

# 1 Simple device noise models for analog circuit design

## 1.1 MOSFETs noise

Noise is modeled considering an ideal (noiseless) MOSFET with a noise current source (equivalent noise current) connected across the drain and source terminals, as shown in Fig.1.1. This schematization, that involves a single noise source, is valid at low frequencies. For very high frequencies an input current source should be added. The input current source is not completely dependent on the output current source ( $i_n$  in Fig. 1.1) but a certain degree of correlation exists. The single-source noise schematization can be used up to a frequency limit that depends on the technological process. In the next part of this document, we will assume that the single noise source model of Fig. 1.1 is valid.

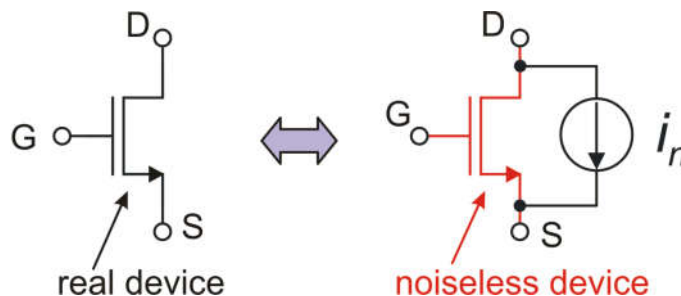


Fig. 1.1. Modeling of the Mosfet noise with a current source placed across drain and source.

### Noise current spectral density (MOSFETs in saturation region)

The current noise of MOSFETs is characterized by two main contributions: thermal noise and flicker noise.

**Thermal Noise.** Equation (1.1) represents a commonly used expression for the thermal component of the noise power spectral density (PSD).

$$S_{i_n-T}(f) = \frac{8}{3} kT g_m \cdot m \quad (1.1)$$

where  $m=1+g_{mb}/g_m$  and  $g_{mb}$  is the body effect transconductance. Note that  $g_{mb}$  is generally in the order of 0.2  $g_m$ , thus assuming  $m=1$  leads only to underestimating the noise density by nearly 20 %. In some articles and textbooks, the following alternative expression is used for the PSD of the MOSFET thermal noise current:

$$S_{i_n-T}(f) = \gamma_n 4kT g_m \quad (1.2)$$

where  $\gamma_n$  is a multiplying factor. Considering that, in a resistor, the equivalent current noise is given by  $4kTG$ , where  $G=1/R$ , then the MOSFET current noise PSD is larger than that of a resistor of conductance  $G=g_m$  by a factor  $\gamma_n$ .

Note that expression (1.2) is equivalent to (1.1) if  $\gamma_n$  is equal to:

$$\gamma_n = \frac{2}{3} m \quad (1.3)$$

**Flicker noise.** Several different expressions are used to model the flicker component of the MOSFET equivalent noise current. An expression that yields a sufficient accuracy and is very practical for hand calculations is the following:

$$S_{In-F}(f) = \frac{N_f}{WL} \frac{1}{f} g_m^2 \quad (1.4)$$

where  $N_f$  is a process parameter that we will indicate with  $N_{fn}$  for n-MOSFETs and  $N_{fp}$  for p-MOSFETs.

A more general equation for the PSD of the flicker noise current is given by Eq.(1.5). This expression is used in some SPICE models.

$$S_{In-F}(f) = \frac{k_{fi} I_D^\alpha}{C_{OX} L_{eff}^2} \frac{1}{f^\gamma} \quad (1.5)$$

where  $k_{fi}$  is a process dependent parameter. It can be easily demonstrated that (1.5) is equivalent to (1.4) with the following parameter choice:

$$\gamma = 1, \quad \alpha = 1, \quad N_f = \frac{k_{fi}}{2C_{OX}^2 \mu} \quad (1.6)$$

The particular case represented by (1.4) is generally preferred by analog designers since it allows obtaining a good tradeoff between simplicity and accuracy.

### *Equivalent gate noise*

In frequent cases, it is more convenient to represent the device noise by a voltage source in series with the gate. The equivalence with the circuit of Fig.1.1 should hold in all possible configurations, In particular, if we short circuit the gate to the source and calculate the drain noise current produced by  $v_n$ , then the result should be identical to  $i_n$ . Imposing this condition, the following relationship is found:

$$g_m v_n = i_n \quad (1.7)$$

Therefore, the PSD of the equivalent drain noise current ( $S_{In}$ ) and of the equivalent voltage source ( $S_{Vn}$ ) are tied together by the following equality:

$$S_{In}(f) = g_m^2 S_{Vn}(f) \quad (1.8)$$

Unfortunately, a voltage source is not sufficient to replace the noise current  $i_n$ . To understand this, consider the case where the gate is left open. In these conditions, no current is flowing in the input mesh so that, due to the gate-source capacitance,  $v_{gs}$  is zero. Thus, the current source  $v_n$  is unable to produce a noise drain current. Conversely, with the equivalent circuit of Fig.1.1 the drain noise current

is always equal to  $i_n$ , regardless of the input termination. For this reason, a gate-referred circuit capable of representing the effects of  $i_n$  for all gate connections must include also a gate current source, as shown in Fig. 1.2.

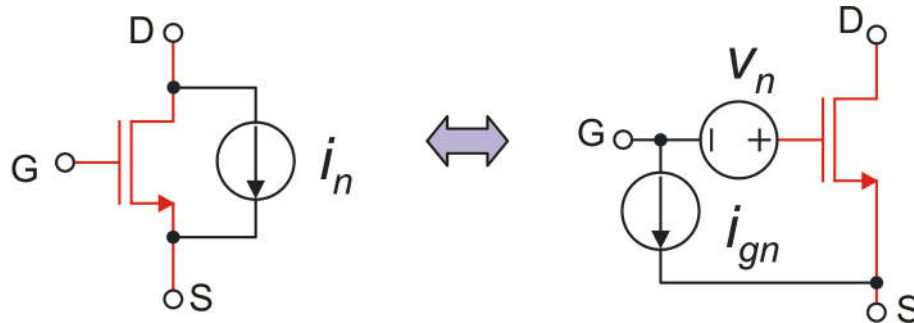


Fig.1.2. Gate-referred noise sources.

By imposing the equivalence between the two noise equivalent circuits in the case of open gate, it is possible to find the following relationship between  $i_n$  and  $i_{gn}$ :

$$i_{gn} = i_n \frac{j\omega C_{gs}}{g_m} = -v_n j\omega C_{gs} \tag{1.9}$$

If the path from the gate to the source is characterized by a finite resistance indicated with  $R_s$  (see Fig.1.3), then the  $v_{gs}$  of the noiseless MOSFET (in red in Fig.1.3) is given by:

$$v_{gs} = \frac{1}{1 + j\omega R C_{gs}} v_n - \frac{R}{1 + j\omega R C_{gs}} i_n \tag{1.10}$$

Considering that  $i_n$  depends on  $v_n$  through (1.9), then:

$$v_{gs} = \frac{1}{1 + j\omega R C_{gs}} v_n + \frac{j\omega R C_{gs}}{1 + j\omega R C_{gs}} v_n = v_n \tag{1.11}$$

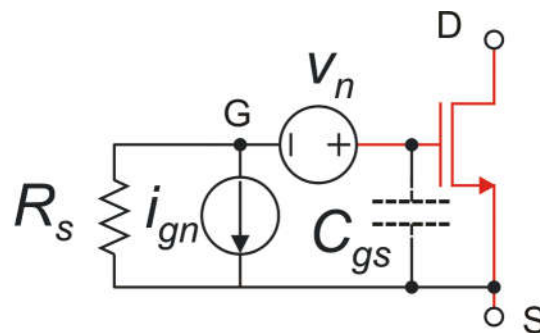


Fig.1.3. Input referred noise sources with termination of the gate on a resistor.

The result is that, for any possible gate termination, the combined effect of noise sources  $v_n$  and  $i_{gn}$  cause  $v_{gs}$  to be equal to  $v_n$ .

The PSD of  $v_n$  in the case of thermal and flicker noise can be simply obtained applying (1.8) to (1.1) and (1.4), respectively.

Then:

$$S_{v_{n-T}} = \frac{8}{3} kT \frac{1}{g_m} \cdot m \quad (\text{Gate-referred thermal noise PSD}) \quad (1.12)$$

$$S_{v_{n-F}} = \frac{N_f}{WL} \frac{1}{f} \quad (\text{Gate-referred flicker noise PSD}) \quad (1.13)$$

## 1.2 Noise in bipolar transistors

Differently from MOSFETs, bipolar transistor has a non-zero input bias current (the base current) which, as every currents, includes a noise component. As a result, the BJT noise model includes two current sources, as shown in Fig. 1.4

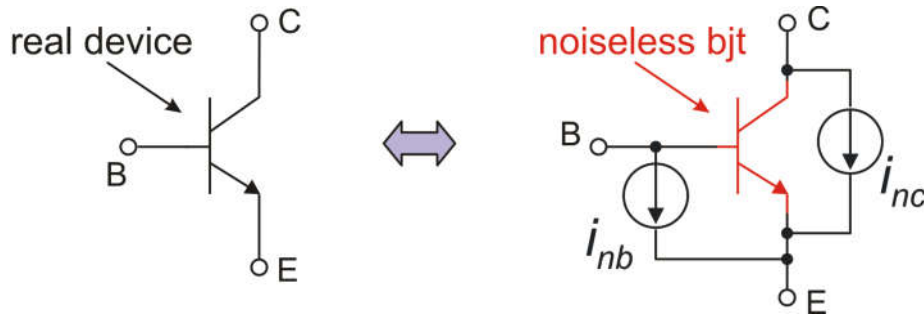


Fig.1.4. Modelling of the BJT noise with a drain ( $i_{nc}$ ) and base ( $i_{nb}$ ) equivalent noise current sources.

The collector noise current is considered to be affected only by shot noise with a spectral density given by:

$$S_{i_{nC}} = 2qI_C \quad (1.14)$$

The base current includes also a significant flicker noise contribution. Its total spectral density is given by:

$$S_{i_{nB}} = 2qI_B + k_{fB} \frac{I_B^\alpha}{f} \quad (1.15)$$

Where  $k_{fB}$  and exponent  $\alpha$  are process dependent parameters.

The collector current noise dominates when the path between the base and the emitter is marked by a low resistance. Conversely, at low enough frequencies the flicker component of the base current begins to give a non-negligible contribution even when the gate-to-emitter resistance is kept properly low. However, typical flicker corner frequencies are, at most, of the order of a few hundred Hertz, while MOSFETs of moderate gate area may exhibit corner frequencies up to several MHz.

The expression of the collector noise current can be re-arranged as follows, in order to make the expression similar to (1.1), allowing easy comparison between MOSFETs and BJTs.

$$S_{I_{nC}} = 2qg_mV_T = 2kTg_m \quad (1.16)$$

where we have used the simple relationship:  $I_C = g_mV_T$ , with  $V_T = kT/q$ . Note that (1.16) is equivalent to (1.2) with  $\gamma_n = 1/2$ . Since  $\gamma_n$  is close to one for MOSFETs, a BJT produces nearly half the noise of a MOSFET with the same  $g_m$ . This is true if the base current noise can be neglected, i.e. when the base of the BJT is driven by a source with a small enough output impedance.

The input referred noise voltage of a BJT, consisting to a voltage source in series with the base that substitutes the current source  $i_{nc}$ , can be simply found by dividing the expression in Eq. by  $(g_m)^2$ . Then we have:

$$S_{V_n} = 2kT \frac{1}{g_m} = 2kT \left( \frac{V_T}{I_C} \right) \quad (1.17)$$

### 1.3 Noise efficiency factor.

The fact that the best tradeoff between current consumption and input voltage noise is obtained with a BJT device makes the latter suitable to be used as a reference to compare the efficiency of more complex circuits or amplifiers. A figure of merit (FOM) which is based on this idea is the NEF (Noise Efficiency Factor), introduced by M. Steyaert in Ref. [1]. This figure is given by Eq. (1.18):

$$NEF = \frac{V_{rms-rti}}{\sqrt{4kT \left( \frac{V_T}{I_{supply}} \right) B \frac{\pi}{2}}} \quad (1.18)$$

where  $V_{rms-rti}$  and  $I_{tot}$  are the input referred noise (*rms* value) and total current consumption of the amplifier the NEF refer to. The factor  $B$  indicates the -3 dB bandwidth of the amplifier. Considering (1.17), it can be easily found that the denominator of the NEF is proportional to the input referred *rms* noise of a single BJT biased with a collector current equal to the total current used by the amplifier ( $I_{tot}$ ). The noise is calculated over the effective bandwidth of the amplifier, which is considered to be equal to  $B\pi/2$ . This is the effective noise bandwidth corresponding to the -3dB bandwidth when the amplifier is characterized by a first-order low-pass behavior (20 dB/decade). More precisely, the denominator exactly equal to the cited single-BJT noise multiplied by a factor of  $\sqrt{2}$ . The larger the NEF, the less efficient is the amplifier to produce a low input noise voltage for a given current consumption.

### 1.4 References.

- [1] M. Steyaert, et al., "A micropower low-noise monolithic instrumentation amplifier for medical purposes," IEEE J. Solid State Circuits, vol. 22, no. 6, pp. 1163–1168, Dec. 1987.

Example of MOSFET parameters for a 0.25  $\mu\text{m}$  CMOS process.

Parametro	n-MOS	p-MOS
$\mu_n C_{ox}, \mu_p C_{ox}$	$240 \times 10^{-6} \text{ A/V}^2$	$50 \times 10^{-6} \text{ A/V}^2$
$V_{tn}, V_{tp}$	0.43 V	-0.56 V
$\gamma$ (effetto body)	$0.44 \text{ V}^{1/2}$	$0.59 \text{ V}^{1/2}$
$k_{\lambda}$	50 V/ $\mu\text{m}$	50 V/ $\mu\text{m}$
$\alpha$ (coeff. termico della $V_t$ )	-1 mV / $^{\circ}\text{C}$	1 mV / $^{\circ}\text{C}$
$N_{fn}, N_{fp}$ (fattore rumore flicker)	$6 \times 10^{-10} \text{ V}^2 \mu\text{m}^2$	$2 \times 10^{-10} \text{ V}^2 \mu\text{m}^2$
$C_{Vt}$ (matching $V_t$ )	8.5 mV· $\mu\text{m}$	8.5 mV· $\mu\text{m}$
$C_{\beta}$ (matching beta)	0.03 $\mu\text{m}$	0.03 $\mu\text{m}$
$C_{ox}$	6.2 fF/ $\mu\text{m}^2$	6.2 fF/ $\mu\text{m}^2$
$L_C$ (lunghezza minima D/S)	1.2 $\mu\text{m}$	1.2 $\mu\text{m}$
$C_J$	1.8 fF/ $\mu\text{m}^2$	1.8 fF/ $\mu\text{m}^2$
$C_{gdo}$	0.6 fF/ $\mu\text{m}$	0.6 fF/ $\mu\text{m}$
$t_{ox}$	5.6 nm	5.6 nm