# 1 Output voltage of BJT Cascode and Wilson current mirrors

## 1.1 Cascode mirror

The topology of the cascode current mirror and the small signal equivalent circuit used to calculate the output resistance are shown in Fig. 1.1. The circuit is probed by the voltage source $v_p$, so that the output resistance is $v_p/i_p$. In order to simplify the calculations, the simple mirror formed by $Q_1$ and $Q_2$ is modeled with its small signal equivalent circuit.

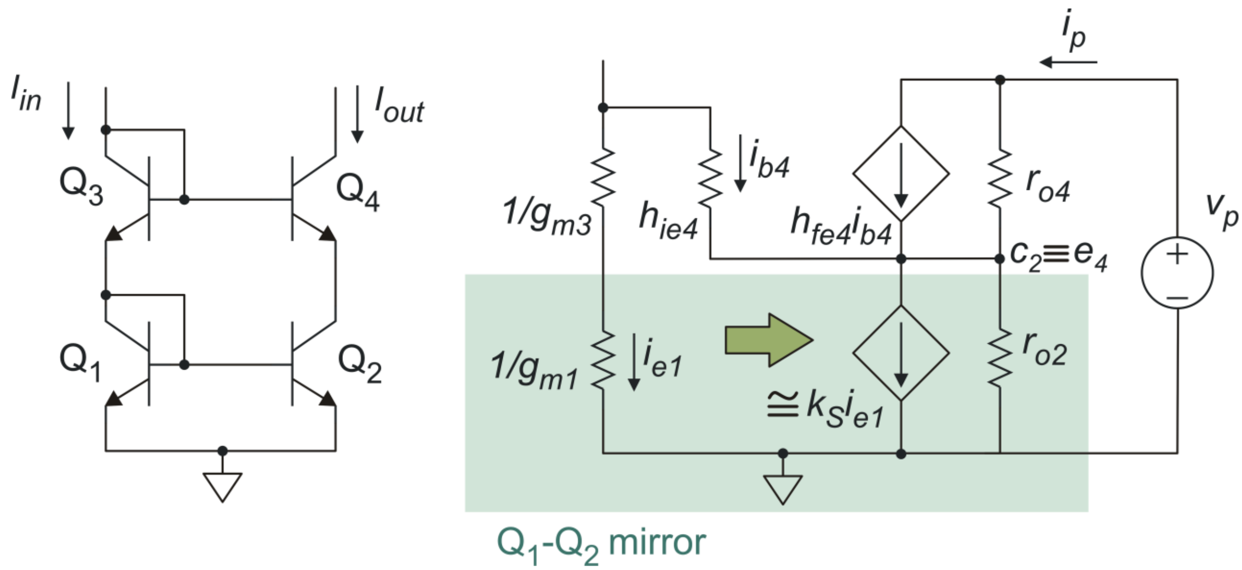


Fig.1.1. BJT Cascode mirror topology (left); Small signal equivalent circuit for output resistance calculation (right).The arrow indicates the input-to-output direction of the $Q_1$-$Q_2$ simple current mirror.

Current balance at $Q_2$ collector node ($Q_4$ emitter) gives:

$$i_{b4} + i_p = k_S i_{e1} + \frac{v_{c2}}{r_{o2}} = -k_S i_{b4} + \frac{v_{c2}}{r_{o2}} \tag{1.1}$$

with $v_{c2}$ given by:

$$v_{c2} = v_{e4} = -i_{b4}\left(\frac{1}{g_{m1}} + \frac{1}{g_{m3}} + h_{ie4}\right) \cong -i_{b4}h_{ie4} \tag{1.2}$$

Substituting $v_{c2}$ from (1.2) into (1.1) we find:

$$i_p \cong -b i_{b4} \tag{1.3}$$

Where coefficient $b$ is defined as:

$$b = \left(1 + k_S + \frac{h_{ie4}}{r_{o2}}\right) \tag{1.4}$$

Current balance at the output node of the mirror ($Q_4$ collector) gives:

$$i_p = h_{fe} i_{b4} + \frac{v_p - v_{e4}}{r_{o4}} \tag{1.5}$$

Substituting $v_{e4}$ from (1.2) into (1.5) we get:

$$i_p = h_{fe} i_{b4} + \frac{v_p + i_{b4} h_{ie4}}{r_{o4}} \tag{1.6}$$

Using (1.3):

$$i_p = -h_{fe} \frac{i_p}{b} + \frac{v_p}{r_{o4}} - \frac{i_p}{b} \frac{h_{ie4}}{r_{o4}} \tag{1.7}$$

From (1.7) , the output resistance of the cascode mirror is given by:

$$R_{out} = \frac{v_p}{i_p} = r_{o4} \left(1 + \frac{h_{fe}}{b} + \frac{1}{b} \frac{h_{ie4}}{r_{o4}}\right) \tag{1.8}$$

Considering that, generally, for a BJT operating in active region, $h_{ie} << r_o$, we find the approximation:

$$R_{out} = \frac{v_p}{i_p} \cong r_{o4} \left(1 + \frac{h_{fe}}{k_S + 1}\right) \tag{1.9}$$

For a current mirror designed to provide a unity gain ($k_S$=1), we get:

$$R_{out} \cong r_{o4} \left(1 + \frac{h_{fe}}{2}\right) \tag{1.10}$$

## 1.2 Wilson Current Mirror

The topology of a 4-transistor current mirror and the small-signal equivalent circuit used for calculation of the output resistance is shown in Fig. 1.2. The circuit is probed by the voltage source $v_p$. The output resistance is $v_p/i_p$. In order to simplify the calculations, the simple mirror formed by $Q_1$ and $Q_2$ is modeled with its small signal equivalent circuit. Note that the $Q_2$-$Q_1$ mirror transmits the currents in the opposite direction with respect to the case of the cascode mirror in Fig. 1.1 (see the arrow in both figures). The current gain of the $Q_2$-$Q_1$ mirror is equal to $1/k_S$, which is the inverse of the overall current gain of the Wilson mirror ($k_S$). Calculations will be carried out considering a generic $k_S$. However, it should be observed that the Wilson current mirror provides base current compensation only for $k_S$=1.

Neglecting the current into $r_{o1}$ resistor, it is possible to find that $i_{b4} = -i_{e2}/k_S$. This approximation can be easily motivated considering the mesh formed by resistors $1/g_{m2}$, $h_{ie4}$, $1/g_{m3}$ and $r_{o1}$. Voltage balance gives:

$$i_{e2}\frac{1}{g_{m2}} = -i_{b4}\left(h_{ie4}+\frac{1}{g_{m3}}\right)+\left(-i_{b4}-\frac{i_{e2}}{k_S}\right)r_{o1} \tag{1.11}$$

The resulting $i_{e2}$ vs $i_{b4}$ relationship is:

$$i_{e2} = -i_{b4}\frac{\left(h_{ie4}+\frac{1}{g_{m3}}+r_{o1}\right)}{\left(\frac{1}{g_{m2}}+\frac{r_{o1}}{k_S}\right)} = -i_{b4}\frac{\left(1+\frac{h_{ie4}}{r_{o1}}+\frac{1}{r_{o1}g_{m3}}\right)}{\left(\frac{1}{k_S}+\frac{1}{r_{o1}g_{m2}}\right)} \cong -k_S i_{b4} \;\Rightarrow\; i_{b4} = -\frac{1}{k_s}i_{e2} \tag{1.12}$$

The approximation is due to the fact that, generally, for BJTs operating in active region, $h_{ie} << r_o$, and $g_m r_o >> 1$.

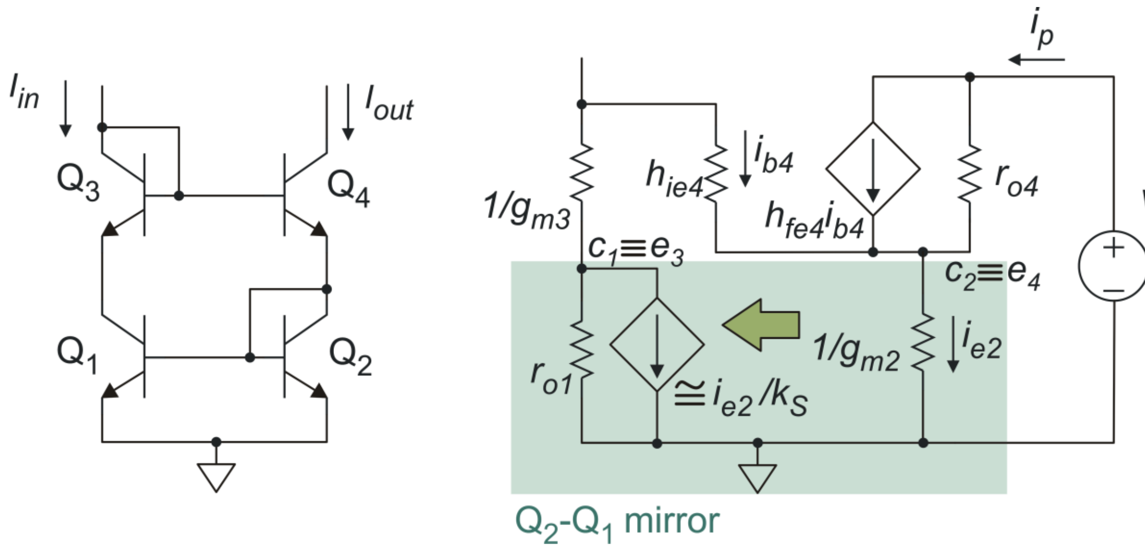


Fig. 1.2. Topology of a 4-BJT Wilson current mirror (left) and small signal equivalent circuit used for calculation of the output resistance. The arrow indicates the input-to-output direction of the $Q_1$-$Q_2$ simple current mirror.

Current balance at node $c_2$, gives:

$$i_p = i_{e2} - i_{b4} \cong -\left(k_S+1\right)i_{b4} \cong \left(1+\frac{1}{k_S}\right)i_{e2} \tag{1.13}$$

Current balance at the output node gives:

$$i_p = h_{fe4}i_{b4} + \frac{v_p - v_{e4}}{r_{o4}} = h_{fe4}i_{b4} + \frac{v_p - \frac{i_{e2}}{g_{m2}}}{r_{o4}} \tag{1.14}$$

Using (1.13) for $i_{e2}$ and $i_{b4}$, we finally find an equation where only variables $i_p$ and $v_p$ are present.

$$i_p = -h_{fe4}\frac{i_p}{1+k_S} + \frac{v_p}{r_{o4}} - \frac{i_p}{1+k_S^{-1}}\frac{1}{g_{m2}r_{o4}} \tag{1.15}$$

which gives the output resistance:

$$R_{out} = \frac{v_p}{i_p} = r_{o4}\left(1+\frac{h_{fe4}}{1+k_S}+\frac{1}{\left(1+k_S^{-1}\right)g_{m2}r_{o4}}\right) \cong r_{o4}\left(1+\frac{h_{fe4}}{1+k_S}\right) \tag{1.16}$$

Since the Wilson current mirror is generally used with $k_S$=1, $R_{out}$ becomes:

$$R_{out} \cong r_{o4}\left(1+\frac{h_{fe4}}{2}\right) \quad (\text{for } k_S = 1) \tag{1.17}$$