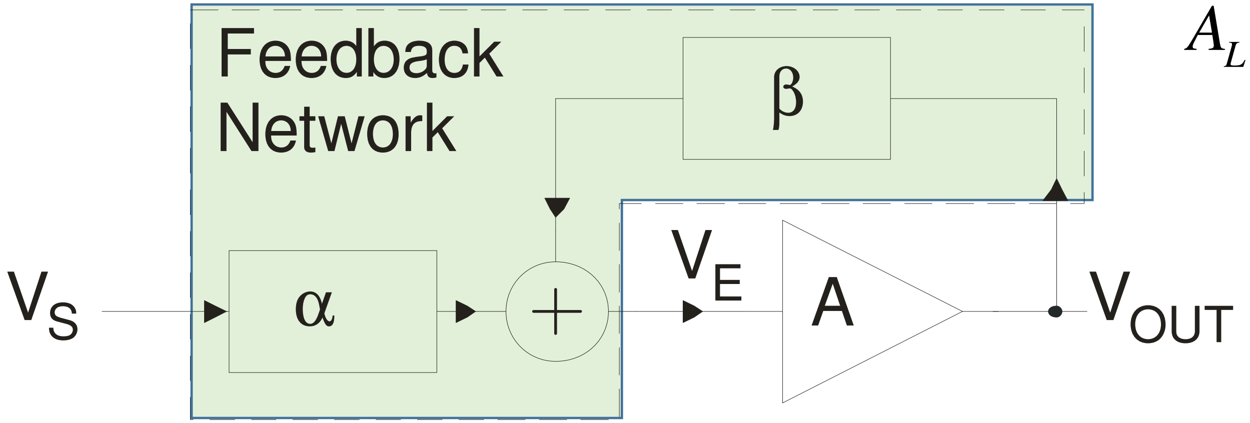
Use of Pellegrini's cut-insertion theorem for the design of feedback systems

Block diagram of a feedback system



$$A_L \equiv \frac{V_{OUT}}{V_S} = -\frac{\alpha}{\beta}\frac{\beta A}{\beta A - 1}$$

$$A_{L-ideal} \cong -\frac{\alpha}{\beta}$$

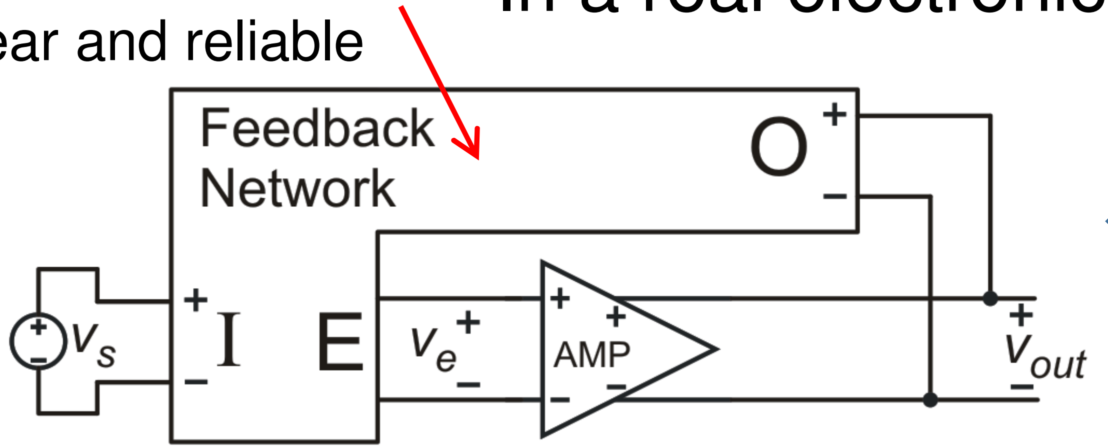
$$\Delta A_L = A_L - A_{L-ideal}$$

$$\Delta A_L = -\frac{\alpha}{\beta}\left(\frac{\beta A}{\beta A - 1}\right) + \frac{\alpha}{\beta} = -\frac{\alpha}{\beta}\left(\frac{\beta A}{\beta A - 1} - 1\right) = -\frac{\alpha}{\beta}\left(\frac{1}{\beta A - 1}\right) \cong -\frac{\alpha}{\beta}\frac{1}{\beta A} \qquad (|\beta A| >> 1)$$

$$|\varepsilon_{AL}| \equiv \left|\frac{\Delta A_L}{A_L}\right| \cong |\beta A|^{-1}$$

Passive network:
linear and reliable

In a real electronic network:



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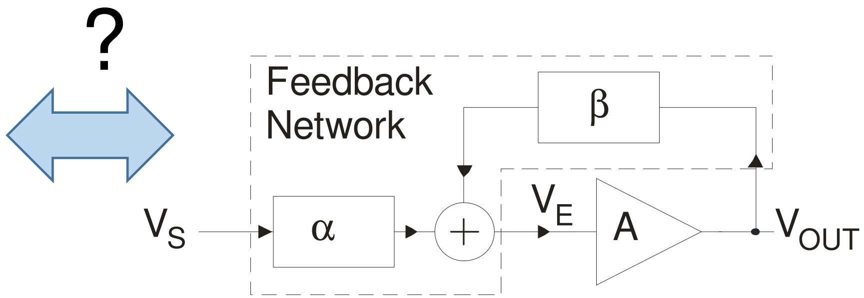
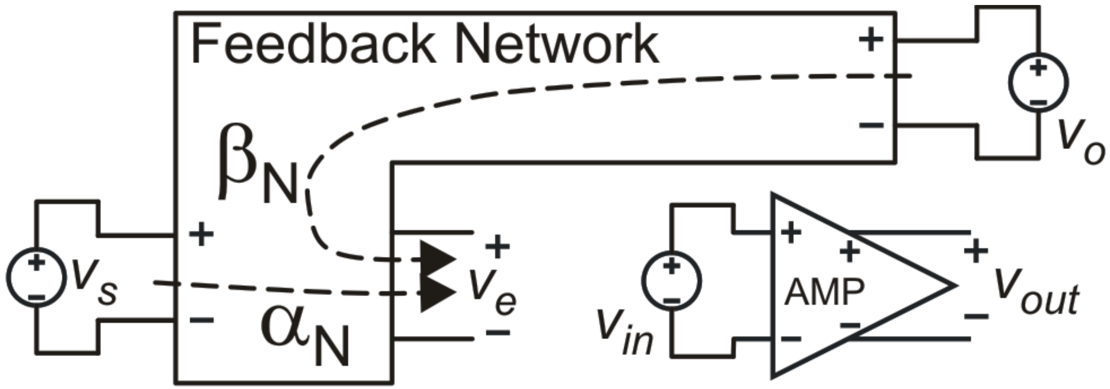


Desired correspondence

$$\alpha_N \rightarrow \alpha$$

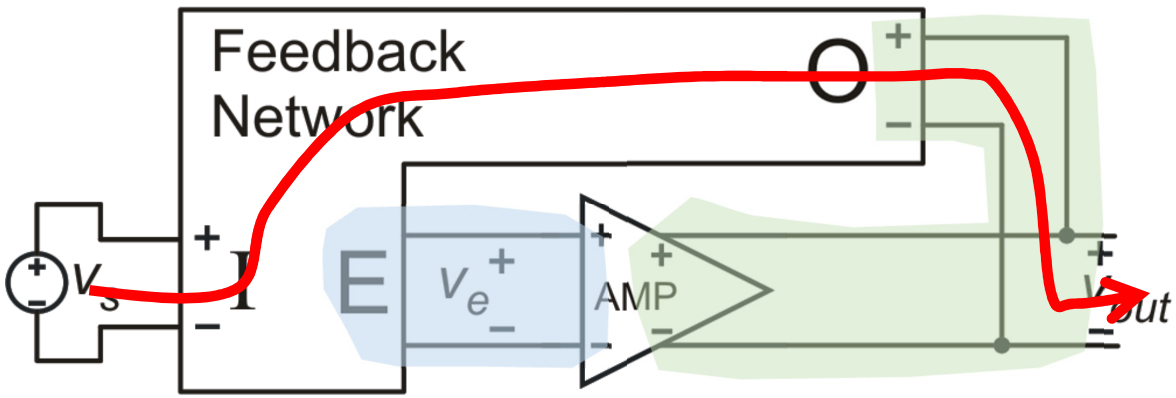
$$\beta_N \rightarrow \beta$$

$$A_{OL} = \frac{V_{out}}{V_{in}} \rightarrow A$$



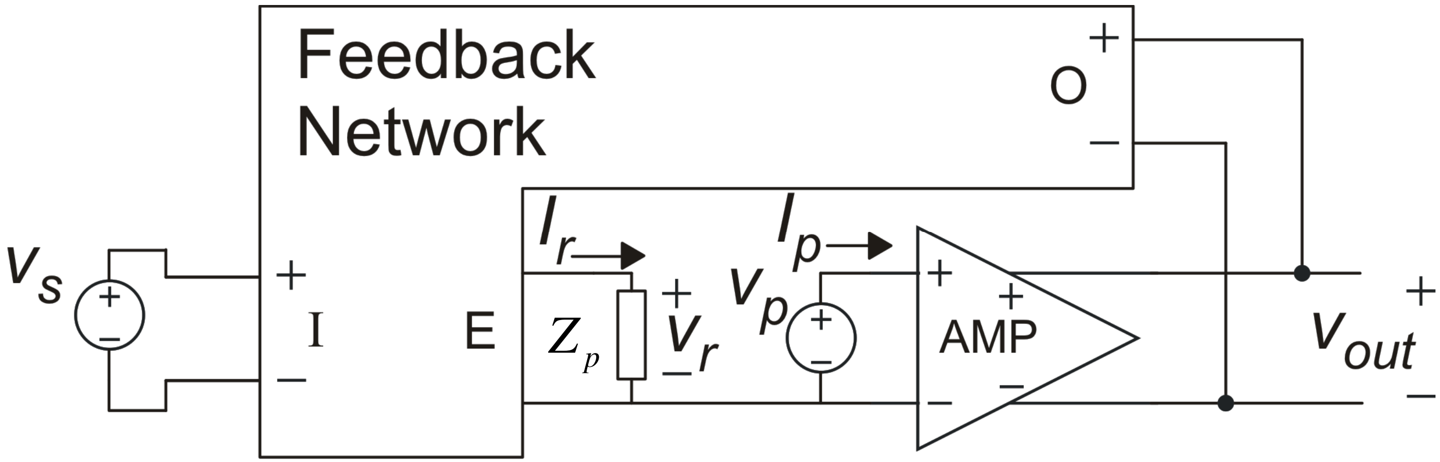
Separation of feedback network and gain element (AMP)

Problems that prevent a direct correspondence between the electrical circuit and the simplified block diagram of a feedback system



- Loading effects of the feedback network on AMP
- Loading effect of the amplifier input impedance on the feedback network
- Direct signal path from the input signal V_S and the output voltage through the feedback network (occurs if the amplifier has a non-zero output impedance).

Pellegrini's cut-insertion theorem



A, β, α

now are calculated taking into account loading effects

γ

takes into account the feed-forward path through the feedback network

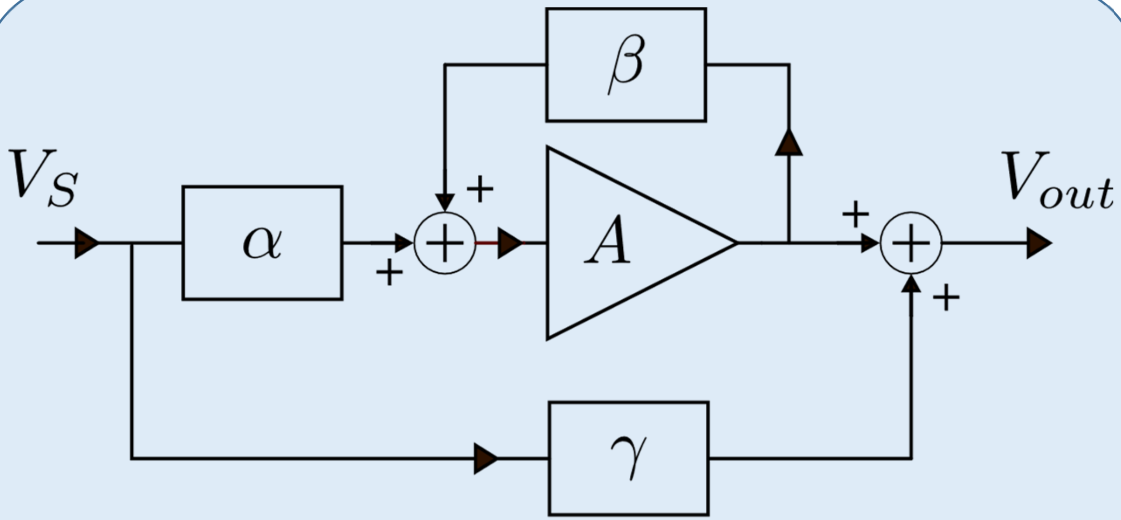
$$Z_i = \left(\frac{v_p}{i_p}\right)_{v_s=0} \; ; \; \beta \equiv \left(\frac{v_r}{v_{out}}\right)_{v_s=0} \; ; \; A = \left(\frac{v_{out}}{v_p}\right)_{v_s=0}$$

$$\frac{1}{Z_p} = \frac{1}{Z_i} + \frac{\rho}{\alpha}(1-\beta A)$$

$$\gamma = \left(\frac{v_{out}}{v_s}\right)_{v_p=0} \; ; \; \alpha \equiv \left(\frac{v_r}{v_s}\right)_{v_p=0} \qquad \rho = \left(\frac{i_p}{v_s}\right)_{v_p=0}$$

ρ is zero if the op-amp is unidirectional

Block diagram corresponding to the cut-insertion schematization

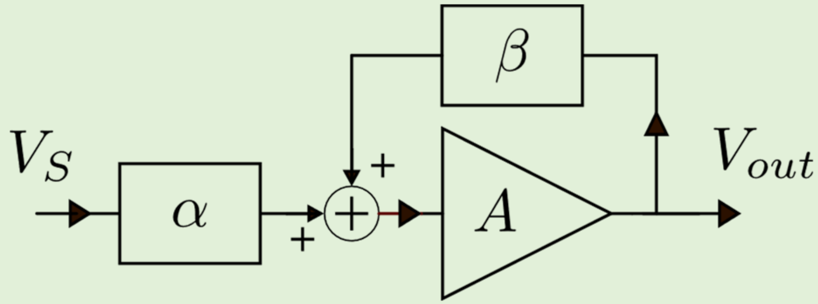


$$\frac{v_{out}}{v_s} = A_L = -\frac{\alpha}{\beta}\frac{\beta A}{\beta A - 1} + \gamma$$

$$\lim_{|\beta A| \to \infty} A_L = -\frac{\alpha}{\beta} + \gamma$$

$$\frac{v_{out}}{v_s} = A_L = -\frac{\alpha}{\beta}\frac{\beta A}{\beta A - 1}$$

$$A_{L-ideal} = \lim_{|\beta A| \to \infty} A_L = -\frac{\alpha}{\beta}$$



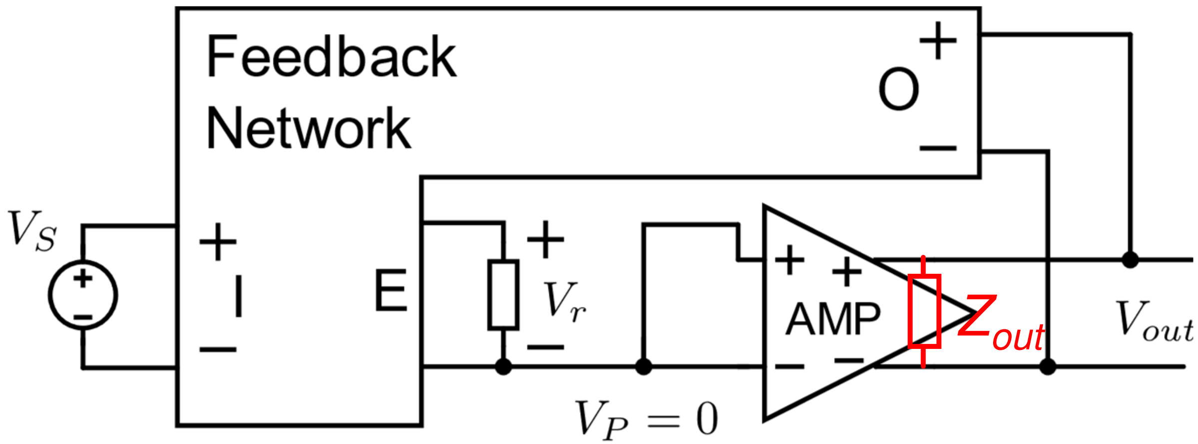
Ideal feedback system

Problems when using the cut-inserion theorem for design purposes

1. I have to design the feedback network for three transfer functions (α, β and γ) instead of only two.
2. α and γ depend on the output impedance of the amplifier, Z_{out}, which is a parameter that varies much and cannot be predicted reliably.
3. Only in the case that Z_{out} is much smaller than the typical impedances of the feedback network, we can design the network for α, β and γ) with port "O" short circuited (γ=0).

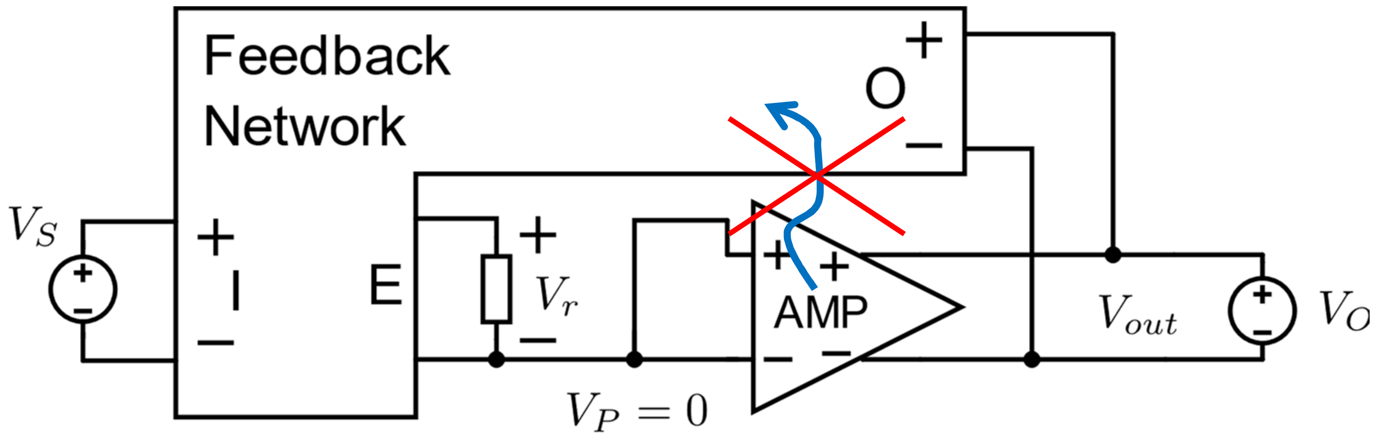
Typically, this does not occur in integrated circuits.

$$\frac{v_{out}}{v_s} = -\frac{\alpha}{\beta} + \gamma$$



Configuration for α, γ

Cut insertion theorem with modified definition of α



If AMP communicates with the feedback network only through the input and output ports (no intermediate signal paths) ......

then $\beta^* = \beta$

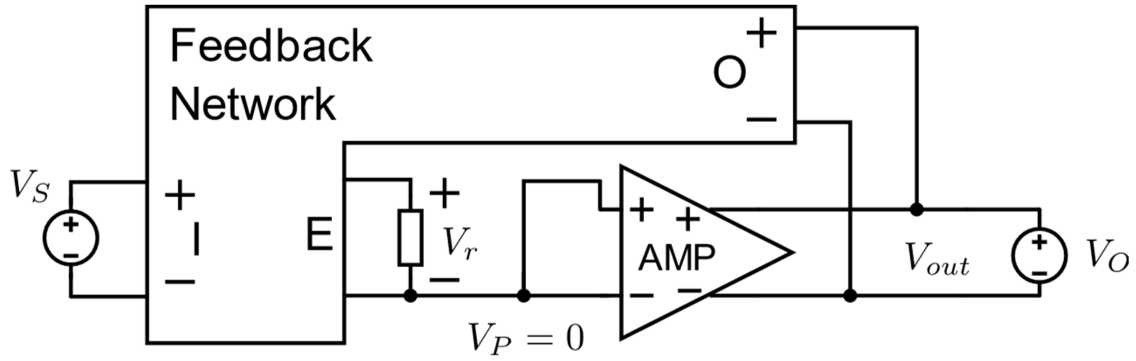
and:

$$\frac{v_{out}}{v_s} = -\frac{\alpha^*}{\beta}\frac{\beta A}{\beta A - 1} + \frac{\gamma}{1-\beta A}$$

A new definition of α and β (renamed α^* and β^*)

$$\alpha^* \equiv \left(\frac{v_r}{v_s}\right)_{v_p, v_o = 0} \; ; \; \beta^* \equiv \left(\frac{v_r}{v_o}\right)_{v_p, v_s = 0}$$

Cut insertion theorem with modified definition of α



$$\frac{v_{out}}{v_s} = -\frac{\alpha^*}{\beta}\frac{\beta A}{\beta A - 1} + \frac{\gamma}{1-\beta A}$$

Now γ is divided by the loop gain βA

$$\lim_{|\beta A|\to\infty}\frac{v_{out}}{v_s} = -\frac{\alpha^*}{\beta} = -\frac{\alpha^*}{\beta^*}$$

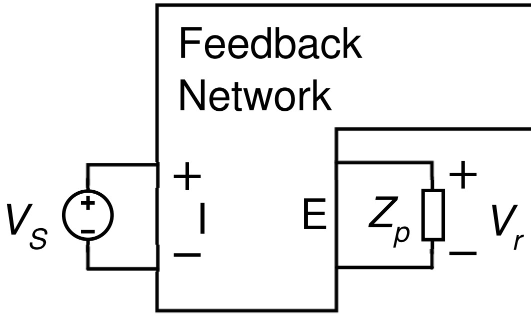
The definition of α* is deeply different from that of α.

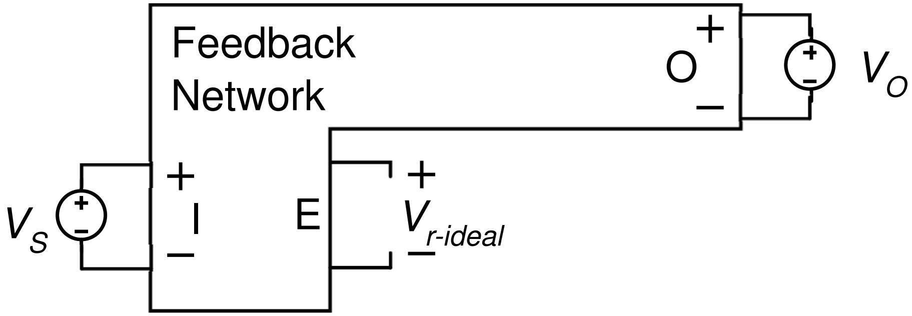
Also the definition of β* is different from that of β. but the two values coincides in most practical cases.

α* and β* are very similar to α_N and β_N that are the functions for which we design the feedback network

Ideal transfer function and network functions

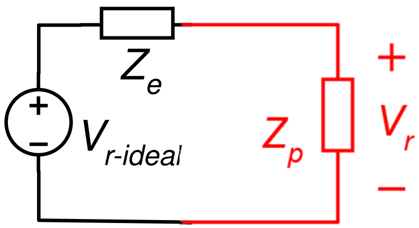
$$\lim_{|\beta A|\to\infty} \frac{v_{out}}{v_s} = -\frac{\alpha^*}{\beta} = -\frac{\alpha^*}{\beta^*}$$





$$V_r = \alpha^* V_S + \beta^* V_O$$

$$V_{r-ideal} = \alpha_N V_S + \beta_N V_O$$



$$V_r = (\alpha_N V_S + \beta_N V_O)\frac{Z_p}{Z_p + Z_e}$$

$$V_r = \alpha_N \frac{Z_p}{Z_p + Z_e} V_S + \beta_N \frac{Z_p}{Z_p + Z_e} V_O$$

$$\alpha^* = \alpha_N \frac{Z_p}{Z_p + Z_e}$$

$$\beta^* = \beta_N \frac{Z_p}{Z_p + Z_e}$$

$$\frac{\alpha^*}{\beta^*} = \frac{\alpha_N}{\beta_N}$$

Finite gain error

$$A_{L-ideal} = -\frac{\alpha^*}{\beta^*}$$

$$A_L = -\frac{\alpha^*}{\beta}\frac{\beta A}{\beta A - 1} + \frac{\gamma}{1-\beta A}$$

$$\Delta A_L = A_L - A_{L-ideal}$$

$$\Delta A_L = -\frac{\alpha^*}{\beta}\frac{\beta A}{\beta A - 1} + \frac{\gamma}{1-\beta A} - \left(-\frac{\alpha^*}{\beta}\right)$$

$$\Delta A_L = -\frac{\alpha^*}{\beta}\left(\frac{\beta A}{\beta A - 1} - 1\right) + \frac{\gamma}{1-\beta A}$$

$$\Delta A_L = -\frac{\alpha^*}{\beta}\left(\frac{1}{\beta A - 1}\right) + \frac{\gamma}{1-\beta A}$$

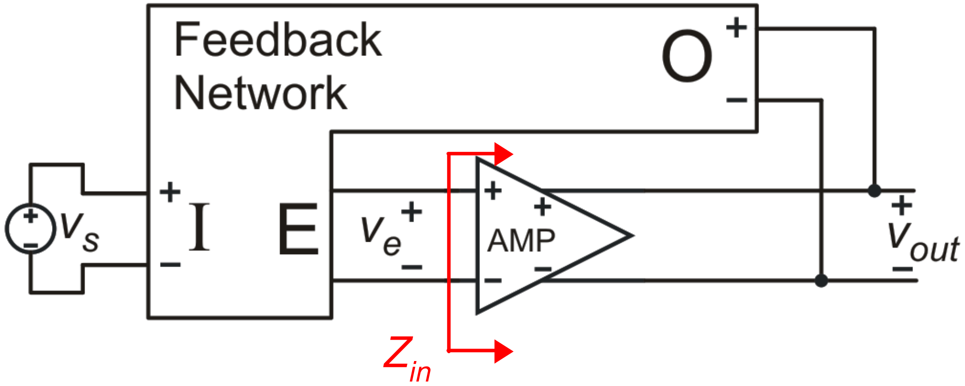
$$\frac{\Delta A_L}{A_{L-ideal}} = \left(\frac{1}{\beta A - 1}\right)\left(1 + \frac{\gamma}{A_{L-ideal}}\right)$$

$$e_r = \left|\frac{\Delta A_L}{A_{L-ideal}}\right| \cong \frac{1}{|\beta A|}\left|1 + \frac{\gamma}{A_{L-ideal}}\right|$$

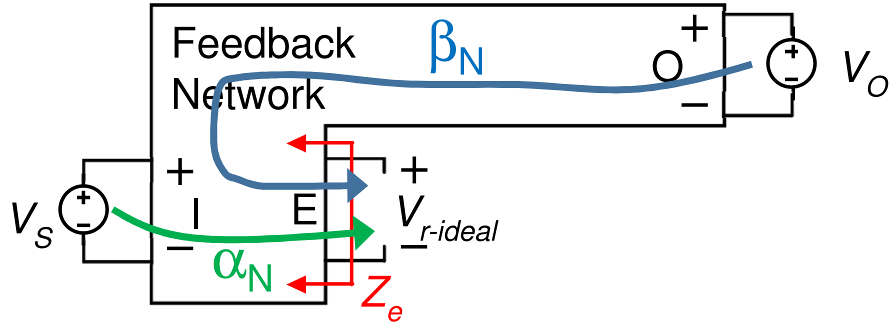
From the Schwarz inequality:

$$e_r \le \frac{1}{|\beta A|}\left(1 + \left|\frac{\gamma}{A_{L-ideal}}\right|\right)$$

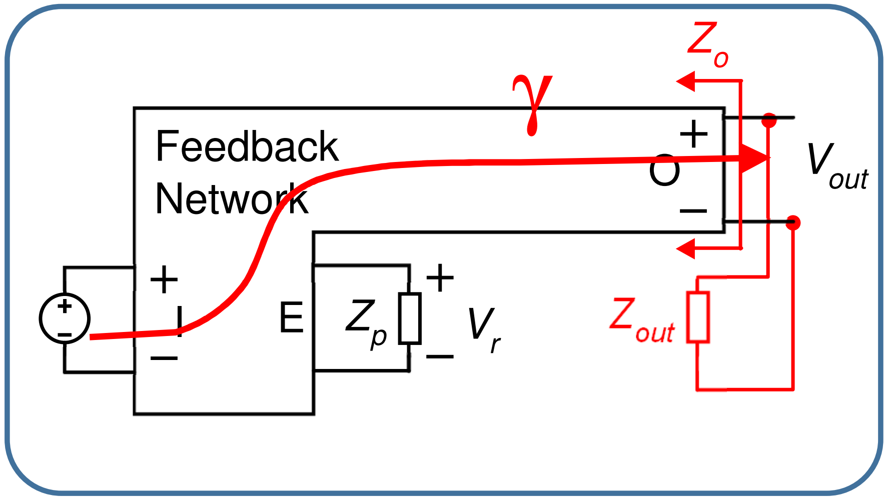
Final recap



$$A_{L-ideal} = -\frac{\alpha^*}{\beta^*} = -\frac{\alpha_N}{\beta_N}$$



$$e_r \le \frac{1}{|\beta A|}\left(1+\left|\frac{\gamma}{A_{L-ideal}}\right|\right)$$



$$\beta = \beta^* = \beta_N \frac{Z_{in}}{Z_{in}+Z_e}$$

$$A = A_{VOL}\frac{Z_o}{Z_o+Z_{out}}$$