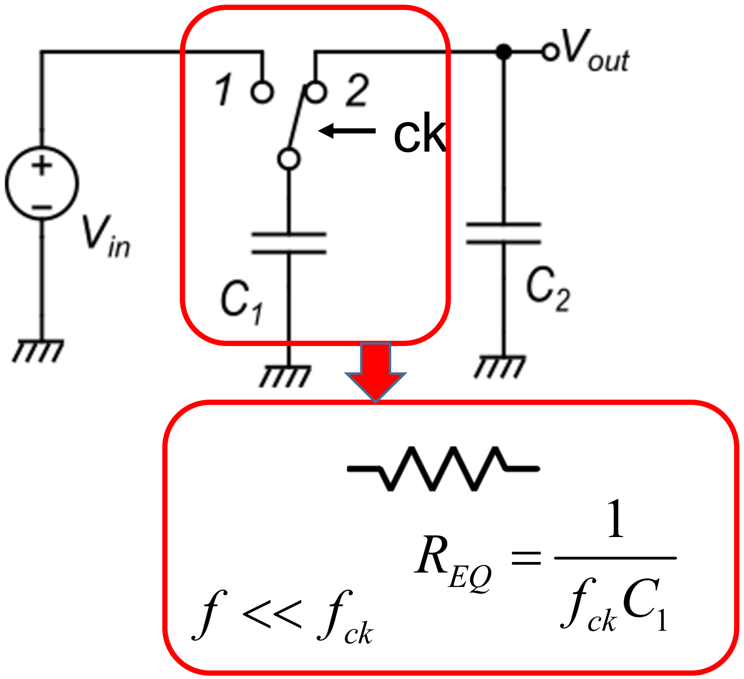
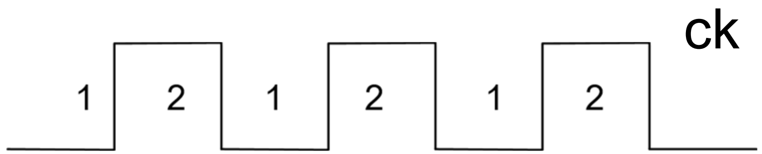
Switched Capacitor (SC) circuits: general considerations

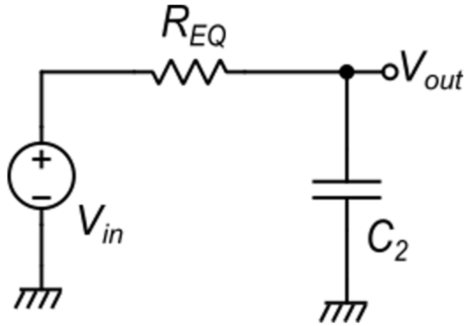
The SC equivalent resistance



$$R_{EQ} = \frac{1}{f_{ck} C_1}$$

$$f << f_{ck}$$



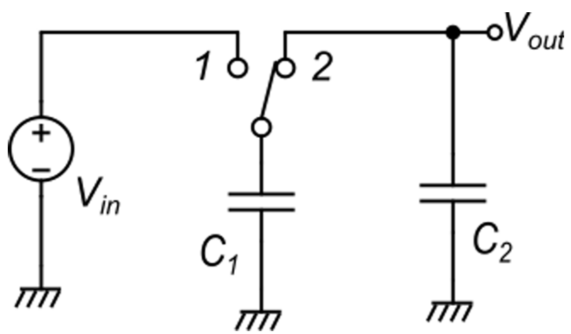


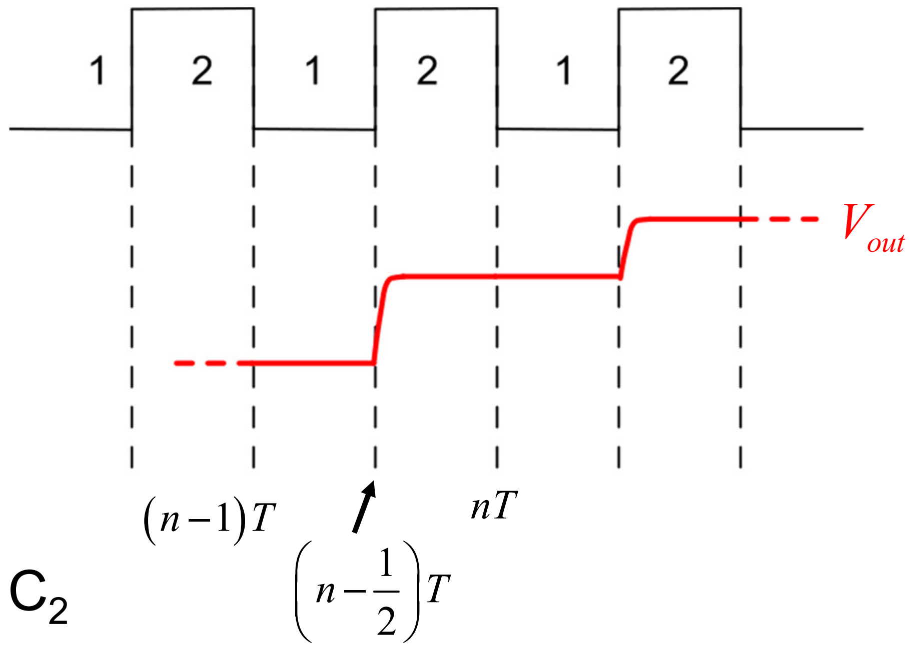
$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + \frac{j\omega}{\omega_p}}$$

$$\omega_p = \frac{1}{R_{EQ} C_2}$$

Switched Capacitor (SC) circuits: general considerations

The discrete time nature of SC circuits





Phase 1: V_{out} holds, $V_{C1}=V_{IN}$

Phase 2: C_1 **samples** V_{IN} and discharges into C_2

$$V_{out}(nT)=\frac{C_1 V_{in}\left(\left(n-\frac{1}{2}\right)T\right)+C_2 V_{out}\left((n-1)T\right)}{C_1+C_2}$$

This discrete-time equation is valid at any frequency

~~$f \ll f_{ck}$~~

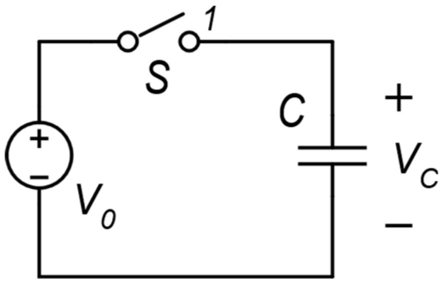
Non-idealities in (SC) circuits

Sampling of a voltage on a capacitor: errors

- *kT/C* noise
- Charge injection

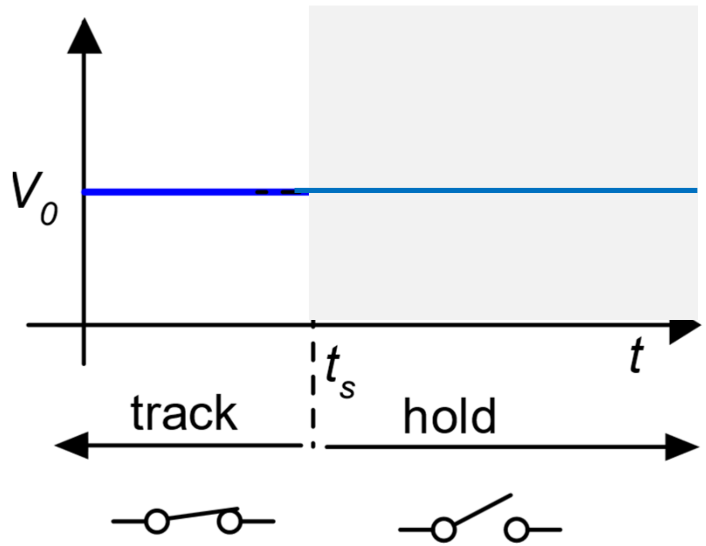
The *kT/C* noise:

a simple example



Phase 1: track
Phase 2: hold

Simple example:
V_0 is constant

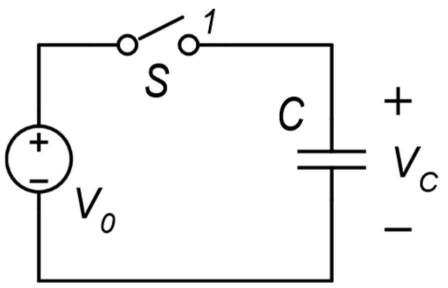


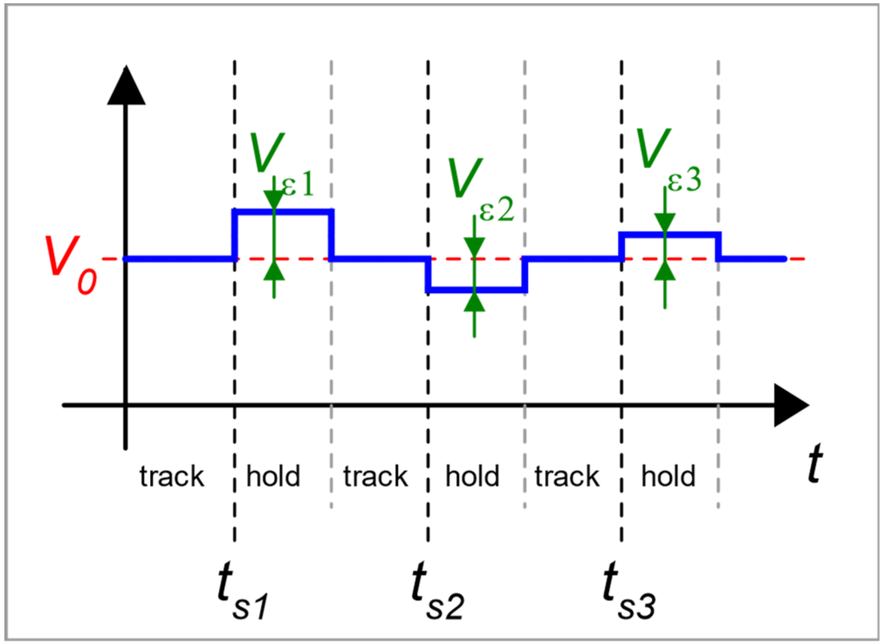
Vε is present even if the switch is perfectly ideal

What is the origin of Vε ?

Random nature of Vε

Repeating the sampling operation several time, the error is everytime different both in terms of magnitude and sign: it is a **random process**



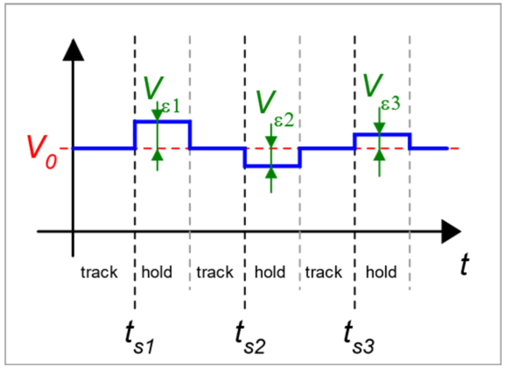


Its mean square value is determined only by:

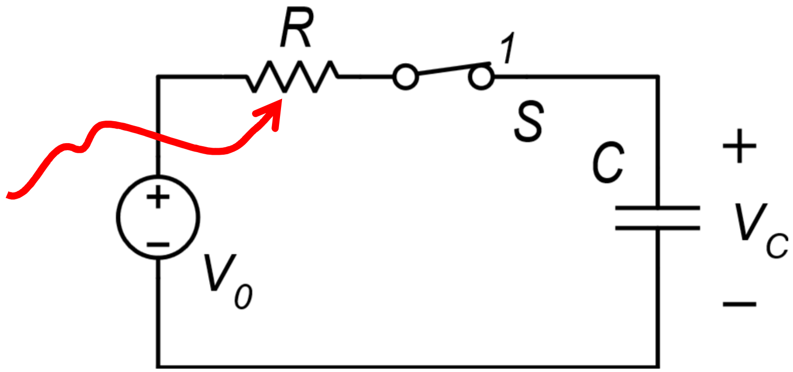
- capacitor value
- temperature

$$< v_{\varepsilon}^{2} > = \frac{kT}{C}$$

Origin of the kT/C noise

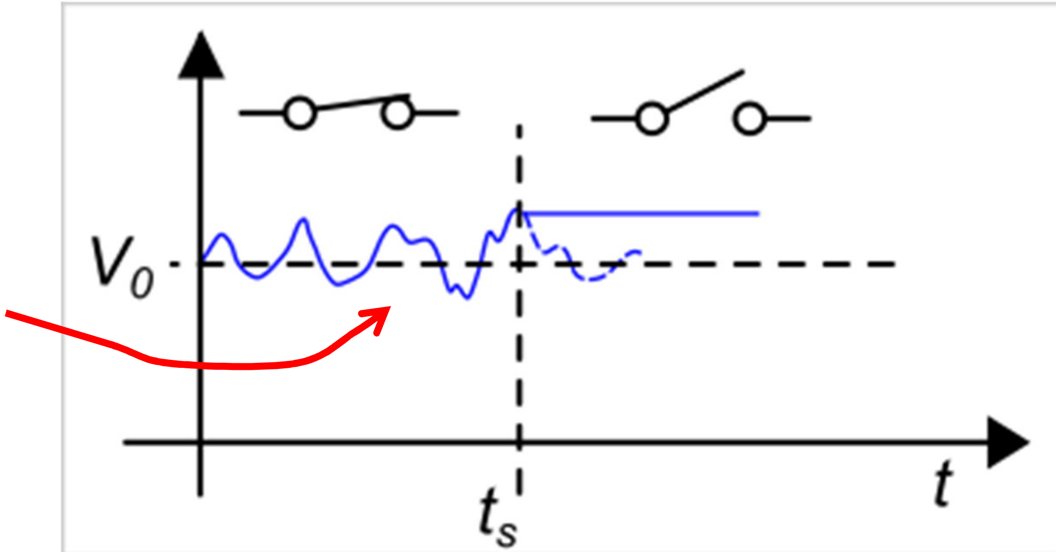


Let us add a series resistance: it can be due to the switch on-resistance and to the equivalent resistance of voltage source V_0.

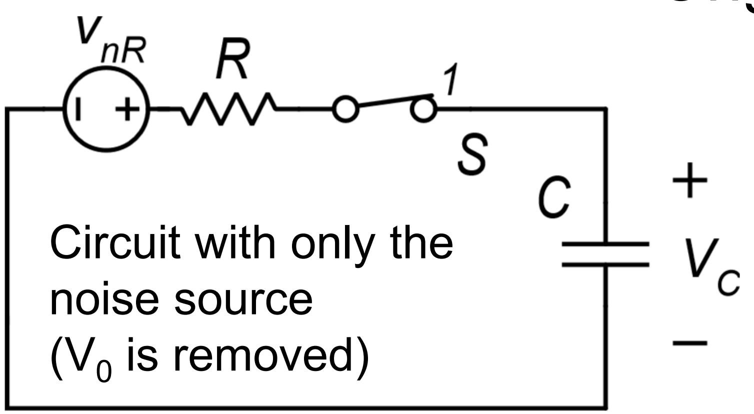


$$\langle v_{\varepsilon}^{2} \rangle = \frac{kT}{C}$$

Now it is clearer: voltage V_C was already fluctuating due to the noise of the resistor and we simply sample V_0 together with the noise sampled at t_S



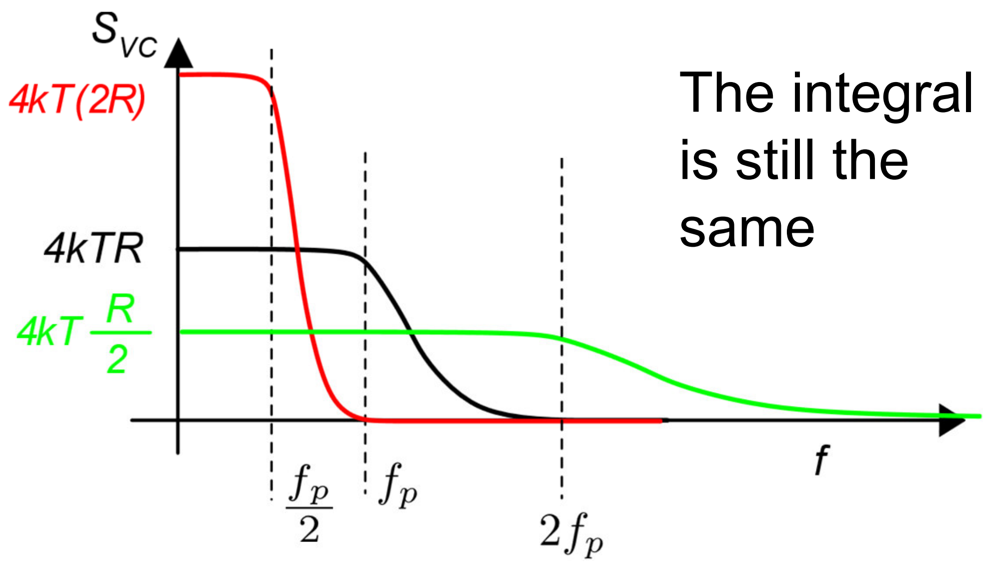
Origin of the kT/C noise



$$S_{VC} = 4kTR \frac{1}{1+\left(\frac{f}{f_p}\right)^2} \qquad f_p = \frac{1}{2\pi RC}$$

$$< v_{nc}^2 > = \int_0^\infty S_{VC}(f)df = \frac{kT}{C}$$

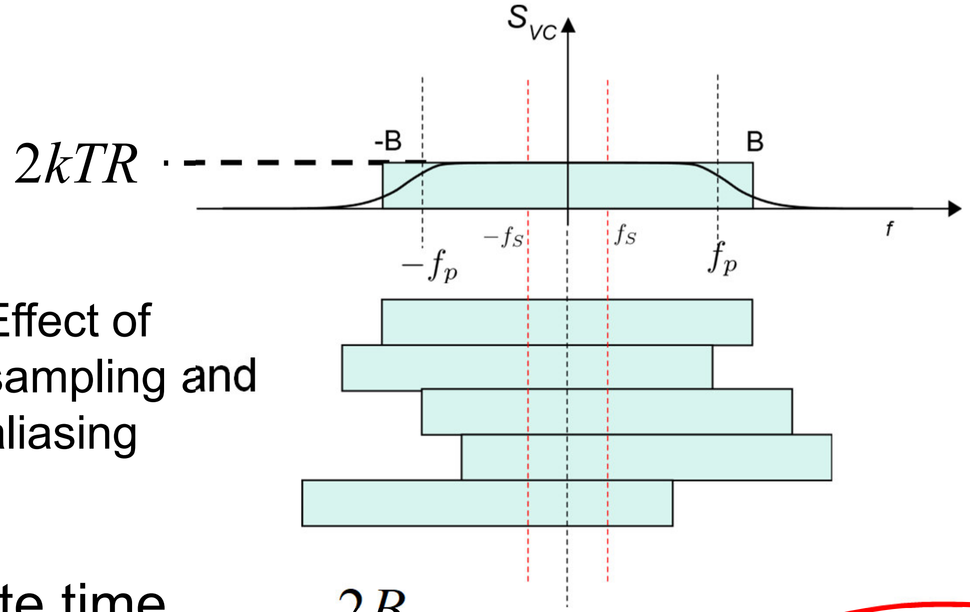
R-> 2R R-> R/2



The integral is still the same

The ideal case (R=0) represents a limit where $4kTR$ tends to zero and f_p tends to infinity. Again, the integral is still the same

kT/C noise as a result of continuous time noise sampling



Effect of sampling and aliasing

Two-sided spectral density of voltage V_C

The green rectangle is a simplified representation of the PSD

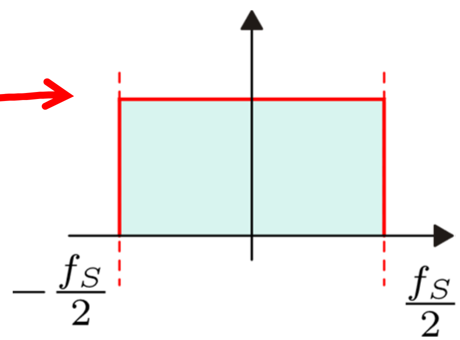
For the integral (i.e. mean square voltage of V_C) to be the same, B must be the effective noise BW:

$$B = f_p \frac{\pi}{2}$$

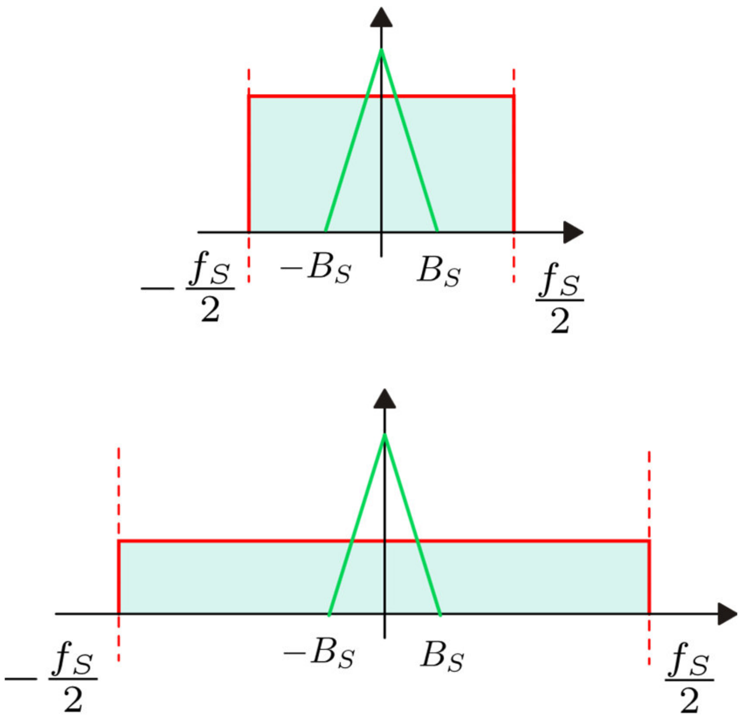
Discrete time PSD

$$\text{if } \frac{2B}{f_S} >> 1$$

$$S_{n-KTC} = \left(\cancel{1+}\frac{2B}{f_S}\right)2kTR \cong \left(2\cdot\frac{1}{2\pi RC}\frac{\pi}{2}\frac{1}{f_S}\right)2kTR = \frac{kT}{C}\frac{1}{f_S}$$



Changing the sampling frequency



The integral of the kT/C noise PSD is equal to *kT/C*, independently of the sampling frequency

Increasing the sampling frequency, the PSD is reduced proportionally to maintain the integral constant

In this way, filtering the output sequence in the discrete time domain (either by digital or analog processing), we get less noise in the signal bandwidth

kT/C noise in summary

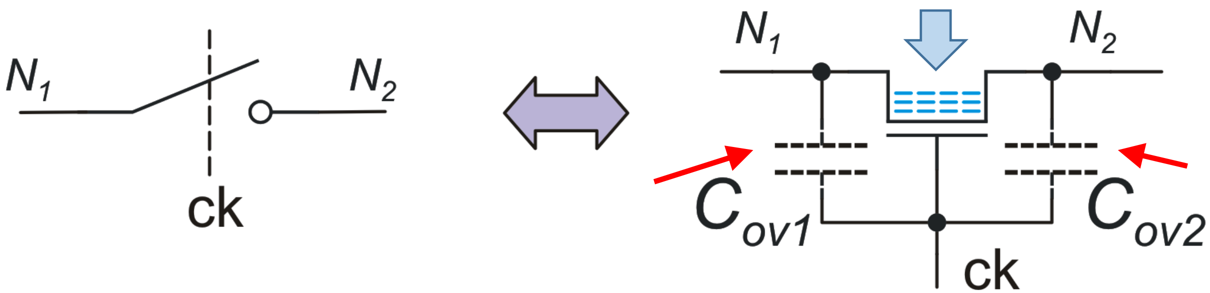
- *kT/C* noise is always present when we sample a voltage on a capacitor. The mean-square voltage of the samples is equal to *kT/C* and is independent of the series resistances of the switch, voltage source and capacitor.
- The result of sampling with a uniform sampling period produces a discrete time sequence affected by kT/C noise.
- If the sampling frequency f_S is << f_p, (i.e. $B >> f_s$), then the PSD of the *kT/C* noise is constant over the DT-frequency interval $-f_S/2$, $f_S/2$.
- Sometimes it is convenient to refer to the noise in terms of charge accumulated into the capacitor. In this case:

$$Q_\varepsilon = V_\varepsilon C \qquad \left\langle Q_\varepsilon^2 \right\rangle = \left\langle V_\varepsilon^2 \right\rangle C^2 = \frac{kT}{C} C^2 = kTC$$

The **charge injection** phenomenon: a systematic error in SC circuits

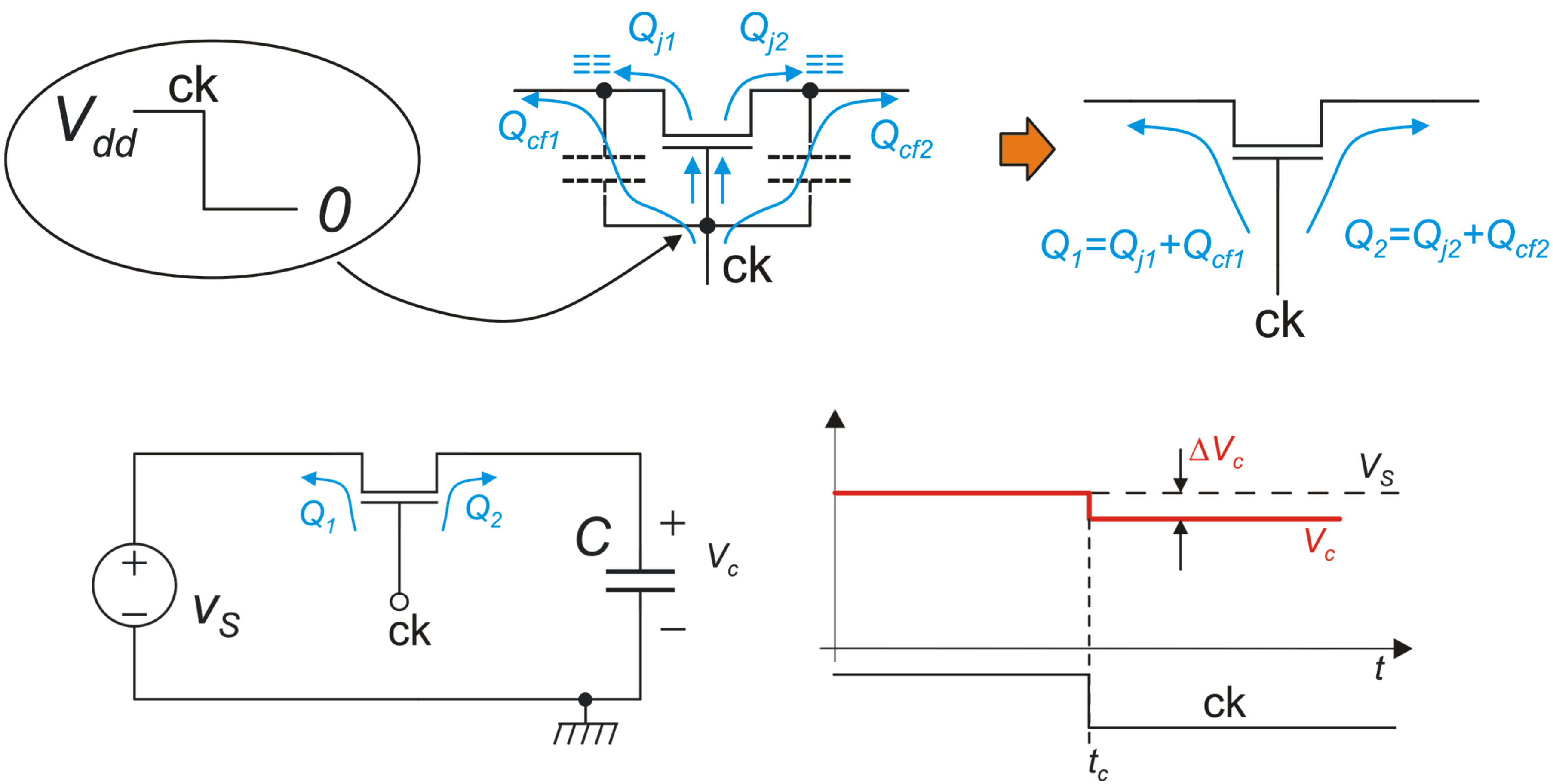
Differently from the kT/C noise, charge injection is due to switch **non-idealities**

The n-MOS switch

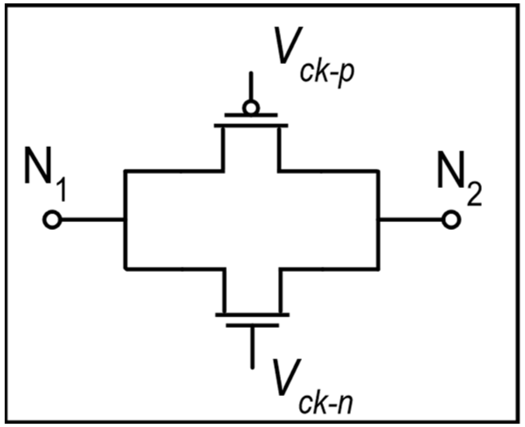


- Presence of overlap capacitance between the switch terminals and the control voltage
- In the on-state, there is charge accumulated into the channel (the mobile charge) that have to be drawn from the drain and source where it has to be pushed back when the switch is turned off

Charge injection during the on-to-off transient

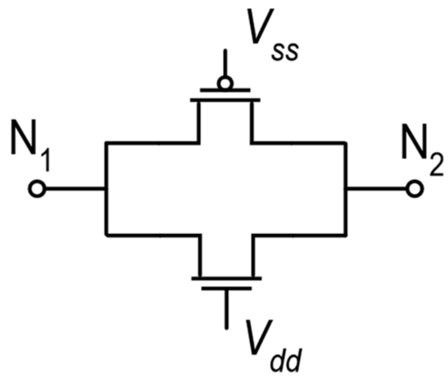


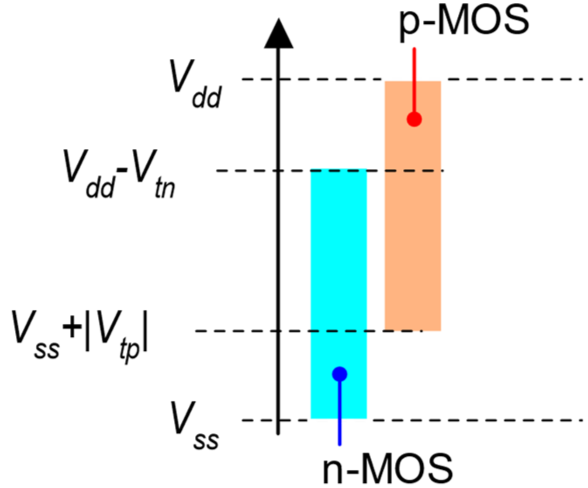
Charge injection in the pass-gate (transmission gate)



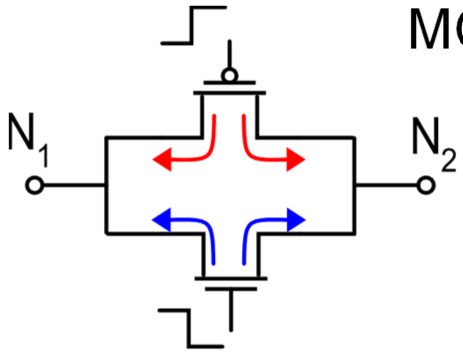
pass gate

on-state



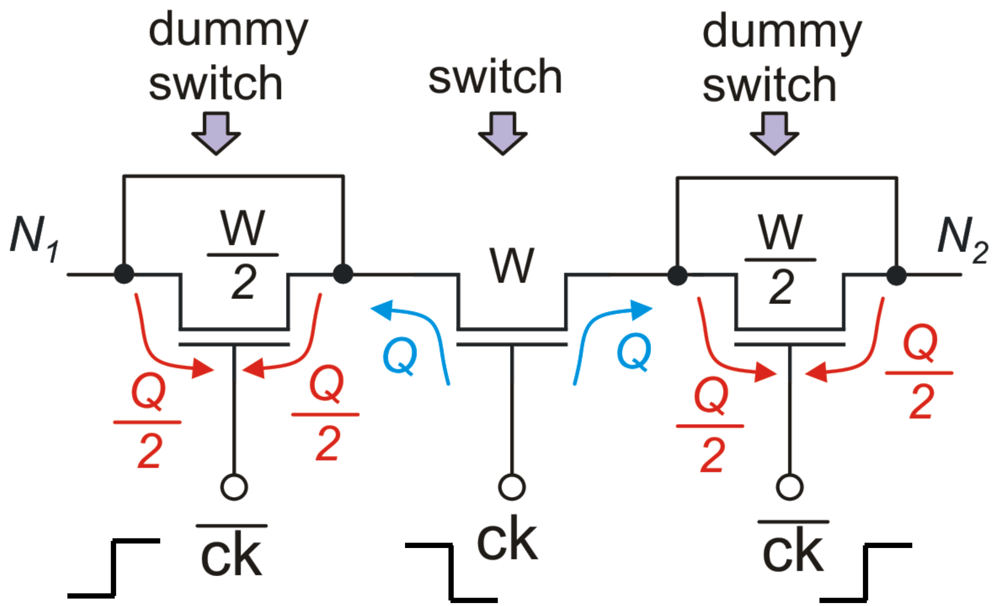


Voltage levels that are correctly transmitted by the n-MOS and p-MOS switches in the on-state



Charge injection in the on-to-off transient: charges are of opposite sign, but compensation is **imperfect** since the mobile charge depends on the V_{GS} and then on voltages at the N_1, N_2 nodes

Signal independent charge injection compensation

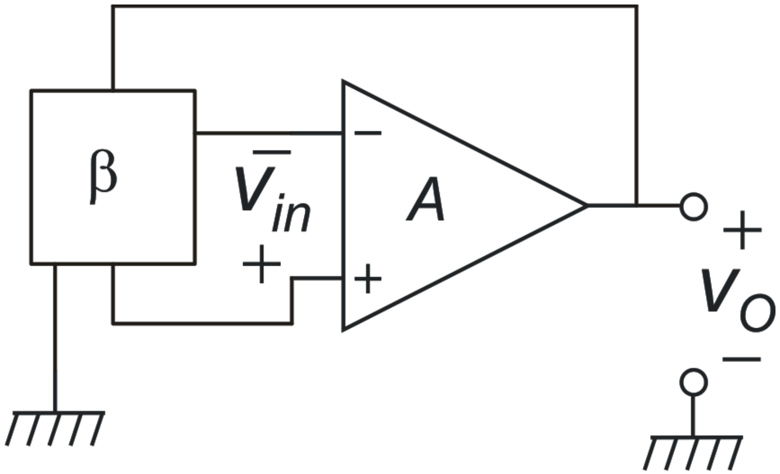


The dummy-switch does not affect the switching function, since it is short-circuited.

Its injected charge is opposite to that of the main switch because it is driven by the complementary control signal

- The dummy switches have the same length of the main switch but they have half the width.
- Dummy switches can be applied to both the p-mos and n-mos of a pass-gate

The input voltage of op-amps in closed loop configuration and in presence of noise / offset



Amplifier: $v_O = A\left(v_{in} - v_n\right)$

Feedback network: $v_{in} = \beta v_O + V_k$

$$v_{in} = \beta A(v_{in} - v_n) + V_k$$

$$v_{in}\left(1 - \beta A\right) = -v_n \beta A + V_k$$

$$v_{in} = \frac{-\beta A}{1-\beta A} v_n + \frac{V_k}{1-\beta A}$$

The input voltage of op-amps in closed loop configuration and in presence of noise / offset

$$v_{in} = \frac{-\beta A}{1-\beta A} v_n - \frac{V_k}{1-\beta A}$$

$$\beta = -\beta_0 < 0 \ (\beta_0 > 0, \ \text{indepent of } s)$$

$$A = \frac{A_0}{1 + \frac{s}{\omega_p}}$$

Let us focus on the effect of noise:

$$v_{in} = \frac{\beta_0 \frac{A_0}{1+\frac{s}{\omega_p}}}{1 + \beta_0 \frac{A_0}{1+\frac{s}{\omega_p}}} v_n = \frac{\beta_0 A_0}{1 + \frac{s}{\omega_p} + \beta_0 A_0} v_n$$

The fact that this term can be non-negligible is the origin of the finite gain error, We will not consider it for now.

The input voltage of op-amps in the presence of noise

$$v_{in} = \frac{\beta_0 A_0}{1 + \dfrac{s}{\omega_p} + \beta_0 A_0} v_n \qquad v_{in} = \underbrace{\frac{\beta_0 A_0}{1 + \beta_0 A_0}}_{\cong 1} \frac{1}{1 + \dfrac{s}{\underbrace{\omega_p \left(1 + \beta_0 A_0\right)}_{\cong \omega_p \beta_0 A_0 \cong \beta_0 \omega_0}}} v_n$$

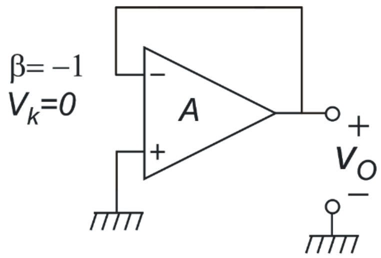
At the input of the op-amp we find a low-pass filtered version of the input referred noise

The cut-off frequency of the filter is nearly $\boldsymbol{\beta_0 f_0}$

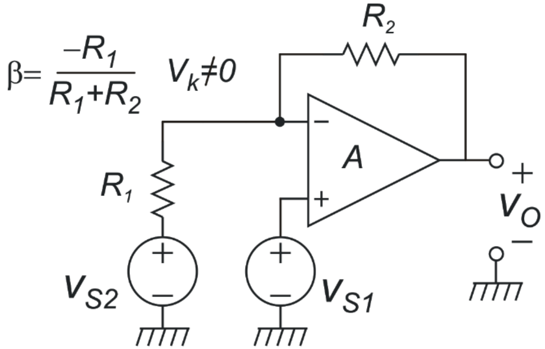
A few examples

$$v_{in} = \beta v_O + V_k$$

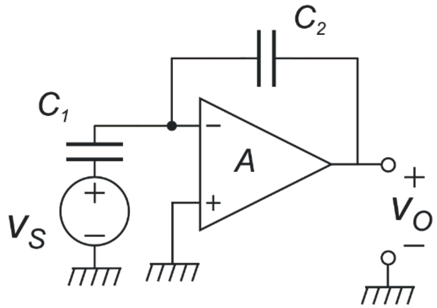
$$v_{in} = \frac{-\beta A}{1-\beta A} v_n + \frac{V_k}{1-\beta A}$$



Unity Gain configuration

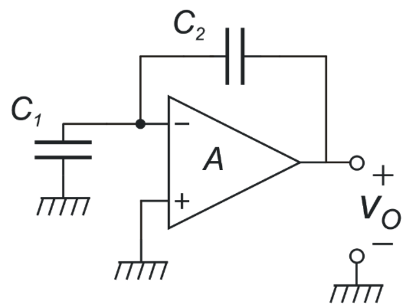


Inverting / non-Inverting Amplifier



$\beta = \frac{-C_2}{C_1+C_2}$ $V_k \neq 0$

Inverting Amplifier with capacitive fedback

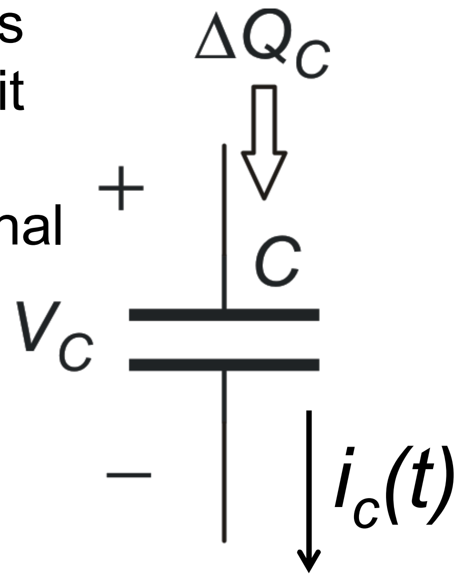


$\beta = \frac{-C_2}{C_1+C_2}$ $V_k \neq 0$

Inverting Amplifier with capacitive fedback for V_S=0

Charges through capacitors: conventions

Conventions on signs:
A charge is positive if it enters the plus terminal



Charge that pass through the capacitor from time t_i to time t_f

$$\Delta Q = \int_{t_i}^{t_f} i_c(t)dt = C\left[V_C(t_f) - V_C(t_i)\right]$$

$$V_C(t_f) = V_C(t_i) + \frac{\Delta Q}{C}$$