Frequency response of two stage operational amplifiers with Miller compensation.

The small signal equivalent circuit of a two-stage op-amp is shown in Fig.1. There are three independent capacitors, thus there will be three poles. Since all capacitors are interacting, it is not possible to separate the poles with a rigorous analytical procedure. Nevertheless, it can be useful to find approximate expressions, based on the physical behavior of the network.

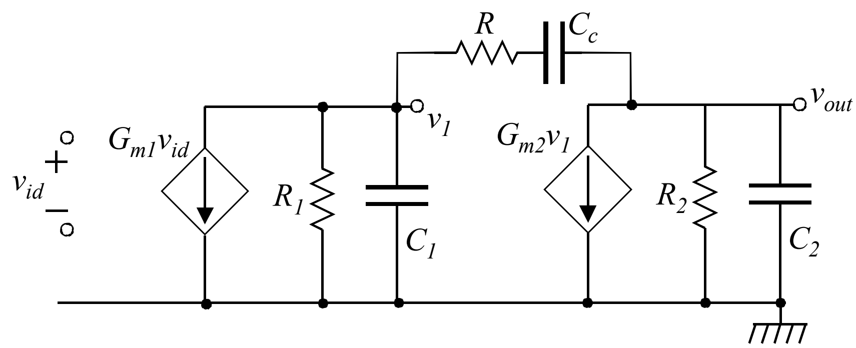

Fig.1. Small signal equivalent circuit of a two stage operational amplifier.

First, it is possible to observe that a zero a present in the frequency response. To find it, it is possible to calculate the ratio v_{out}/v_1 with the circuit of Fig.2.

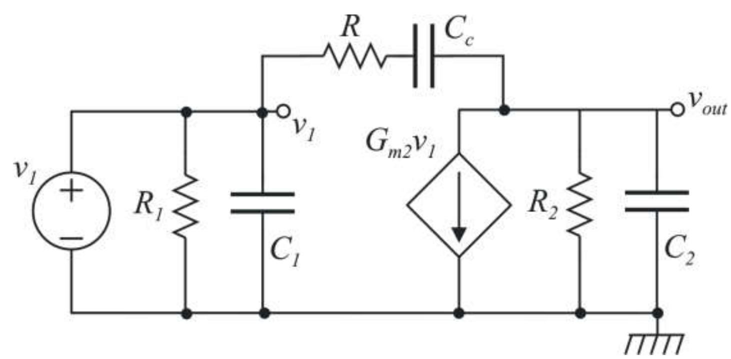

Fig.2. Circuit used to calculate the v_{out}/v_1 ratio. .

To facilitate the calculation, it is convenient to find first the output short circuit current, as shown in Fig.3(a), obtaining:

$$i_{out-sc} = -G_{m2}v_1 + \frac{v_1}{R + \frac{1}{sC_C}} = -G_{m2}\frac{1 + sC_C\left[R - 1/G_{m2}\right]}{1 + sC_C R}v_1 \tag{1}$$

The output voltage is then:

$$\frac{v_{out}}{v_1} = \frac{i_{out-sc}}{v_1}Z \tag{2}$$

where Z is the impedance seen across the output nodes, when the source v1 is turned off, as in Fig.3(b).

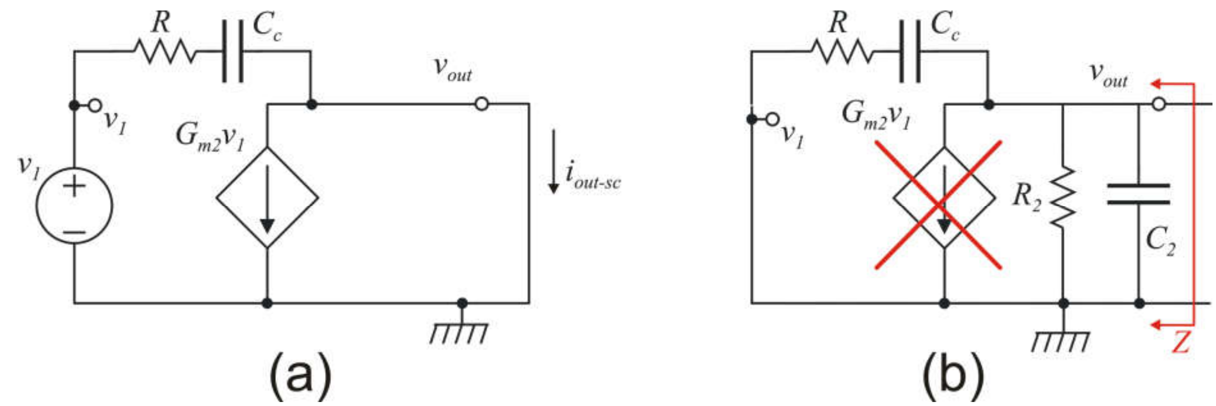

Fig.3. Calculation the v_{out}/v_1 ratio: (a) output short circuit current; (b) impedance Z seen when v_1 is turned off.

With simple calculations, we get:

$$Z=\left[\frac{1}{R+\frac{1}{sC_C}}+\frac{1}{R_2}+sC_2\right]^{-1}=\frac{R_2(1+sC_CR)}{1+s(C_CR+C_CR_2+C_2R_2)+s^2C_CC_2R_2R} \quad (3)$$

From (1-3), we find:

$$\frac{v_{out}}{v_1}=-G_{m2}R_2\frac{1+sC_C(R-1/G_{m2})}{1+s(C_CR+C_CR_2+C_2R_2)+s^2C_CC_2R_2R} \quad (4)$$

Equation (4) demonstrates that a zero is present. The exact value of zero is:

$$s_Z=-\frac{1}{C_C(R-1/G_{m2})} \quad (5)$$

Note that the role of resistor R is to modify the value of the zero. A typical choice is pushing the zero to infinity, i.e. cancelling it. Another possible choice is choosing the value of R such that the zero is negative ($R>1/G_{m2}$) and its value is equal to the first non-dominant pole. In any case, R should be of the same order of magnitude as $1/G_{m2}$. Thus, R is much smaller than R_2 (which is comparable with r_d). Considering $R<<R_2$, it is possible to find the following approximate radices for the denominator of transfer function (4):

$$s_{a1}=-\frac{1}{R_2(C_C+C_2)} \qquad s_{a2}=-\frac{1}{R\frac{C_CC_2}{C_C+C_2}} \quad (6)$$

No one of these poles coincides with the zero given in (5) and therefore, the zero is not cancelled and is actually present in the v_{out}/v_1 transfer function. Since the signal from v_d to v_{out} pass through the intermediate voltage v_1, this zero is also a zero of the overall amplifier gain (v_{out}/v_{id}). On the contrary, it can be shown that singularities s_{a1} and s_{a2} do not appear in the overall transfer function, since they are cancelled by zeros in the v_1/v_{id} transfer functions that exactly match s_{a1} and s_{a2}.

Now, let us consider the input mesh, formed by source $G_{m1}v_{id}$, resistor R_1 and capacitor C_1. The impedance formed by the series of C_C and R load the input mesh, through the Miller effect. It is possible to consider this loading effect, by placing an equivalent impedance in parallel to R_1 and C_1. This impedance is given by:

$$Z_{eq} = \frac{R + \dfrac{1}{j\omega C_C}}{1 - k_M} \tag{7}$$

where k_M is the v_{out}/v_1 ratio given by (4). Although k_M varies with frequency, we can consider it constant up to the frequencies of the singularities that affect its frequency response. The singularity of k_M at lower frequency is the pole s_{a1}. The DC value of k_M can be derived from (4) by letting s go to zero. It turns out to be $k_M = -G_{m2}R_2$. We consider also that the numerator in (4) is practically equal to the sole capacitive component up to angular frequencies equal to $1/(RC_C)$, which are much higher than s_{a1}, since $R \ll R_2$ for the reasons exposed above. Then, at low frequencies, the effect on the input mesh is placing a capacitor equal to $C_C(1+G_{m2}R_2)$ in parallel to C_1, as shown in Fig.4. Since, by design, C_C is made equal or, more frequently, higher than C_1, and $G_{m2}R_2 \gg 1$, C_1 can be neglected in the parallel. Then the angular frequency of the pole associated to the input mesh will be:

$$\omega_p \cong \frac{1}{R_1 C_C (1 + G_{m2} R_2)} \cong \frac{1}{R_1 C_C G_{m2} R_2} \tag{8}$$

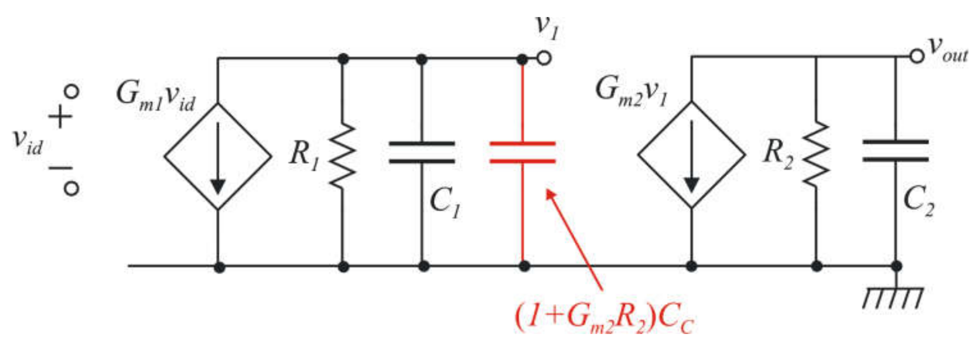

Fig.4. Calculation of the dominant pole due to magnification of the compensation capacitors through the Miller effect.

The analysis performed using a constant $k_M = v_{out}/v_1$ ratio is valid up to an angular frequency equal to s_{a1}. From (6) and (8), we find that:

$$\frac{s_{a1}}{\omega_p} = \frac{R_1 C_C G_{m2} R_2}{R_2 (C_C + C_2)} = R_1 G_{m2} \frac{C_C}{(C_C + C_2)} \tag{9}$$

Considering that C_C is generally chosen to be of the same order of magnitude as C_2, and that $R_1 G_{m2} \gg 1$, then: $\omega_p \ll s_{a1}$. Therefore, at the frequency of the pole ω_p, k_M is really very close to the DC value, confirming the validity of the approximations used to derive (8).

It is important to observe that the compensation capacitor, magnified by a factor nearly $G_{m2}R_2$ shifts he pole of the input mesh back to low frequencies.
On the other hand, the negative feedback introduced by capacitor C_C lowers the output resistance, shifting the pole of the output mesh (output pole) to high frequencies. In the absence of feedback, the dominant output pole would be given by s_{a1}. Note that s_{a1} is obtained by connecting an ideal voltage source across v_1, interrupting in this way the signal transfer from the output (v_{out}) to the input (v_1) of the second stage. During normal operation of the amplifier, this voltage source is not present. For frequencies so high that the impedance of capacitors C_1 and C_2 is much lower than R_1 and R_2, respectively, we have the situation depicted in Fig.5, where R_1 and R_2 have been removed. The voltage source v_p has been introduced in Fig.5 in order to evaluate the resistance seen by capacitors C_1,C_C and C_2. Resistance R has been neglected, since, for the frequency interval taken into account, we consider that R is still much lower than the impedance of C_C.

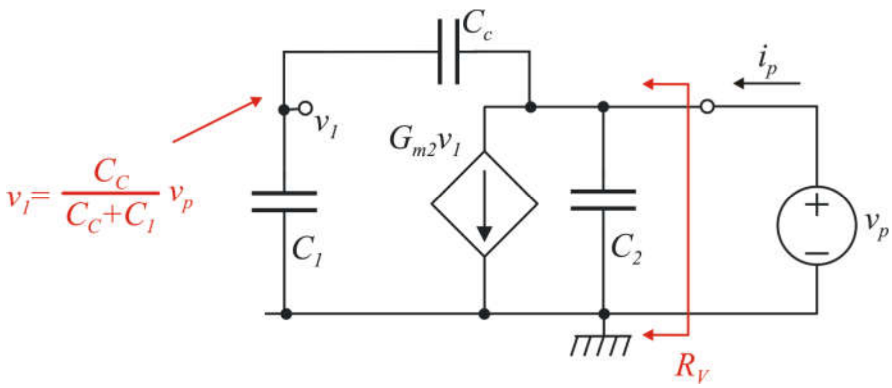

Fig.5. Approximate calculation of the first non-dominant pole, pushed to high frequencies by the reduction of the output resistance induced by negative feedback.

The resistance is related to the voltage controlled current source, since the voltage that controls it (v_1) is a fraction of the voltage across it (v_p). More precisely:

$$v_1 = \frac{C_C}{C_1 + C_C} v_p \quad \Rightarrow \quad \mathrm{Re}(i_p) = G_{m2} \frac{C_C}{C_1 + C_C} v_p \tag{10}$$

where Re(i_p) is the real part of current ip. The resistance seen by the capacitors is then:

$$R_V = \frac{v_p}{\mathrm{Re}(i_p)} = \frac{C_C + C_1}{C_C} \frac{1}{G_{m2}} \tag{11}$$

The output resistance is then of the same order of magnitude as $1/G_{m2}$. In practical cases, $C_C \gg C_1$ by design, so that the output resistance is very close to $1/G_{m2}$. Reduction of the output resistance occurring at high frequencies is a beneficial effect that is particularly useful to reduce the gain penalty introduced by resistive loads. Resistance R_V is in parallel with the capacitor obtained by the parallel of C_2 and the series of C_C and C_1. This capacitor is given by:

$$C_{eq} = C_2 + \frac{C_1 C_C}{C_1 + C_C} \tag{12}$$

The angular frequency of the output pole is then given by

$$\omega_2 = \frac{1}{C_{eq} R_V} = \frac{1}{\left(C_2 + \dfrac{C_1 C_C}{C_1 + C_C}\right)\dfrac{1}{G_{m2}}} = \frac{G_{m2}}{\left(\dfrac{C_2 C_1 + C_2 C_C + C_1 C_C}{C_1 + C_C}\right)\dfrac{C_1 + C_C}{C_C}} =$$
$$= \frac{G_{m2}}{\left(\dfrac{C_2 C_1 + C_C (C_2 + C_1)}{C_C}\right)} = \frac{G_{m2}}{(C_2 + C_1)} \frac{1}{1 + \dfrac{C_2 C_1}{(C_2 + C_1)} \dfrac{1}{C_C}} \tag{13}$$

Indicating with C_S a capacitor equal to the series of C_1 and C_2, the expression of the output pole can be simplified as:

$$\omega_2 = \frac{G_{m2}}{(C_2 + C_1)} \left(1 + \frac{C_S}{C_C}\right)^{-1} \tag{14}$$

As we have previously stated, this result was obtained by neglecting R. It can be demonstrated that the presence of R can be taken into account by introducing a high frequency pole given by:

$$\omega_3 = \frac{1}{R\left(\dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_C}\right)^{-1}} \tag{15}$$

This can be intuitively justified considering that, when frequency tends to infinity, C_1 and C_2 impedances tend to zero, and so do voltages v_1 and v_{out}. Therefore, currents in R_1, R_2 and current source $G_{m2}v_1$ gets lower and lower. This does not occur to the current into the capacitors, since their impedance also tends to zero with frequency. Then, at sufficiently high frequencies, the only elements that remains in the circuit are the capacitors. Then, *R* sees the series of the three capacitors, producing the pole in (15).