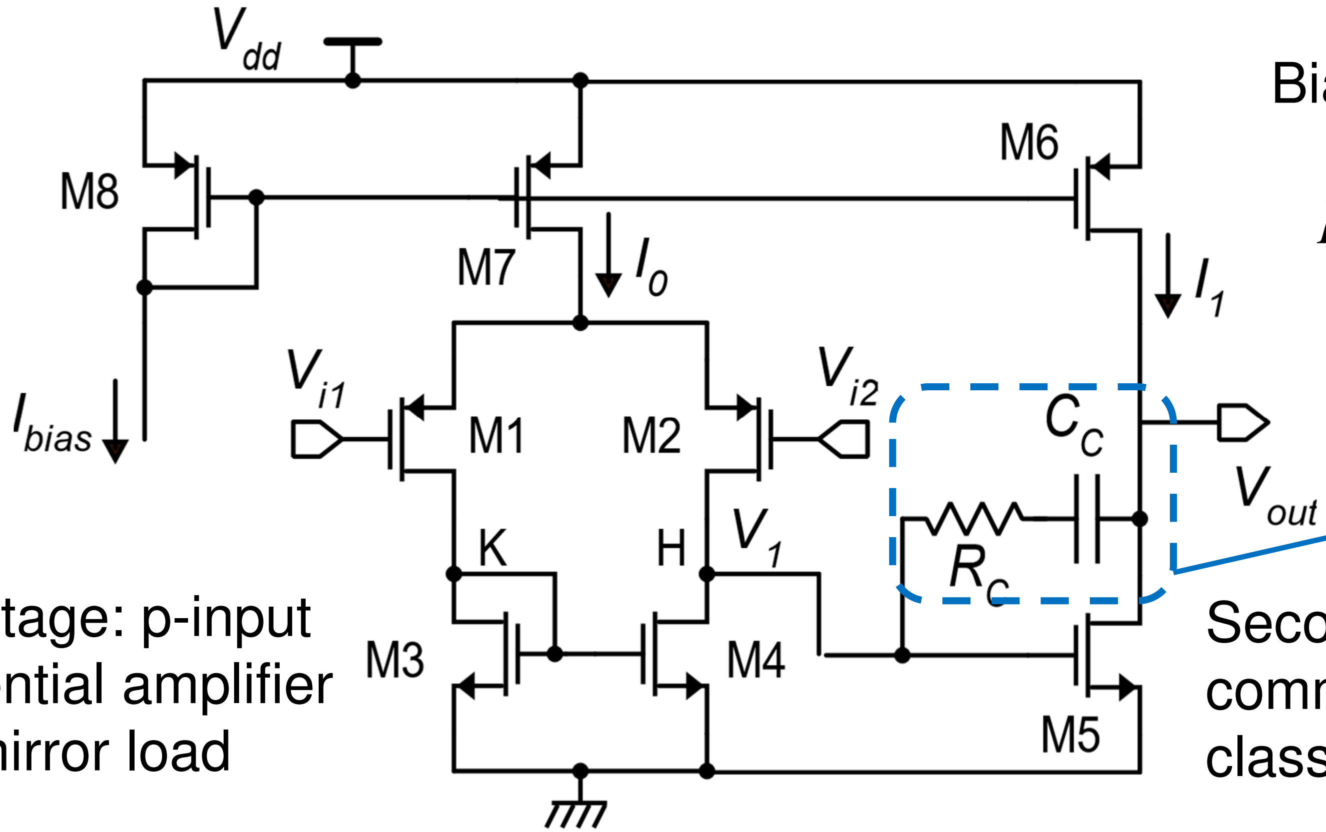
The simplest CMOS two-stage op-amp



Bias currents of the op-amp

$$I_0 = \frac{\beta_7}{\beta_8} I_{bias} \qquad I_1 = \frac{\beta_6}{\beta_8} I_{bias}$$

C_C, R_C: frequency compensation network

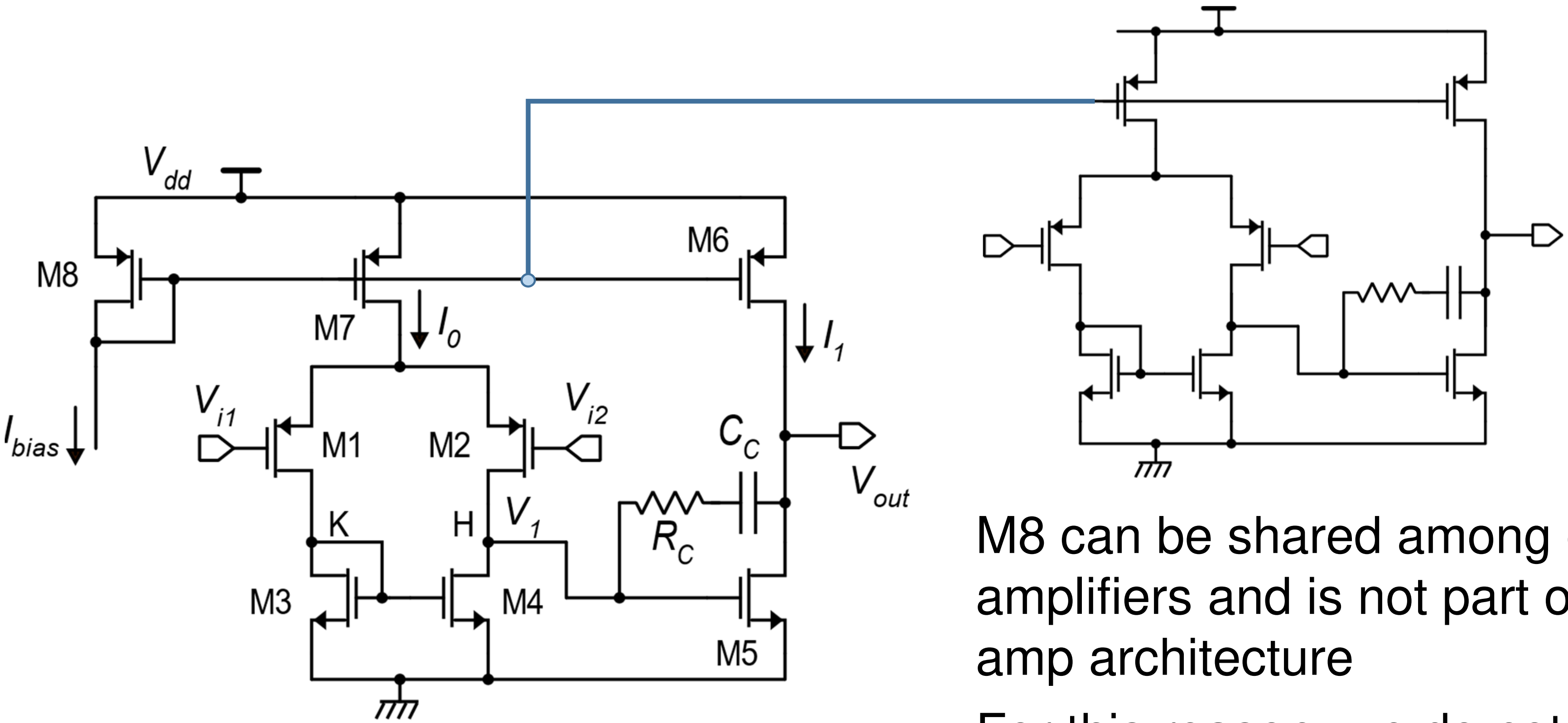
First stage: p-input differential amplifier with mirror load

Second stage: *n*-input common source class-A stage

The output of the first stage is "referred to *gnd*"

The input of the second stage is "referred to *gnd*"

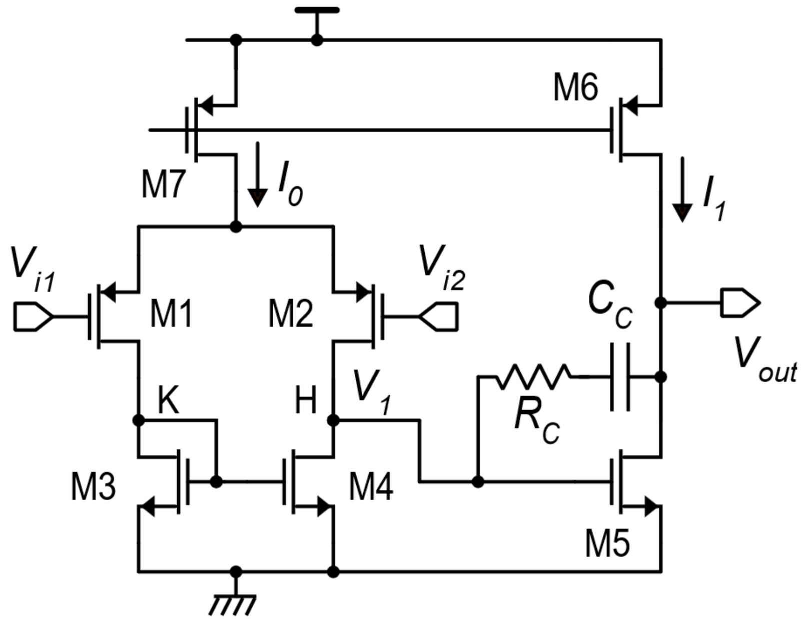
Use of a single M8 device to bias multiple op-amp



M8 can be shared among different amplifiers and is not part of the op-amp architecture

For this reason, we do not consider M8 in the amplifier topology

Degrees Of Freedom (DOFs)



Possible DOFs:

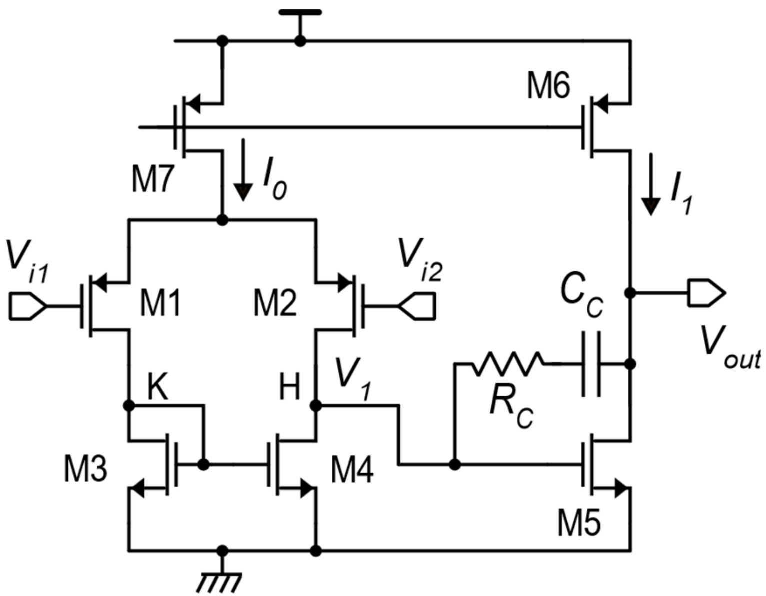
W, L of all devices (14 DOFs)
I_0, I_1
C_C, R_C

First estimate: Total number of DOFS: 18

But ...

- Not all DOFs are independent.
- It is necessary to choose a set of independent DOFs

Constraints



Constraints are relationships among DOFs

Two types of constraints:

- Equality constraints:

E.g. $$\frac{I_1}{I_0} = \frac{\beta_6}{\beta_7}$$

- Inequality constraints:

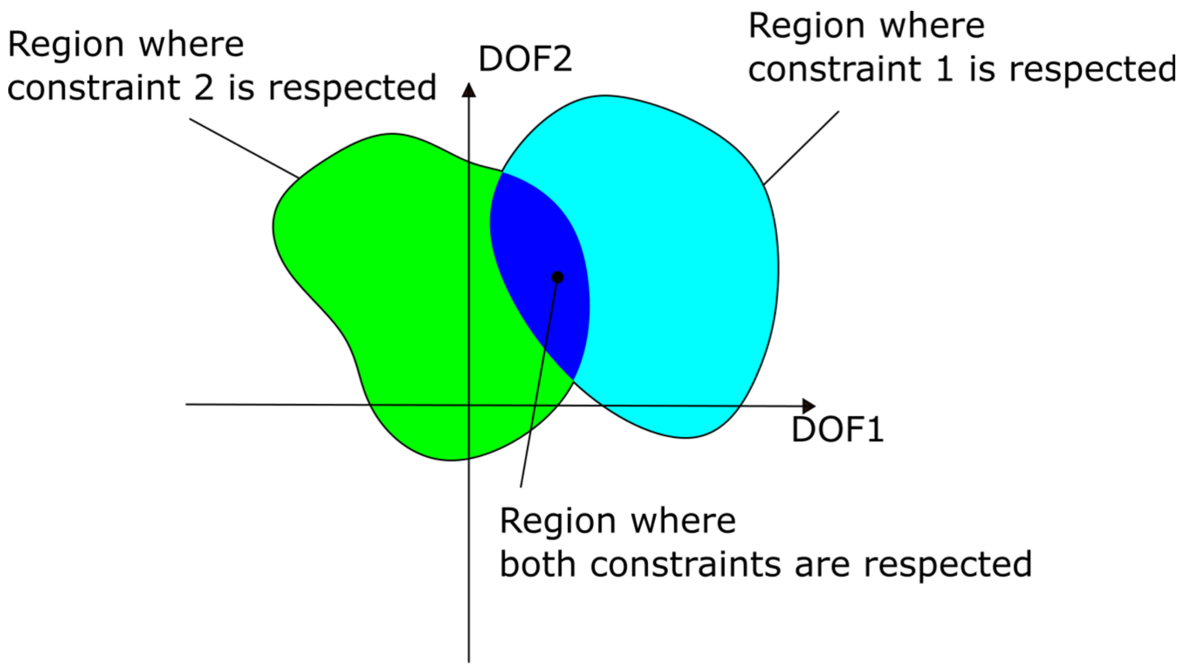
E.g. $$GBW\left(DOFs\right) \geq GBW_{\min}$$

Constraints

- Every **equality constraint** reduces the dimension of the DOF space. Equality constraints represent exact conditions that has to be fulfilled in order to guarantee correct operation of the circuit. Some equality constraints derive from simple considerations, such as symmetry: M1=M2, M3=M4. With a few exceptions, equality constraints are specific of the topology and does not depend on the specifications
- Inequality constraints are derived from the circuit specifications. They do not reduce the dimension of the DOF space but select regions of the DOF space where the specs are met.

The sizing process: role of multiple inequality constraints

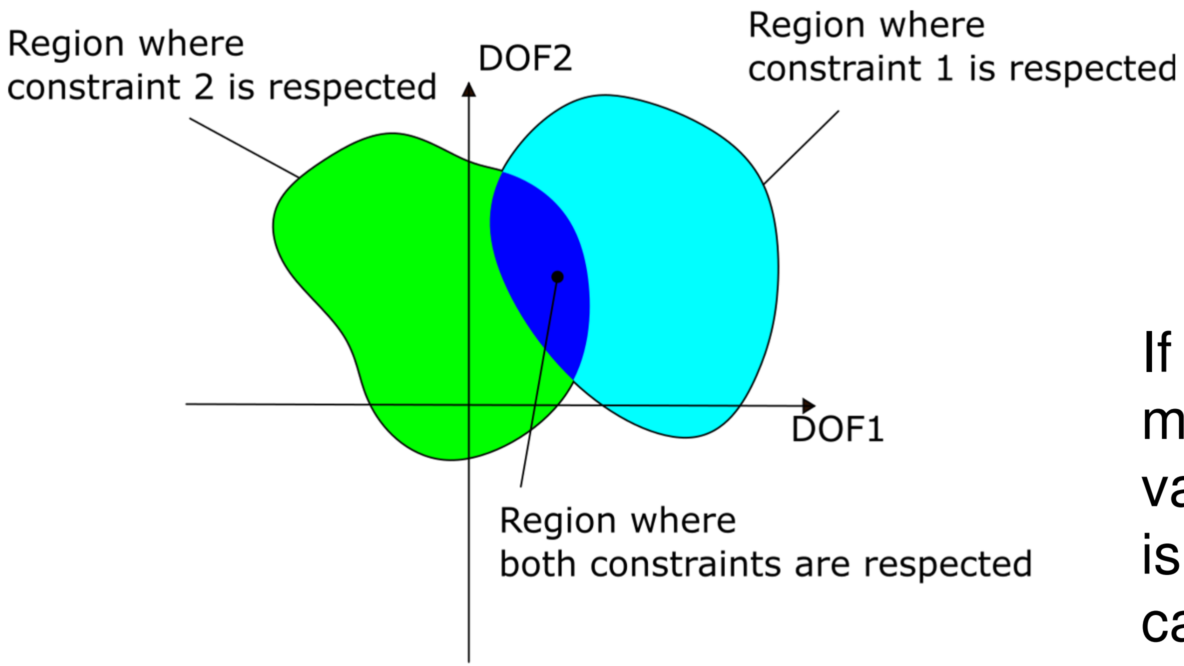
A very simple case with only two independent DOFs and two inequality constraints



Combining the various inequality constraints, we find a domain (the intersection of all regions) where all points satisfy all constraints. All points in the domain are valid solutions.

If such region does not exist (null intersection), the sizing problem is: "unfeasible".

The sizing process: automatic algorithms



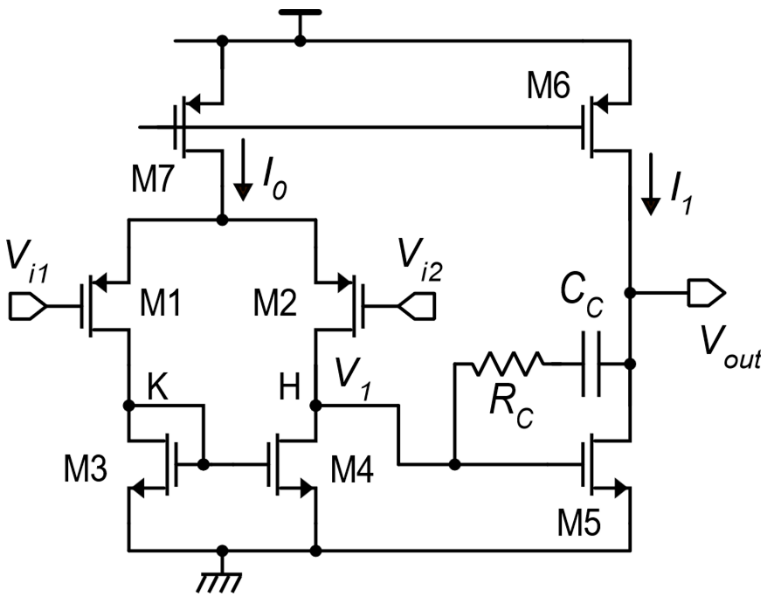
Computer programs that perform automatic sizing, are not compatible with an infinite number of feasible solutions. To find a single solution, an optimization condition is often added.

If the design is performed manually, any point (set of DOFs values) in the intersection domain is a good solution. Also in this case, optimization or arbitrary techniques can be used to operate the choice

Sizing of a new topology: steps

1. Find **equality constraints** to reduce the number of independent DOFs. These constraints will be of two types:
   (a) Strictly necessary constraints (if not respected the circuit does not work properly)
   (b) Arbitrary constraints: they are added to further reduce the DOF set and simplify the design. These constraints should be motivated.
2. Choose a set of DOFs that have the following properties:
   (a) the remaining dependent DOFs can be easily derived from this set;
   (b) the specifications (**inequality constraints**) can be written easily and in a simple form as a function of the selected DOFs
3. Write the specifications in terms of the selected DOFs and try to find general design rules.

Equality constraints for the simple 2-stage op-amp



Symmetry (necessary to obtain low offset and high CMRR

N. of equality constraints

M1=M2 (W_1=W_2, L_1=L_2) ------ 2

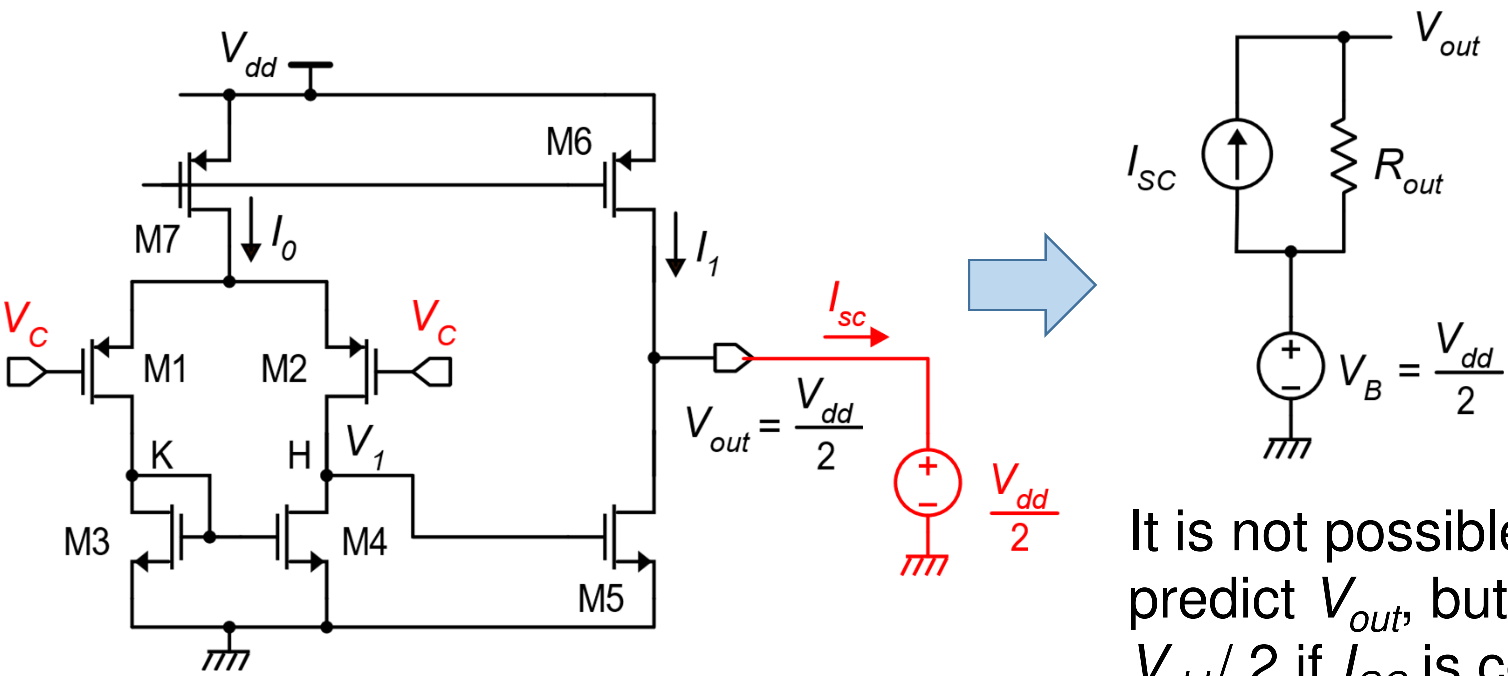
M3=M4 (W_3=W_4, L_3=L_4) ------ 2

Current ratios

$$\frac{I_1}{I_0} = \frac{\beta_6}{\beta_7} = \frac{W_6 / L_6}{W_7 / L_7}$$ ------------- 1

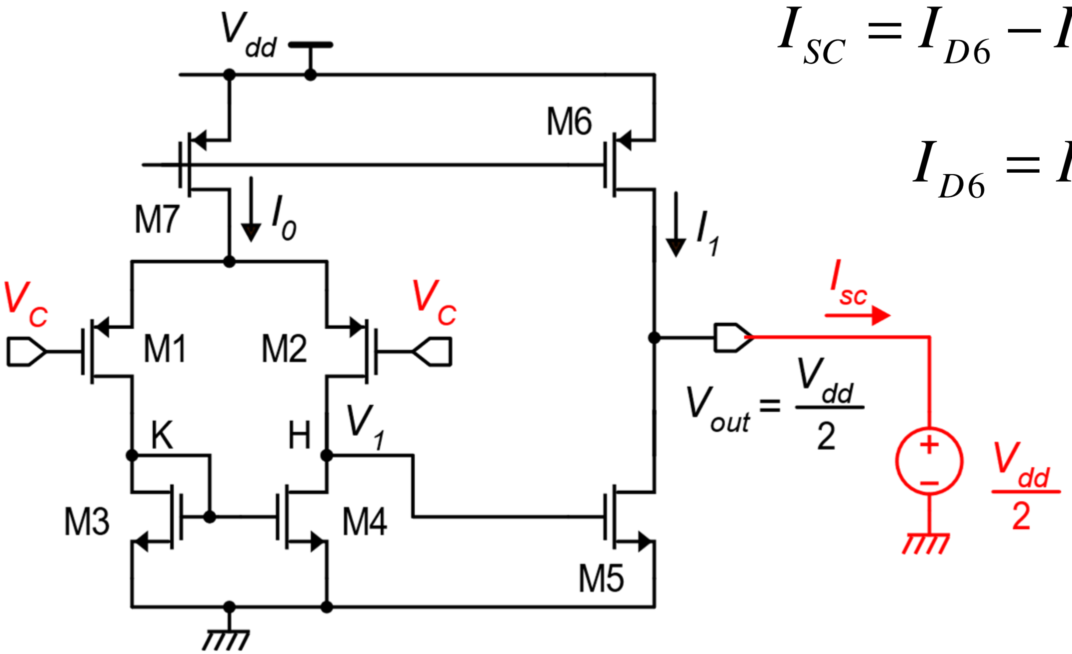
Initial DOF number: 18 , Resulting DOFs after reduction: 18-5=13

Necessary constraint: null systematic offset



It is not possible to exactly predict V_{out}, but it will be far from V_{dd}/ 2 if I_{SC} is consistently different from zero

Necessary constraint: null systematic offset



$$I_{SC} = I_{D6} - I_{D5} = 0 \Rightarrow I_{D6} = I_{D5}$$

$$I_{D6} = I_0 \frac{\beta_6}{\beta_7}$$

Since only a common mode voltage is applied to the input:

$$V_H = V_K \Rightarrow V_{GS5} = V_{GS3}$$

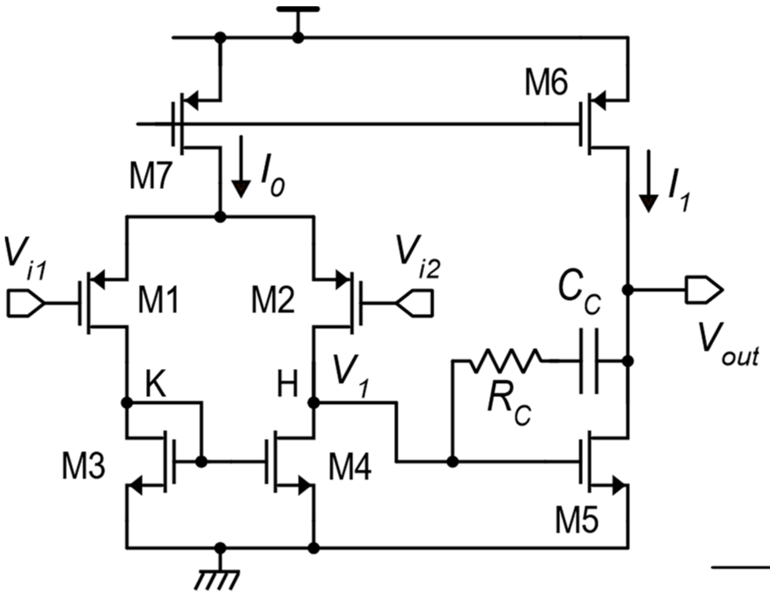
$$I_{D3} = I_{D4} = \frac{I_0}{2}$$

$$I_{D5} = \frac{I_0}{2} \frac{\beta_5}{\beta_3}$$

$$I_{D5} = I_{D6} \Rightarrow \frac{I_0}{2} \frac{\beta_5}{\beta_3} = I_0 \frac{\beta_6}{\beta_7}$$

$$\boxed{\frac{1}{2} \frac{\beta_5}{\beta_3} = \frac{\beta_6}{\beta_7}}$$

More constraints



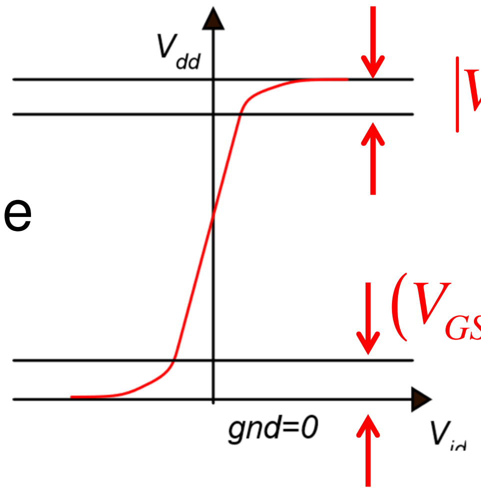
$$\frac{1}{2}\frac{\beta_5}{\beta_3} = \frac{\beta_6}{\beta_7}$$ Null (small) systematic offset

Arbitrary constraints

Good matching M5-M3: $L_5=L_3$

Good matching M6-M7: $L_6=L_7$

Let's now consider the output characteristic



Symmetric output swing (same margins to Vdd and gnd)

$$\left(V_{GS}-V_t\right)_5 = \left|V_{GS}-V_t\right|_6$$

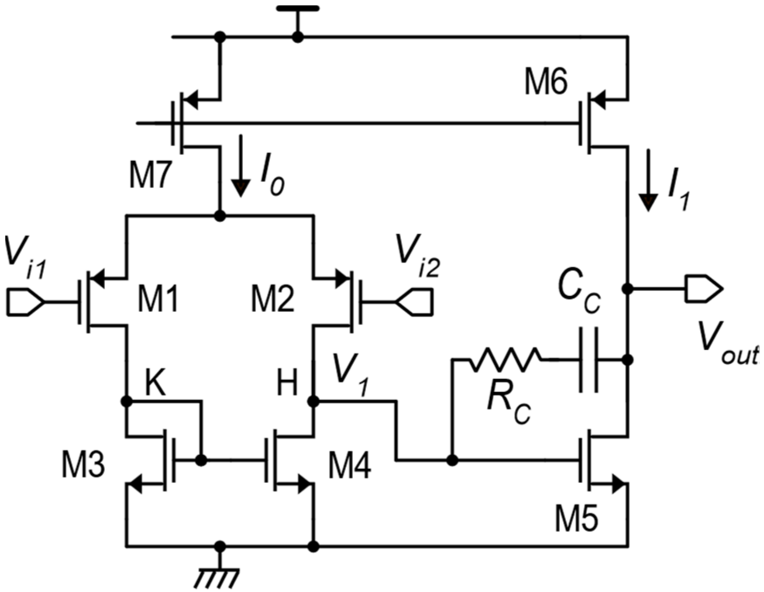
Residual number of DOFs

13 - 4 = 9 Of these residual DOFs we can separate two ones (C_C and R_C) that do not affect the dc performances and the operating point. We will come back to them later. Then we will focus on **7** DOFs (bias current I_0 and device size) that affect the operating point and we will call them "static" DOFs).

We could select 7 DOFs within the original set (16 DOFs, R_C and C_C are not included) and then try to derive the remaining ones using the equations that tie them (equality constraints).

It is more useful to choose a set of DOFs that may not necessarily include the original 18, in a way that the other ones can be easily derived.

Selection of the 7 DOFs



Rationale: the most important MOSFETs of the circuits are M1 (=M2) and M5, since these are the devices that are at the heart of the two stages, where they perform the V-to-I conversion.

We include all possible DOFs of M1 and M5 into the selected set

$$\mathrm{M1}:\ W_1,\ L_1,\ \left|V_{GS}-V_t\right|_1$$

$$\mathrm{M5}:\ W_5,\ L_5,\ (V_{GS}-V_t)_5$$

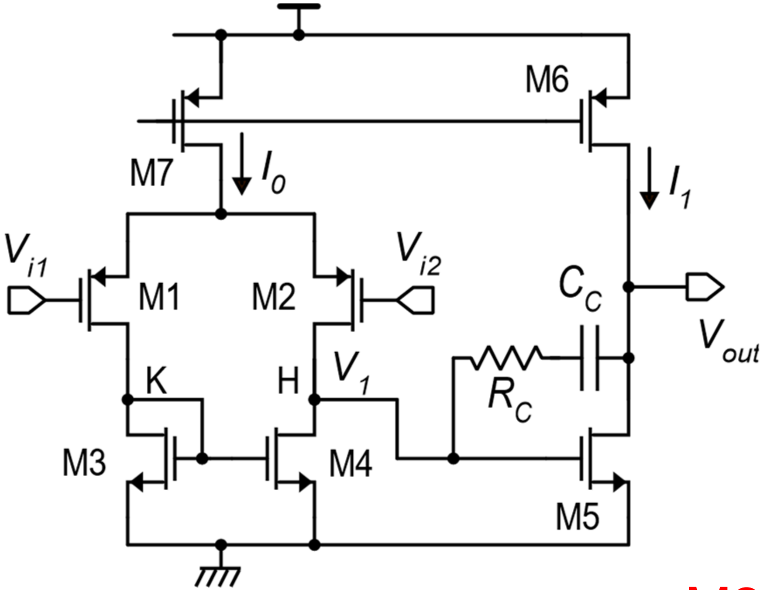
6 DOFs

To complete the set, let us include also L_6 into the DOFs

Final set of static DOFs: $\left\{W_1,\ L_1,\ \left|V_{GS}-V_t\right|_1,\ W_5,\ L_5,\ (V_{GS}-V_t)_5,\ L_6\right\}$

Derivation of all the op-amp parameters from the 7 DOFs

All conditions will refer to the operating point ($V_{id}=0$)



M2 is dentical to M1, then M1 DOFs specify also M2 parameters

$$I_0 = 2I_{D1}, \quad I_1 = I_{D5}$$

$$I_{D1} = f_p(\underline{W_1, L_1, |V_{GS} - V_t|_1}) = \frac{\mu_p C_{OX}}{2} \frac{\underline{W_1}}{\underline{L_1}} \underline{(V_{GS} - V_t)}_1^2$$

$$I_{D5} = f_n\left[\underline{W_5, L_5, (V_{GS} - V_t)}_5\right] = \frac{\mu_n C_{OX}}{2} \frac{\underline{W_5}}{\underline{L_5}} \underline{(V_{GS} - V_t)}_5^2$$

M3=M4:

$$L_3 = \underline{L_5}$$

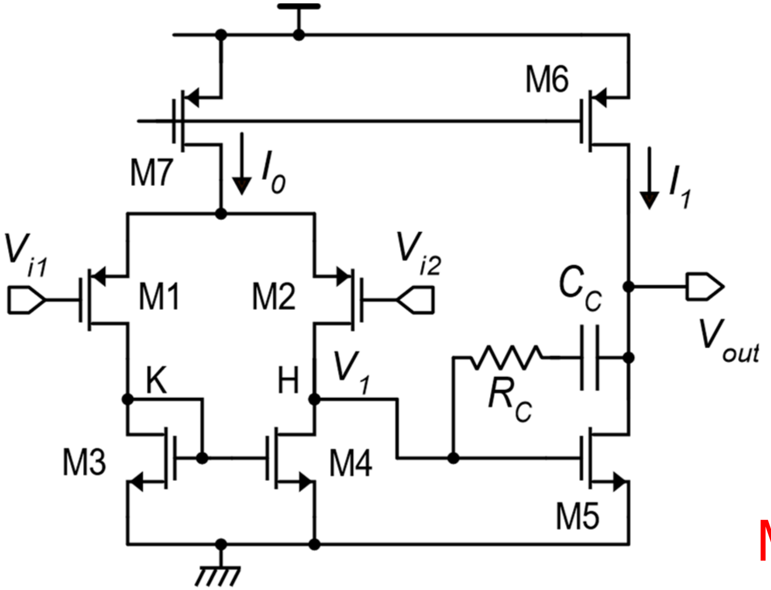
$$I_{D3} = I_{D1}$$

$$(V_{GS} - V_t)_3 = \underline{(V_{GS} - V_t)_5}$$

$$\frac{\mu_n C_{OX}}{2} \frac{W_3}{L_3} = \frac{I_{D3}}{(V_{GS} - V_t)_3^2} \Rightarrow \frac{W_3}{L_3}$$

$$\boxed{W_3, L_3}$$

Derivation of all the op-amp parameters from the 7 DOFs



M6:

$$I_{D6} = I_{D5}$$

$$\left|V_{GS} - V_t\right|_6 = \underline{\left(V_{GS} - V_t\right)_5}$$

$$\frac{\mu_p C_{OX}}{2}\frac{W_6}{L_6} = \frac{I_{D6}}{\left(V_{GS} - V_t\right)_6^2} \Rightarrow \frac{W_6}{L_6}$$

$$\underline{L_6}$$

$$\boxed{W_6,\, L_6}$$

M7:

$$I_{D7} = 2I_{D1}$$

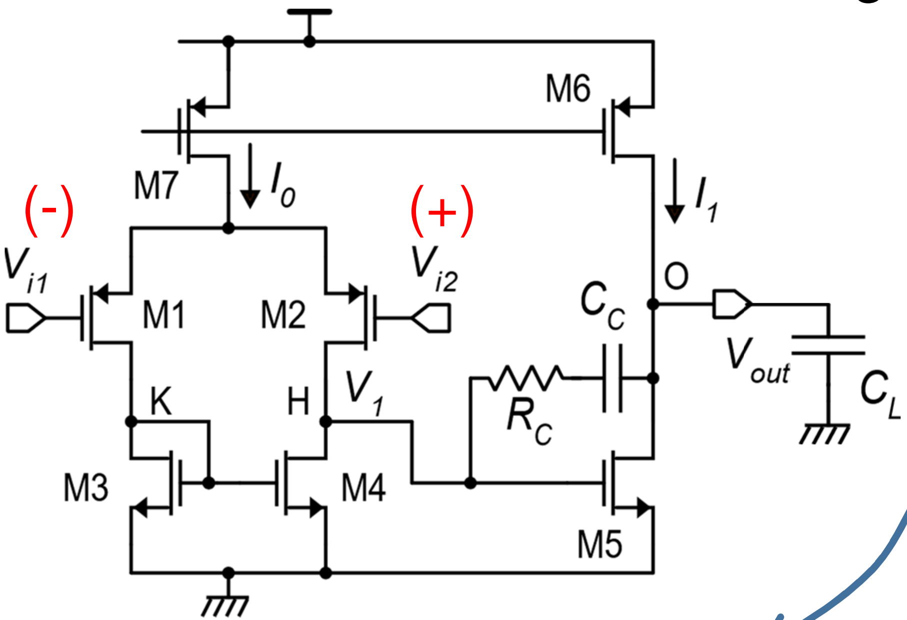
$$\left|V_{GS} - V_t\right|_7 = \left|V_{GS} - V_t\right|_6$$

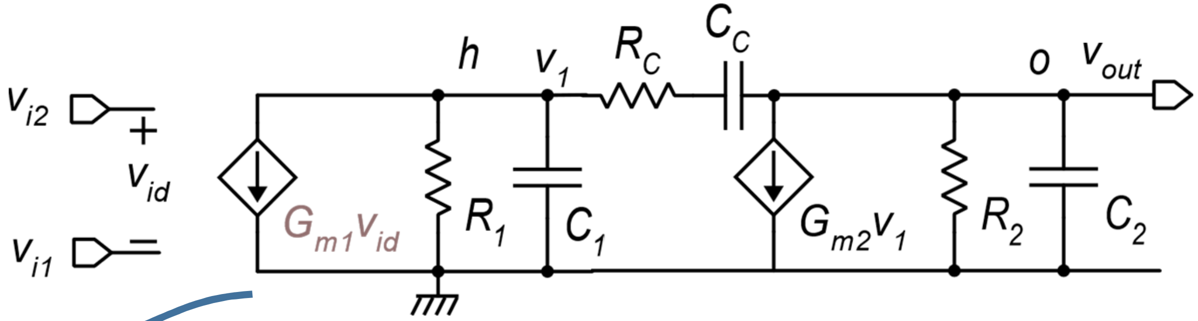
$$\frac{W_7}{L_7}$$

$$L_7 = \underline{L_6}$$

$$\boxed{W_7,\, L_7}$$

Small-signal equivalent circuit





Note: the equivalent circuit can be used to represent the behavior of most two-stage topologies, not just the simple amplifier of the figure.

$$G_{m1} = g_{m1} \qquad R_1 = r_{d4} \| r_{d2} = r_{d1} \| r_{d3}$$

$$G_{m2} = g_{m5} \qquad R_2 = r_{d5} \| r_{d6}$$

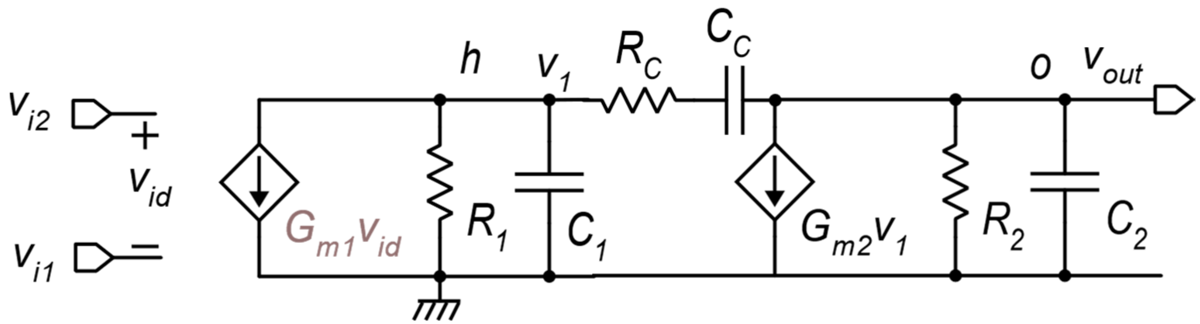
$$C_1 = C_{DB2} + C_{DB4} + C_{GS5}$$

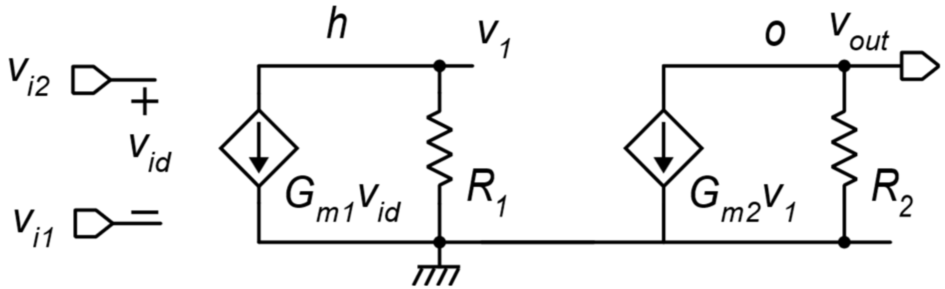
$$C_2 = C_2' + C_L$$

$$C_2' = C_{DB5} + C_{DB6}$$

All these values are functions of the DOFs

dc gain



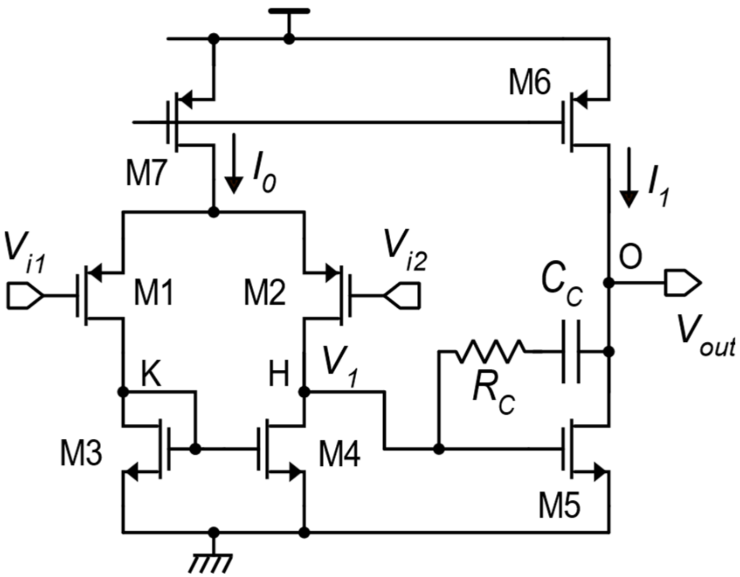


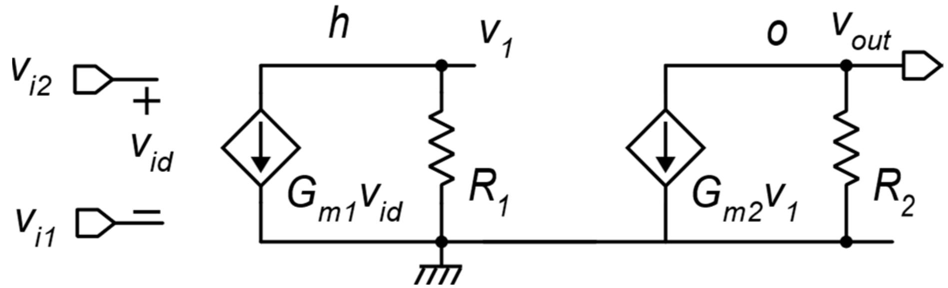
$$v_1 = -G_{m1}v_{id} \cdot R_1$$

$$v_{out} = -G_{m2}v_1 \cdot R_2$$

$$v_{out} = -G_{m2}\left(-G_{m1}v_{id} \cdot R_1\right)R_2 = G_{m1}R_1G_{m2}R_2v_{id} \Rightarrow A_0 = G_{m1}R_1G_{m2}R_2$$

dc gain as a function of the DOFs





$$A_0 = G_{m1} R_1 G_{m2} R_2$$

$$G_{m1} = g_{m1}$$

$$G_{m2} = g_{m5}$$

$$R_1 = r_{d4} \| r_{d2} = r_{d1} \| r_{d3}$$

$$R_2 = r_{d5} \| r_{d6}$$

$$A_0 = g_{m1} \frac{1}{\dfrac{1}{r_{d1}} + \dfrac{1}{r_{d3}}} g_{m5} \frac{1}{\dfrac{1}{r_{d5}} + \dfrac{1}{r_{d6}}}$$

$$g_m = \frac{I_D}{V_{TE}}$$

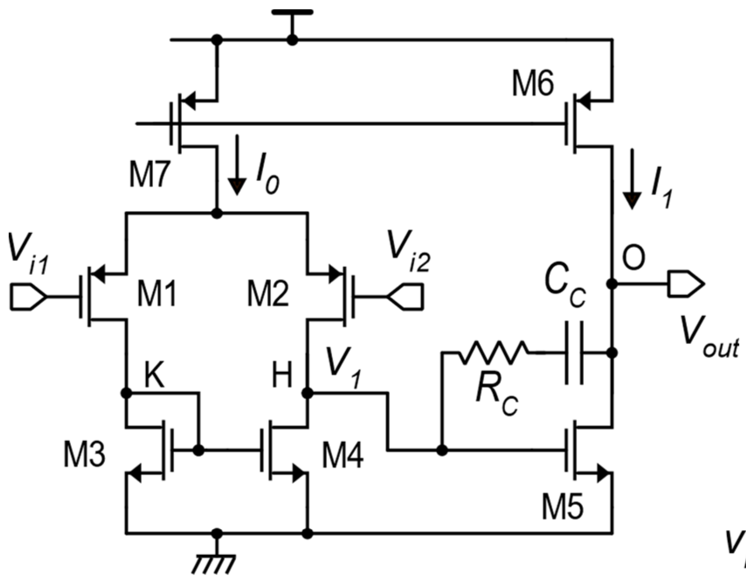
$$\frac{1}{r_d} = g_d = \lambda I_D$$

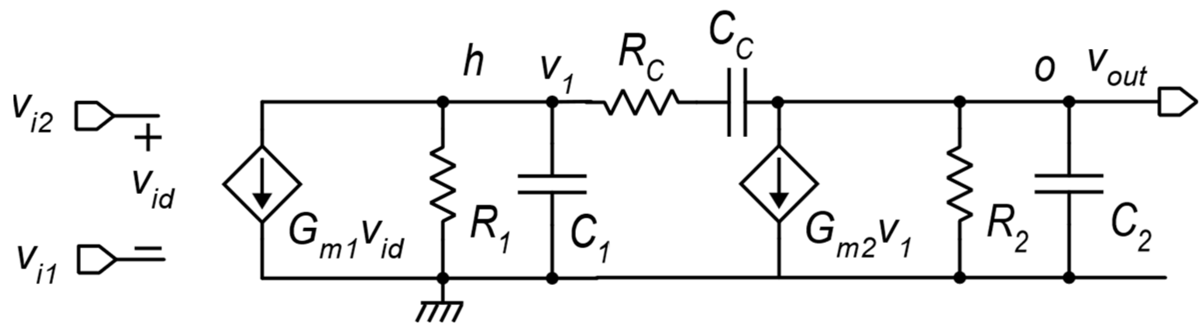
$$I_{D1} = I_{D3}$$

$$I_{D5} = I_{D6}$$

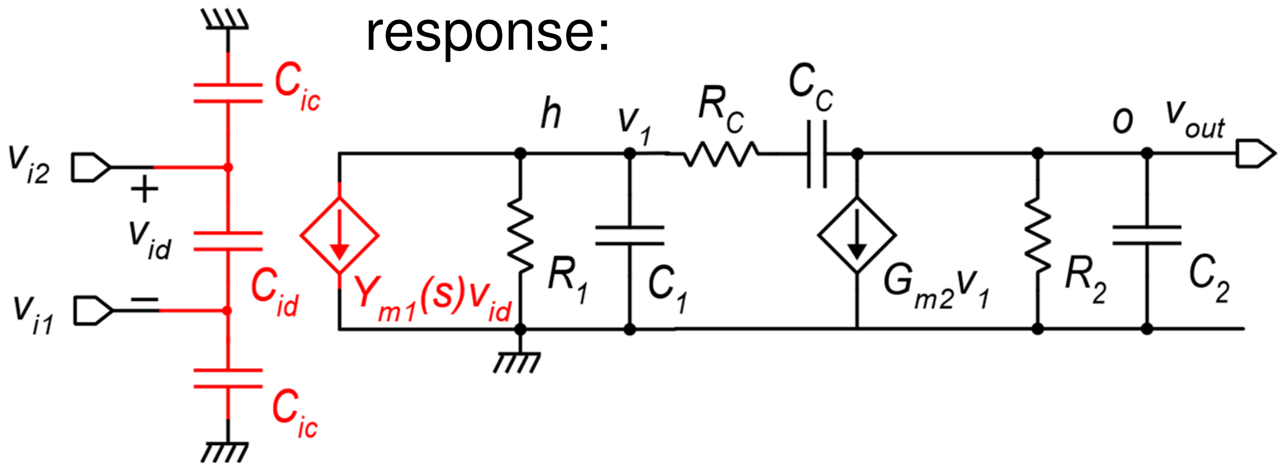
$$\boxed{A_0 = \frac{1}{V_{TE1}} \frac{1}{V_{TE5}} \frac{1}{(\lambda_1 + \lambda_3)} \frac{1}{(\lambda_5 + \lambda_6)}}$$

Frequency response of a two-stage op-amp





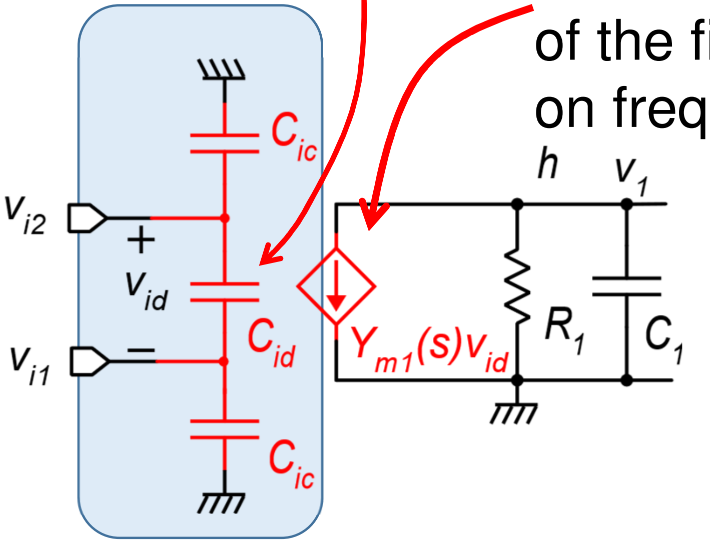
This circuit does not include all components that affect the frequency response:



Frequency response

$$C_{id} = \frac{c_{gs1}}{2}$$

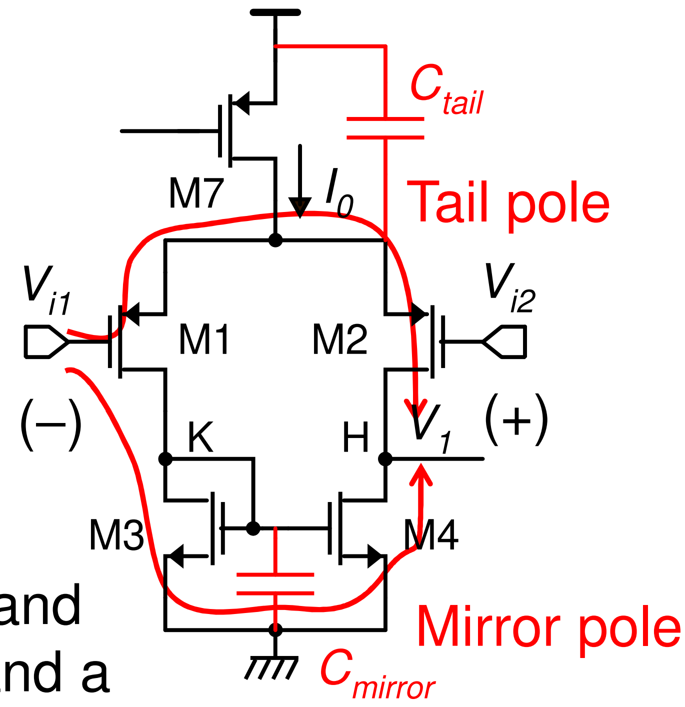
The short circuit output current of the first stage also depends on frequency



These capacitances determine the load presented by the amplifier to the signal source. We will suppose that v_{i1} and v_{i2} are produced by ideal voltage sources, thus no loading effect will be considered

$$-\beta V_{out}$$

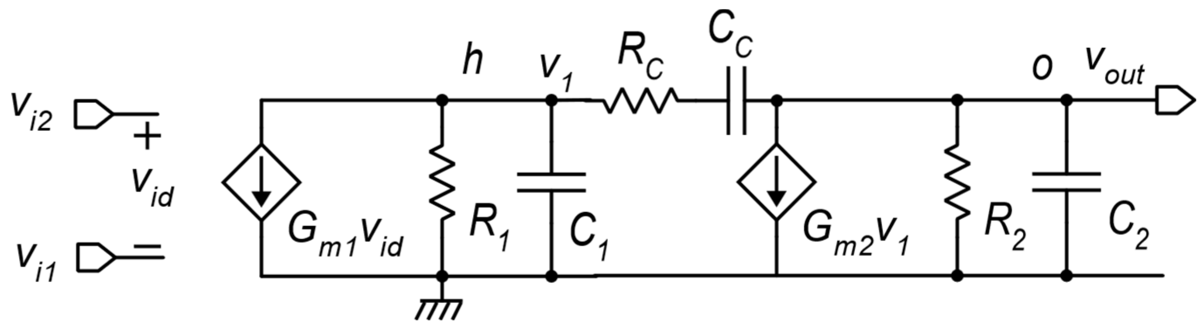
First stage



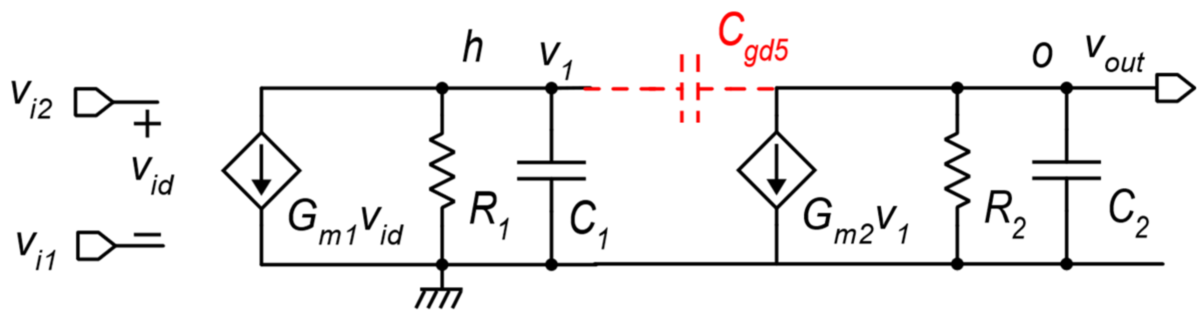
The tail pole and mirror pole (and a zero created by their combination) affect $Y_m(s)$. Frequencies are of the order of

$$f \sim \frac{1}{2\pi}\frac{g_m}{C_p}$$

Frequency response, simplified small-signal circuit

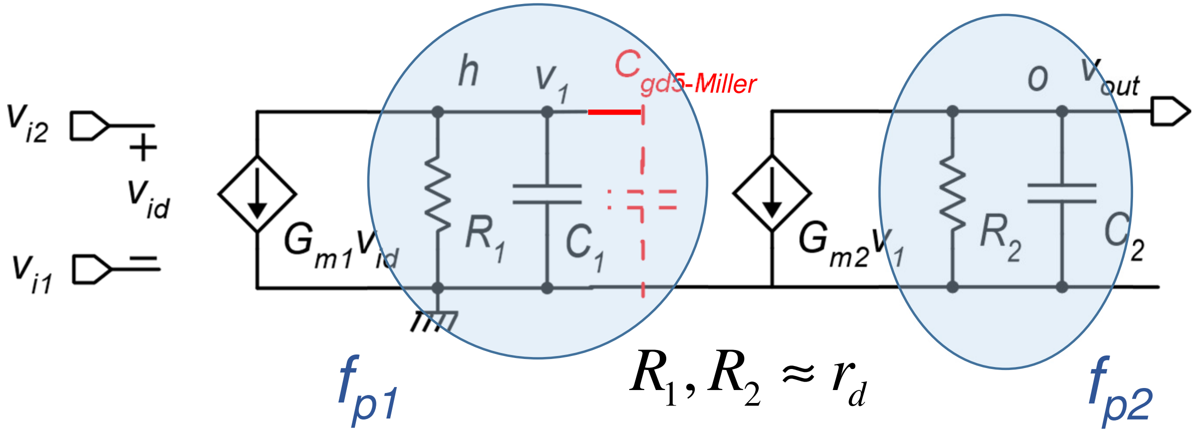


If the compensation group C_C-R_C is not present:



We still have C_{gd5} parasitic capacitance, which is not sufficient to produce the compensation effect

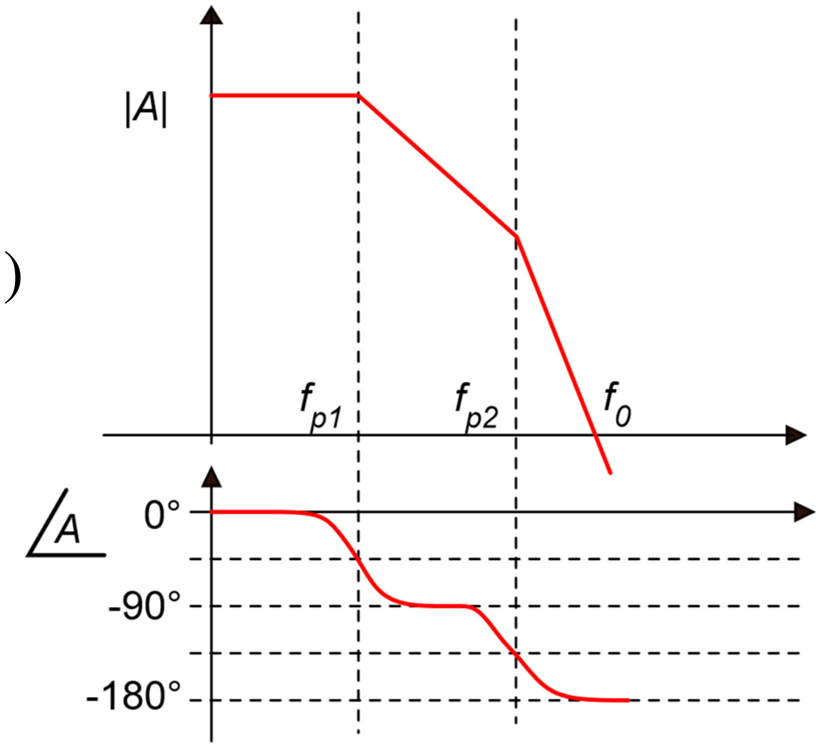
Uncompensated frequency response



$$R_1, R_2 \approx r_d$$

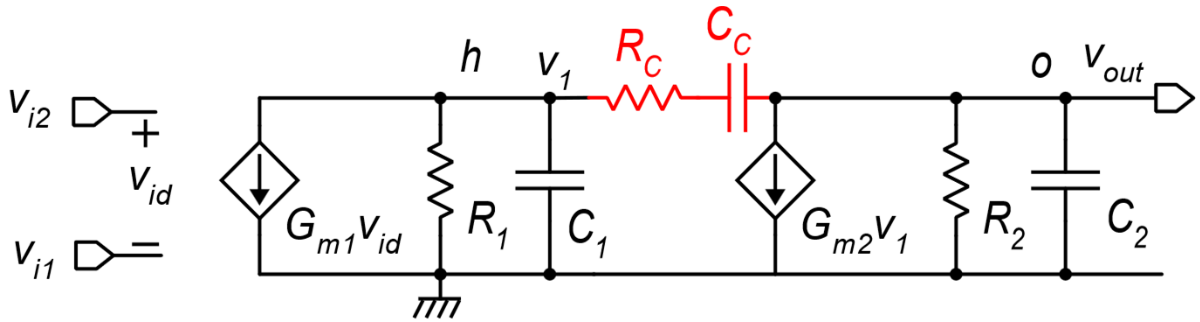
$$C_1, C_2, C_{gd5} \approx C_p \ \ (parasitic\ cap.)$$

$$f_{p1}, f_{p2} \approx \frac{1}{C_p r_d} = \frac{g_d}{C_p} << \frac{g_m}{C_p}$$



Without compensation, we have two poles at frequencies f_{p1}, f_{p2}, which are of the same order of magnitude and none of them is dominant.
The result is a very small or even negative (= instability) phase margin.

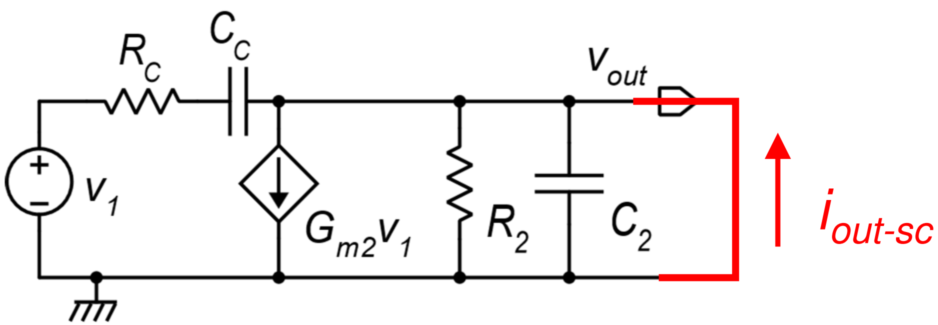
Miller compensation = **Pole splitting**



It is convenient to divide the bridge impedance R_C-C_C into two impedances by means of the Miller theorem

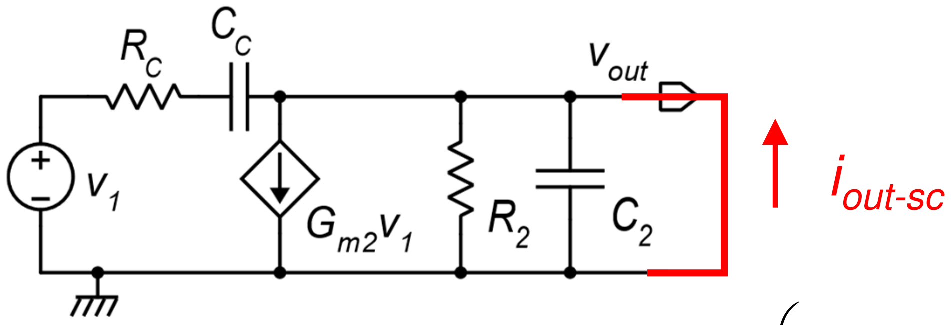
To do this, we need to calculate the Miller factor $K=v_{out}/v_1$. We force voltage v_1 and use the Norton equivalent model of the output port.

$$i_{out-sc} = G_{m2}v_1 - \frac{v_1}{R_C + \frac{1}{sC_C}} = v_1\left(G_{m2} - \frac{sC_C}{1+sC_CR_C}\right)$$



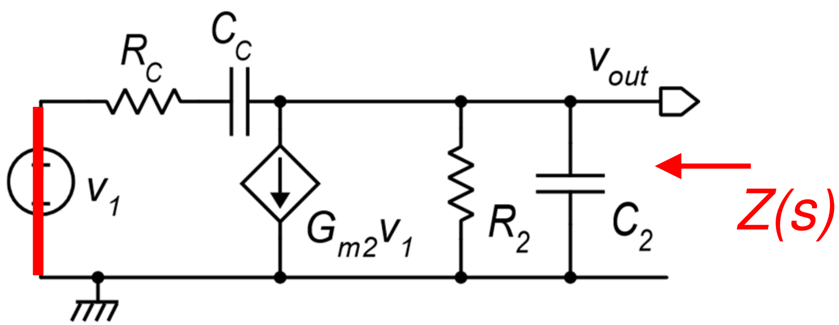
$$= v_1\left(\frac{G_{m2} + sC_CR_CG_{m2} - sC_C}{1+sC_CR_C}\right)$$

Miller factor



$$i_{out-sc} = v_1\left(\frac{G_{m2} + sC_C R_C G_{m2} - sC_C}{1 + sC_C R_C}\right)$$

$$i_{out-sc} = v_1 G_{m2}\left(\frac{1 + sC_C R_C - s\dfrac{C_C}{G_{m2}}}{1 + sC_C R_C}\right) = v_1 G_{m2}\left(\frac{1 + sC_C\left(R_C - \dfrac{1}{G_{m2}}\right)}{1 + sC_C R_C}\right)$$

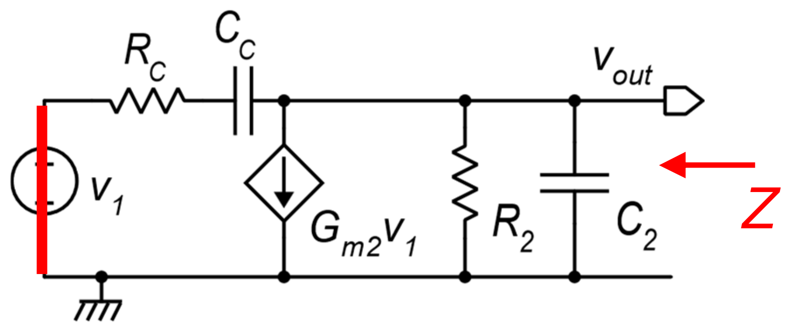


$$v_{out} = -Z(s) i_{out-sc} = -Z(s) v_1 G_{m2}\left(\frac{1 + sC_C\left(R_C - \dfrac{1}{G_{m2}}\right)}{1 + sC_C R_C}\right)$$

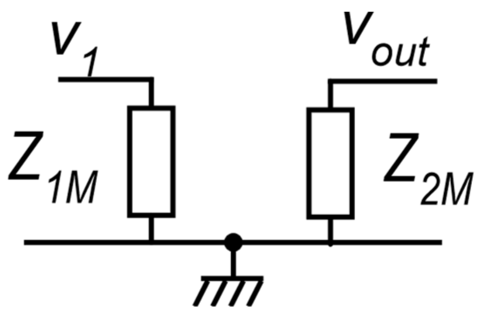
Miller factor: $K(s) = \dfrac{v_{out}}{v_1}$

Transformation of the bridge impedance Z_C by the Miller Theorem

$$K=\frac{v_{out}}{v_1}=-ZG_{m2}\left(\frac{1+sC_C\left(R_C-\dfrac{1}{G_{m2}}\right)}{1+sC_CR_C}\right)$$

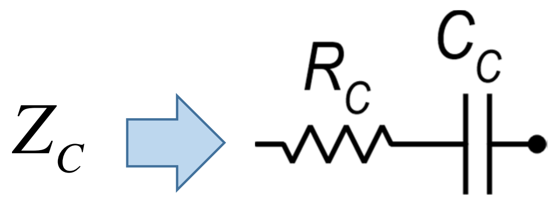


The low frequency limit of the K factor: $Z(f\to 0)=R_2$ $\quad K(f\to 0)=-G_{m2}R_2$



$$Z_{1M}=\frac{Z_C}{1-K}=\frac{1}{j\omega C_C(1-K)}$$

$$Z_{2M}=\frac{-KZ_C}{1-K}=\frac{-K}{j\omega C_C(1-K)}$$



$$Z_C\left(f\ll\frac{1}{2\pi C_CR_C}\right)=\frac{1}{j\omega C_C}$$

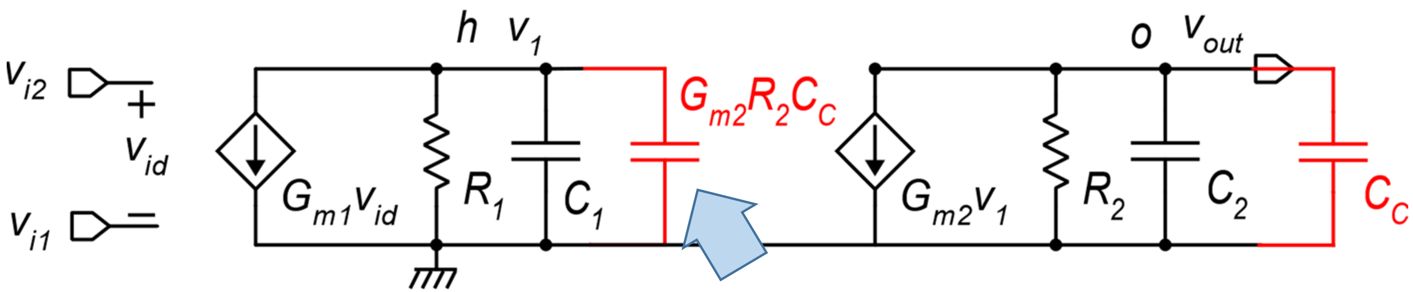
Miller transformations: shifting the input pole to very low frequencies

$$Z_{1M} = \frac{Z_C}{1-K} = \frac{1}{j\omega C_C(1-K)} \qquad Z_{2M} = \frac{-KZ_C}{1-K} = \frac{-K}{j\omega C_C(1-K)}$$

Using

$$K = K(0) = -G_{m2}R_2$$

$$Z_{1M} = \frac{1}{j\omega C_C(1+G_{m2}R_2)} \cong \frac{1}{j\omega C_C G_{m2}R_2} \qquad Z_{2M} \cong \frac{1}{j\omega C_C}$$

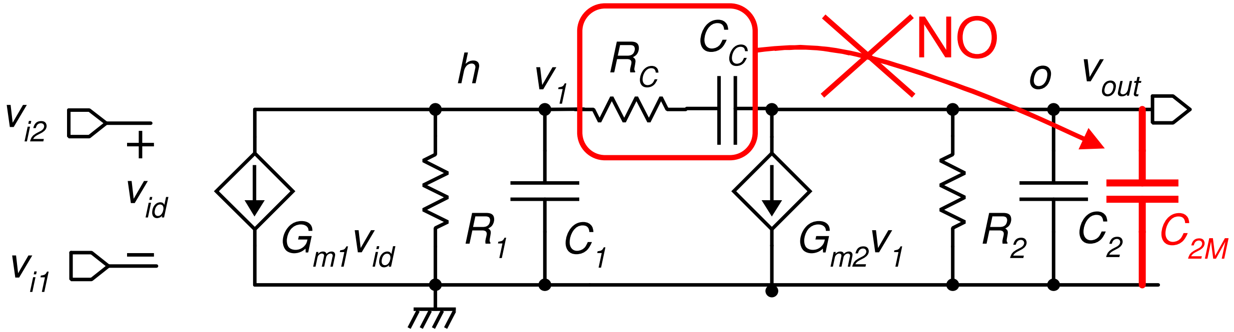


This sets the dominant pole:

$$\omega_p \cong \frac{1}{R_1 G_{m2}R_2C_C}$$

First effect of Miller compensation: a very large capacitor $G_{m2}R_2C_C$ is brought back to the input mesh, shifting the input pole back to low frequencies:

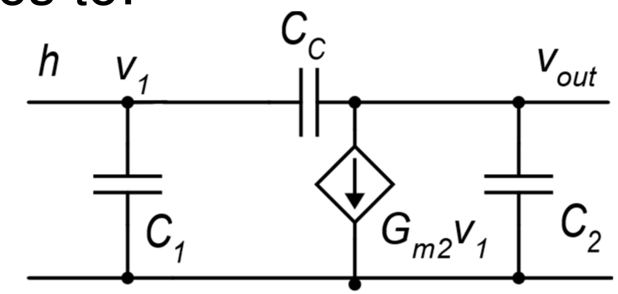
Second effect of Pole Splitting: shifting the output pole to high frequencies



We cannot use the Miller theorem again, because the resulting pole would fall at frequencies where K is very different from **K(0)**.

At frequencies such that: $\frac{1}{\omega C_2} << R_2 \ , \ \frac{1}{\omega C_1} << R_1$ and still: $\frac{1}{\omega C_C} >> R_C$

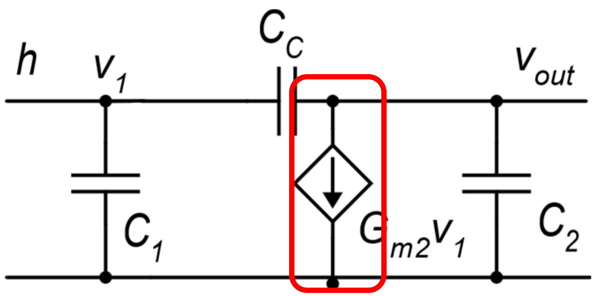
The equivalent circuit reduces to:



Current source $G_{m2}v_1$ is controlled by the voltage across it:

$$v_1 = v_{out} \frac{C_C}{C_1 + C_C}$$

Second effect of Miller Compensation: shifting the output pole to high frequencies



Then, it is equivalent to a resistance: $R_V = \dfrac{v_{out}}{i}$

$$v_1 = v_{out}\frac{C_C}{C_1+C_C} \qquad i = G_{m2}v_1 = v_{out}G_{m2}\frac{C_C}{C_1+C_C}$$

$$R_V = \frac{v_{out}}{i} = \frac{1}{G_{m2}}\frac{C_1+C_C}{C_C}$$

This resistance "sees" a capacitance: $C_V = C_2 + \dfrac{C_1C_C}{C_1+C_C}$

This sets a pole at: $\omega_2 = \dfrac{1}{R_V C_V}$

$$\omega_2 = \frac{1}{\dfrac{1}{G_{m2}}\dfrac{C_1+C_C}{C_C}\left(C_2+\dfrac{C_1C_C}{C_1+C_C}\right)}$$

Note: R_V is actually the op-amp output resistance at medium and high frequencies. It is of the order of $1/G_{m2}$ and is much smaller than the value in dc (order of r_d).

Second effect of Miller Compensation: shifting the output pole to high frequencies

$$\omega_2 = \frac{1}{R_V C_V} = \frac{1}{\dfrac{1}{G_{m2}}\dfrac{C_1+C_C}{C_C}\left(C_2+\dfrac{C_1 C_C}{C_1+C_C}\right)} = \frac{G_{m2}}{\dfrac{C_1C_2+C_CC_2+C_1C_C}{C_C}}$$

$$\omega_2 = \frac{G_{m2}}{\left(\dfrac{C_1C_2}{C_C}+C_2+C_1\right)} = \frac{G_{m2}}{(C_1+C_2)\left(1+\dfrac{1}{C_C}\cdot\dfrac{C_1C_2}{C_1+C_2}\right)}$$

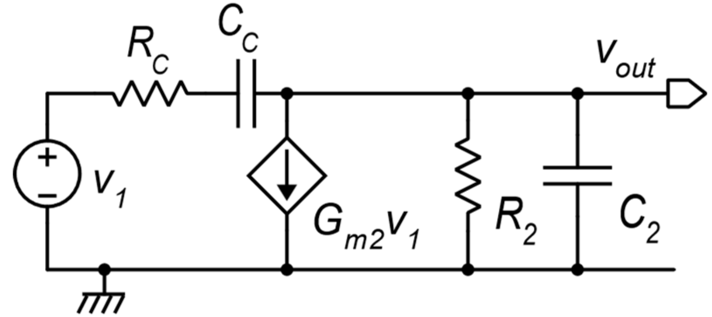
$$C_S = \frac{C_1C_2}{C_1+C_2}$$

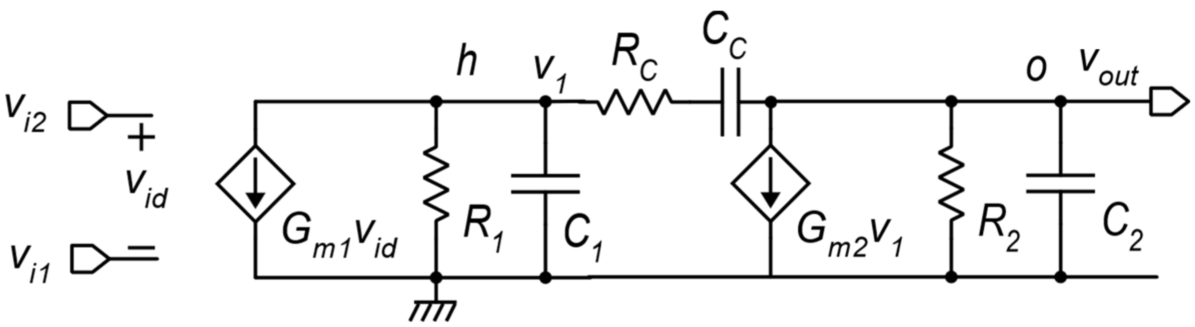
Series of C_1 and C_2

$$\boxed{\omega_2 = \frac{G_{m2}}{(C_1+C_2)\left(1+\dfrac{C_S}{C_C}\right)}}$$

Miller Factor and overall transfer function

Miller factor: $K(s)=\dfrac{v_{out}}{v_1}=-ZG_{m2}\left(\dfrac{1+sC_C\left(R_C-\dfrac{1}{G_{m2}}\right)}{1+sC_CR_C}\right)$





Factorizing *A*: $$A=\frac{v_{out}}{v_{id}}=\frac{v_1}{v_{id}}\frac{v_{out}}{v_1}=\frac{v_1}{v_{id}}K(s)$$

The zero introduced by R_C-C_C

$$K(s)=\frac{v_{out}}{v_1}=-ZG_{m2}\left(\frac{1+sC_C\left(R_C-\dfrac{1}{G_{m2}}\right)}{1+sC_CR_C}\right)$$

$$A=\frac{v_{out}}{v_{id}}=\frac{v_1}{v_{id}}\frac{v_{out}}{v_1}=\frac{v_1}{v_{id}}K(s)$$

There is a zero in K(s):

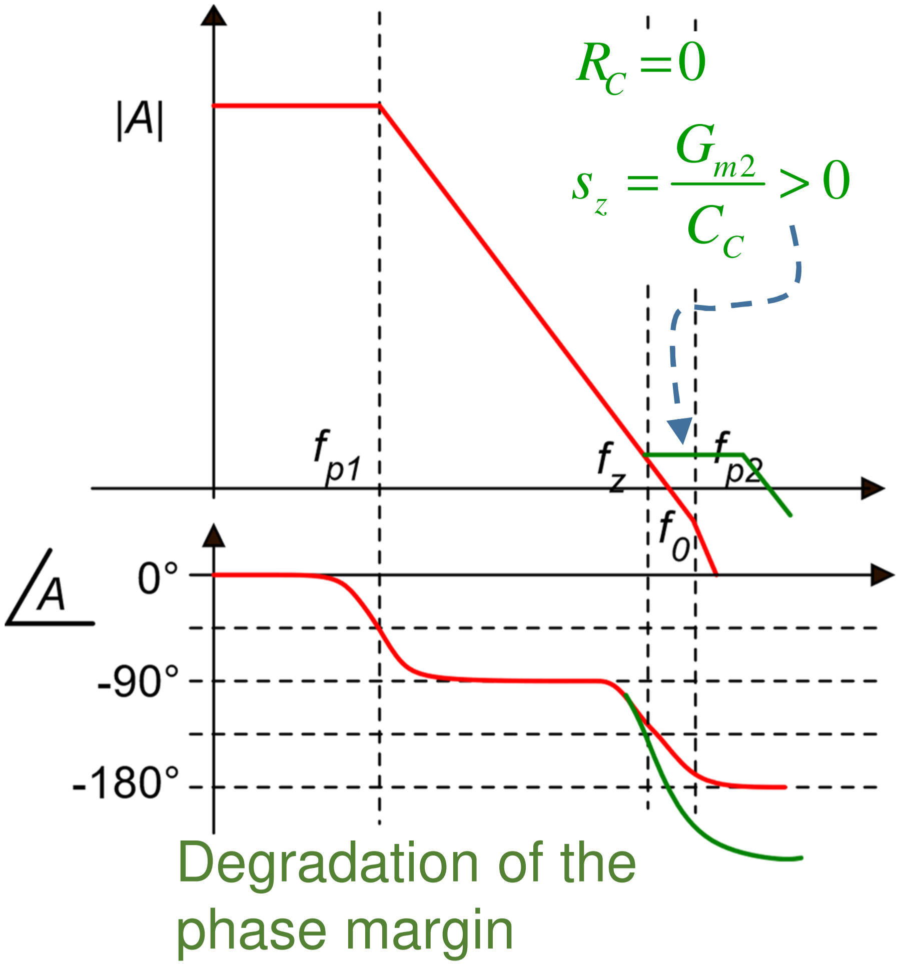
$$s_z=\frac{-1}{C_C\left(R_C-\dfrac{1}{G_{m2}}\right)}$$

If R_C=0
(no R_C)

$$s_z=\frac{-1}{C_C\left(\dfrac{-1}{G_{m2}}\right)}=\frac{G_{m2}}{C_C}>0$$

This zero occurs also in the overall transfer-function of the amplifier (A). It cannot be cancelled by an equal zero, since an unstable pole (>0) would be required

Effects of the zero in the transfer function



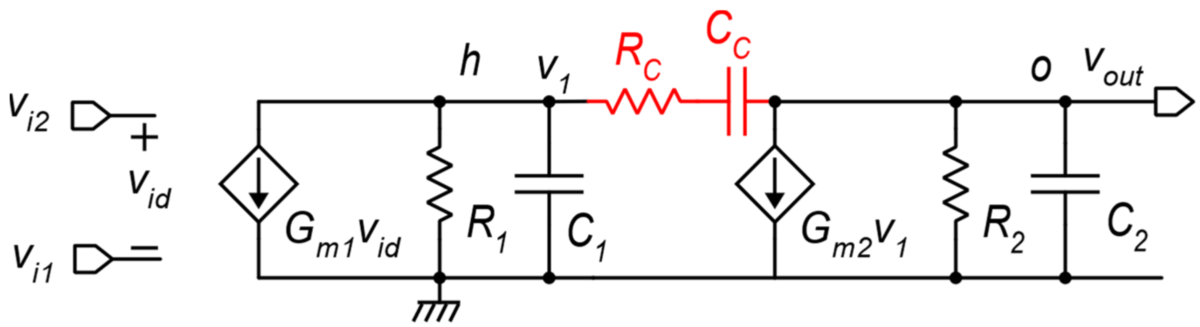
Thanks to R_C:

$$s_z = \frac{-1}{C_C\left(R_C - \frac{1}{G_{m2}}\right)}$$

$$R_C = \frac{1}{G_{m2}} \qquad s_z \rightarrow \infty$$

With this choice for R_C, we can eliminate the zero and cancel its bad effect on the phase delay.
Other choices are possible:
for $R_c > 1/G_{m2}$ it is possible to change the positive zero into a negative one and use it to compensate f_{p2}.

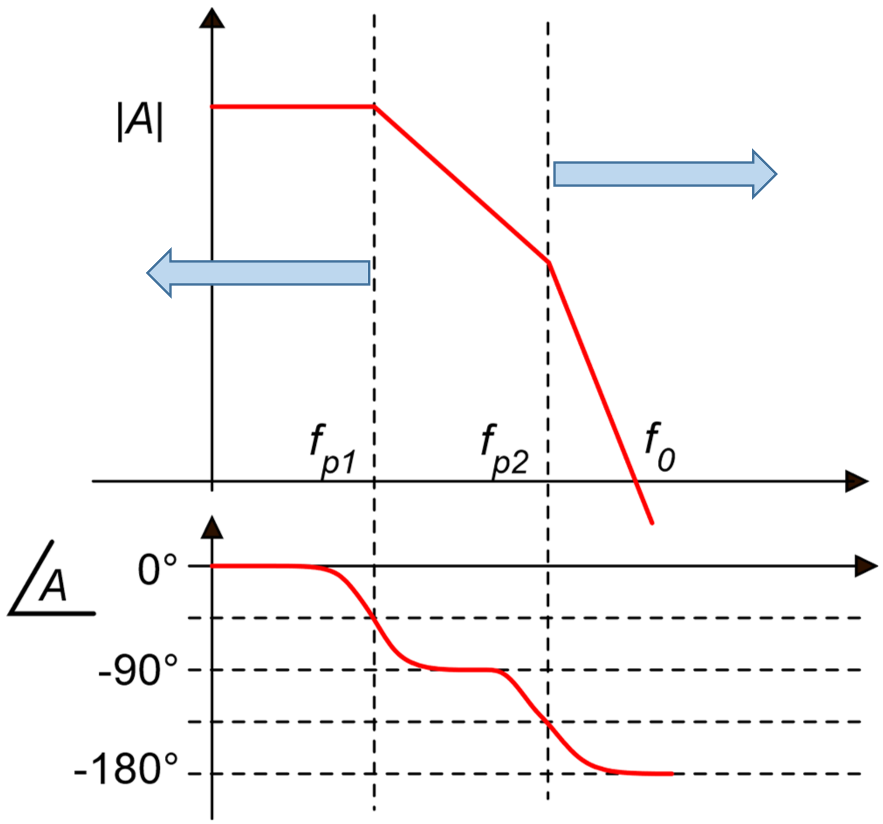
Summary of pole splitting



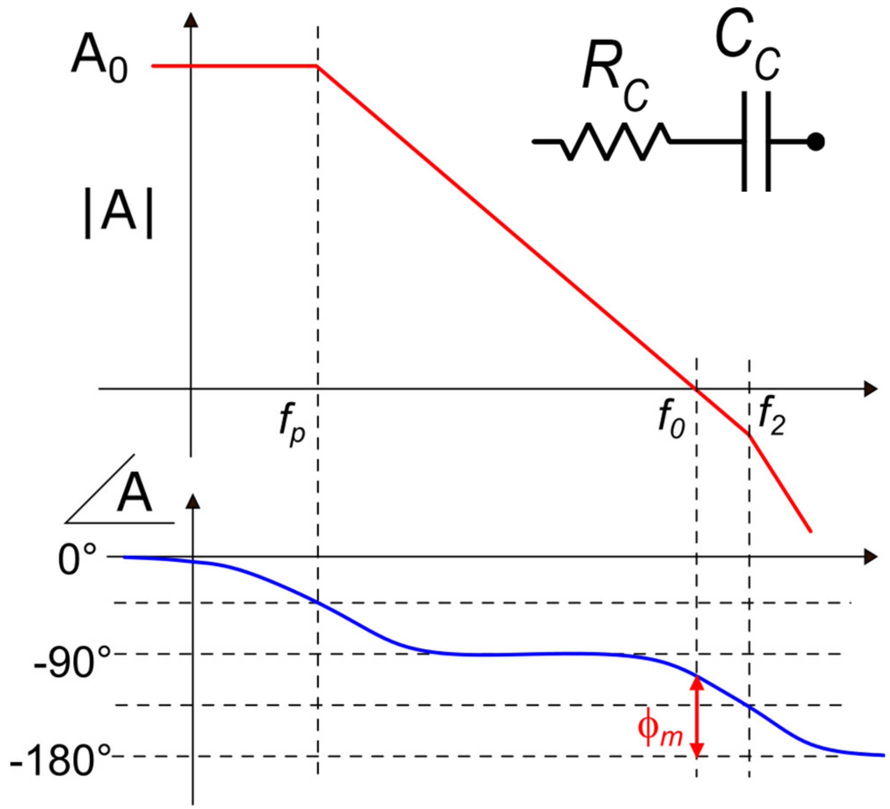
Capacitor C_C introduces a feedback across the second stage that:

1. Puts an equivalent large capacitor ($C_C G_{m2} R_2 >> C_1$) across the output resistance of the first stage (R_1) shifting the first pole back to very low frequencies
2. Reduces the output resistance (R_V) at medium/high frequencies from R_2 to a value close to $1/G_{m2}$. This shifts the output pole to much higher frequencies.
3. **Resistor R_C** is significant only at high frequencies and "shapes" the zero, either cancelling it or turning it into a negative zero

Pole splitting, graphical view

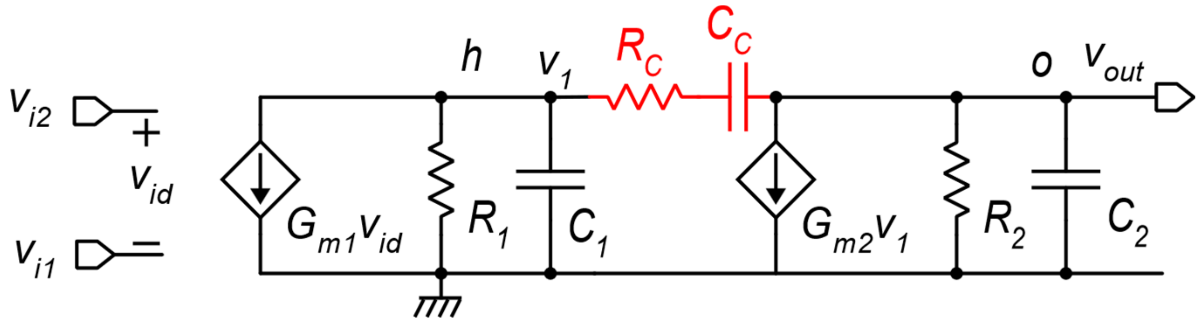


Before compensation



After compensation

Summary of singularities



$$\omega_p \cong \frac{1}{R_1 G_{m2} R_2 C_C}$$

$$s_z = \frac{-1}{C_C\left(R_C - \frac{1}{G_{m2}}\right)}$$

$$\omega_2 \cong \frac{G_{m2}}{(C_1+C_2)\left(1+\frac{C_S}{C_C}\right)} = \frac{G_{m2}}{(C_1+C_2)}\left(1+\frac{C_S}{C_C}\right)^{-1} \qquad C_S = \frac{C_1 C_2}{C_1+C_2}$$

$$\omega_3 \cong \frac{1}{R_C\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_C}\right)^{-1}} >> \omega_2$$

This third pole ($s_3 = -\omega_3$), can be guessed considering that at very high frequencies the whole network reduces to the three capacitors and resistor R_C.