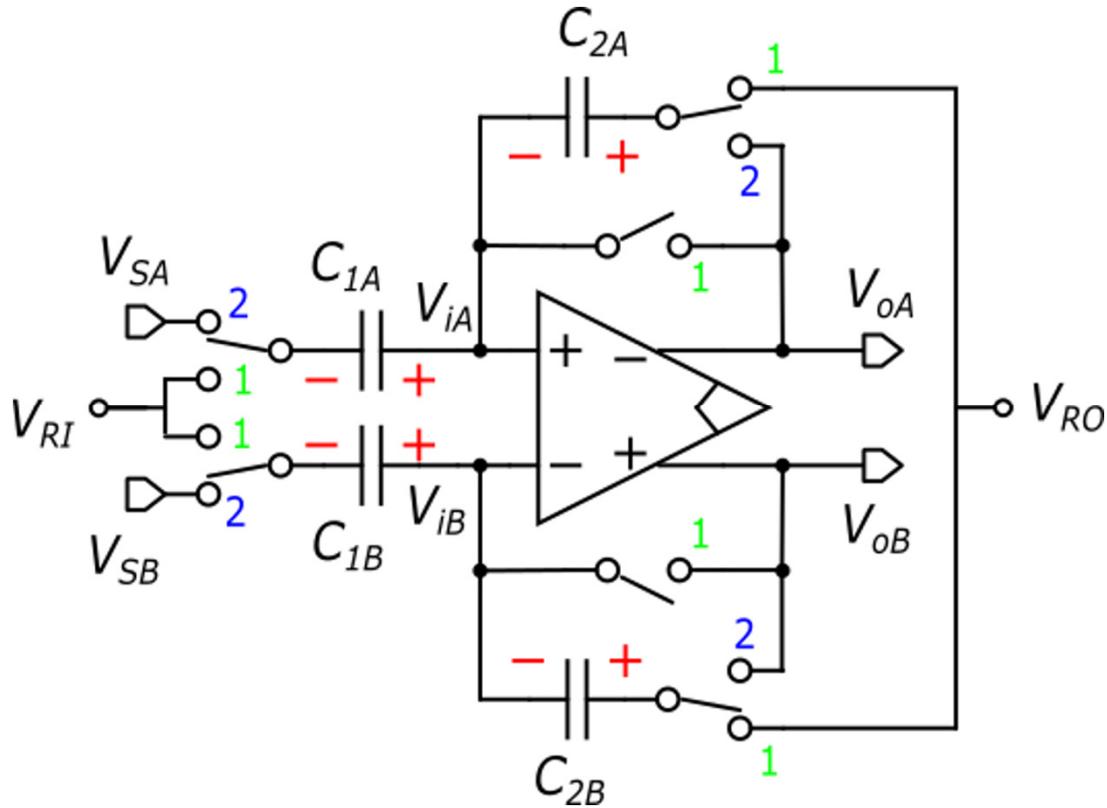


# Fully-differential switched capacitor amplifier



$$V_{Sd} = V_{SA} - V_{SB} \quad \text{Input. diff. voltage}$$

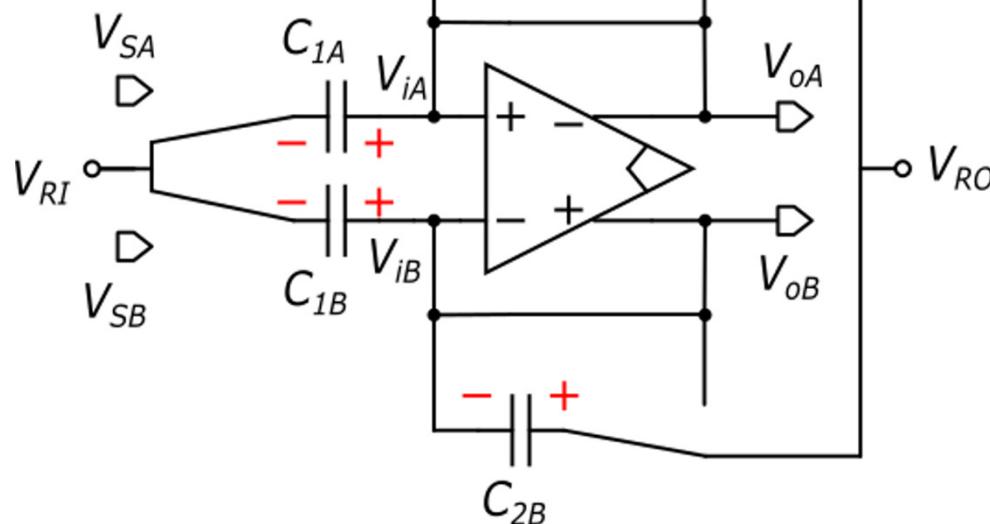
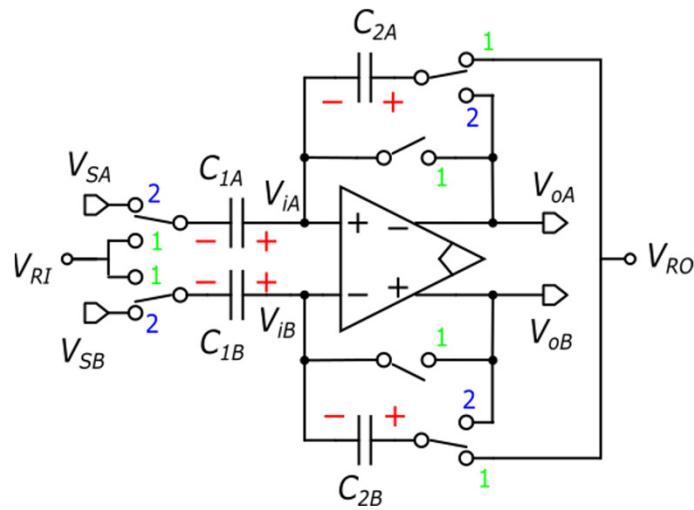
$$V_{od} = V_{oB} - V_{oA} \quad \text{Output. diff. voltage}$$

$V_{RO}$  and  $V_{RI}$  are constant voltages used to periodically discharge capacitor pairs. They substitute *gnd* in a single-supply circuit

We will consider the nominal case for the capacitors:

$$\begin{cases} C_{1A} = C_{1B} \equiv C_1 \\ C_{2A} = C_{2B} \equiv C_2 \end{cases}$$

## Phase 1



Due to the unity-gain connection

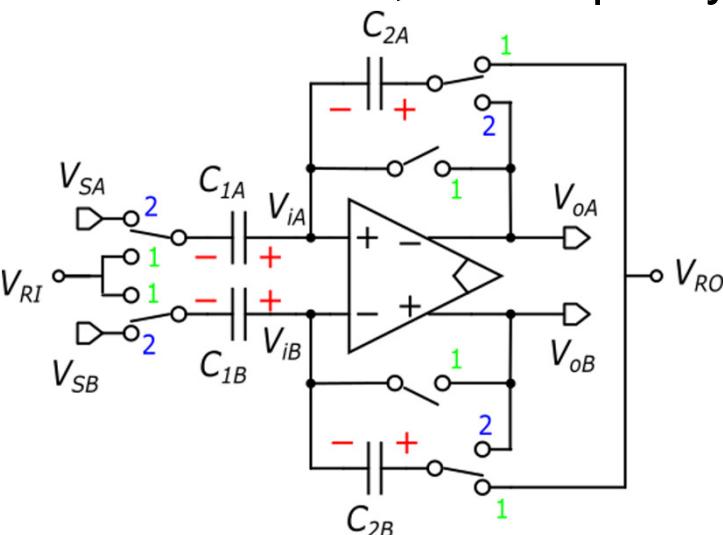
$$v_{iA}^{(1)} = V_{CMO} + \frac{v_n^{(1)}}{2}; \quad v_{iB}^{(1)} = V_{CMO} - \frac{v_n^{(1)}}{2}$$

Let us write the voltages across each capacitor

$$\left\{ \begin{array}{l} v_{C1A}^{(1)} = V_{CMO} + \frac{v_n^{(1)}}{2} - V_{RI} \\ v_{C1B}^{(1)} = V_{CMO} - \frac{v_n^{(1)}}{2} - V_{RI} \\ v_{C2A}^{(1)} = V_{RO} - V_{CMO} - \frac{v_n^{(1)}}{2} \\ v_{C2B}^{(1)} = V_{RO} - V_{CMO} + \frac{v_n^{(1)}}{2} \end{array} \right.$$

## Transition to phase 2

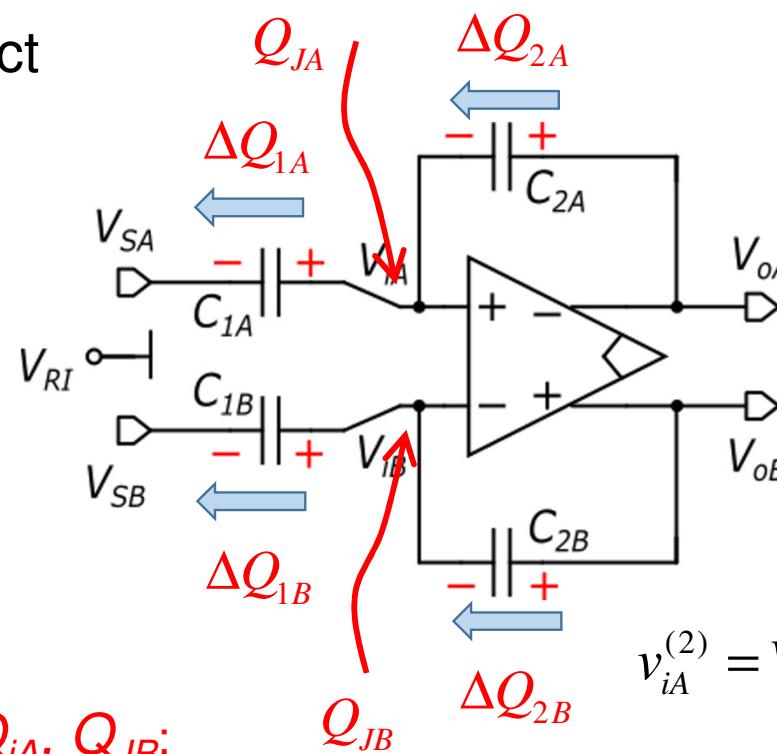
In this analysis we will neglect the kT/C noise, for simplicity



$$v_{C1A}^{(2)} = V_{ic}^{(2)} + \frac{v_n^{(2)}}{2} - V_{SA}$$

$$v_{C1B}^{(2)} = V_{ic}^{(2)} - \frac{v_n^{(2)}}{2} - V_{SB}$$

$Q_{jA}, Q_{JB}$ : caused by charge-injection and other mechanisms



$$\begin{cases} v_{C2A}^{(2)} = v_{C2A}^{(1)} + \frac{\Delta Q_{2A}}{C_2} \\ v_{C2B}^{(2)} = v_{C2B}^{(1)} + \frac{\Delta Q_{2B}}{C_2} \end{cases} \rightarrow V_{RO}$$

$$v_{iA}^{(2)} = V_{ic}^{(2)} + \frac{v_n^{(2)}}{2};$$

$$v_{iB}^{(2)} = V_{ic}^{(2)} - \frac{v_n^{(2)}}{2}$$

$$\begin{cases} \Delta Q_{2A} = \Delta Q_{1A} - Q_{JA} = C_1 (v_{C1A}^{(2)} - v_{C1A}^{(1)}) - Q_{JA} \\ \Delta Q_{2B} = \Delta Q_{1B} - Q_{JB} = C_1 (v_{C1B}^{(2)} - v_{C1B}^{(1)}) - Q_{JB} \end{cases}$$

$$\begin{cases} v_{C2A}^{(2)} = v_{C2A}^{(1)} + \frac{\Delta Q_{2A}}{C_2} \\ v_{C2B}^{(2)} = v_{C2B}^{(1)} + \frac{\Delta Q_{2B}}{C_2} \end{cases}$$

$$\Delta Q_{2A} = C_1 \left( v_{C1A}^{(2)} - v_{C1A}^{(1)} \right) - Q_{JA}$$

$$\Delta Q_{2B} = C_1 \left( v_{C1B}^{(2)} - v_{C1B}^{(1)} \right) - Q_{JB}$$

$$v_{C2A}^{(2)} = V_{RO} - V_{CMO} - \frac{v_n^{(1)}}{2} + \frac{C_1}{C_2} \left( V_{ic}^{(2)} + \frac{v_n^{(2)}}{2} - V_{SA} - V_{CMO} - \frac{v_n^{(1)}}{2} + V_{RI} \right) - \frac{Q_{JA}}{C_2}$$

$$v_{C2B}^{(2)} = V_{RO} - V_{CMO} + \frac{v_n^{(1)}}{2} + \frac{C_1}{C_2} \left( V_{ic}^{(2)} - \frac{v_n^{(2)}}{2} - V_{SB} - V_{CMO} + \frac{v_n^{(1)}}{2} + V_{RI} \right) - \frac{Q_{JB}}{C_2}$$

**Phase 2**

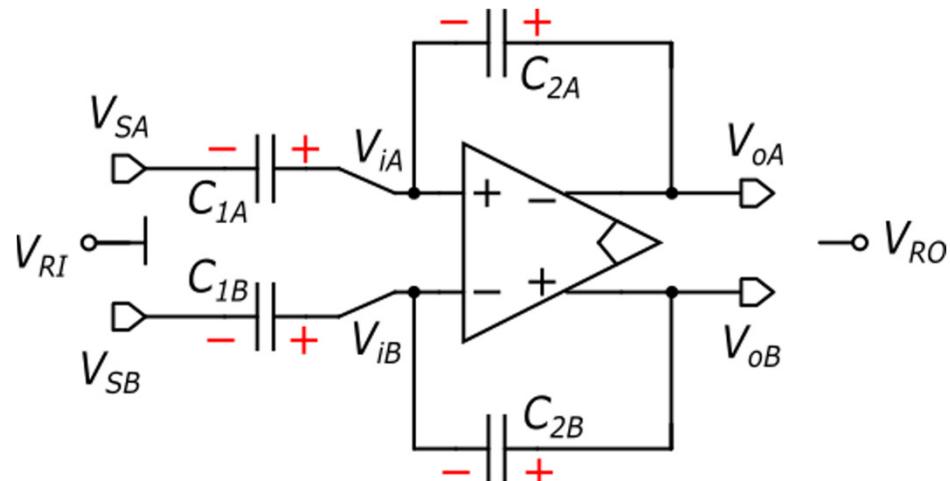
$$\begin{aligned} v_{C1A}^{(2)} &= V_{ic}^{(2)} + \frac{v_n^{(2)}}{2} - V_{SA} \\ v_{C1B}^{(2)} &= V_{ic}^{(2)} - \frac{v_n^{(2)}}{2} - V_{SB} \end{aligned}$$

**Phase 1**

$$\begin{cases} v_{C1A}^{(1)} = V_{CMO} + \frac{v_n^{(1)}}{2} - V_{RI} \\ v_{C1B}^{(1)} = V_{CMO} - \frac{v_n^{(1)}}{2} - V_{RI} \\ v_{C2A}^{(1)} = V_{RO} - V_{CMO} - \frac{v_n^{(1)}}{2} \\ v_{C2B}^{(1)} = V_{RO} - V_{CMO} + \frac{v_n^{(1)}}{2} \end{cases}$$

$$v_{C2A}^{(2)} = V_{RO} - V_{CMO} - \frac{v_n^{(1)}}{2} + \frac{C_1}{C_2} \left( V_{ic}^{(2)} + \frac{v_n^{(2)}}{2} - V_{SA} - V_{CMO} - \frac{v_n^{(1)}}{2} + V_{RI} \right) - \frac{Q_{JA}}{C_2}$$

$$v_{C2B}^{(2)} = V_{RO} - V_{CMO} + \frac{v_n^{(1)}}{2} + \frac{C_1}{C_2} \left( V_{ic}^{(2)} - \frac{v_n^{(2)}}{2} - V_{SB} - V_{CMO} + \frac{v_n^{(1)}}{2} + V_{RI} \right) - \frac{Q_{JB}}{C_2}$$



$$\begin{cases} v_{oA}^{(2)} = v_{iA}^{(2)} + v_{C2A}^{(2)} \\ v_{oB}^{(2)} = v_{iB}^{(2)} + v_{C2B}^{(2)} \end{cases}$$

$$v_{iA}^{(2)} = V_{ic}^{(2)} + \frac{v_n^{(2)}}{2}; \quad v_{iB}^{(2)} = V_{ic}^{(2)} - \frac{v_n^{(2)}}{2}$$

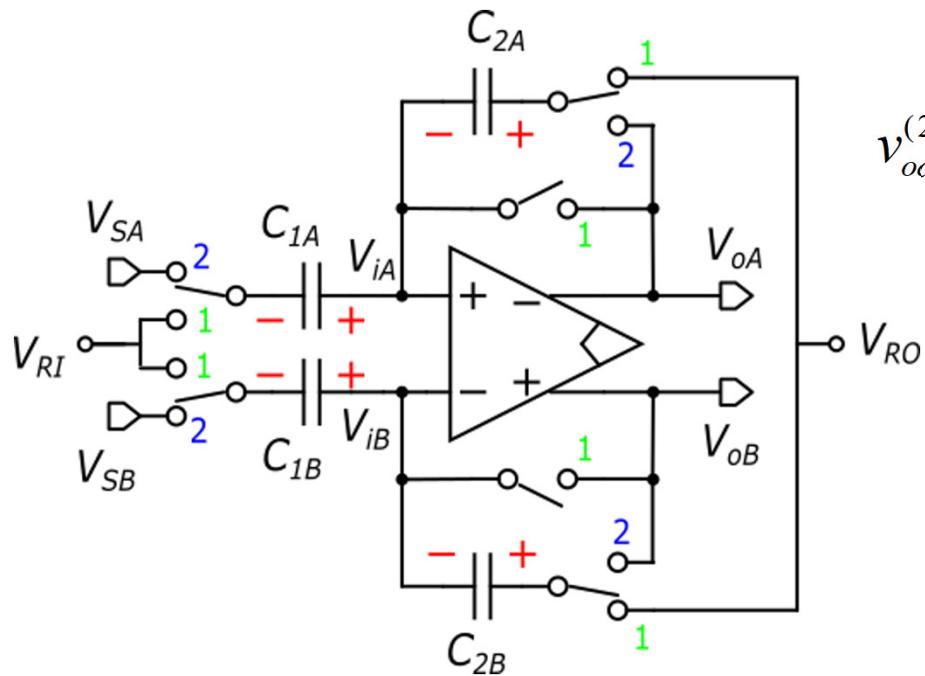
$$\begin{cases} v_{oA}^{(2)} = V_{ic}^{(2)} + \frac{v_n^{(2)}}{2} + V_{RO} - V_{CMO} - \frac{v_n^{(1)}}{2} + \frac{C_1}{C_2} \left( V_{ic}^{(2)} + \frac{v_n^{(2)}}{2} - V_{SA} - V_{CMO} - \frac{v_n^{(1)}}{2} + V_{RI} \right) - \frac{Q_{JA}}{C_2} & - \\ v_{oB}^{(2)} = V_{ic}^{(2)} - \frac{v_n^{(2)}}{2} + V_{RO} - V_{CMO} + \frac{v_n^{(1)}}{2} + \frac{C_1}{C_2} \left( V_{ic}^{(2)} - \frac{v_n^{(2)}}{2} - V_{SB} - V_{CMO} + \frac{v_n^{(1)}}{2} + V_{RI} \right) - \frac{Q_{JB}}{C_2} & + \end{cases}$$

$$v_{od}^{(2)} = v_{oB}^{(2)} - v_{oA}^{(2)}$$

Differential mode analysis

$$v_{od}^{(2)} = -v_n^{(2)} + v_n^{(1)} + \frac{C_1}{C_2} \left( -v_n^{(2)} + V_{SA} - V_{SB} + v_n^{(1)} \right) - \frac{Q_{JB} - Q_{JA}}{C_2}$$

$$v_{od}^{(2)} = \frac{C_1}{C_2} \left( V_{SA} - V_{SB} \right) - \left( v_n^{(2)} - v_n^{(1)} \right) \left( \frac{C_1}{C_2} + 1 \right) - \frac{Q_{JB} - Q_{JA}}{C_2}$$



$$v_{od}^{(2)} = \frac{C_1}{C_2} (V_{SA} - V_{SB}) - (v_n^{(2)} - v_n^{(1)}) \left( \frac{C_1}{C_2} + 1 \right) - \frac{Q_{JB} - Q_{JA}}{C_2}$$

$V_{Sd}$       output noise due to the amplifier      output error due to charge injection

$$A_{dd} = \frac{C_1}{C_2} \equiv A$$

$$\text{Errors referred to the input } -(v_n^{(2)} - v_n^{(1)}) \left( \frac{A+1}{A} \right) - \frac{Q_{JB} - Q_{JA}}{C_1}$$

Note that CDS is applied to the amplifier noise

## Common mode components

$$\begin{cases} v_{oA}^{(2)} = V_{ic}^{(2)} + \frac{v_n^{(2)}}{2} + V_{RO} - V_{CMO} - \frac{v_n^{(1)}}{2} + \frac{C_1}{C_2} \left( V_{ic}^{(2)} + \frac{v_n^{(2)}}{2} - V_{SA} - V_{CMO} - \frac{v_n^{(1)}}{2} + V_{RI} \right) - \frac{Q_{JA}}{C_2} & \times \frac{1}{2} \\ v_{oB}^{(2)} = V_{ic}^{(2)} - \frac{v_n^{(2)}}{2} + V_{RO} - V_{CMO} + \frac{v_n^{(1)}}{2} + \frac{C_1}{C_2} \left( V_{ic}^{(2)} - \frac{v_n^{(2)}}{2} - V_{SB} - V_{CMO} + \frac{v_n^{(1)}}{2} + V_{RI} \right) - \frac{Q_{JB}}{C_2} & \times \frac{1}{2} \end{cases}$$

Neglecting the charge injection (calculation of the CM components require less accuracy)

$$V_{CMO} = V_{ic}^{(2)} + V_{RO} - V_{CMO} + \frac{C_1}{C_2} \left( V_{ic}^{(2)} - \frac{V_{SA} + V_{SB}}{2} - V_{CMO} + V_{RI} \right)$$

$$V_{CMO} = V_{ic}^{(2)} (1 + A) + V_{RO} - V_{CMO} (1 + A) + \frac{C_1}{C_2} (-V_{SC} + V_{RI})$$

## Common mode components

$$V_{CMO} = V_{ic}^{(2)} (1 + A) + V_{RO} - V_{CMO} (1 + A) + \frac{C_1}{C_2} (-V_{SC} + V_{RI})$$

$$V_{CMO} - V_{RO} + V_{CMO} (1 + A) - \frac{C_1}{C_2} (-V_{SC} + V_{RI}) = V_{ic}^{(2)} (1 + A)$$

$$V_{ic}^{(2)} = V_{CMO} + \frac{V_{CMO} - V_{RO}}{A+1} + \frac{A}{A+1} (V_{SC} - V_{RI})$$

Since in phase 1  $V_{ic}$  is equal to  $V_{CMO}$ , it is desirable that it does not change passing into phase 2.

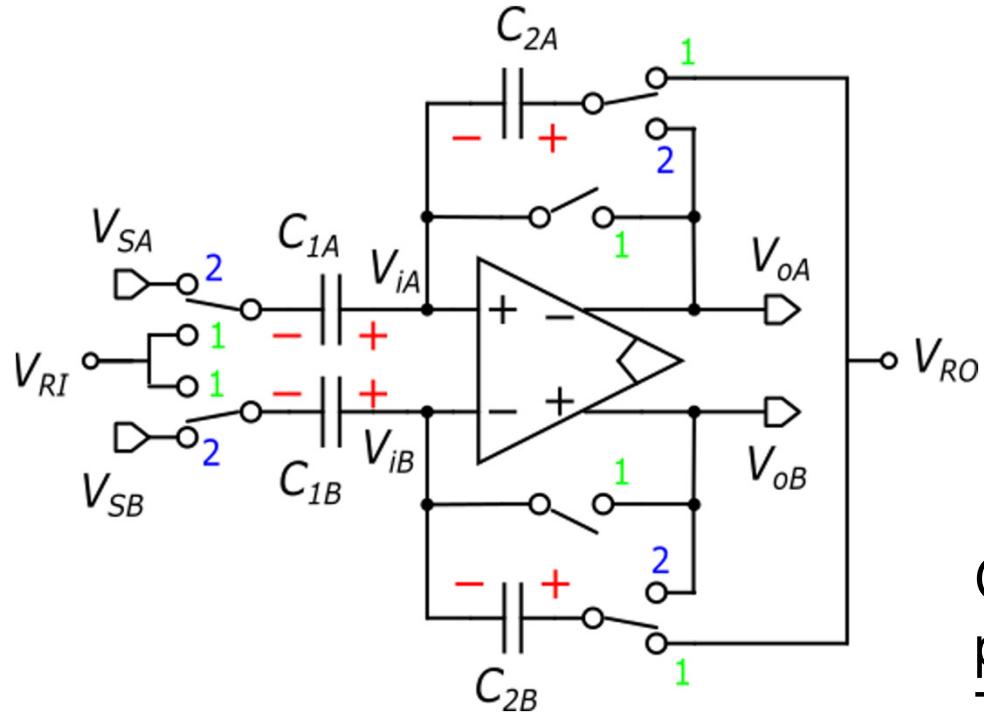
To obtain this, we set:

$$V_{RO} = V_{CMO}$$

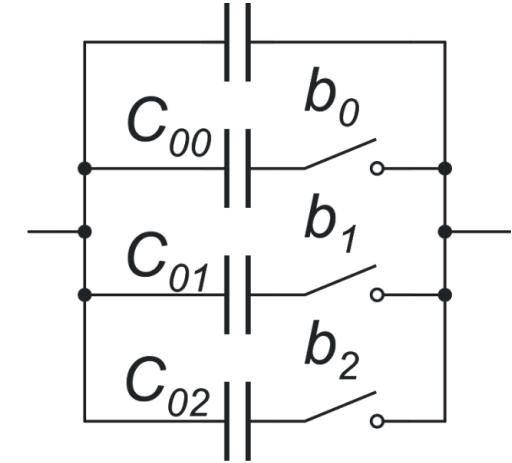
$$V_{RI} = V_{SC}$$

This is not always possible to guarantee, since the common mode of the source may be not predictable

# Programmable gain amplifier (PGA)



$$A_L = \frac{C_1}{C_2}$$



Capacitors can be made easily programmable.  
The advantage with respect of resistive feedback amplifier is that the switch resistance does not affect accuracy (it affects only speed)

## Additional charge injection from non-constant $V_{ic}$

$$V_{ic}^{(1)} = V_{CMO}$$

$$V_{ic}^{(2)} = V_{CMO} + \frac{V_{CMO} - V_{RO}}{A+1} + \frac{A}{A+1}(V_{SC} - V_{RI})$$

In this simplified picture, switches and feedback capacitors are not represented

$$\Delta Q_A = \Delta V_{ic} C_{pA}$$

$$\Delta Q_B = \Delta V_{ic} C_{pB}$$

These charges contribute to  $Q_{JA}$  and  $Q_{JB}$  together with charge injection from the switches

Since parasitic capacitance may have some mismatch, a differential voltage may result.

