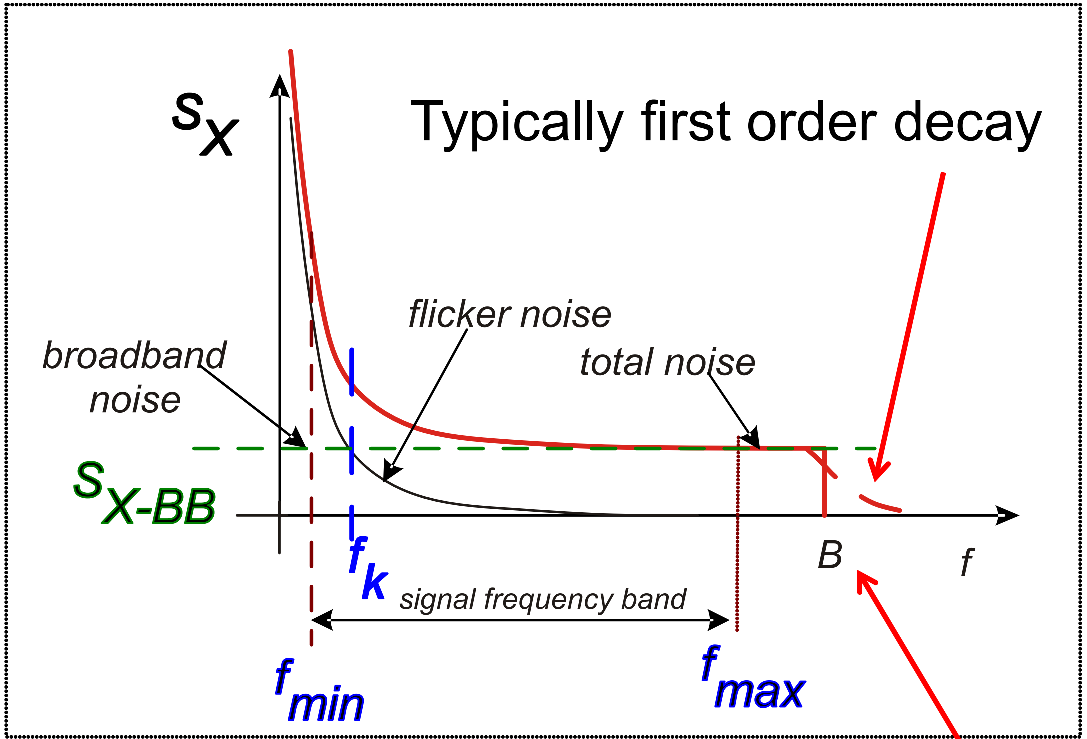
Practical Rules for noise calculations

Idealized amplifier noise density

Noise PSD in Low-Pass Systems



B=equivalent noise bandwidth

PSD: Power Spectral Density

$$S_{XF}(f) = \frac{k_F}{f^{\gamma}} \qquad \gamma \cong 1$$

$$S_{X-BB}(f) = \text{constant} = S_{X-BB}$$

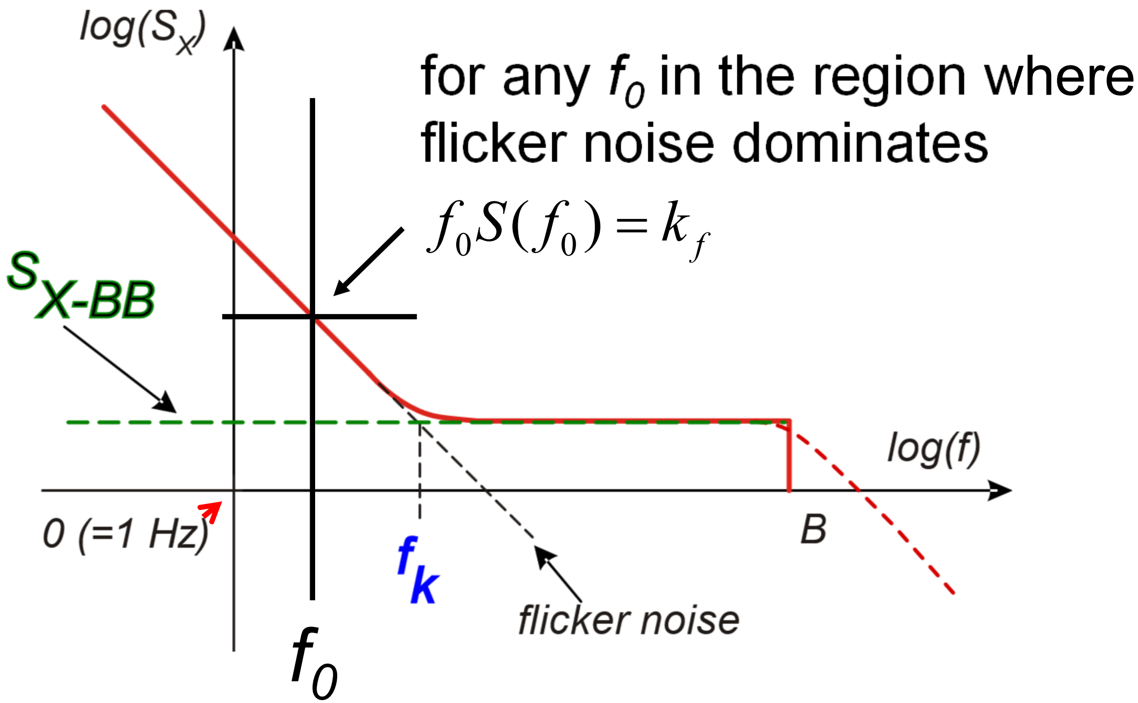
Definition of flicker corner frequency

$$f_k : \quad S_{XF}(f_k) = S_{X-BB}(f_k)$$

$$\Downarrow$$

$$f_k S_{X-BB} = k_F$$

Amplifier noise spectrum in logarithmic axes



Total rms noise in the signal bandwidth

$$x_{rms} = \sqrt{\int_{f_{\min}}^{f_{\max}} S_X(f)df} = \sqrt{\int_{f_{\min}}^{f_{\max}} S_{XBB}(f)df + \int_{f_{\min}}^{f_{\max}} S_{XF}(f)df}$$

Broad-band

$$\int_{f_{\min}}^{f_{\max}} S_{XBB}(f)df = S_{XBB}\left(f_{\max} - f_{\min}\right)$$

$$S_{XF}(f) = \frac{k_F}{f^{\gamma}}$$

Flicker

$$\int_{f_{\min}}^{f_{\max}} S_{XF}(f)df = k_F \ln\left(\frac{f_{\max}}{f_{\min}}\right) = k_F \underbrace{\log_{10}\left(\frac{f_{\max}}{f_{\min}}\right)} \frac{1}{\log_{10}(e)} \cong k_F 2.3 \cdot n_{dec}$$

n_{dec} =number of decades from f_{min} to f_{max}

Flicker noise for signal bands that include *dc*

$$\int_{f_{\min}}^{f_{\max}} S_{XF}(f)df = k_F \ln\left(\frac{f_{\max}}{f_{\min}}\right)$$

If f_{min} =0, the integral is infinity

Many signals of interest include dc. This is true for practically most signals produced by sensors like temperature, pressure, acceleration etc.

The solution to this paradox is that, in practical cases, speaking of a real dc component is meaningless, since it would be constant across an infinite interval of time.

For every practical scenario, there is always a **finite** "observation time period", across which we require a signal to be constant to state that this is a dc component.

Flicker noise for signal bands that include *dc*

Then, we use the flicker noise expression:

$$\int_{f_{\min}}^{f_{\max}} S_{XF}(f)\,df = k_F \ln\left(\frac{f_{\max}}{f_{\min}}\right) \qquad \text{with} \qquad f_{\min} \approx \frac{1}{T_{obs}}$$

Where T_{obs} is the observation time.

If the signal band includes dc, we generally set T_{obs} = 10s-100s, resulting in f_{min}= 0.1-0.01 Hz.

Example

$$\int_{f_{\min}}^{f_{\max}} S_{XF}(f)df = k_F\, 2.3 \cdot n_{dec}$$

Specifications: f_{max}=1kHz, f_{min}=0 (dc)

For T_{obs}=100 s, f_{min}=0.01 Hz

$n_{dec}=5$

For $T_{obs}=10^5$ s (> 1 day) s, f_{min}=10 μHz

$n_{dec}=8$

The flicker component to $<(x_n)^2>$ is increased by 60% and the *rms* component by 26%

In terms of resolution, this is quite a negligible increase. To have an increase of 1 unit in the ENOB associated to the DR we need an increase of 100 % in x_{rms} , i.e. a factor of 4 in $<(x_n)^2>$. The presence of a significant contribution from S_{XBB} makes this flicker increment even less important.

The choice of T_{obs} (f_{min}) is not critical !

Something more about the broad-band component

$$\int_{f_{\min}}^{f_{\max}} S_{XBB}(f)\,df = \underline{S_{XBB}}\left(f_{\max} - f_{\min}\right)$$

S_{XBB} has units $[X]^2/Hz$ where [X] are the units of quantity X.
For example if X is a voltage, we have **V²/Hz**
This is not the specification that is generally used in practical cases (e.g. amplifier datasheets).

What is generally given, is the square root of the PSD:

$$\sqrt{S_X} \qquad \text{Units:} \qquad [X]/\sqrt{Hz}$$

Example

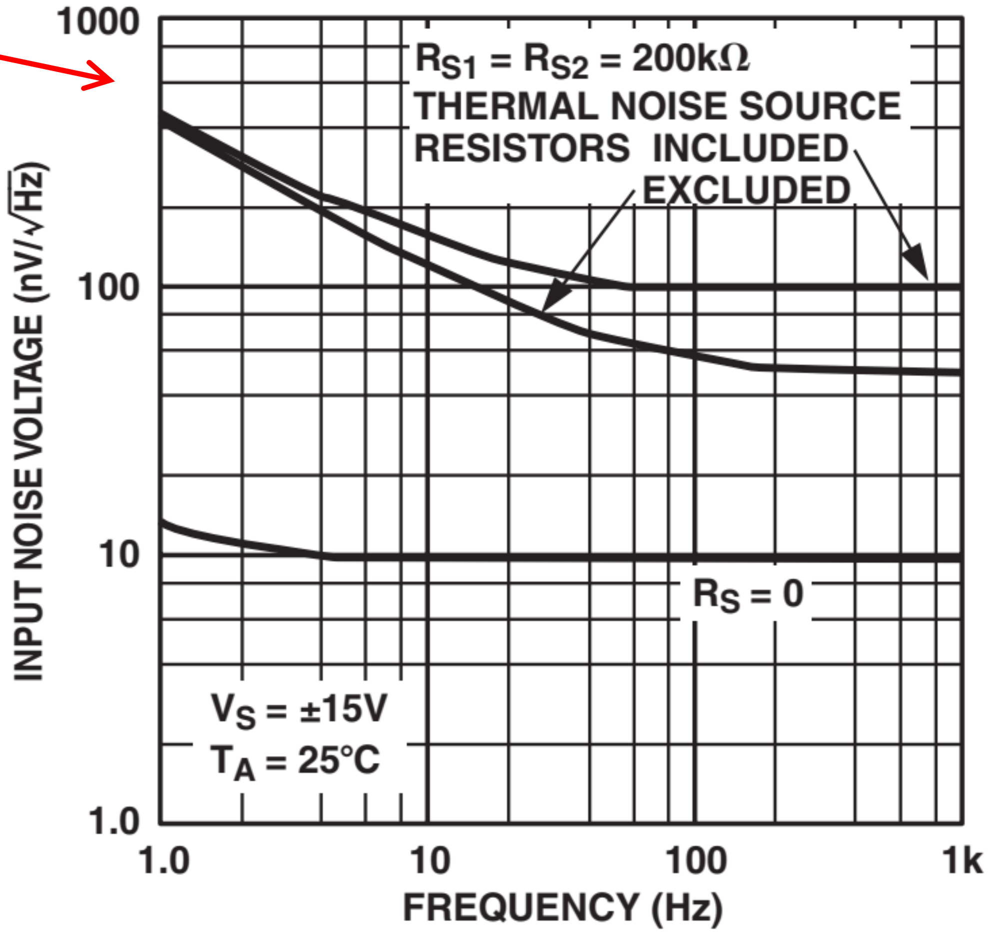
$$\sqrt{S_X}$$

$$nV/\sqrt{Hz}$$

or nV/sqrt(Hz)

«noise density»

(instead of :
noise **power** spectral density)



Practical *rms* noise calculation:

$$x_{rms} = \sqrt{\int_{f_{\min}}^{f_{\max}} S_X(f)df} = \sqrt{\int_{f_{\min}}^{f_{\max}} S_{XBB}(f)df + \int_{f_{\min}}^{f_{\max}} S_{XF}(f)df}$$

$$x_{rms-BB} = \sqrt{\int_{f_{\min}}^{f_{\max}} S_{XBB}(f)df}\,; \qquad x_{rms-F} = \sqrt{\int_{f_{\min}}^{f_{\max}} S_{XF}(f)df}$$

$$x_{rms} = \sqrt{x_{rms-BB}^2 + x_{rms-F}^2}$$

$$\sqrt{1+\left(\frac{1}{5}\right)^2} = 1.0198$$

It is sufficient that one of the two contribution is 5 times smaller than
the other to get practically negligible (with a 2 % error)

Example

Includes both flicker and BB noise

Input Noise Voltage	e_n p-p	0.1 Hz to 10 Hz[3]		0.38	0.65	μV p-p
Input Noise Voltage Density	e_n	f_O = 10 Hz		10.5	20.0	nV/√Hz

$$x_{rms-BB} = \sqrt{\int_{f_{\min}}^{f_{\max}} S_{XBB}(f)df} = \sqrt{S_{XBB}\left(f_{\max} - f_{\min}\right)} = \sqrt{S_{XBB}}\sqrt{\left(f_{\max} - f_{\min}\right)}$$

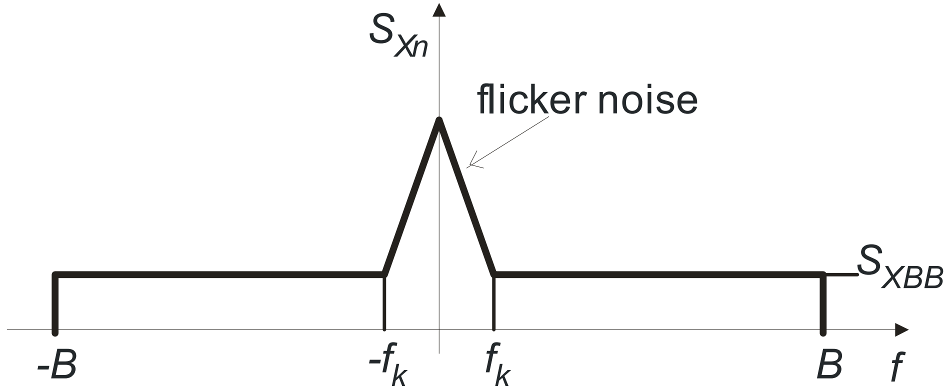
$$x_{rms-F} = \sqrt{\int_{f_{\min}}^{f_{\max}} S_{XF}(f)df} = \sqrt{2.3k_F n_{dec}} \cong 1.5\sqrt{k_F}\sqrt{n_{dec}}$$

> X_{pp-F} over two decades

$$x_{n-pp} = 4x_{n-rms} = 4\sqrt{x_{rms-BB}^2 + x_{rms-F}^2} = \sqrt{\left(4x_{rms-BB}\right)^2 + \left(4x_{rms-F}\right)^2}$$

$$x_{n-pp} = \sqrt{x_{pp-BB}^2 + x_{pp-F}^2}$$

Schematic two-sided representation of amplifier noise

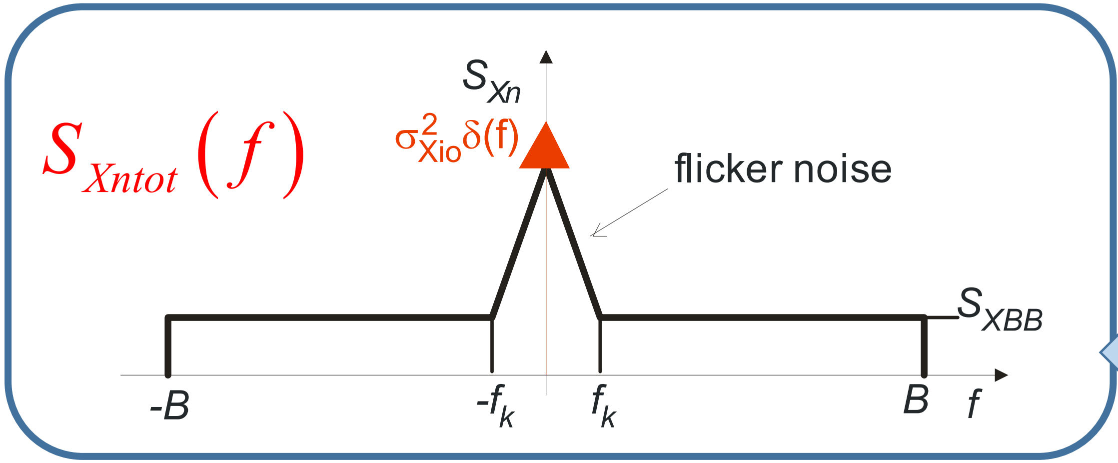


$$x_{ntot} = x_n + x_{io}$$

Total additive error: offset + noise

x_{io} is a stationary, non-ergodic stochastic process.
Noise and offset are independent processes

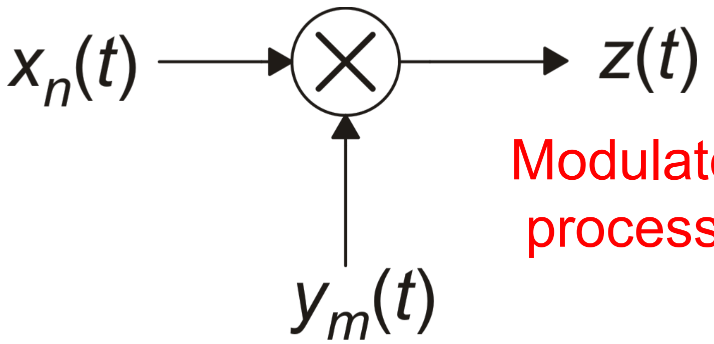
$$R_{Xntot}(\tau) = R_{Xn}(\tau) + \sigma_{Xio}^2$$



Generalized spectrum that represents noise and offset together

Modulation of a stochastic process

Stationary
stochastic process
(e.g. noise)



Modulated
process

Modulating signal (e.g. local oscillator)

We can consider y_m a stochastic process by adding some randomness. It is sufficient to introduce a random delay to make *z(t)* stationary. Or we can refer to the formalism of <u>cyclostationary</u> processes

If y_m is a periodic deterministic signal, *Z(t)* is generally no more a stationary process

The auto-correlation function is no more a simple function of the delay τ and a PSD cannot be calculated

But ...

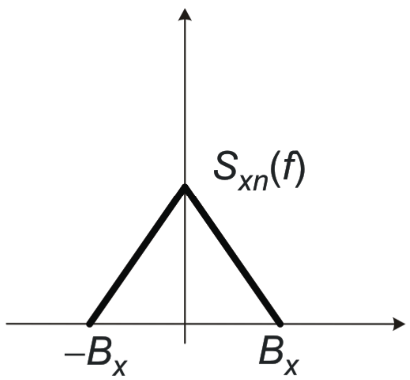
Modulation of stochastic processes

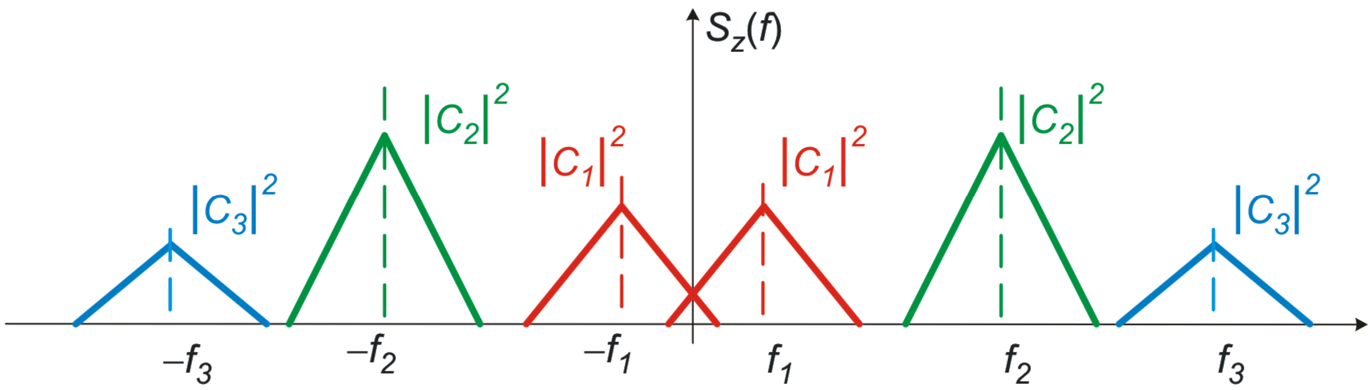
$$y_m(t) = \sum_{k=-n}^{n} C_k e^{j\omega_k t}$$

Complex Fourier series representation of the modulating signal $y_m(t)$

$$C_{-k} = C_k^* \qquad \omega_k = k\omega_0$$

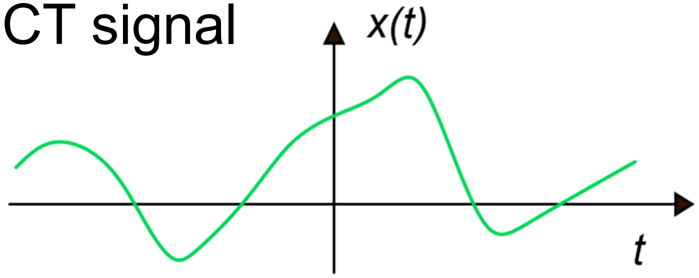
$$S_Z(f) = \sum_{i=-\infty}^{\infty} \left|C_i\right|^2 S_{Xn}(f - k\omega_i)$$



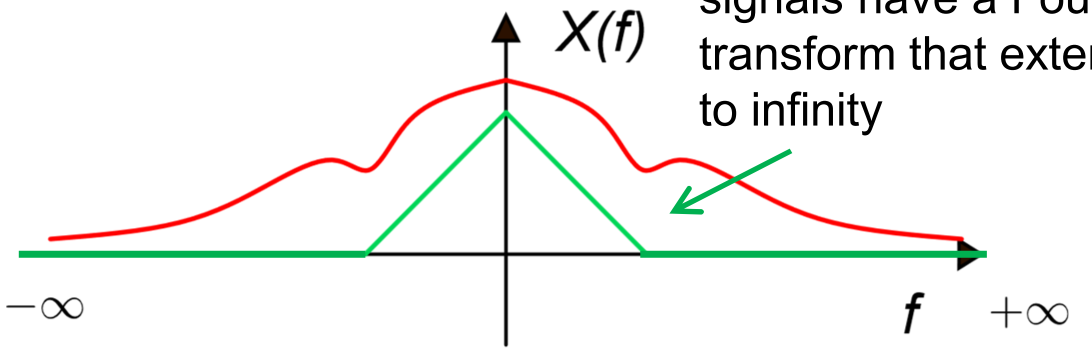


Discrete-time (DT) and continuous-time (CT) signals

CT signal

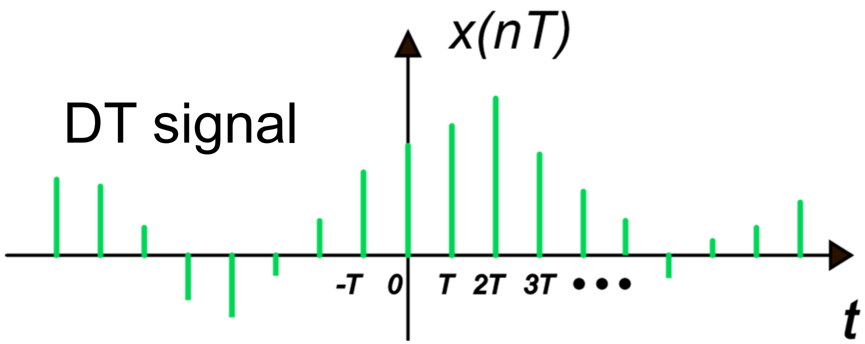


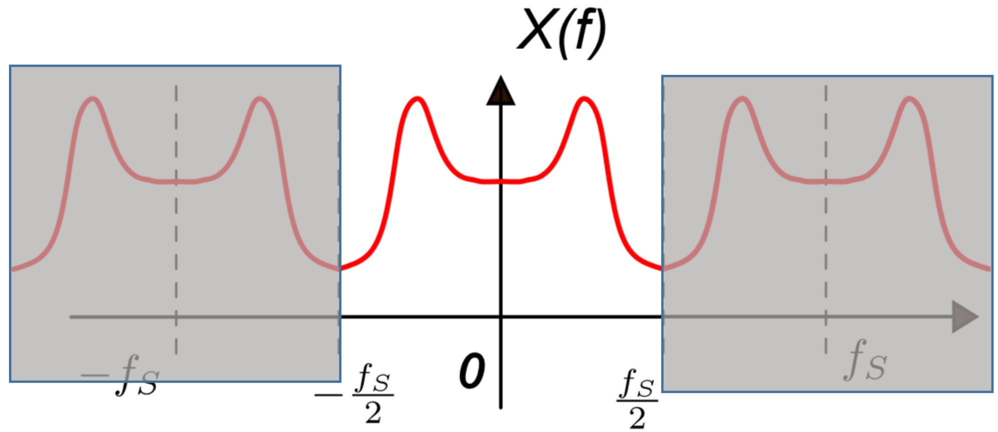
Even band-limited signals have a Fourier transform that extends to infinity



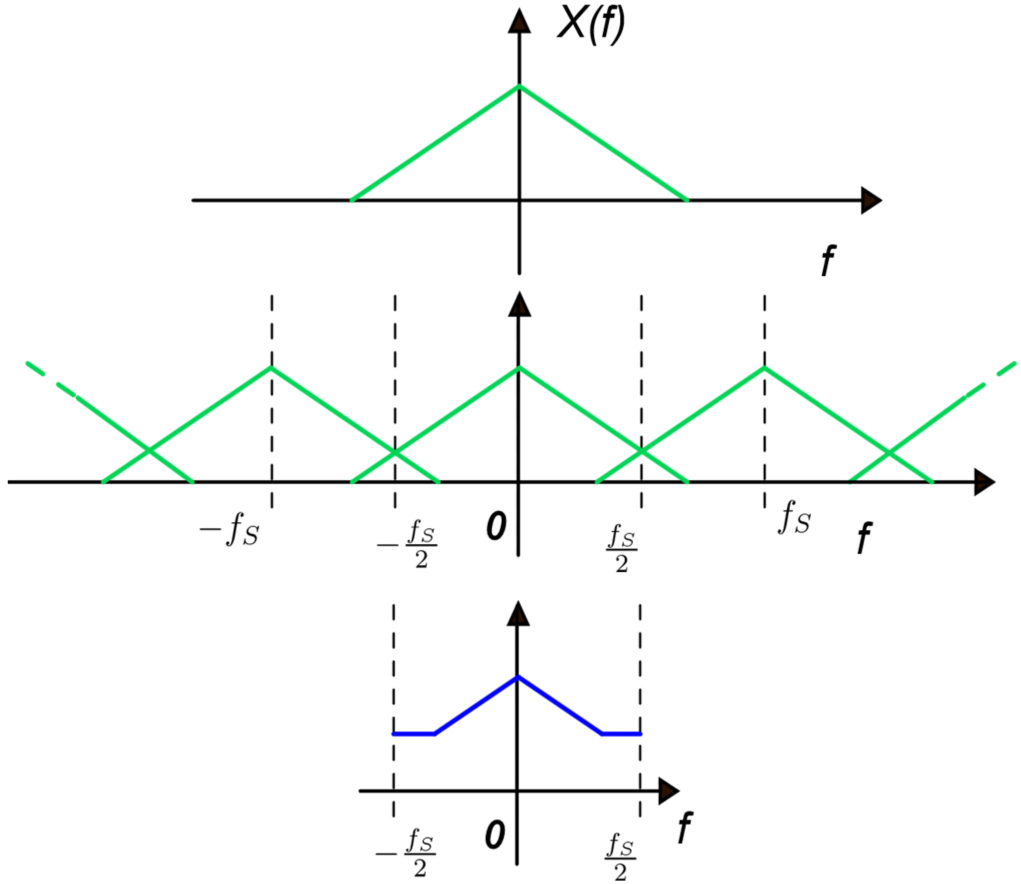
…. and their Fourier transform

DT signal





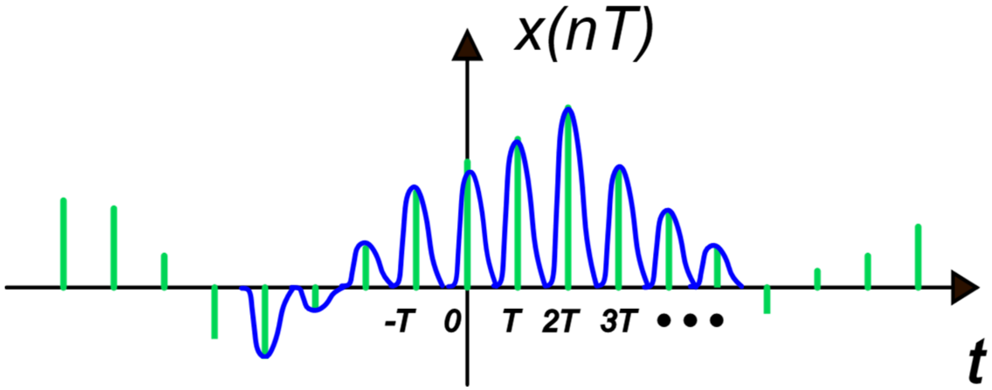
DT signals from sampling of CT signals



Place a replica of the original spectrum across each multiple of f_S

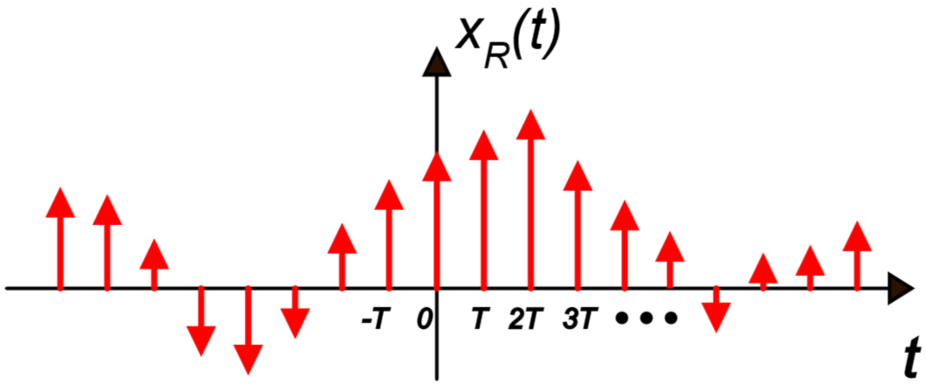
Add the replicas only across the DT frequency interval

CT signals from DT ones

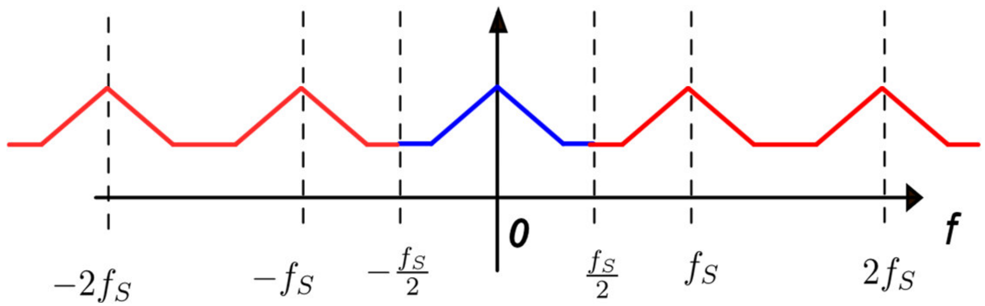


For our purposes, we are interested in:

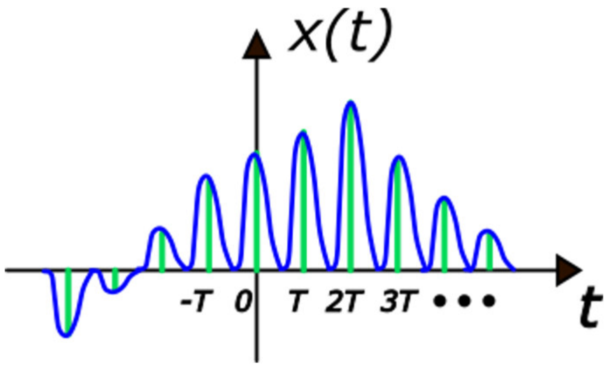
- Reconstruction with delta functions δ(t)
- Hold - reconstruction

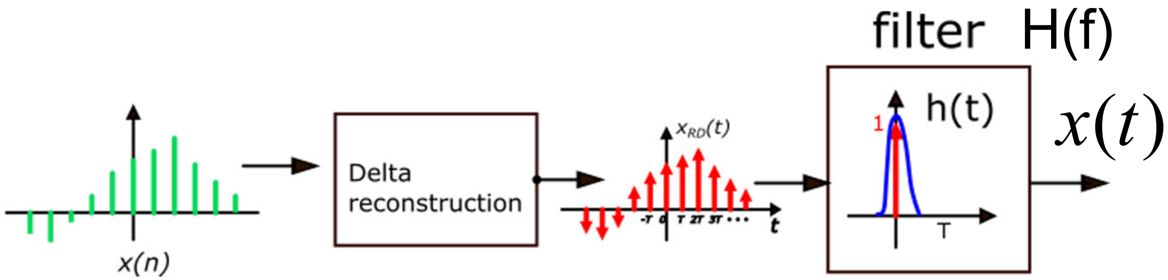


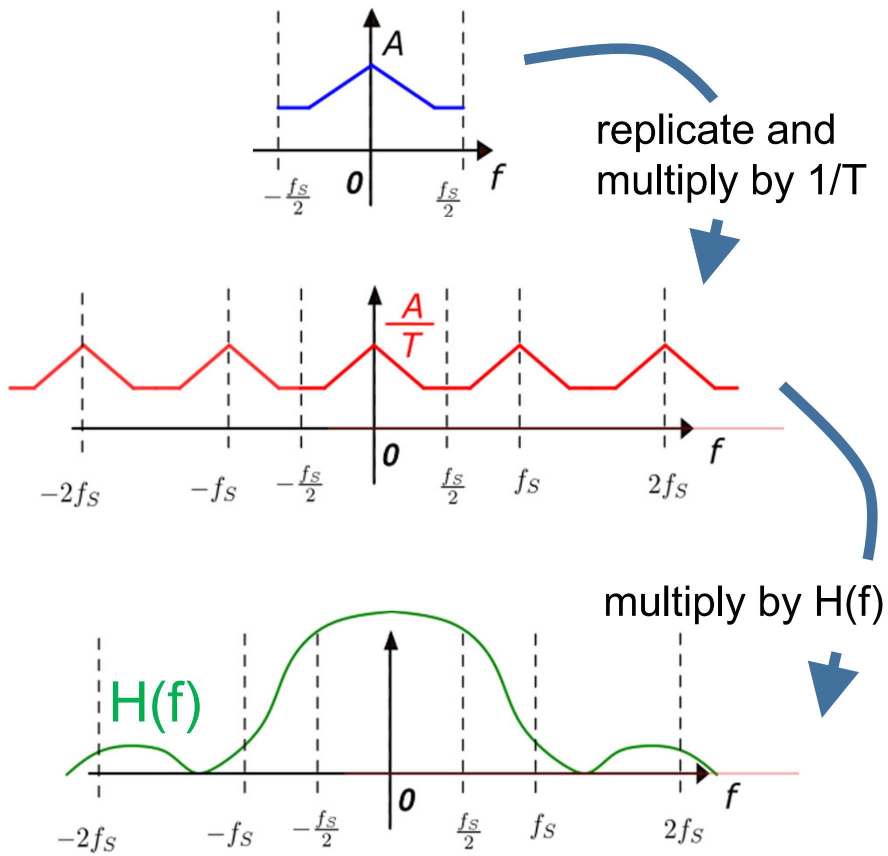
Replicas are multiplied by $\frac{1}{T}$



DT-CT reconstruction with an arbitrary pulse

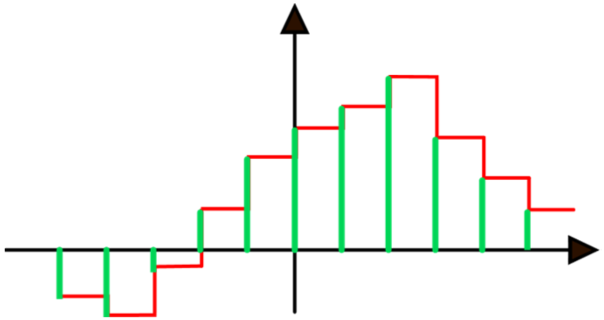






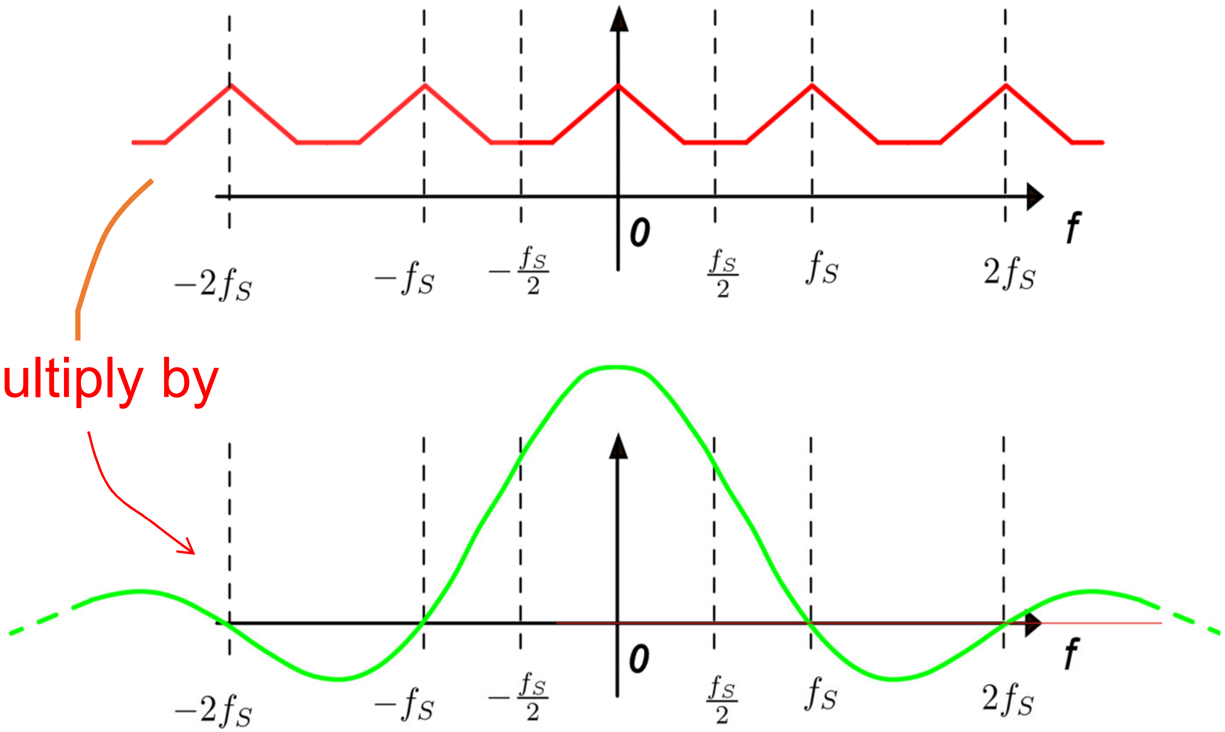
Theoretical (non real) process

Hold reconstruction

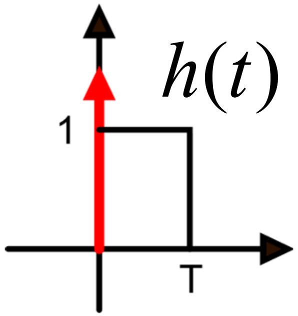


It is the simpler way to convert a digital signal (data sequence) into an analog one by means of a DAC

$$\text{replicas} \times \frac{1}{T}$$

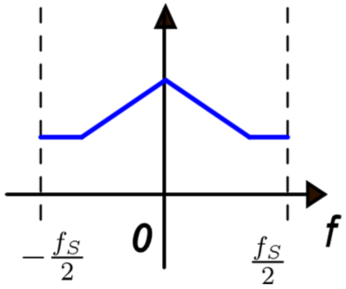


Reconstruction pulse

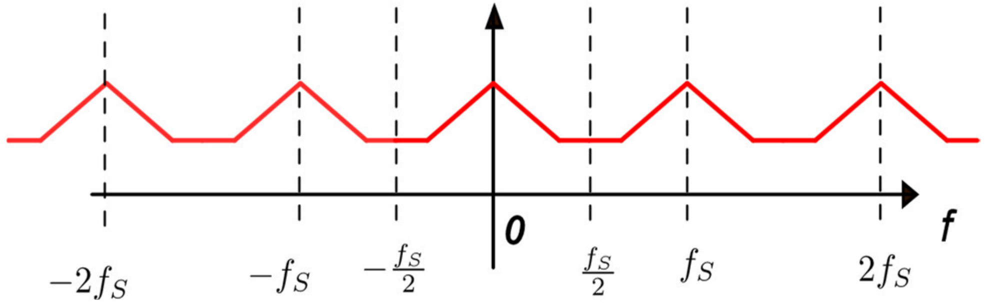


$$H(f) = T \cdot e^{-j\pi fT} \operatorname{sinc}(\pi fT) \qquad \operatorname{sinc}(x) = \frac{\sin(x)}{x}$$

Hold - reconstruction: summary

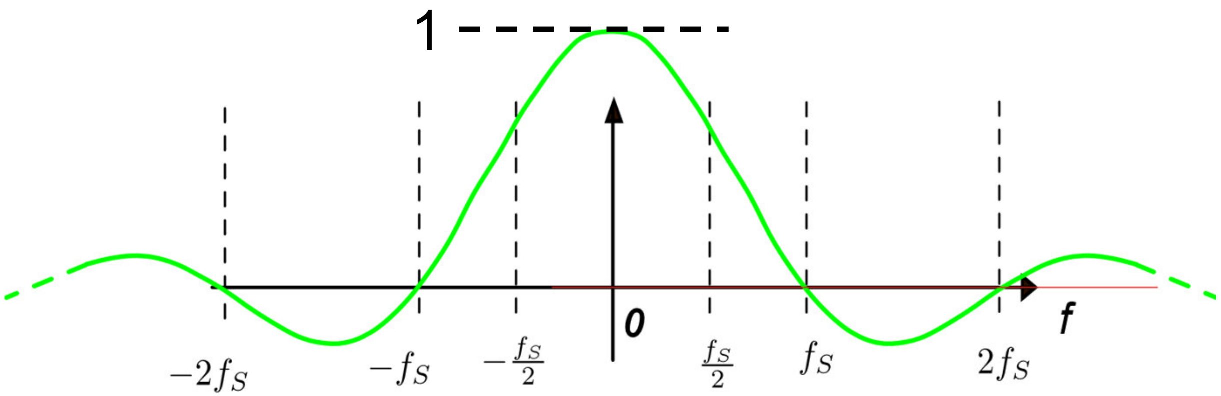


Start from the discrete time fourier transform



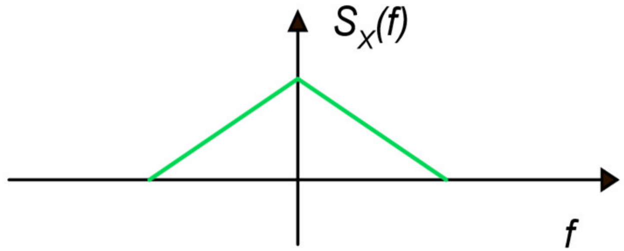
Multiply by: $e^{-j\pi fT}\mathrm{sinc}\left(\pi fT\right)$

Replicate the spectrum (non scaled) across any multiple of f_s

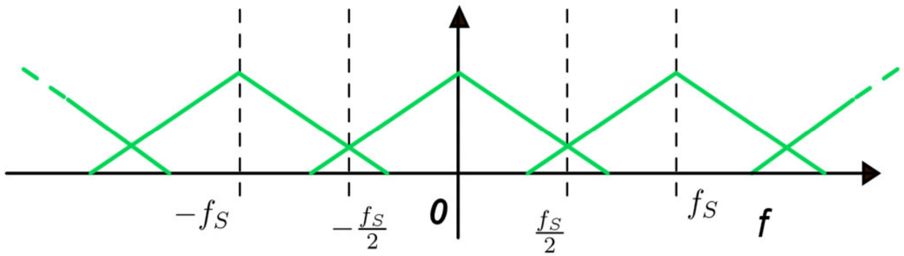


Sampling and holding a stochastic process

PSD of the CT stochastic process

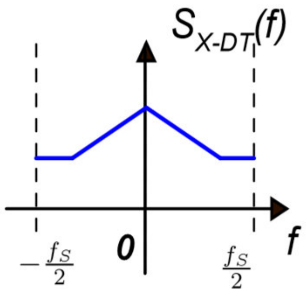


The PSD is replicated and the replicas are added

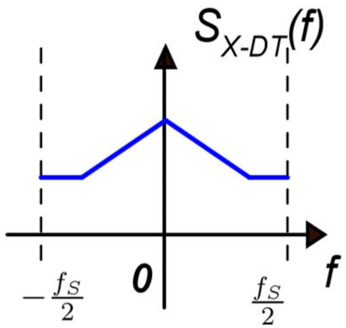


The procedure is similar to the case of a deterministic signal but for a stochastic process it is the PSD to be involved

Only the spectrum in the $-f_S/2$, $+f_S/2$ interval is retained

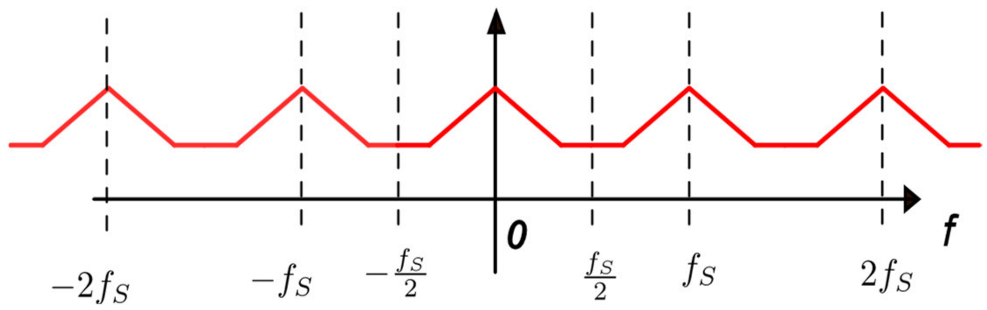


Reconstruction by hold-operation



PSD of the DT stochastic process

Replicate with f_s step



multiply by $|H(s)|^2 = \mathrm{sinc}^2(\pi f T)$