Determination of the effect of the common mode feedback control in the case of mismatch between the output resistances.

Defining:

$$R = \frac{R_{out1} + R_{out2}}{2} \quad \Delta R = R_{out1} - R_{out2} \tag{1}$$

and:

$$I_{\varepsilon 1} = \frac{I_{\varepsilon 1} + I_{\varepsilon 2}}{2} \quad \Delta I_{\varepsilon 1} = I_{\varepsilon 1} - I_{\varepsilon 2} \tag{2}$$

and referring the output offset short circuit current to an input offset:

$$\Delta I_{\varepsilon 1} = g_{m1} V_{io}, \tag{3}$$

we can calculate the output voltages considering the presence of the CMFB term $-g_m^*(V_{oc} - V_{CMO})$ on both the output short circuit currents, where V_{CMO} is the target output common mode voltage:

$$\begin{aligned} V_{o1} &= V_{CMO} + \left(R + \frac{\Delta R}{2}\right)\left[-g_m^*(V_{oC} - V_{CMO}) - \frac{g_{m1}}{2}(V_d - V_{io}) + I_\varepsilon\right] \\ V_{o2} &= V_{CMO} + \left(R - \frac{\Delta R}{2}\right)\left[-g_m^*(V_{oC} - V_{CMO}) + \frac{g_{m1}}{2}(V_d - V_{io}) + I_\varepsilon\right] \end{aligned} \tag{4}$$

which can be expanded as in the following equation:

$$\begin{aligned} V_{o1} &= V_{CMO} + R\left[-g_m^*(V_{oC} - V_{CMO}) - \frac{g_{m1}}{2}(V_d - V_{io}) + I_\varepsilon\right] + \frac{\Delta R}{2}\left[-g_m^*(V_{oC} - V_{CMO}) - \frac{g_{m1}}{2}(V_d - V_{io}) + I_\varepsilon\right] \\ V_{o2} &= V_{CMO} + R\left[-g_m^*(V_{oC} - V_{CMO}) + \frac{g_{m1}}{2}(V_d - V_{io}) + I_\varepsilon\right] + \frac{\Delta R}{2}\left[g_m^*(V_{oC} - V_{CMO}) - \frac{g_{m1}}{2}(V_d - V_{io}) - I_\varepsilon\right] \end{aligned} \tag{5}$$

The output differential and common mode voltages are then:

$$V_{od} = V_{o2} - V_{o1} = g_{m1} R (V_d - V_{io}) + \Delta R\left[g_m^*(V_{oC} - V_{CMO}) - I_\varepsilon\right] \tag{6}$$

$$V_{oC} = \frac{V_{o1} + V_{o2}}{2} = V_{CMO} - R g_m^*(V_{oC} - V_{CMO}) + R I_\varepsilon - \frac{\Delta R}{2}\frac{g_{m1}}{2}(V_d - V_{io}) \tag{7}$$

Solving the common mode equation (7) we find:

$$(V_{oC} - V_{CMO}) = \frac{R I_\varepsilon}{(1 + R g_m^*)} - \frac{\Delta R}{4}\frac{g_{m1}}{(1 + R g_m^*)}(V_d - V_{io}) \tag{8}$$

From which we can calculate the CMFB term:

$$g_m^*(V_{oC} - V_{CMO}) = \frac{g_m^* R I_\varepsilon}{(1 + R g_m^*)} - \frac{\Delta R}{4}\frac{g_{m1} g_m^*}{(1 + R g_m^*)}(V_d - V_{io}) \tag{9}$$

Substituting this quantity into V_{od}, we get:

$$V_{od} = g_{m1}R(V_d - V_{io}) + \Delta R\left[+\frac{g_m^* R I_\varepsilon}{(1+Rg_m^*)} - \frac{\Delta R}{2}\frac{g_m^*}{(1+Rg_m^*)}\frac{g_{m1}}{2}(V_d - V_{io}) - I_\varepsilon\right] =$$
$$= g_{m1}R(V_d - V_{io}) - \Delta R\frac{\Delta R}{2}\frac{g_m^*}{(1+Rg_m^*)}\frac{g_{m1}}{2}(V_d - V_{io}) - \Delta R I_\varepsilon\left[1 - \frac{g_m^* R}{(1+Rg_m^*)}\right] = \tag{10}$$
$$= g_{m1}R(V_d - V_{io})\left[1 - \frac{\Delta R}{2R}\frac{\Delta R}{2}\frac{g_m^*}{(1+Rg_m^*)}\right] - \Delta R I_\varepsilon\frac{1}{(1+Rg_m^*)}$$

With reasonable approximations:

$$V_{od} = g_{m1}R(V_d - V_{io})\left[1 - \frac{1}{4}\left(\frac{\Delta R}{R}\right)^2\right] - \frac{\Delta R}{R}\frac{I_\varepsilon}{g_m^*} \tag{11}$$

Conclusions

Regarding the output common mode stabilization, equation (8) indicates that there is an additional error term with respect to the case of equal output resistances. This term can be written as:

$$\frac{\Delta R}{4}\frac{g_{m1}}{(1+Rg_m^*)}(V_d - V_{io}) \cong \frac{\Delta R}{4R}\frac{g_{m1}}{g_m^*}(V_d - V_{io}) \tag{12}$$

This error is of the same order of magnitude as the input signal $V_d - V_{io}$. This implies a coupling between the input differential signal and the output common mode voltage. However, due to the large gain of the amplifier, the input signal is generally in the sub-millivolt range, so that the effects on the output common mode voltage are negligible.

As far as the output differential mode is concerned, Eq.(11) shows that the output resistance mismatch (ΔR), alters the differential gain, which now is given by:

$$A_{dd} = g_{m1}R\left[1 - \frac{1}{4}\left(\frac{\Delta R}{R}\right)^2\right] \tag{13}$$

instead of simply $g_{m1}R$. For moderate resistance mismatch, this effect is not important. For example, for ΔR/R=0.2 (20 % mismatch), the gain is loss is only 1 %, which can be neglected since we are dealing with operational amplifiers, where the gain should be high but no gain precision is required.

In addition to the effect on the gain, an additional offset term is also present in Eq. (11). This term is given by (see also Chap 3.3B):

$$V_{od-offset} = \frac{\Delta R}{R}\frac{I_\varepsilon}{g_m^*} = \frac{\Delta R}{R}\frac{I_\varepsilon}{I_D^*}V_{TE}^* \tag{14}$$

Even in the case of strong output resistance mismatch, this offset term can be expected to be the order of a few mV. Once referred to the input, it falls into the microvolt range or below. Therefore, it can be always neglected with respect to the main offset term V_{io}.