## 1 Design of a CMOS operational amplifier with Offset and GBW/stability specifications.

### 1.1 Amplifier topology



### 1.2 Process parameters

| Parametro | $\mathrm{n}-\mathrm{MOS}$ | $\mathrm{p}-\mathrm{MOS}$ |
| :--- | :--- | :--- |
| $\mu_{\mathrm{n}} \mathrm{Cox}, \mu_{\mathrm{p}} \mathrm{Cox}$ | $240 \times 10^{-6} \mathrm{~A} / \mathrm{V}^{2}$ | $50 \times 10^{-6} \mathrm{~A} / \mathrm{V}^{2}$ |
| $\mathrm{~V}_{\mathrm{tn}}, \mathrm{V}_{\mathrm{tp}}$ | 0.43 V | -0.56 V |
| $\gamma$ (effetto body) | $0.44 \mathrm{~V}^{1 / 2}$ | $0.59 \mathrm{~V}^{1 / 2}$ |
| $\mathrm{k}_{\lambda}$ | $50 \mathrm{~V} / \mu \mathrm{m}$ | $50 \mathrm{~V} / \mu \mathrm{m}$ |
| $\alpha$ (coeff. termico della Vt) | $-1 \mathrm{mV} /{ }^{\circ} \mathrm{C}$ | $1 \mathrm{mV} /{ }^{\circ} \mathrm{C}$ |
| $\mathrm{N}_{\mathrm{fn}}, \mathrm{N}_{\mathrm{fp}}$ (fattore rumore flicker) | $6 \times 10^{-10} \mathrm{~V}^{2} \mu \mathrm{~m}^{2}$ | $2 \times 10^{-10} \mathrm{~V}^{2} \mu \mathrm{~m}^{2}$ |
| $\mathrm{C}_{\mathrm{Vt}}$ (matching Vt) | $8.5 \mathrm{mV} \cdot \mu \mathrm{m}$ | $8.5 \mathrm{mV} \cdot \mu \mathrm{m}$ |
| $\mathrm{C}_{\beta}$ (matching beta) | $0.03 \mu \mathrm{~m}$ | $0.03 \mu \mathrm{~m}$ |
| $\mathrm{C}_{\text {ox }}$ | $6.2 \mathrm{fF} / \mu \mathrm{m}^{2}$ | $6.2 \mathrm{fF} / \mu \mathrm{m}^{2}$ |
| $\mathrm{~L}_{\mathrm{C}}$ (lunghezza minima $\mathrm{D} / \mathrm{S}$ ) | $1.2 \mu \mathrm{~m}$ | $1.2 \mu \mathrm{~m}$ |
| $\mathrm{C}_{\mathrm{J}}$ | $1.8 \mathrm{fF} / \mu \mathrm{m}^{2}$ | $1.8 \mathrm{fF} / \mu \mathrm{m}^{2}$ |
| $\mathrm{Cgdo}^{2}$ | $0.6 \mathrm{fF} / \mu \mathrm{m}$ | $0.6 \mathrm{fF} / \mu \mathrm{m}$ |
| $\mathrm{t}_{\text {ox }}$ | 5.6 nm | 5.6 nm |

### 1.3 Design specifications

The goal is designing an operational amplifier with:

- An offset voltage (absolute value) smaller than 3 mV
- A GBW of 10 MHz for a load capacitance $\left(\mathrm{C}_{\mathrm{L}}\right)$ up to 10 pF .
- A phase margin around $70^{\circ}$ in unity gain configuration


### 1.4 Solution

## Offset condition

In order to have an offset voltage that is smaller than 3 mV for the most part of the fabricated devices ( $99.7 \%$ ) we need to impose that:

$$
\begin{equation*}
3 \sigma_{v i o}=3 \mathrm{mV} \Rightarrow \sigma_{v i o}=1 \mathrm{mV} \tag{1.1}
\end{equation*}
$$

We can express the standard deviation with the following relationship:

$$
\begin{equation*}
\sigma_{v i o}^{2}=\frac{A}{W_{1} L_{1}}+\frac{B}{W_{3} L_{3}} \tag{1.2}
\end{equation*}
$$

where:

$$
\begin{equation*}
A=C_{V t p}^{2}+\left[\frac{\left(V_{G S}-V_{t}\right)_{1}}{2} C_{\beta p}\right]^{2} \quad B=F^{2} C_{V t n}^{2}+\left[\frac{\left(V_{G S}-V_{t}\right)_{1}}{2} C_{\beta n}\right]^{2} \tag{1.3}
\end{equation*}
$$

and

$$
\begin{equation*}
F=\frac{g_{m 3}}{g_{m 1}}=\frac{\left(V_{G S}-V_{t}\right)_{1}}{\left(V_{G S}-V_{t}\right)_{3}} \tag{1.4}
\end{equation*}
$$

With the process parameters in paragraph $1.2, p$ and $n$ devices have the same matching coefficients, then $\mathrm{C}_{\beta \mathrm{p}}=\mathrm{C}_{\beta \mathrm{n}}$ and $\mathrm{C}_{\mathrm{vtp}}=\mathrm{C}_{\mathrm{vtn}}$. Inspection of the previous equations suggests that, the smaller coefficients $A$ and $B$, the smaller will be the area $\left(\mathrm{W}_{1} \mathrm{~L}_{1}\right.$ and $\mathrm{W}_{3} \mathrm{~L}_{3}$ gate areas) required to obtain the desired $\sigma_{\text {vio }}$. We can reduce A and B by choosing a small value for $\left(\mathrm{V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{t}}\right)_{1}$. In addition, we can choose a small value for $F$, in order to reduce coeff. $B$. We choose:

$$
\begin{equation*}
\left(V_{G S}-V_{t}\right)_{1}=100 \mathrm{mV},\left(V_{G S}-V_{t}\right)_{3}=300 \mathrm{mV} \Rightarrow F=\frac{1}{3} \tag{1.5}
\end{equation*}
$$

With these overdrive voltages, coefficients $A$ and $B$ are:
$\mathrm{A}=74.5 \times 10^{-6} \mathrm{~V}^{2} \mathrm{\mu m}^{2}$
$\mathrm{B}=10.3 \times 10^{-6} \mathrm{~V}^{2} \mu^{2}$
In spite of the coincidence of $p$-MOS and $n$-MOS matching parameters, $A$ and $B$ are very different. This is the effect of the coefficient $F$. Note that $F$ affects only the threshold voltage mismatch term $\left(\mathrm{C}_{\mathrm{t}}\right)$, which, with small value of $\left(V_{G S}-V_{t}\right)_{1}$, is by far the dominant component of $A$ and $B$.

At this point, we have one equation (1.2) and two unknowns ( $\mathrm{W}_{1} \mathrm{~L}_{1}$ and $\mathrm{W}_{3} \mathrm{~L}_{3}$ ). We can chose the solution that minimizes the total gate area $\left(\mathrm{W}_{1} \mathrm{~L}_{1}+\mathrm{W}_{3} \mathrm{~L}_{3}\right)$. This is obtained for:

$$
\begin{equation*}
\frac{W_{3} L_{3}}{W_{1} L_{1}}=\sqrt{\frac{B}{A}}=0.37 \tag{1.6}
\end{equation*}
$$

from which we find:

$$
\begin{align*}
& W_{1} L_{1}=\frac{1}{\sigma_{v i o}^{2}}(A+\sqrt{A B}) \cong 102 \mu \mathrm{~m}^{2}  \tag{1.7}\\
& W_{3} L_{3}=W_{1} L_{1} \sqrt{\frac{B}{A}} \cong 38 \mu \mathrm{~m}^{2}
\end{align*}
$$

At this point, we have determined the gate areas of $\mathrm{M}_{1}$ and $\mathrm{M}_{3}$, but we are still unable to find their W and L . If the offset specification is the only requirement, than this problem remains undetermined.

## GBW and phase margin

In our example, we have also a GBW specification and this allow determining the W/L. With the hypotheses:

$$
\begin{equation*}
C_{1} \ll C_{c}, C_{2} \quad \text { and } \quad C_{2} \cong C_{L} \tag{1.8}
\end{equation*}
$$

We have:

$$
\begin{equation*}
G B W=\frac{1}{2 \pi \sigma} \frac{g_{m 5}}{C_{L}} \Rightarrow g_{m 5}=2 \pi \sigma C_{L} \cdot G B W \cong 1.88 \mathrm{mS} \tag{1.9}
\end{equation*}
$$

where we choose $\sigma=3$ to obtain a phase margin of nearly $70^{\circ}$.
With the value of $g_{m 5}$, we can calculate the value of resistor $R$ necessary to shift the RHP (Right HalfPlane) zero to infinity:

$$
\mathrm{R}=1 / \mathrm{g}_{\mathrm{m} 5}=532 \Omega
$$

From $g_{m}$ we can find $\mathrm{M}_{5}$ aspect ratio:

$$
\begin{equation*}
g_{m 5}=\mu_{n} C_{O X} \frac{W_{5}}{L_{5}}\left(V_{G S}-V_{t}\right)_{5} \Rightarrow \frac{W_{5}}{L_{5}}=\frac{g_{m 5}}{\mu_{n} C_{O X}\left(V_{G S}-V_{t}\right)_{5}}=26.1 \tag{1.10}
\end{equation*}
$$

where we have used the property: $\left(\mathrm{V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{t}}\right)_{5}=\left(\mathrm{V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{t}}\right)_{3}=300 \mathrm{mV}$
We can propagate the result found for $g_{m 5}$ back to the first stage:

$$
\begin{equation*}
g_{m 1}=\frac{1}{\sigma} \frac{C_{C}}{C_{L}} g_{m 5} \tag{1.11}
\end{equation*}
$$

Using the practical rule $\mathrm{C}_{\mathrm{C}}=\mathrm{C}_{\mathrm{L}}=10 \mathrm{pF}$ and $\sigma=3$, we obtain:

$$
\begin{equation*}
g_{m 1}=\frac{g_{m 5}}{3} \cong 0.63 \mathrm{mS} \tag{1.12}
\end{equation*}
$$

## Design completion

From $g_{m l}$, we can determine $g_{m 3}$, considering that, from the definition of parameter $F$ :

$$
\begin{equation*}
g_{m 3}=F \cdot g_{m 1}=0.21 \mu \mathrm{~S} \tag{1.13}
\end{equation*}
$$

Again, since we have fixed the $\mathrm{M}_{1}$ overdrive voltage to 100 mV , we can find $\mathrm{W}_{1} / \mathrm{L}_{1}$ :

$$
\begin{equation*}
\frac{W_{1}}{L_{1}}=\frac{g_{m 1}}{\mu_{p} C_{O X}\left(V_{G S}-V_{t}\right)_{1}}=126 \tag{1.14}
\end{equation*}
$$

From $\left(V_{G S}-V_{t}\right)_{3}=300 \mathrm{mV}$, we can find $\mathrm{W}_{3} / \mathrm{L}_{3}$ :

$$
\begin{equation*}
\frac{W_{3}}{L_{3}}=\frac{g_{m 3}}{\mu_{n} C_{O X}\left(V_{G S}-V_{t}\right)_{3}}=2.92 \tag{1.15}
\end{equation*}
$$

Now we can find $\mathrm{W}_{1}$ and $\mathrm{L}_{1}$ individually:

$$
\left.\begin{array}{l}
W_{1} L_{1}=102 \mu \mathrm{~m}^{2}  \tag{1.16}\\
\frac{W_{1}}{L_{1}}=126
\end{array}\right\} \Rightarrow W_{1}=\sqrt{W_{1} L_{1} \cdot \frac{W_{1}}{L_{1}}} \cong 114 \mu \mathrm{~m} \quad L_{1}=W_{1} \cdot\left(\frac{W_{1}}{L_{1}}\right)^{-1} \cong 0.9 \mu \mathrm{~m}
$$

Applying the same procedure to $\mathrm{M}_{3}$ :

$$
\left.\begin{array}{l}
W_{3} L_{3}=38 \mu \mathrm{~m}^{2} \\
\frac{W_{3}}{L_{3}}=2.92
\end{array}\right\} \Rightarrow W_{3}=\sqrt{W_{3} L_{3} \frac{W_{3}}{L_{3}}} \cong 10.5 \mu \mathrm{~m} \quad L_{1}=W_{3} \cdot\left(\frac{W_{3}}{L_{3}}\right)^{-1} \cong 3.6 \mu \mathrm{~m}
$$

In this way, we have determined all is needed for $M_{1}$ and $M_{3}$. We can propagate $L_{3}$ to $M_{5}$, according to the arbitrary choice $L_{3}=L_{5}$, introduced to keep a precise current ratio between $M_{3}$ and $M_{5}$ in rest conditions.
Since we have determined $\mathrm{W}_{5}$ / $\mathrm{L}_{5}$ earlier, we now can find $\mathrm{M}_{5}$ individual width and length:

$$
\left.\begin{array}{l}
L_{5}=3.6 \mu \mathrm{~m}  \tag{1.17}\\
\frac{W_{5}}{L_{5}}=26.1
\end{array}\right\} \Rightarrow W_{5} \cong 94 \mu \mathrm{~m}
$$

Let us now complete the op-amp design. The only parameters that are still missing belong to $\mathrm{M}_{7}$ and $\mathrm{M}_{6}$. Opting for a symmetrical output swing, we set:

$$
\begin{equation*}
\left(V_{G S}-V_{t}\right)_{6}=\left(V_{G S}-V_{t}\right)_{5} \tag{1.18}
\end{equation*}
$$

Since $I_{D 6}=I_{D 5}$, this means that $M_{5}$ and $M_{6}$ should have the same $\beta$. Then:

$$
\begin{equation*}
\mu_{p} C_{O X} \frac{W_{6}}{L_{6}}=\mu_{n} C_{O X} \frac{W_{5}}{L_{5}} \Rightarrow \frac{W_{6}}{L_{6}}=\frac{\mu_{n} C_{O X}}{\mu_{p} C_{O X}} \frac{W_{5}}{L_{5}} \cong 125 \tag{1.19}
\end{equation*}
$$

We have now to individually choose $\mathrm{W}_{6}$ and $\mathrm{L}_{6}$. Let us recall that $\mathrm{L}_{6}$ is one of the DOFs in our model of the op-amp. $\mathrm{L}_{6}$ will affect mainly the dc gain. We can make $\mathrm{L}_{6}$ equal to $\mathrm{L}_{5}$ in order to have a balanced effect of the dc gain of the second stage. Larger $\mathrm{L}_{6}$ values does not result in important advantages (the gain becomes dominated by $\mathrm{r}_{\mathrm{d} 5}$ ); much smaller values begin to have a serious impact on the gain. Setting $\mathrm{L}_{5}=\mathrm{L}_{6}$, we find $\mathrm{W}_{6}$ :

$$
\begin{equation*}
L_{6}=L_{5}=3.6 \mu \mathrm{~m} \Rightarrow W_{6}=L_{6}\left(\frac{W_{6}}{L_{6}}\right)=450 \mu \mathrm{~m} \tag{1.20}
\end{equation*}
$$

Finally, we set $\mathrm{M}_{7}$ parameters. Using the equal-length condition to improve precision of the $\mathrm{I}_{\mathrm{D} 6}$ over $\mathrm{I}_{\mathrm{D} 7}$ ratio, we have:

$$
\begin{equation*}
L_{7}=L_{6}=3.6 \mu \mathrm{~m} \tag{1.21}
\end{equation*}
$$

Let now exploit the condition for null output short-circuit current when $\mathrm{Vd}=0$ (null systematic offset):

$$
\begin{equation*}
\frac{\beta_{6}}{\beta_{7}}=\frac{1}{2} \frac{\beta_{5}}{\beta_{3}} \tag{1.22}
\end{equation*}
$$

Note that the factors $\mu$ Cox cancel each other in both hands of equation (1.22). Then, we obtain a condition on the aspect ratios that allows us to $\mathrm{M}_{7}$ aspect ratio.

$$
\begin{equation*}
\frac{\frac{W_{6}}{L_{6}}}{\frac{W_{7}}{L_{7}}}=\frac{1}{2} \frac{\frac{W_{5}}{L_{5}}}{\frac{W_{3}}{L_{3}}} \Rightarrow \frac{W_{7}}{L_{7}}=2 \frac{W_{6}}{L_{6}} \frac{\frac{W_{3}}{L_{3}}}{\frac{W_{5}}{L_{5}}} \cong 28 \tag{1.23}
\end{equation*}
$$

This allow us to find $\mathrm{M}_{7}$ parameters:

$$
\begin{equation*}
L_{7}=L_{6}=3.6 \mu \mathrm{~m} \Rightarrow W_{7}=L_{7}\left(\frac{W_{7}}{L_{7}}\right)=101 \mu \mathrm{~m} \tag{1.24}
\end{equation*}
$$

Device $\mathrm{M}_{8}$ is not strictly part of the amplifier, since it can be shared among several different op-amps. We consider that We have to design a cell that is to be biased by a current ( $\mathrm{I}_{\mathrm{B}}$ ), than $\mathrm{M}_{8}$ is necessary. To simplify the design, we set $\mathrm{M}_{8}=\mathrm{M}_{7}$, thus $\mathrm{I}_{\mathrm{B}}=\mathrm{I}_{0}$.

## Calculation of the bias currents

$$
\begin{align*}
& I_{0}=2 I_{D 1} \cong 2 g_{m 1} V_{T E 1}=2 g_{m 1} \frac{\left(V_{G S}-V_{t}\right)_{1}}{2} \cong 63 \mu \mathrm{~A} \\
& I_{B}=I_{0}=63 \mu \mathrm{~A}  \tag{1.25}\\
& I_{1} \cong g_{m 5} V_{T E 5}=g_{m 5} \frac{\left(V_{G S}-V_{t}\right)_{5}}{2} \cong 282 \mu \mathrm{~A}
\end{align*}
$$

Final component table

|  | $\mathrm{W}(\mu \mathrm{m})$ | $\mathrm{L}(\mu \mathrm{m})$ |
| :--- | :--- | :--- |
| $\mathrm{M}_{1}, \mathrm{M}_{2}$ | 114 | 0.9 |
| $\mathrm{M}_{3}, \mathrm{M}_{4}$ | 10.5 | 3.6 |
| $\mathrm{M}_{5}$ | 94 | 3.6 |
| $\mathrm{M}_{6}$ | 450 | 3.6 |


| M7 | 101 | 3.6 |
| :--- | :--- | :--- |
| M8 | 101 | 3.6 |
| R | $532 \Omega$ |  |
| $\mathrm{C}_{\mathrm{C}}$ | 10 pF |  |
| $\mathrm{I}_{\mathrm{B}}$ | $63 \mu \mathrm{~A}$ |  |

## Verification of the original hypotheses

$$
\begin{equation*}
C_{1}=C_{G S 5}+C_{D B 2}+C_{D B 4} \tag{1.26}
\end{equation*}
$$

with

$$
\left.\begin{array}{c}
C_{G S 5}=\frac{2}{3} C_{O X} W_{5} L_{5} \cong 1.4 \mathrm{pF} \\
C_{D B 2}=C_{J_{p}} L_{C} W_{2} \cong 0.246 \mathrm{pF} \\
C_{D B 4}=C_{J_{n}} L_{C} W_{4} \cong 0.023 \mathrm{pF}
\end{array}\right\} \Rightarrow C_{1} \cong 1.67 \mathrm{pF}
$$

It can be easily shown that hypotheses (1.8) are verified.

### 1.5 Performance estimation

We have designed the amplifier according to offset and GBW specifications. It is possible to estimate the remaining performance figures using approximate expressions.

## Total current consumption:

Excluding the bias transistor $\mathrm{M}_{8}$, the amplifier is marked by the following current consumption:

$$
\begin{equation*}
I_{\text {sup } p l y}=2 I_{0}+I_{1}=345 \mu \mathrm{~A} \tag{1.29}
\end{equation*}
$$

Considering also M8, to total current consumption is:

$$
\begin{equation*}
I_{\text {tot }}=I_{\text {supply }}+I_{B}=408 \mu \mathrm{~A} \tag{1.30}
\end{equation*}
$$

## Thermal noise density

$$
\begin{equation*}
S_{V t h} \cong 2 \frac{8}{3} k T \frac{1}{g_{m 1}}(1+F) \cong 4.5 \times 10^{-17} V^{2} / H_{z}(6.7 \mathrm{nV} / \sqrt{\mathrm{Hz}}) \tag{1.31}
\end{equation*}
$$

Flicker noise density

$$
\begin{equation*}
k_{F}=2\left(\frac{N_{f p}}{W_{1} L_{1}}+F^{2} \frac{N_{f n}}{W_{3} L_{3}}\right) \cong 7.42 \times 10^{-12} \mathrm{~V}^{2} \tag{1.32}
\end{equation*}
$$

Flicker corner frequency

$$
\begin{equation*}
f_{k}=\frac{k_{F}}{S_{V T h}} \cong 165 \mathrm{kHz} \tag{1.33}
\end{equation*}
$$

Slew rate

$$
\begin{equation*}
s_{R}=\frac{I_{0}}{C_{c}}=6.3 \mathrm{~V} / \mu \mathrm{s} \tag{1.34}
\end{equation*}
$$

