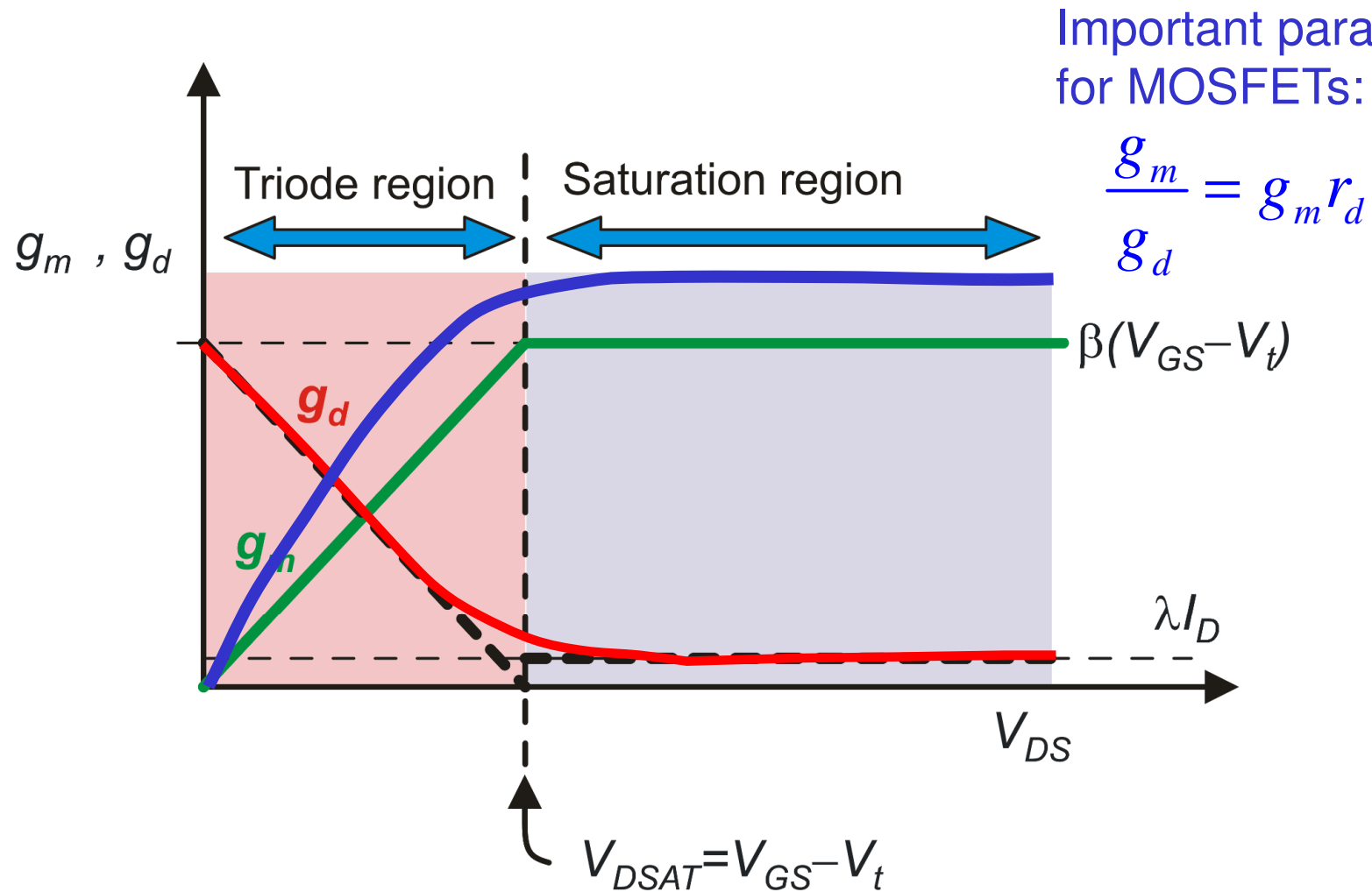
$$g_m \equiv \left(\frac{\partial I_{DS}}{\partial V_{GS}} \right)_{V_{DS}, V_{BS}} = \beta_n (V_{GS} - V_t) [1 + \lambda(V_{DS} - V_{DSAT})] \cong \underline{\beta_n (V_{GS} - V_t)}$$

$$\frac{1}{r_d} = g_{ds} \equiv \left(\frac{\partial I_{DS}}{\partial V_{DS}} \right)_{V_{GS}, V_{BS}} = \lambda \frac{\beta_n}{2} (V_{GS} - V_t)^2 \cong \underline{\lambda I_{DS}}$$

g_m, g_d in strong inversion



g_m, g_d in strong inversion



Transconductance models in saturation

In strong Inversion:

only a few %

acceptable approximation, because we are studying gm, i.e the effect of V_{GS}

$$I_{DS} = \beta_n \frac{(V_{GS} - V_t)^2}{2} \left[1 + \lambda (V_{DS} - V_{DSAT}) \right] \Rightarrow I_{DS} \cong \beta_n \frac{(V_{GS} - V_t)^2}{2}$$

$$(V_{GS} - V_t) = \sqrt{\frac{2I_D}{\beta_n}}$$

$$g_m = \beta_n \sqrt{\frac{2I_D}{\beta_n}} = \sqrt{2I_D \beta_n}$$

$$g_m = \beta_n (V_{GS} - V_t)$$

$$\beta_n = \frac{2I_{DS}}{(V_{GS} - V_t)^2}$$

$$g_m = \frac{2I_{DS}}{(V_{GS} - V_t)} \quad !!!$$

g_m, g_d in weak inversion

$$I_{DS} = I_{SM} e^{\frac{V_{GS} - V_t}{mV_T}} \left(1 - e^{\frac{-V_{DS}}{V_T}} \right) \left[1 + \lambda (V_{DS} - V_{DSAT}) \right]$$

$$g_m = \left(\frac{\partial I_D}{\partial V_{GS}} \right)_{V_{DS}, V_{BS}} = \frac{1}{mV_T} I_{SM} e^{\frac{V_{GS} - V_t}{mV_T}} \left(1 - e^{\frac{-V_{DS}}{V_T}} \right) \left[1 + \lambda (V_{DS} - V_{DSAT}) \right] = \frac{I_D}{mV_T} \quad \text{Exact result}$$

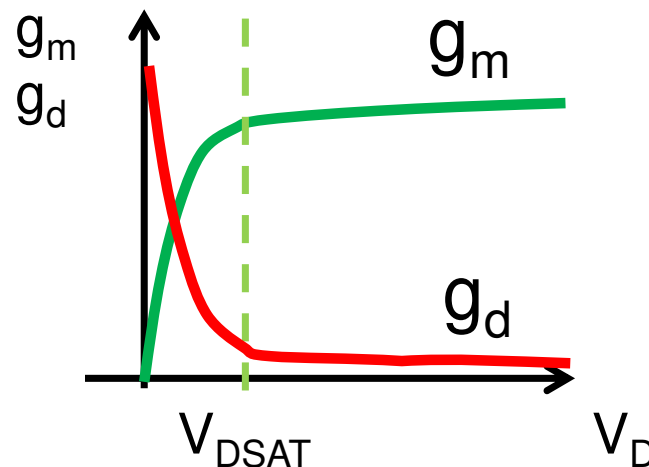
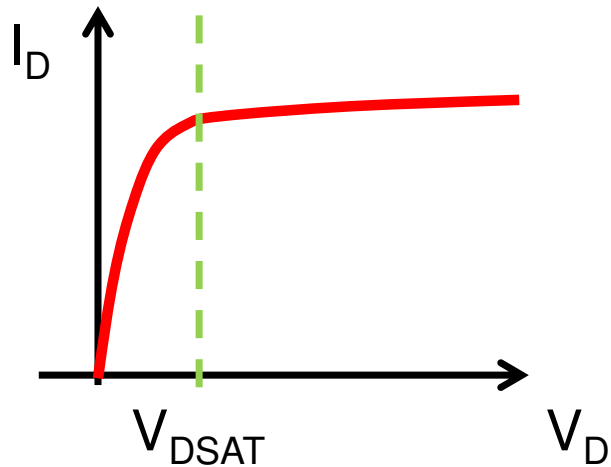
$$\frac{1}{r_d} = g_d = \left(\frac{\partial I_D}{\partial V_{DS}} \right)_{V_{GS}, V_{BS}} =$$

$$= \frac{I_{SM}}{V_T} e^{\frac{V_{GS} - V_t}{mV_T}} e^{\frac{-V_{DS}}{V_T}} \left[1 + \lambda (V_{DS} - V_{DSAT}) \right] + \lambda I_{SM} e^{\frac{V_{GS} - V_t}{mV_T}} \left(1 - e^{\frac{-V_{DS}}{V_T}} \right)$$

g_m, g_d in weak inversion

$$g_m = \frac{I_D}{mV_T}$$

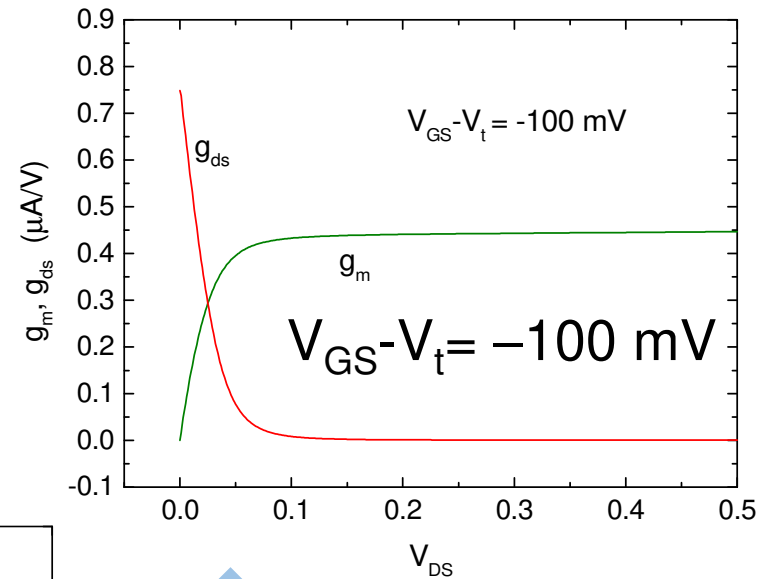
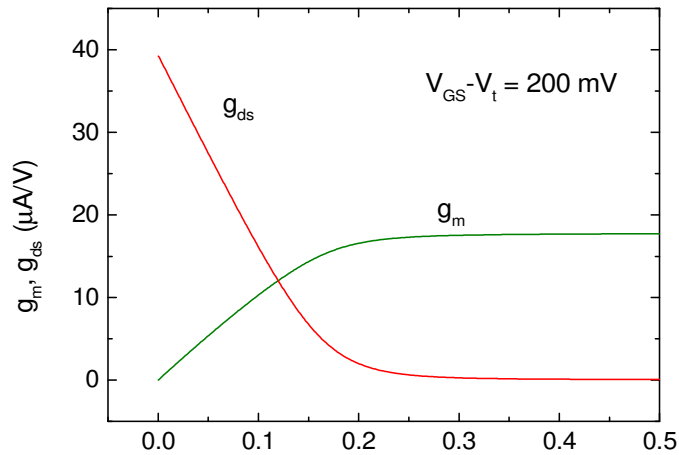
$$g_d = \frac{I_{SM}}{V_T} e^{\frac{V_{GS}-V_t}{mV_T}} e^{\frac{-V_{DS}}{V_T}} \left[1 + \lambda(V_{DS} - V_{DSAT}) \right] + \lambda I_{SM} e^{\frac{V_{GS}-V_t}{mV_T}} \left(1 - e^{\frac{-V_{DS}}{V_T}} \right) \Rightarrow \underline{\underline{g_d \cong \lambda I_D}}$$



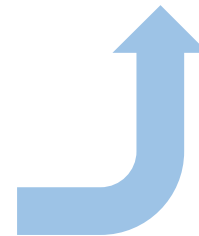
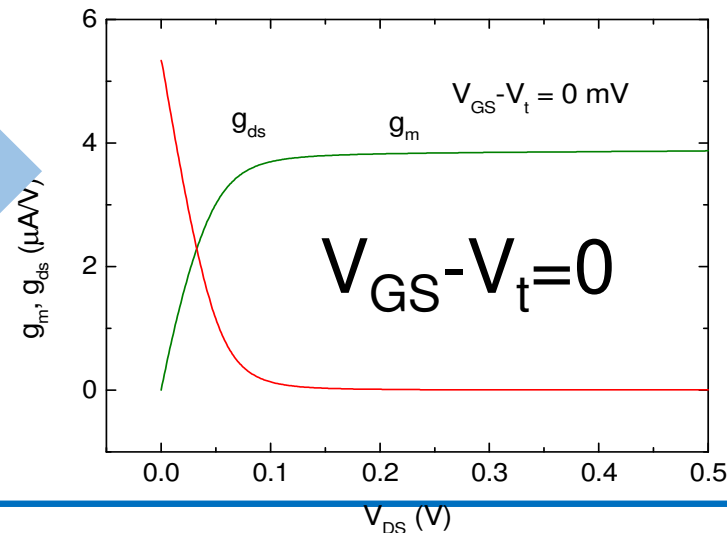
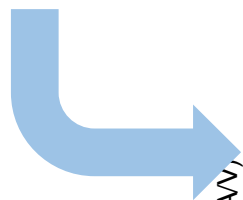
In saturation: $e^{\frac{-V_{DS}}{V_T}} \rightarrow 0$

g_m, g_d everywhere: simulations

Strong inversion



$V_{GS}-V_t = 200$ mV



Unified model for transconductance in saturation

Strong Inversion

$$g_m = \frac{2I_{DS}}{(V_{GS} - V_t)} = \frac{I_{DS}}{\frac{(V_{GS} - V_t)}{2}}$$

$\frac{g_m}{I_D}$ Important parameter for analog circuit design

Weak Inversion

$$g_m = \frac{I_D}{mV_T}$$

BJT

$$g_m = \frac{I_C}{V_T}$$

$$g_m = \frac{I_D}{V_{TE}}$$

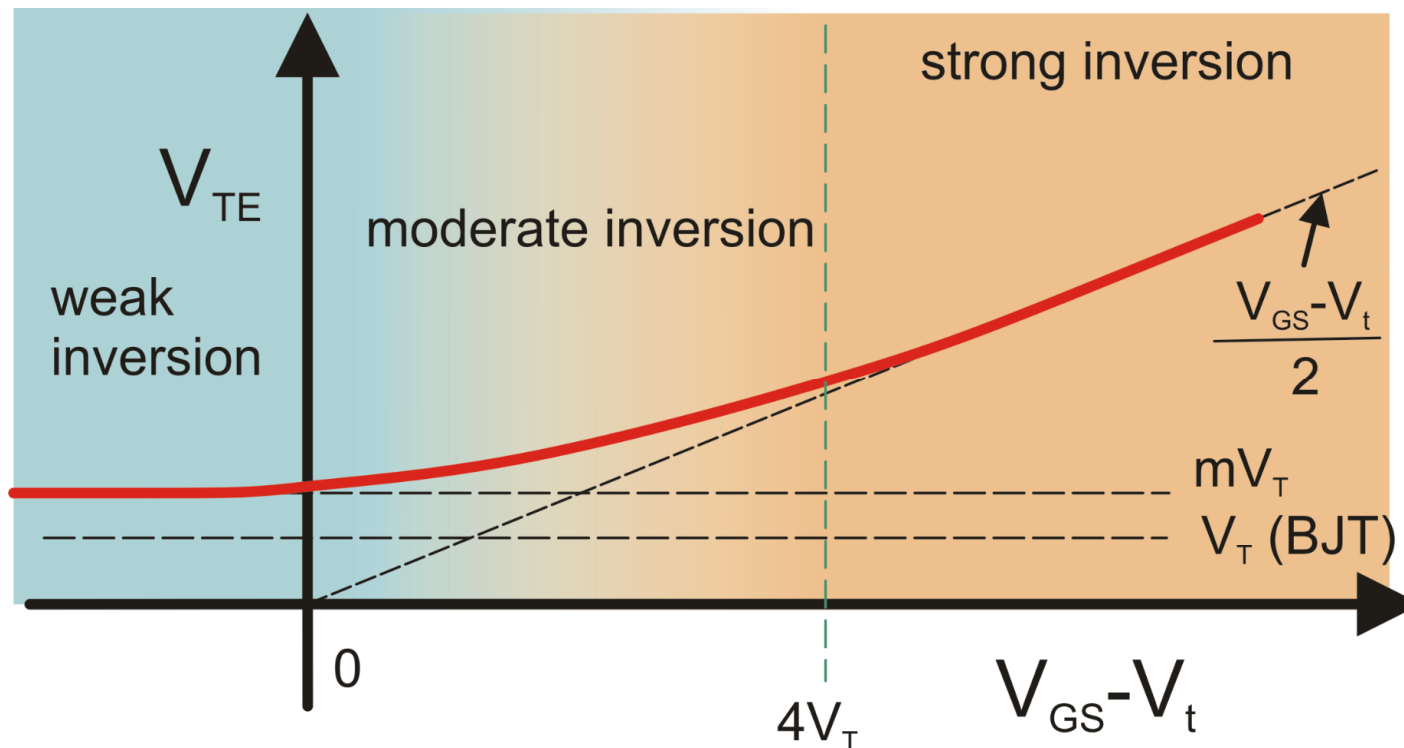
with $V_{TE} = \left(\frac{g_m}{I_D} \right)^{-1}$

Definition of V_{TE}

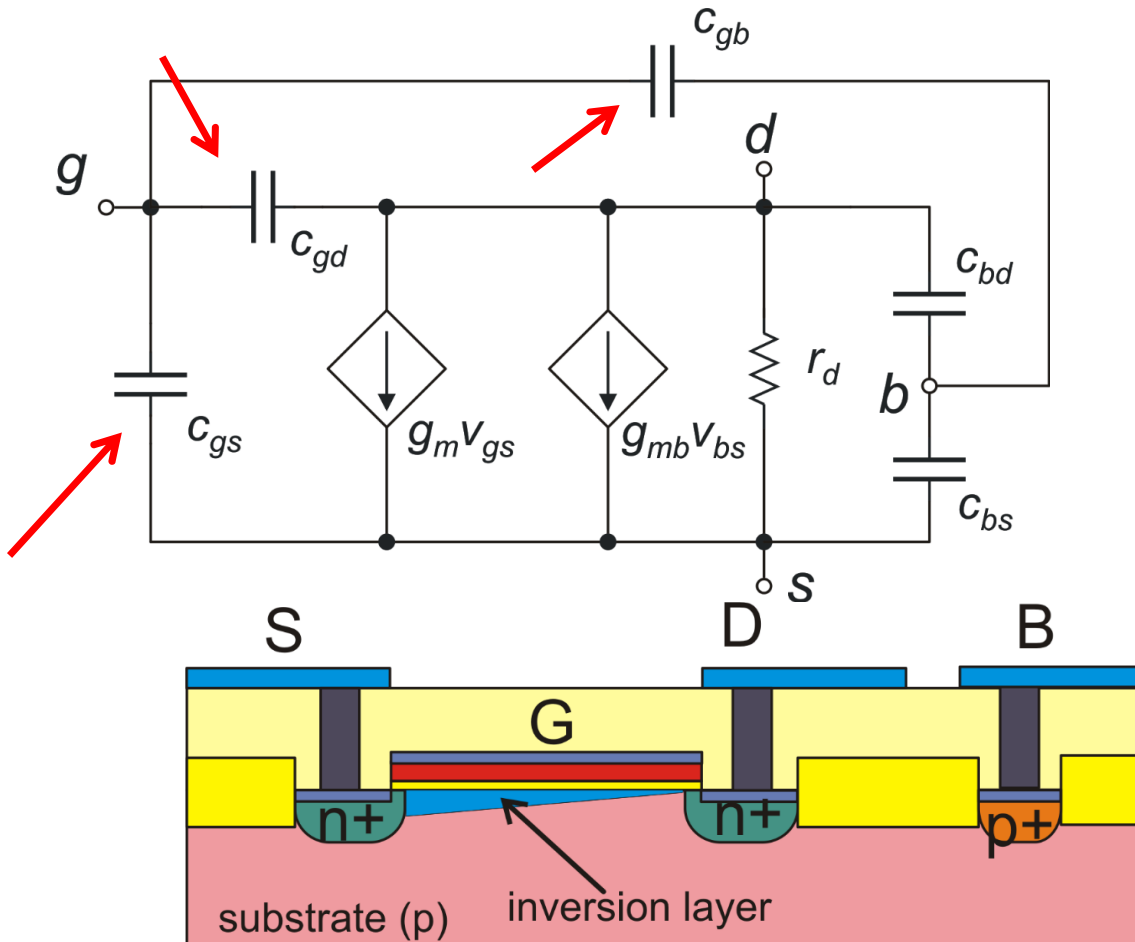
$$= \begin{cases} \frac{(V_{GS} - V_t)}{2} & \text{Strong inversion} \\ mV_T & \text{Weak inversion} \\ V_T & \text{BJT} \end{cases}$$

Effective Thermal Voltage: V_{TE}

The smaller the V_{TE} , the higher the g_m that can be obtained with a given I_D



MOSFET Capacitance Model: gate related capacitances



extrinsic cap.

intrinsic cap..

$$C_{gs} = C_{gs}^{(ov)} + C_{gs}^{(i)}$$

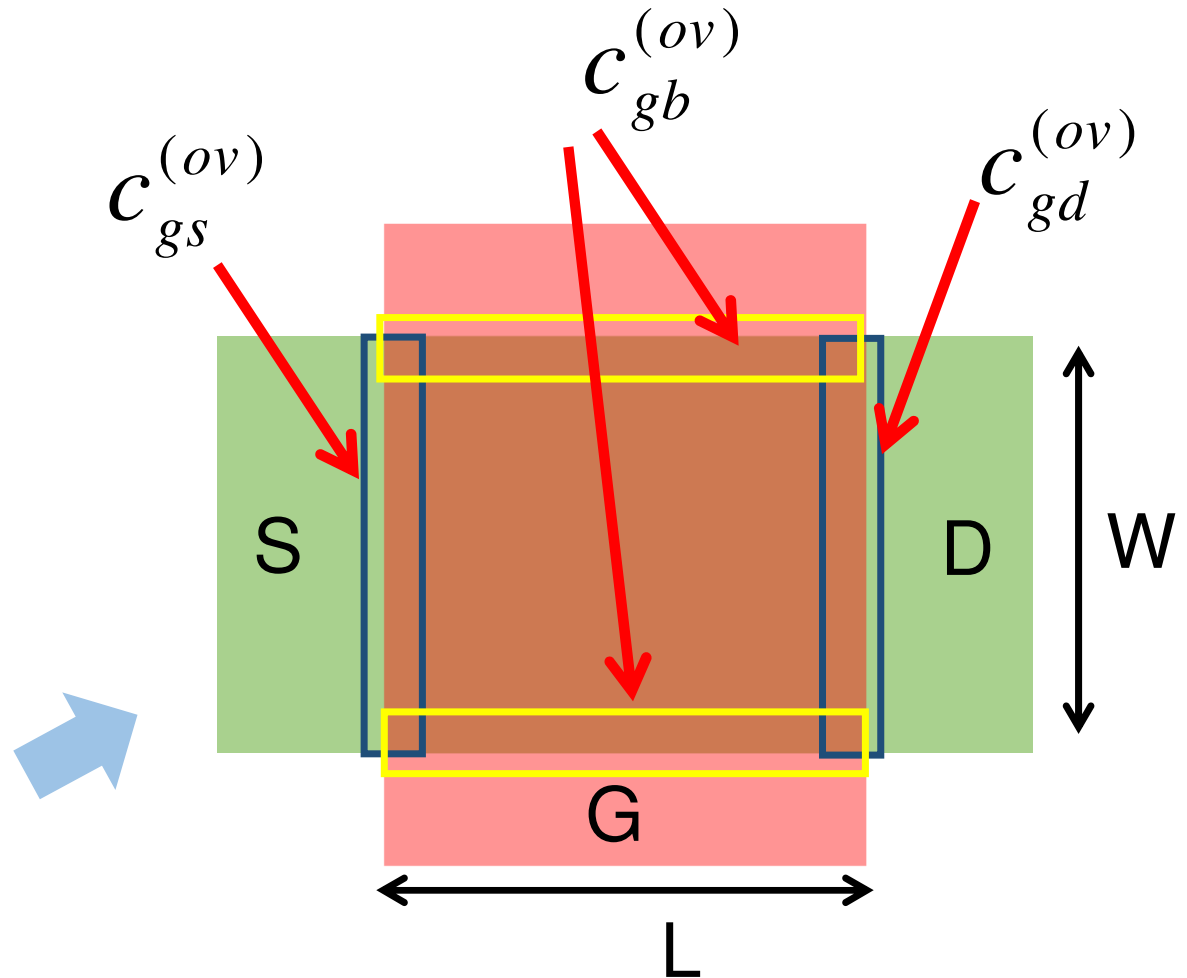
$$C_{gd} = C_{gd}^{(ov)} + C_{gd}^{(i)}$$

$$C_{gb} = C_{gb}^{(ov)} + C_{gb}^{(i)}$$

Estrinsic Capacitances

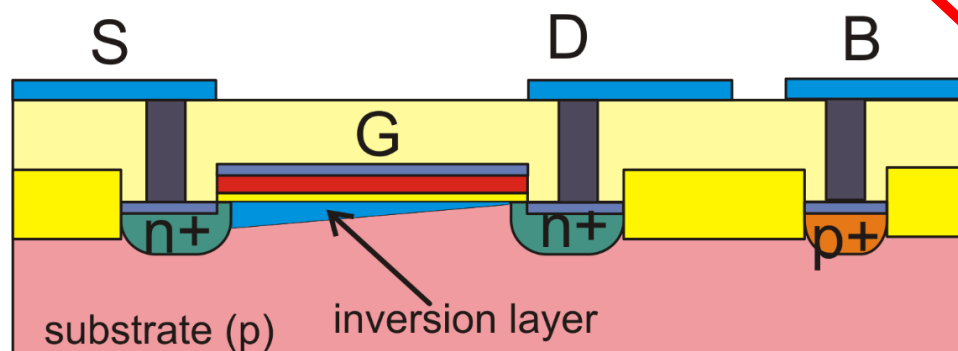
$$\begin{aligned}
 \rightarrow & \left\{ \begin{aligned} C_{gs}^{(ov)} &= C_{gso} \cdot W \\ C_{gd}^{(ov)} &= C_{gdo} \cdot W \end{aligned} \right. \\
 \rightarrow & \\
 \rightarrow & \left\{ \begin{aligned} C_{gb}^{(ov)} &= C_{gbo} \cdot L \end{aligned} \right.
 \end{aligned}$$

Localization of extrinsic capacitances: along the borders of the gate



Intrinsic capacitances: The Meyer Model

	Off ($V_{GS} \ll V_t$)	Triode	Saturation
$C_{gs}^{(i)}$	0	$\frac{1}{2}C_{OX}WL$	$\frac{2}{3}C_{OX}WL$
$C_{gd}^{(i)}$	0	$\frac{1}{2}C_{OX}WL$	0
$C_{gb}^{(i)}$	$\left(\frac{1}{C_{OX}WL} + \frac{1}{C_{dm}} \right)^{-1}$	0	0

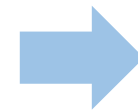


Series of the oxide and depletion layer capacitances. Can be approximated with only the oxide cap $C_{OX}WL$

Charge oriented models (Dutton and Ward model)

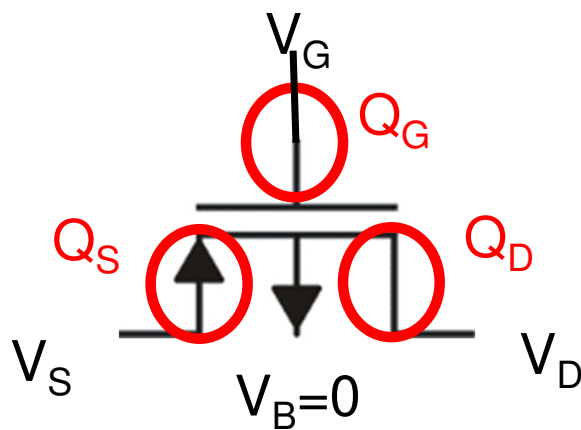
Limits of the Meyer Model:

- Does not guarantee charge conservation
- Capacitances are reciprocal



Important errors in circuits using MOSFETs as switches.

Dutt and Ward model



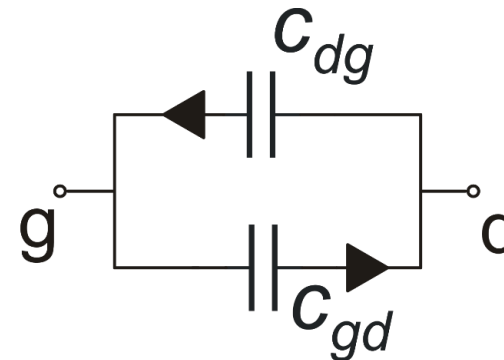
$$C_{ij} = \frac{\partial Q_i}{\partial V_j}$$

Array of 9 capacitances

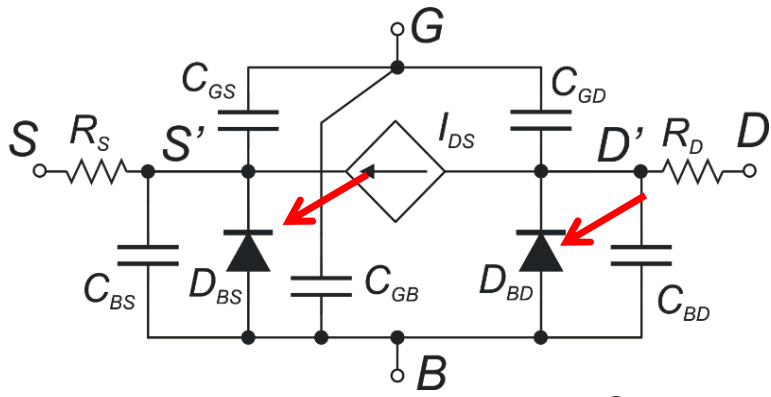
C_{ij} are < 0 for $i \neq j$ (trans-capacitances)

C_{ij} are > 0 for $i = j$ (self capacitances)

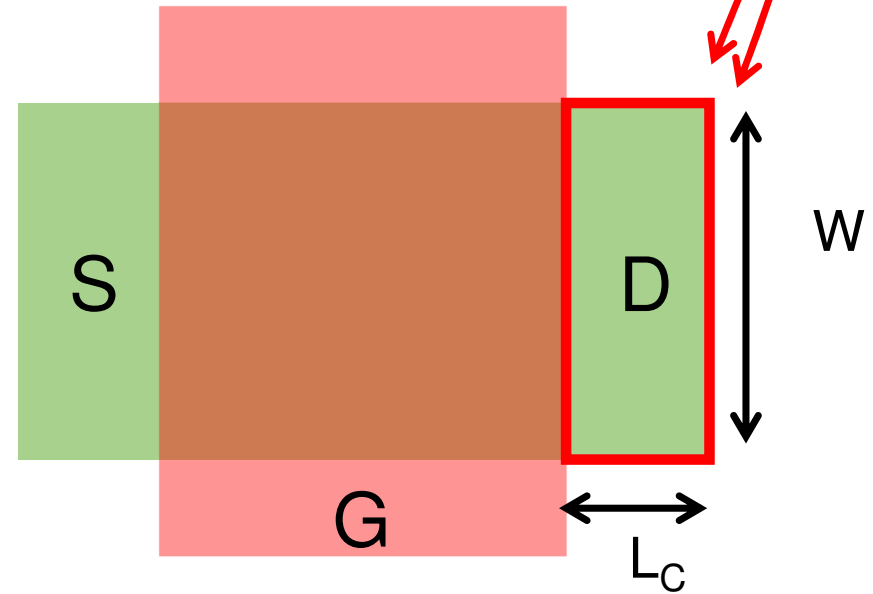
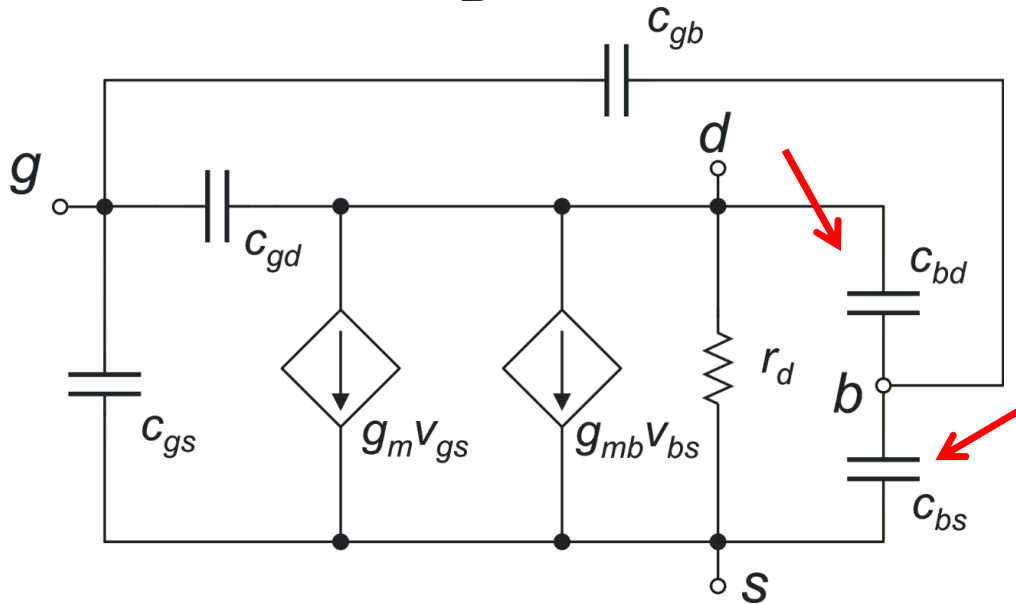
Generally: $C_{ij} \neq C_{ji}$



Junction capacitances



$$C_{bd} = \frac{C_J A_D}{\left(1 + \frac{V_{SB}}{V_0}\right)^{m_j}} + \frac{C_{JSW} P_D}{\left(1 + \frac{V_{SB}}{V_0}\right)^{m_{jsw}}}$$



Other non-idealities of the MOSFET behaviour

Gate-bias dependent mobility → μC_{ox} depends on V_{GS} (decreases at high V_{GS})
(all devices)

Carrier velocity saturation → I_D dependence on V_{GS} in strong inversion tends to become linear (instead of quadratic)
(Short channel devices). Again, appears as a reduction of the μC_{ox} at high V_{GS}

Gate current → May be due to tunneling **(all devices)**
or hot electrons - hot holes **(Short channel devices)**