Integrated **Analog** Filters

Analog (electronic) signals: information is directly tied to the infinite set of values that a **voltage**, current, charge, frequency, phase that may assume over a finite interval (range)

- Usually, voltage signals are more **conveniently** processed by analog filters.

- Analog signals are **continuous magnitude**, while in the time domain:
  - **Continuous Time (CT)**, defined at each instant of time → **CT Filters**
  - **Discrete Time (DT)**, defined only on a "countable set" of time instants, usually related to a sampling process → **DT Filters**

Integrated Analog **Filters**

Filter ideal operation:

- Modify the **magnitude** of different frequency components (commonly intended use)
- Modify the **phase** of different frequency components (i.e. to compensate for an unwanted phase response of a filter of an amplifier)
- **LTI system**: characterized by a H(s) if CT, H(z) if DT (assuming single-input, single-output)

Real filters, however:

- Generally change both the phase and magnitude of a signal
- Are limited by maximum input signal level (given the maximum tolerable distortion), noise, parameters spread due to sensitivity of components to external phenomena (temperature change, process variability, aging, etc.)

Integrated Analog Filters

- Integration of passive components (R, L, C) with active components (ideally VCVS, CCVS, VCCS, CCCS) are possible in order to implement, virtually, any suitable H(s), or H(z).
- Integration (+ proper design and layout) allows to reduce the relative spread of homogeneous components: the target is to avoid the use of external discrete components
- Integrated analog filters can be implemented following well-known design approaches:
  - Passive LC (R) ladder filters
  - **Cascade of Biquadratic (Biquad) and Bilinear cells**
  - State Variable Filters (based on **integrator** primitives)
  - Simulation of LC filters with active RC networks

Integrated Analog Filters

- **Inductors are generally difficult to miniaturize**
  - L = (coil area) x (number of coils)^2 x (magnetic permeability)
  - Integrated inductors limited to a few nH (max), and limited quality factors (<10 at GHz)
  - Stray magnetic field cause unwanted coupling
- Resistors and capacitors can be easily integrated: feasible ranges are much wider than for inductors
- For some applications **resistors may result to big** (expensive) → **Gm/C** and **Switched-cap** approaches, only use capacitors as passive component

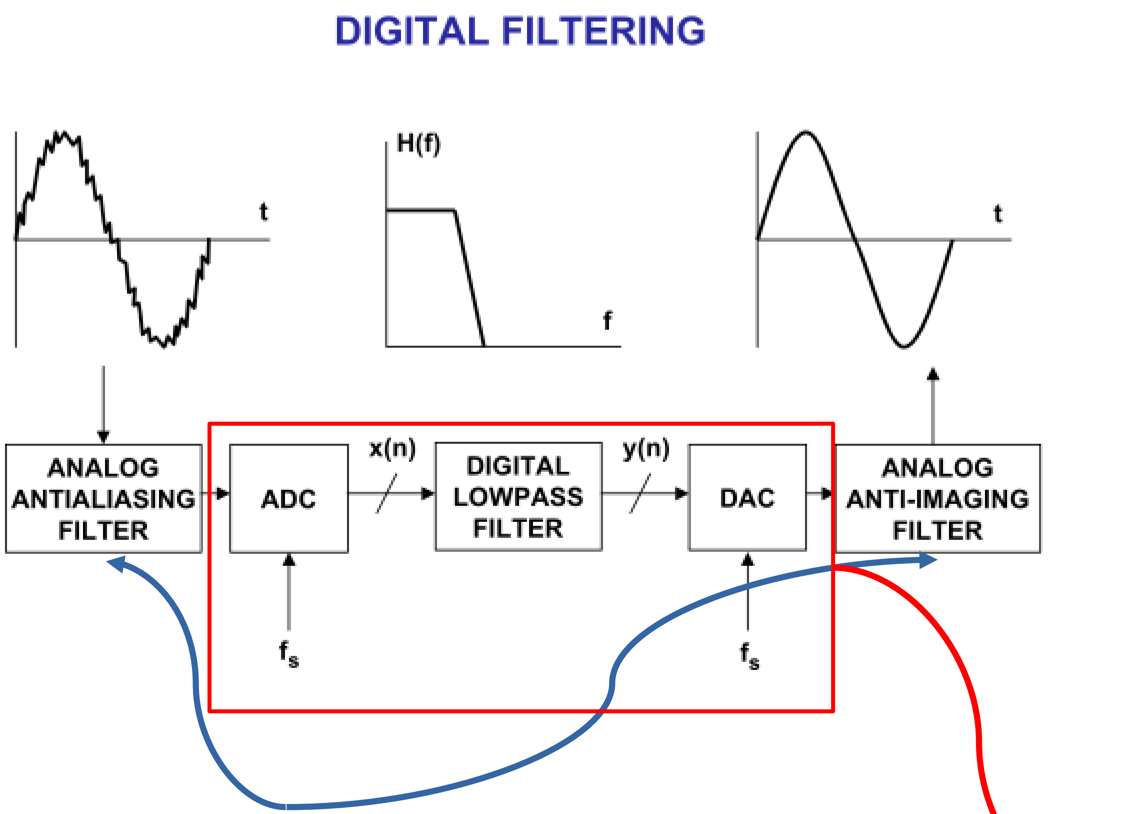
Layer	Sheet resistance	Accuracy	TCR	VCR
	Ω/SQ	%	ppm/°C	ppm/V
poly	**30-200**	25-40	500-1500	**20-200**
N+ or P+ diff.	10-100	20-40	**200-1000**	50-300

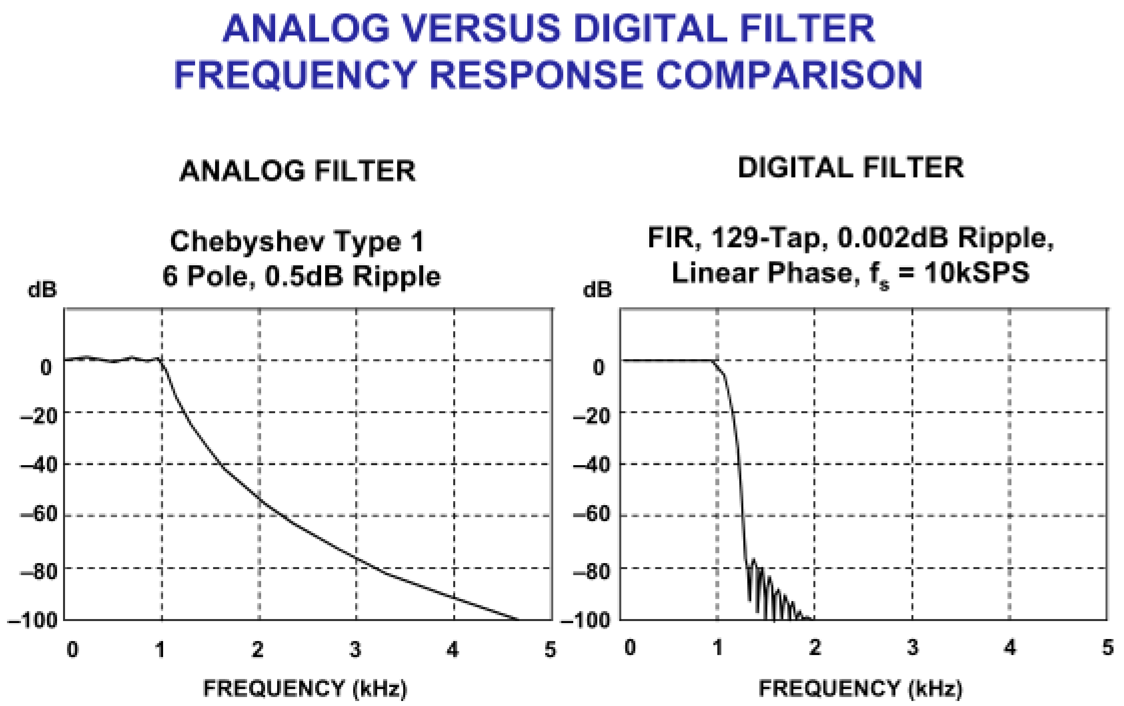
Integrated Analog Filters: why not only digital

DIGITAL FILTERS	ANALOG FILTERS
High Accuracy	Less Accuracy - Component Tolerances
Linear Phase (FIR Filters)	Non-Linear Phase
No Drift Due to Component Variations	Drift Due to Component Variations
Flexible, Adaptive Filtering Possible	Adaptive Filters Difficult
Easy to Simulate and Design	Difficult to Simulate and Design
Computation Must be Completed in Sampling Period - Limits Real Time Operation	Analog Filters Required at High Frequencies and for Anti-Aliasing Filters
Requires High Performance ADC, DAC & DSP	No ADC, DAC, or DSP Required

https://www.analog.com/media/en/training-seminars/design-handbooks/mixedsignal_sect6.pdf

Integrated Analog Filters: why not only digital





Must be CT (analog)

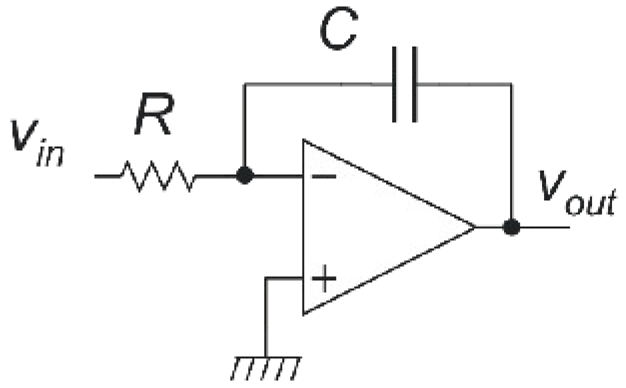
System level choices:

- How complex (expensive) at system level?
- fs > 2 BW! High frequency operation, expensive in hardware and power
- Is reconfigurability needed? (Need for DSP)

https://www.analog.com/media/en/training-seminars/design-handbooks/mixedsignal_sect6.pdf

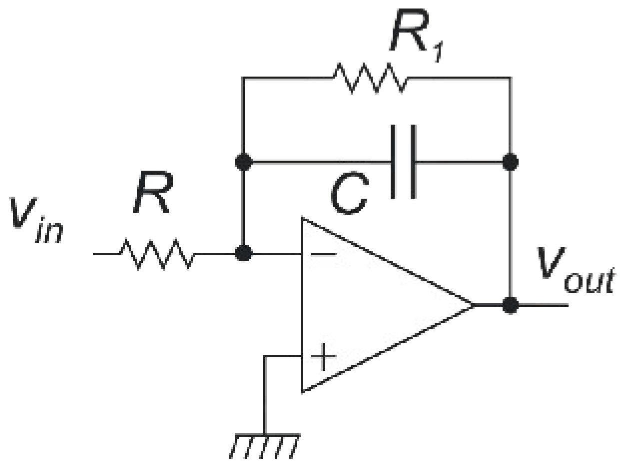
The Opamp-based Integrator

Integrator (inverting)



$$\frac{v_{out}}{v_{in}} = -\frac{1}{RC}\frac{1}{s}$$

Lossy Integrator (inverting)



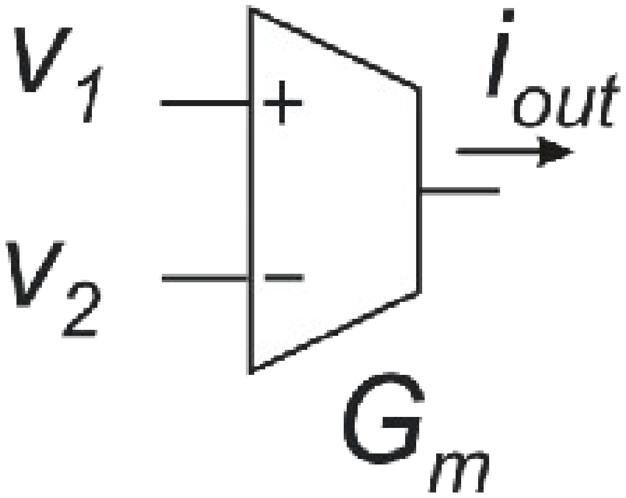
$$\frac{v_{out}}{v_{in}} = -\frac{R_1}{R}\left(\frac{\frac{1}{R_1 C}}{s+\frac{1}{R_1 C}}\right)$$

Inverting-only configuration

Need for R components:
Not convenient for low-frequency applications.

<u>Example:</u>
Pole at 100 Hz, max C=100 pF
→ R_1 = 16 MΩ.
If poly resistor with 200 Ω/SQ is used: SQ = 80000.
Wmin = 0.25 μm.
Min pitch = 0.25 μm
→ L = 20 mm
→ A = L*(W+pitch) = 10 mm^2
→ prototype cost (180 nm CMOS) 4000 Eur/2.25 mm^2
→ Resistor cost: 18 kEur

The Gm-C approach



The Gm block is a **perfectly linear transconductor**, with **infinite Zin and Zout**

An **OTA** approximates the Gm block ideal behaviour.

Typical non-idealities:

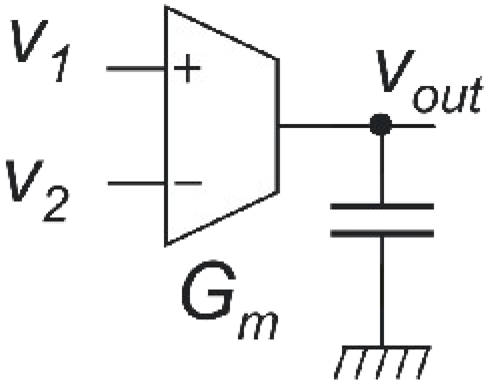
- Finite Rout
- Input Capacitance
- Frequency dependence of Gm
- Input/Output ranges

Ideal operation

$$i_{out} = G_m(v_1 - v_2)$$

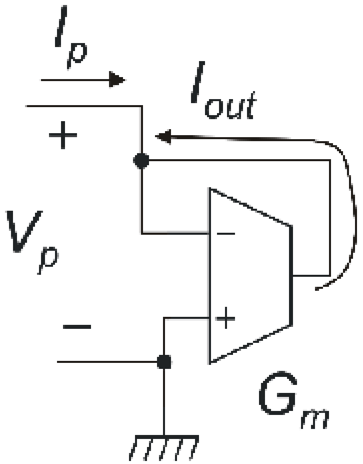
Gm-C filters: basic configurations

OTA-C (Gm-C) Integrator



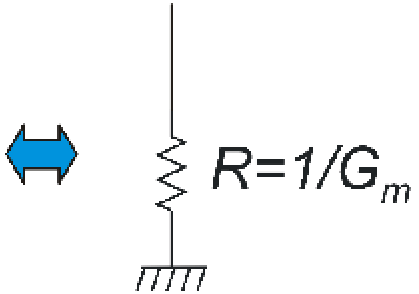
$$v_{out} = \frac{i_{out}}{sC} = \frac{G_m}{sC}\left(v_1 - v_2\right)$$

OTA-C (Gm-C) Eq. Resistor

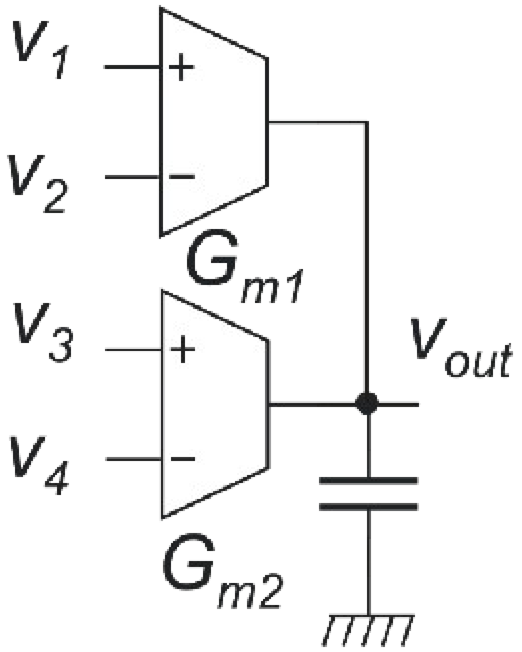


$$I_P = -I_{out} = -G_m(0 - V_P)$$

$$I_P = G_m V_P$$

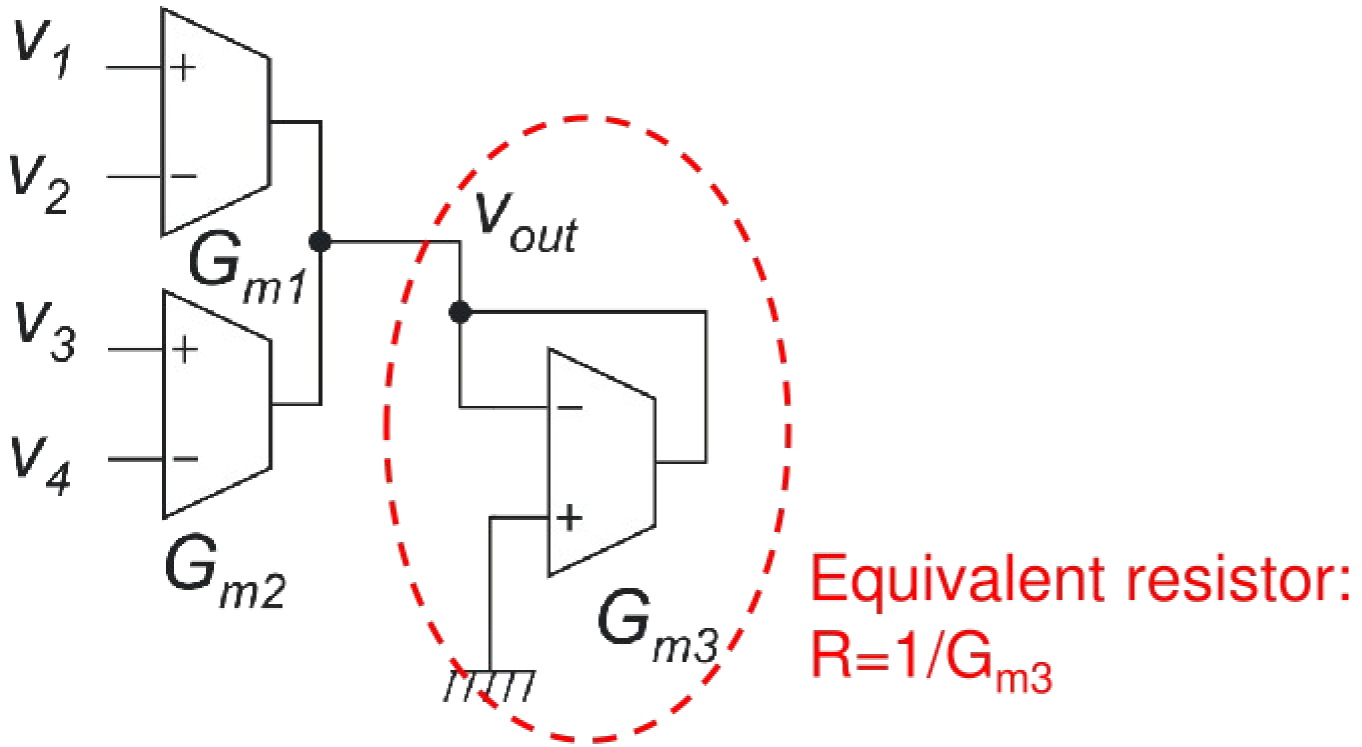


Gm-C filters: basic configurations



Summing Integrator
(inverting / non-inverting)

$$v_{out} = \frac{G_{m1}}{sC}\left[(v_1 - v_2) + \frac{G_{m2}}{G_{m1}}(v_3 - v_4)\right]$$

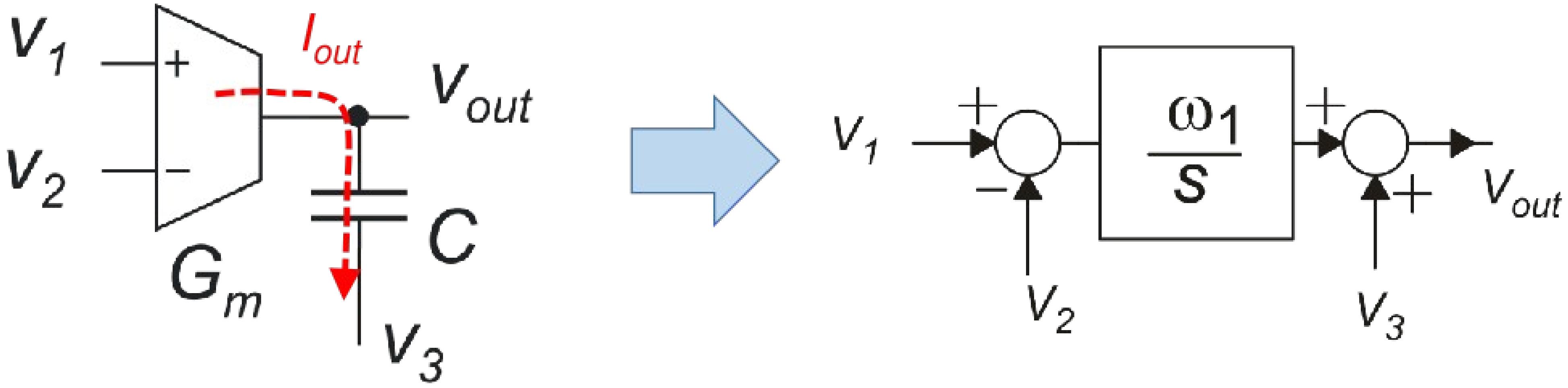


Summing amplifier
(inverting / non-inverting)

$$v_{out} = \frac{G_{m1}}{G_{m3}}(v_1 - v_2) + \frac{G_{m2}}{G_{m3}}(v_3 - v_4)$$

Gm-C filters: basic configurations

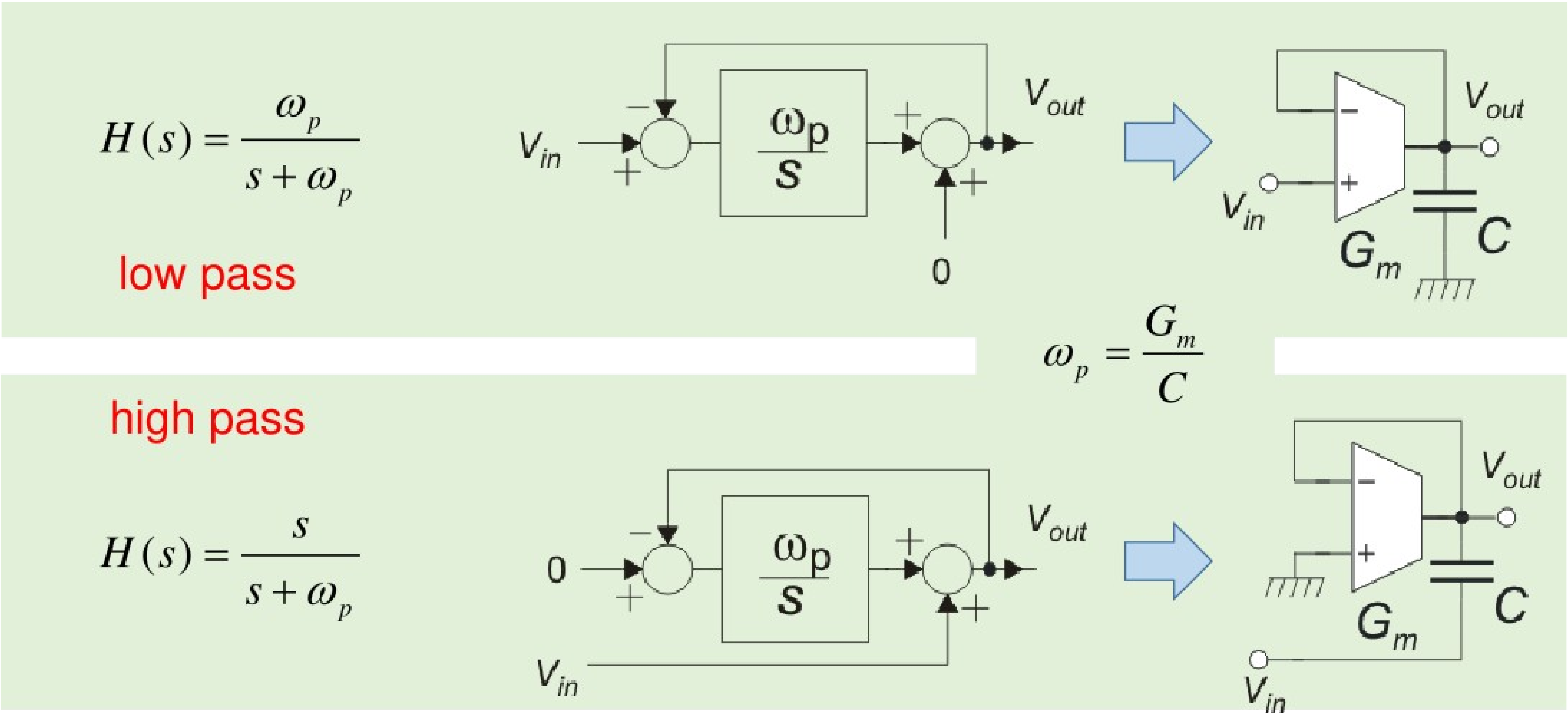
Gm-C integrator with feed-forward input



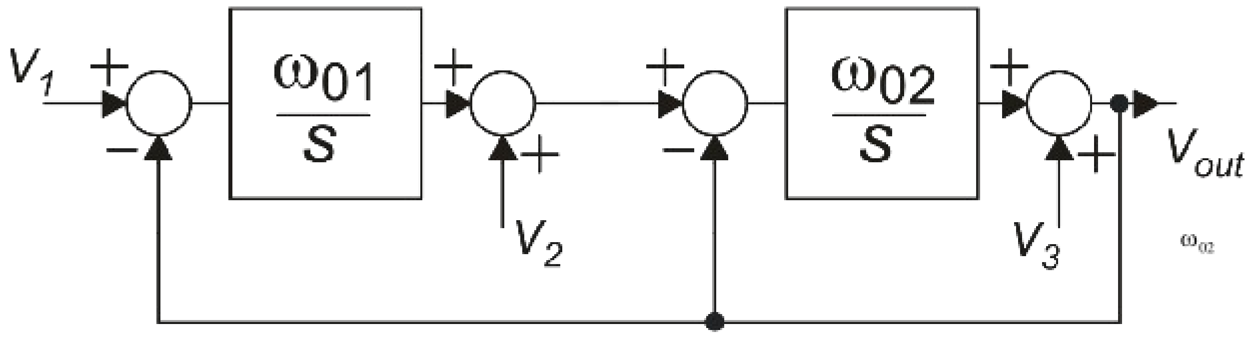
$$v_{out} = \frac{G_m}{sC}\left(v_1 - v_2\right) + v_3$$

$$\omega_1 = \frac{G_m}{C}$$

Gm-C filters: basic configurations



Gm-C filters: the biquadratic cell



$$H(s) = \frac{B_2 s^2 + B_1 \frac{\omega_p}{Q_p} s + B_0 \omega_p^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

$$v_1 = B_0 v_{in}$$

$$v_2 = B_1 v_{in}$$

$$v_3 = B_2 v_{in}$$

$$B_0, B_1, B_2 = \{0, 1\}$$

Flexible Biquad

$$\omega_p = \sqrt{\omega_{01}\omega_{02}}$$

$$Q_p = \sqrt{\frac{\omega_{01}}{\omega_{02}}}$$

Gm-C filters: the biquadratic cell

$$\frac{\omega_p^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

Low pass

$$\frac{s^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

High pass

$$\frac{\frac{\omega_p}{Q_p}s}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

Band pass

$$\frac{s^2 + \omega_p^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

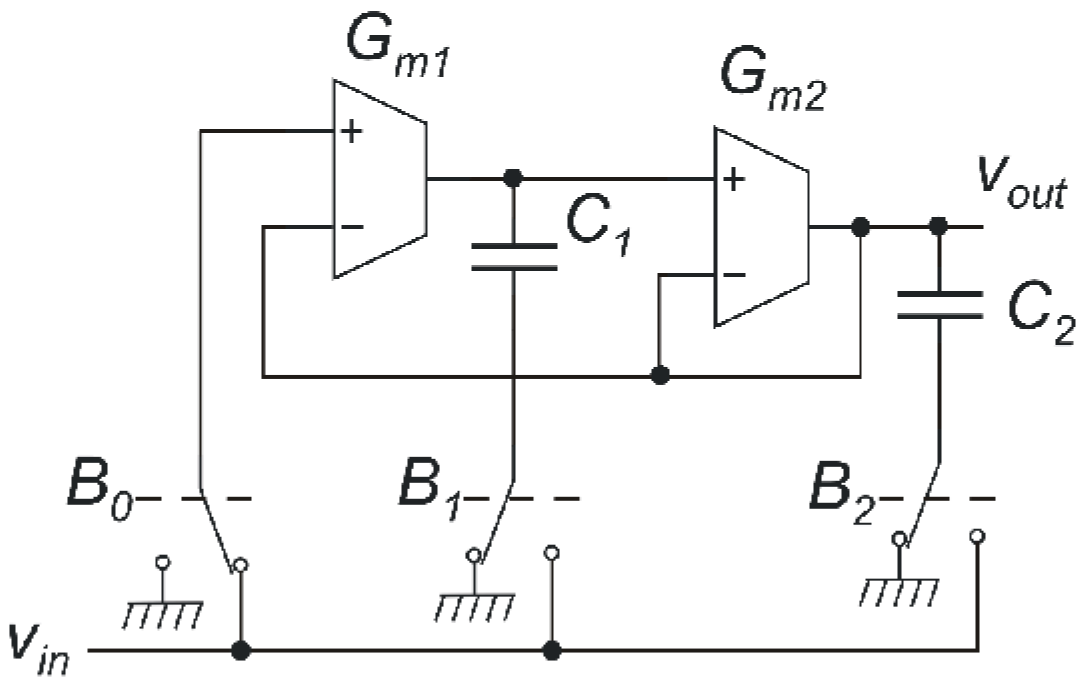
Band Stop

$$\frac{s^2 - \frac{\omega_p}{Q_p}s + \omega_p^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

All pass
(phase equalizer)

All these biquads have unity gain in their respective pass-bands

Gm-C filters: configurable biquadratic cell

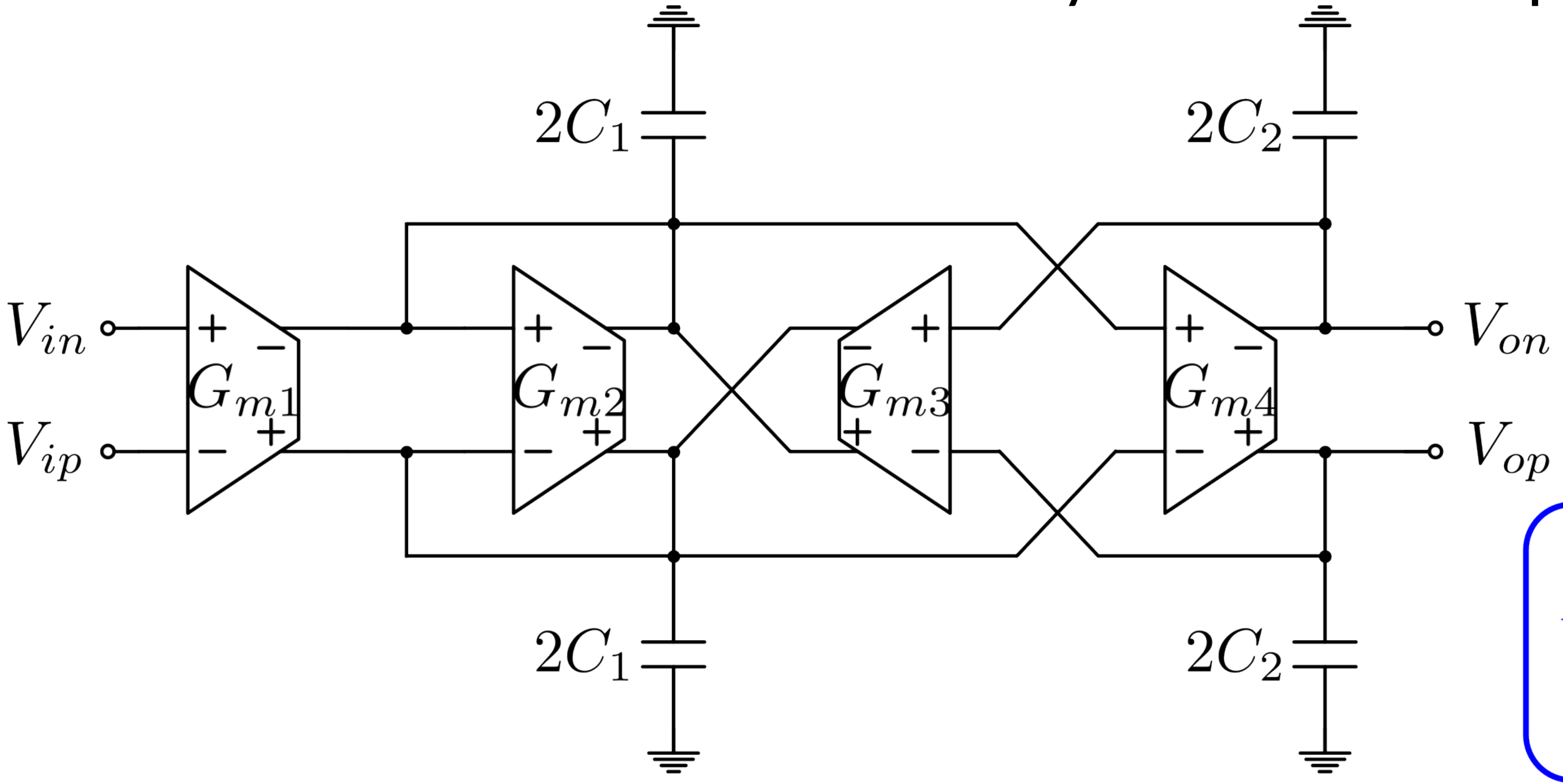


$$\omega_{01} = \frac{G_{m1}}{C_1} \qquad \omega_{02} = \frac{G_{m2}}{C_2}$$

$$\omega_p = \sqrt{\frac{G_{m1}}{C_1}\frac{G_{m2}}{C_2}} \qquad Q_P = \sqrt{\frac{G_{m1}}{G_{m2}}\frac{C_2}{C_1}}$$

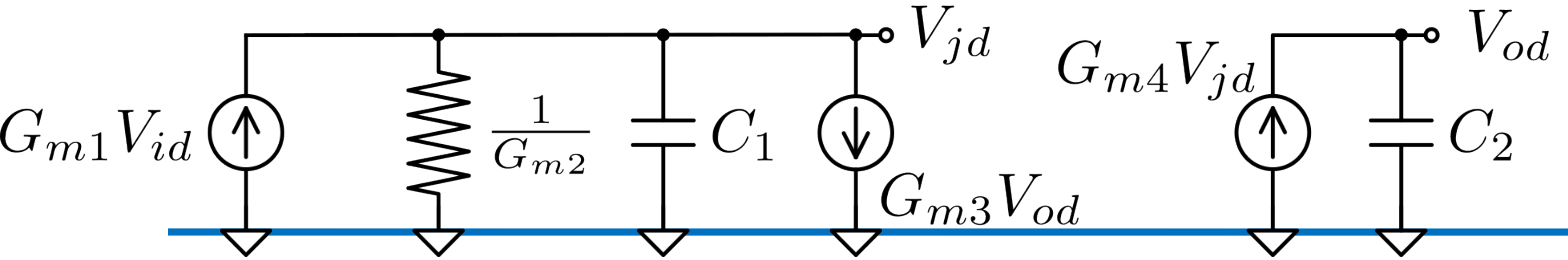
Function	B0	B1	B2
Low pass	1	0	0
High pass	0	0	1
Band-Pass	0	1	0
Notch	1	0	1

Gm-C filters: fully-differential biquadratic cell



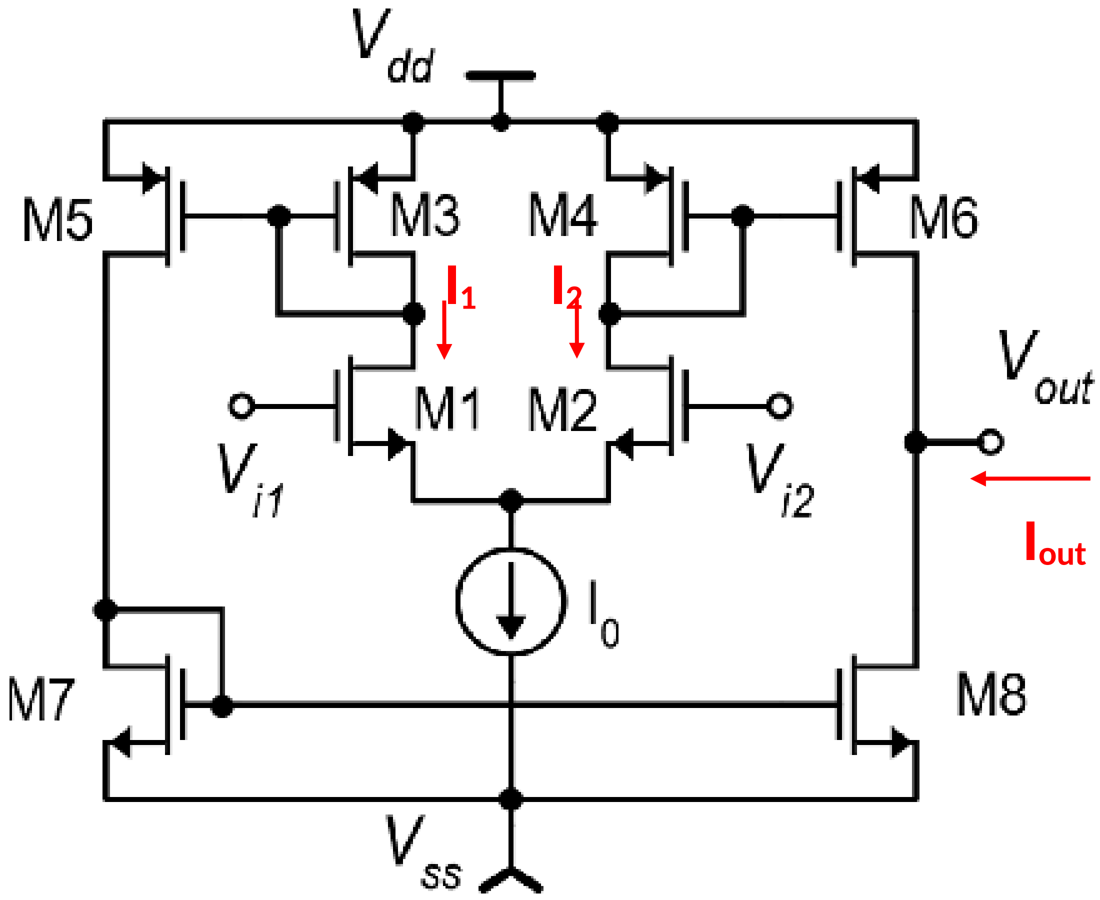
Low-pass configuration

$$H(s) = \frac{G_{m1}}{G_{m3}} \cdot \frac{1}{s^2 \frac{C_1 C_2}{G_{m3} G_{m4}} + s \frac{C_2 G_{m2}}{G_{m3} G_{m4}} + 1}$$



$$\begin{cases} G_{m1} V_{id} = (G_{m2} + sC_1) V_{jd} + G_{m3} V_{od} \\ G_{m4} V_{jd} = sC_2 V_{od} \end{cases}$$

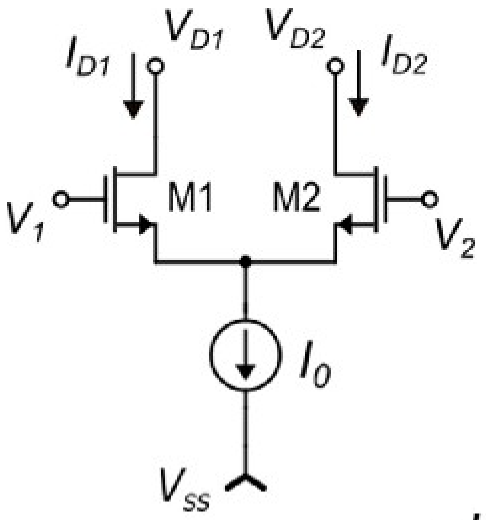
OTA-based Gm transconductor



A simple implementation of a Gm block:

- **Differential couple** (M1-M2): converts the input differential signal into currents I_1, I_2
- **Current mirrors** (M3-M5, M4-M6, M7-M8) convey the current to the output node performing also the current difference

OTA-based Gm transconductor

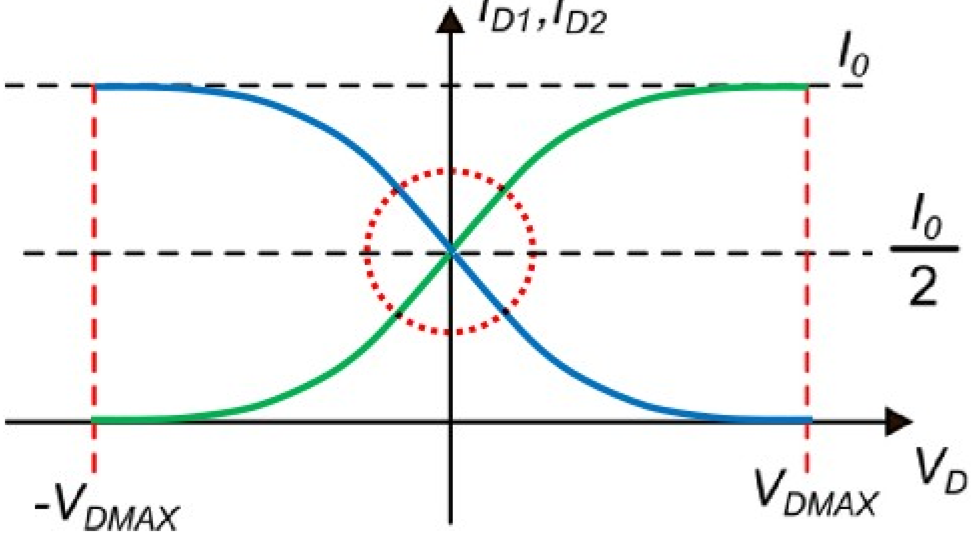


$$I_{D1} = \frac{I_0}{2} + \frac{I_0}{2}\left(\frac{V_D}{V_{DMAX}}\right)\sqrt{2-\left(\frac{V_D}{V_{DMAX}}\right)^2}$$

$$I_{D2} = \frac{I_0}{2} - \frac{I_0}{2}\left(\frac{V_D}{V_{DMAX}}\right)\sqrt{2-\left(\frac{V_D}{V_{DMAX}}\right)^2}$$

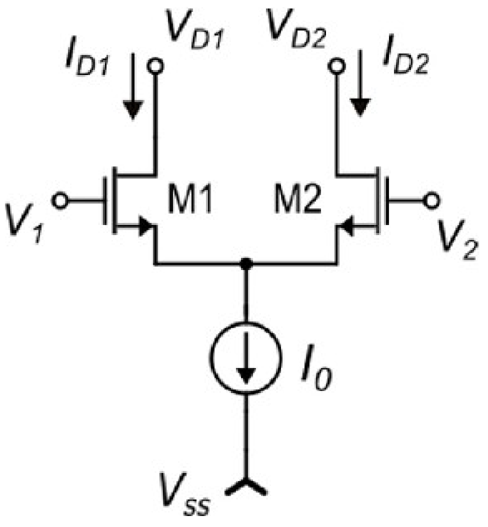
$$I_{D1} \cong \frac{I_0}{2} + v_d \left.\frac{dI_{D1}}{dV_D}\right|_{V_D=0} = \frac{I_0}{2} + \frac{g_m}{2} v_d$$

$$I_{D2} \cong \frac{I_0}{2} + v_d \left.\frac{dI_{D2}}{dV_D}\right|_{V_D=0} = \frac{I_0}{2} - \frac{g_m}{2} v_d$$



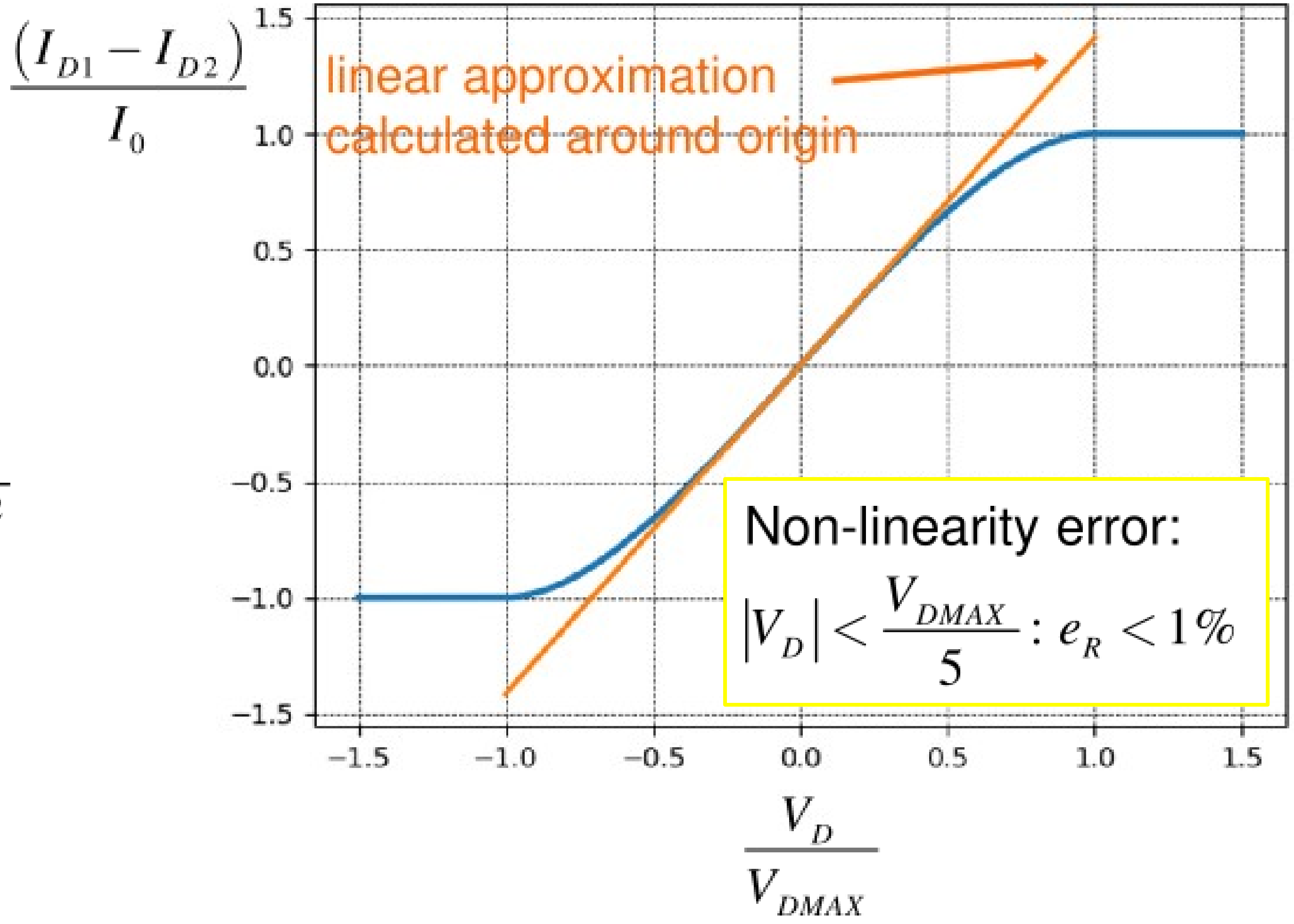
$$g_m = \frac{I_{DQ}}{V_{TE}}; \quad V_{TE} = \frac{V_{GS} - V_t}{2} = \frac{V_{DMAX}}{2\sqrt{2}}$$

OTA-based Gm transconductor



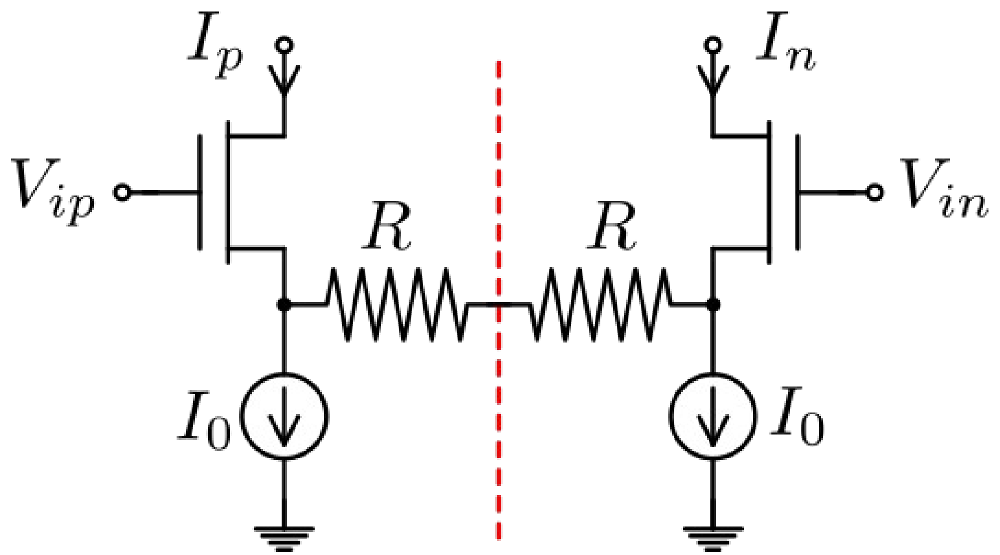
$$I_{D1} - I_{D2} = I_0 \left(\frac{V_D}{V_{DMAX}} \right) \sqrt{2 - \left(\frac{V_D}{V_{DMAX}} \right)^2}$$

$$i_d = i_{d1} - i_{d2} = g_m v_d$$



OTA-based Gm transconductor

Source-degenerated transconductor



$$G_m \approx \frac{g_m}{1+g_m R} \xrightarrow{g_m R \gg 1} \frac{1}{R}$$

$$\begin{cases} I_p \approx I_0 + \frac{g_m}{1+g_m R} v_{ip} \\ I_n \approx I_0 - \frac{g_m}{1+g_m R} v_{in} \end{cases} \longrightarrow I_p - I_n = \frac{g_m}{1+g_m R} v_{id}$$

✓ Under certain conditions, the transconductance appears to be independent of small-signal parameters → **linearity**

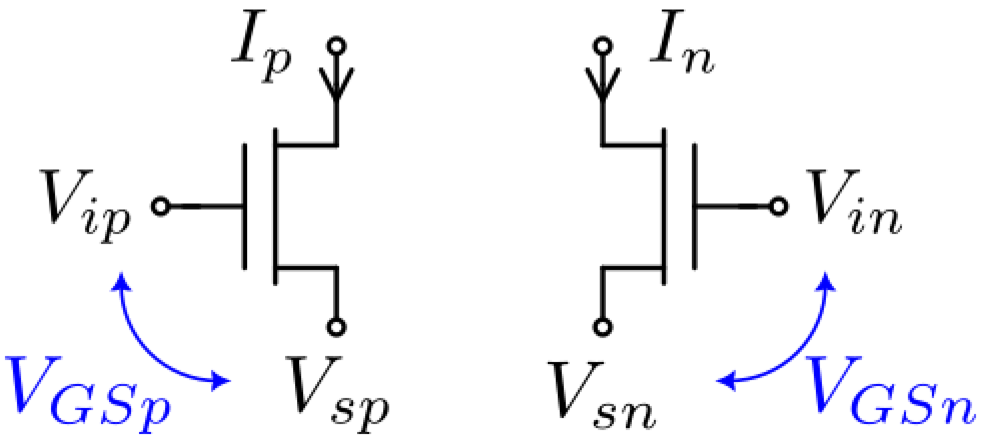
$$R \gg \frac{1}{g_{m,\min}} = \frac{V_{GS}-V_T}{2I_{D,\min}} = \frac{V_{GS}-V_T}{2(I_0 - I_{R,\max})} :$$

$$I_0 \gg I_{R,\max} + \frac{1}{2R}(V_{GS}-V_T) \approx \frac{V_{id,\max} + \frac{1}{2}(V_{GS}-V_T)}{R}$$

✘ High power

✘ R limitations (back to square one)

OTA-based Gm transconductor



Transconductance linearization techniques

Relying on the quadratic expressions (strong inversion):
The difference-of-two-squares rule can be employed:

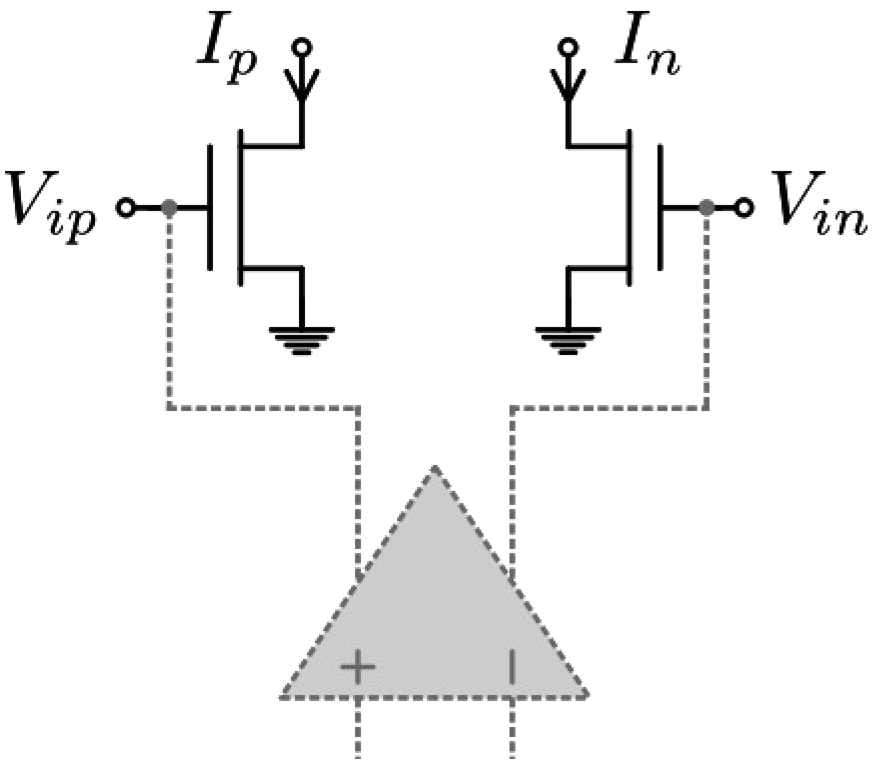
$$\begin{cases} I_p = \frac{\beta}{2}\left(V_{GSp} - V_T\right)^2 \\ I_n = \frac{\beta}{2}\left(V_{GSn} - V_T\right)^2 \end{cases} \longrightarrow I_p - I_n = \beta\left(\frac{V_{GSn} + V_{GSp}}{2} - V_T\right)\left(V_{GSp} - V_{GSn}\right)$$

If:
$$\begin{cases} \frac{V_{GSn}+V_{GSp}}{2} = V_{cmi} + \frac{V_{sn}+V_{sp}}{2} = \mathrm{K}\ (\text{constant}) \\ V_{sn} = V_{sp} \longrightarrow V_{GSp} - V_{GSn} = V_{id} \end{cases}$$

$$\longrightarrow \quad I_p - I_n = \underbrace{\beta\,\mathrm{K}}_{G_m} \cdot V_{id}$$

OTA-based Gm transconductor

Pseudo-differential pair



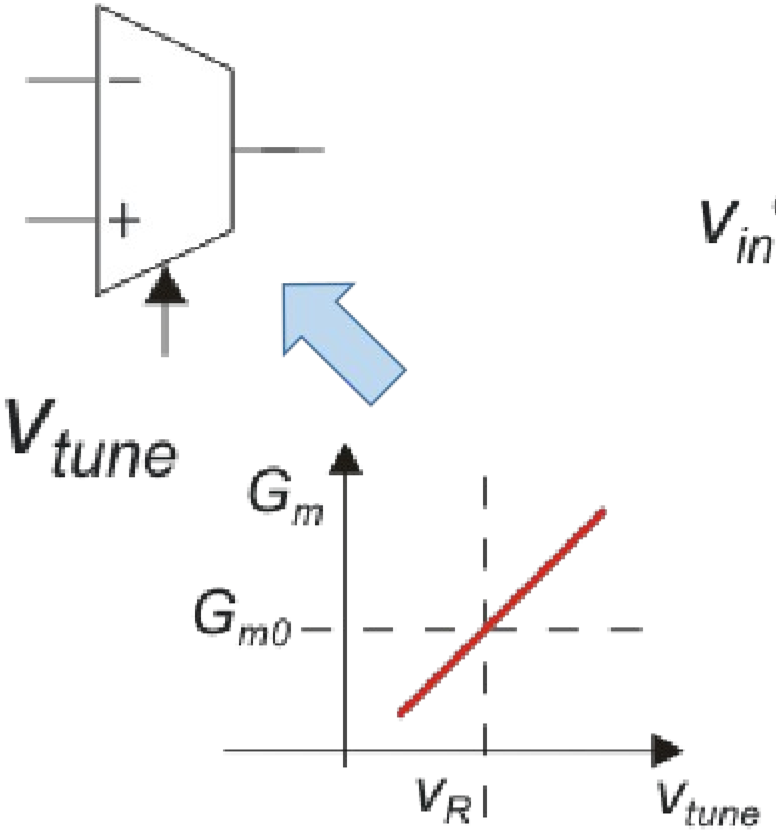
$$\begin{cases} \frac{V_{GSn}+V_{GSp}}{2} = V_{cmi} + \frac{V_{sn}+V_{sp}}{2} = \mathrm{K}\ (\mathrm{constant}) \\ V_{sn} = \quad V_{sp} \longrightarrow V_{GSp} - V_{GSn} = V_{id} \end{cases}$$

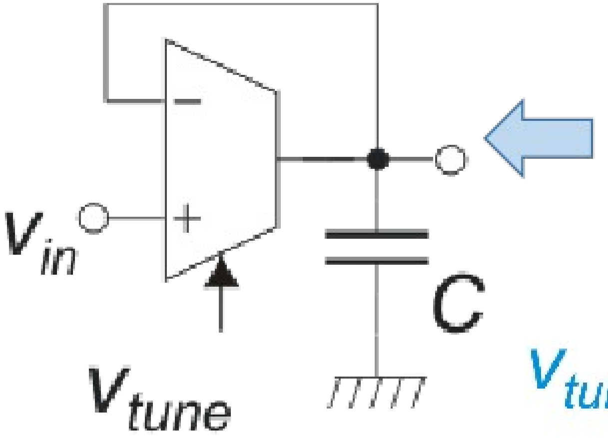
$$I_p - I_n = \underbrace{\beta\,\mathrm{K}}_{G_m} \cdot V_{id}$$

- Both sources are grounded: Vsn = -Vsp = 0
- K = Vcmi, controlled by the CMFB circuit of the preceding fully-differential circuit
- Gm = β Vcmi, the CMFB of the preceding circuit also controls the transconductance, through the VREF terminal

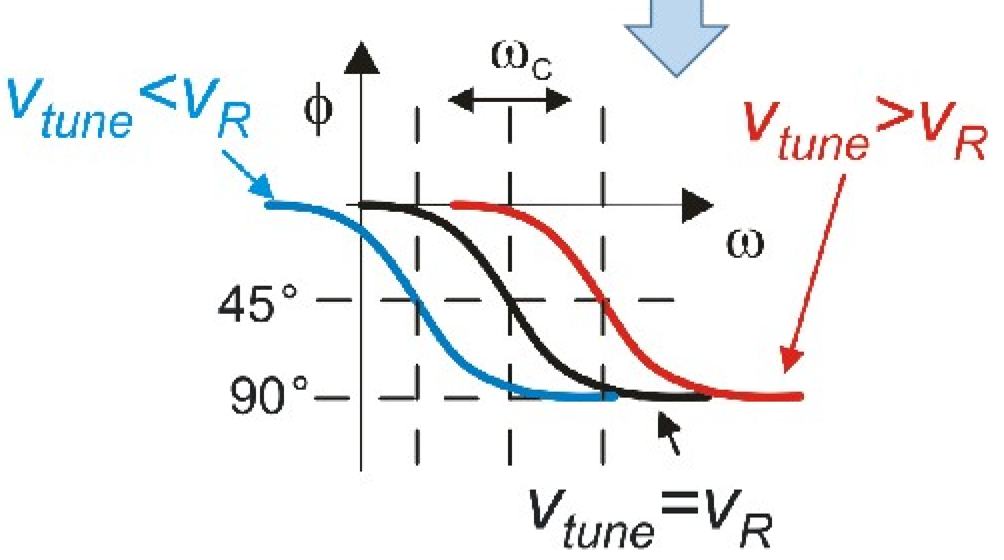
Self-tuning of Gm-C filters

In integrated circuits, Gm's and capacitances are strongly affected by PVT variations (up to ± 30 % variations). For these reasons, in most OTAs the Gm can be controlled by means of a voltage applied to a proper terminal (Vtune). In this way self-tuning of the filter can be accomplished.



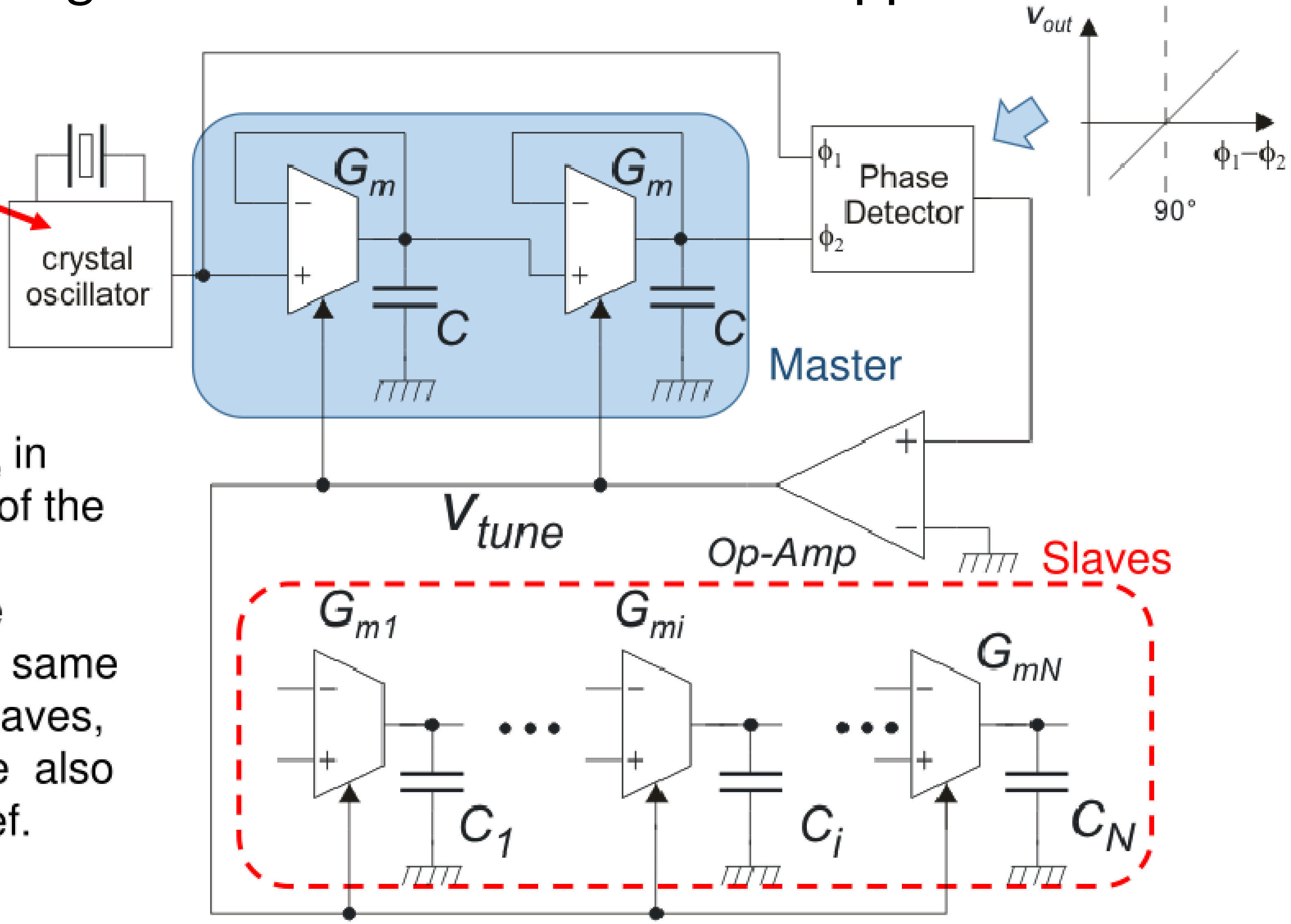


To understand the self-tuning principle, consider the effect of tuning on the phase response of a 1st order LP filter



Self-tuning of Gm-C filters: master-slave approach

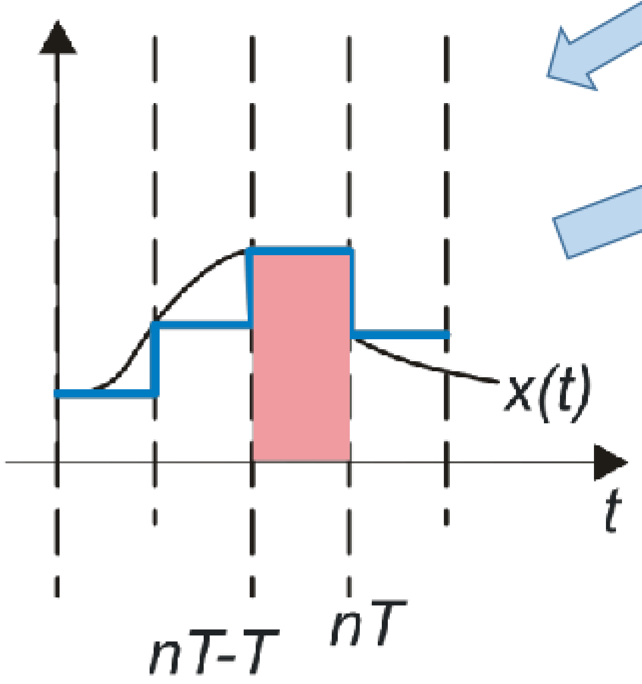
Reference frequency source



The loop varies V_{tune} in such a way that wC of the two master LP filters equals the reference frequency. Since the same Vtune is fed to the slaves, their Gm/C ratios are also proportional to the ref. frequency.

DT integrator: approximation methods

Integral approximation



$$y(t) = \int_0^t x(\tau)d\tau \Leftrightarrow \frac{x(s)}{s}$$

Time continuous

$$y(nT) = y(nT - T) + Tx(nT - T)$$

$$y(n) = y(n-1) + Tx(n-1)$$

$$Y(z) = z^{-1}Y(z) + Tz^{-1}X(z)$$

Forward Euler Integration

$$H_I(z) = T\frac{z^{-1}}{1 - z^{-1}} = T\frac{1}{z - 1}$$

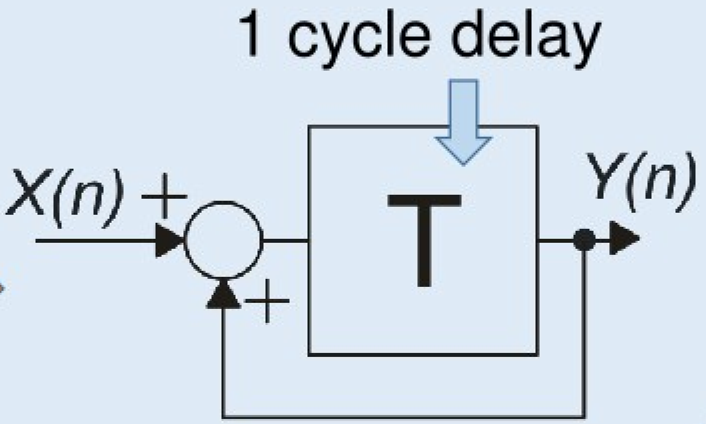
DT integrator: approximation methods

Forward

$$y(nT) = y(nT - T) + Tx(nT - T)$$

$$H_I(z) = T\frac{z^{-1}}{1 - z^{-1}} = T\frac{1}{z - 1}$$

$$\frac{z^{-1}}{1 - z^{-1}}$$

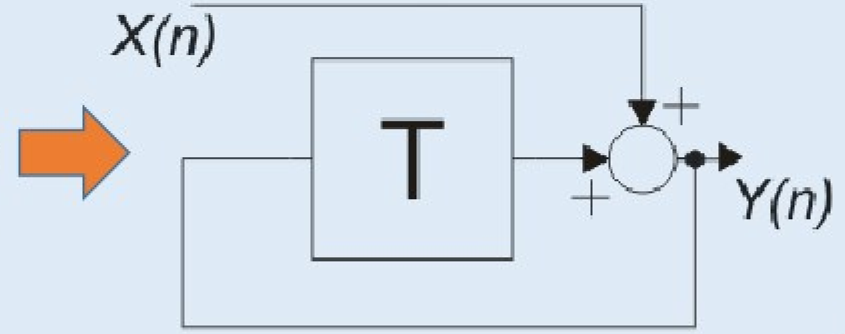


Backward

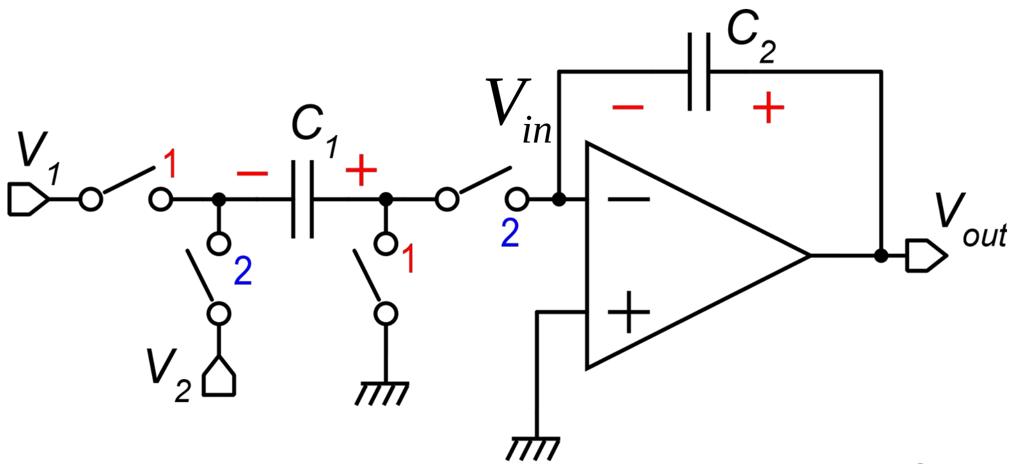
$$y(nT) = y(nT - T) + Tx(nT)$$

$$H_I(z) = T\frac{1}{1 - z^{-1}} = T\frac{z}{z - 1}$$

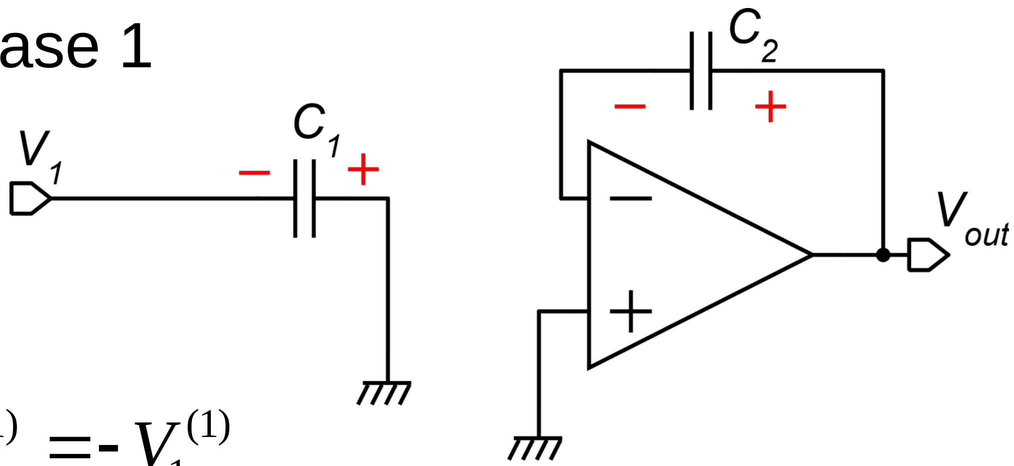
$$\frac{1}{1 - z^{-1}}$$



Parasitic insensitive SC integrator



Phase 1



$$V_{in} = 0$$

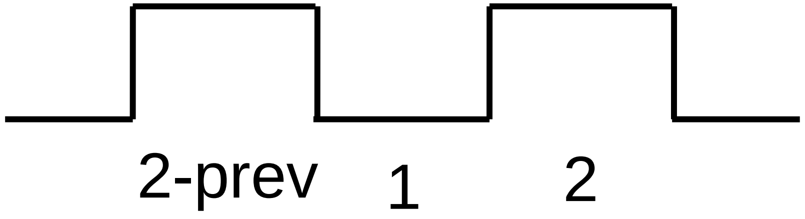
$$V_{C1}^{(1)} = -V_1^{(1)}$$

Simplifying hypotheses:

$$V_{out} = V_{C2}$$

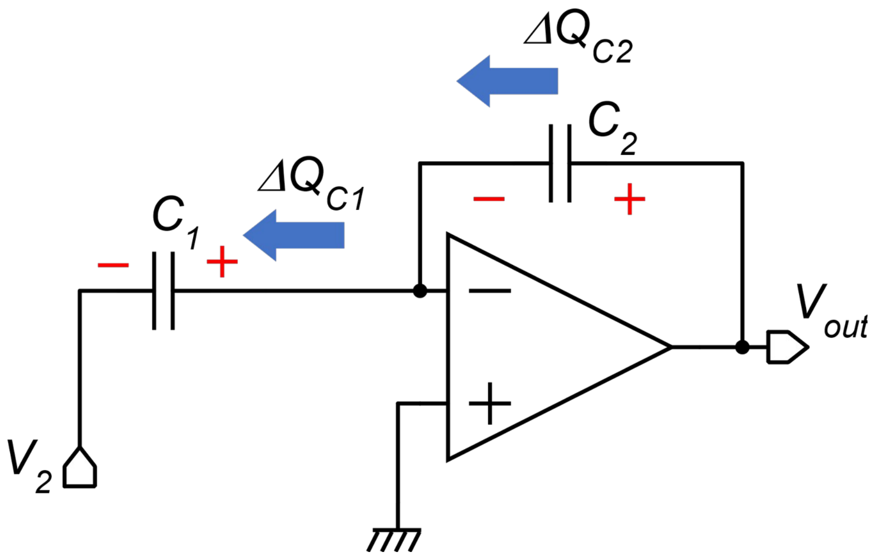
$$V_{C2}^{(1)} = V_{C2}^{(2-prev)}$$

- No offset / noise
- Perfect virtual short circuit
- No charge injection



Parasitic insensitive SC integrator

Phase 2 $\quad V_{C1}^{(2)} = -V_2^{(2)}$



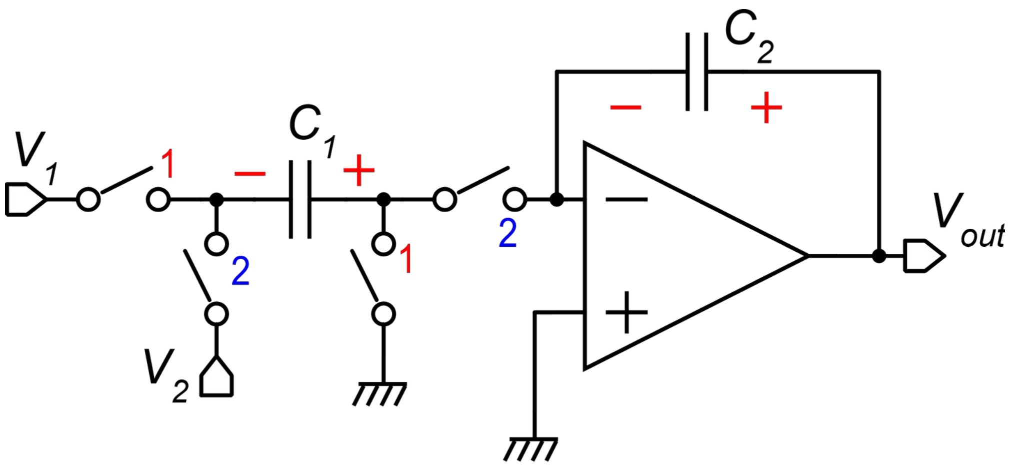
$$V_{out}^{(2)} = V_{C2}^{(2)} = V_{C2}^{(1)} + \frac{\Delta Q_{C2}}{C_2} = V_{out}^{(2-prev)} + \frac{\Delta Q_{C2}}{C_2}$$

$$\Delta Q_{C2} = \Delta Q_{C1} = C_1\left(V_{C1}^{(2)} - V_{C1}^{(1)}\right)$$

$$\Delta Q_{C2} = C_1\left[-V_2^{(2)} - \left(-V_1^{(1)}\right)\right] = C_1\left(V_1^{(1)} - V_2^{(2)}\right)$$

$$V_{out}^{(2)} = V_{out}^{(2-prev)} + \frac{C_1}{C_2}\left(V_1^{(1)} - V_2^{(2)}\right)$$

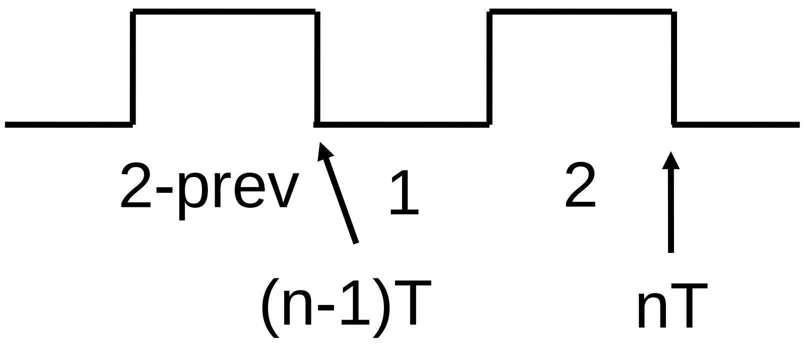
Parasitic insensitive SC integrator



$$V_{out}^{(2)} = V_{out}^{(2-prev)} + \frac{C_1}{C_2}\left(V_1^{(1)} - V_2^{(2)}\right)$$

If we consider that V_1 is sampled at the end of phase 2 and maintained across phase 1:

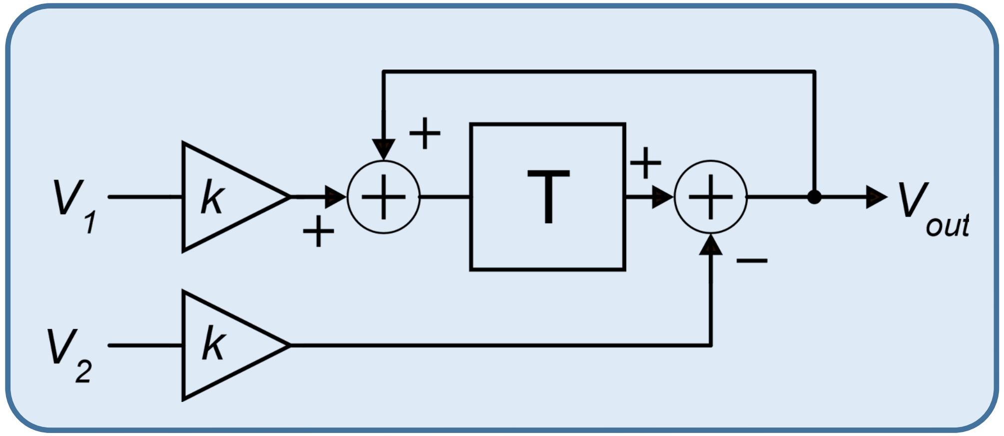
$$V_{out}^{(2)} = V_{out}^{(2-prev)} + \frac{C_1}{C_2}\left(V_1^{(2-prev)} - V_2^{(2)}\right)$$



$$V_{out}(n) = V_{out}(n-1) + \frac{C_1}{C_2}\left[V_1(n-1) - V_2(n)\right]$$

SC integrator: Block diagram and Z-transform

$$V_{out}(n) = V_{out}(n-1) + \frac{C_1}{C_2}\left[V_1(n-1) - V_2(n)\right]$$



Equivalent block diagram

Z-transform

$$V_{out}(z) = V_{out}(z)z^{-1} + \frac{C_1}{C_2}\left[V_1(z)z^{-1} - V_2(z)\right]$$

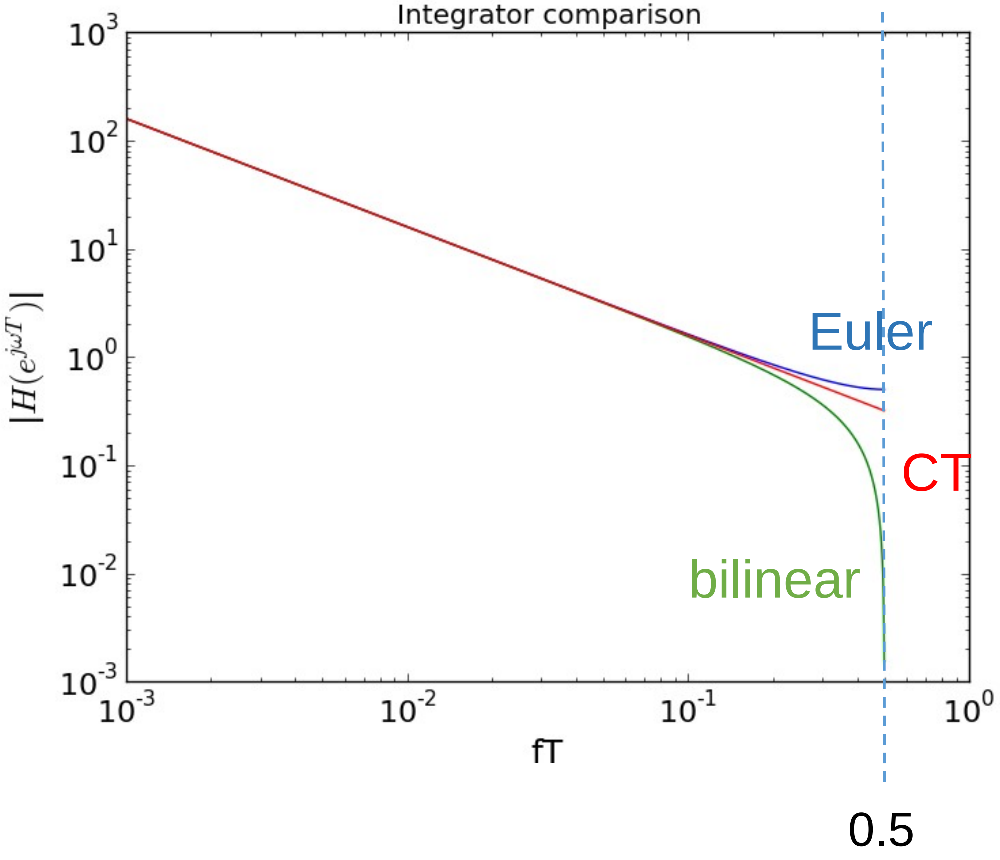
$$V_{out}(z) = +\frac{C_1}{C_2}\frac{z^{-1}}{1-z^{-1}}V_1(z) - \frac{C_1}{C_2}\frac{1}{1-z^{-1}}V_2(z)$$

Now singularities are tied to capacitors' matching and clock frequency:

- Relatively accurate
- Easily tunable

SC integrator: Frequency response

Integrators compared



$$V_{out}(z) = +\frac{C_1}{C_2}\frac{z^{-1}}{1-z^{-1}}V_1(z) - \frac{C_1}{C_2}\frac{1}{1-z^{-1}}V_2(z)$$

$$H_1(z) = \left.\frac{V_{out}(z)}{V_1(z)}\right|_{V_2=0} = \frac{C_1}{C_2}\frac{z^{-1}}{1-z^{-1}}$$ Forward Euler Integrator

$$H_2(z) = \left.\frac{V_{out}(z)}{V_2(z)}\right|_{V_1=0} = -\frac{C_1}{C_2}\frac{1}{1-z^{-1}}$$ Backward Euler Integrator

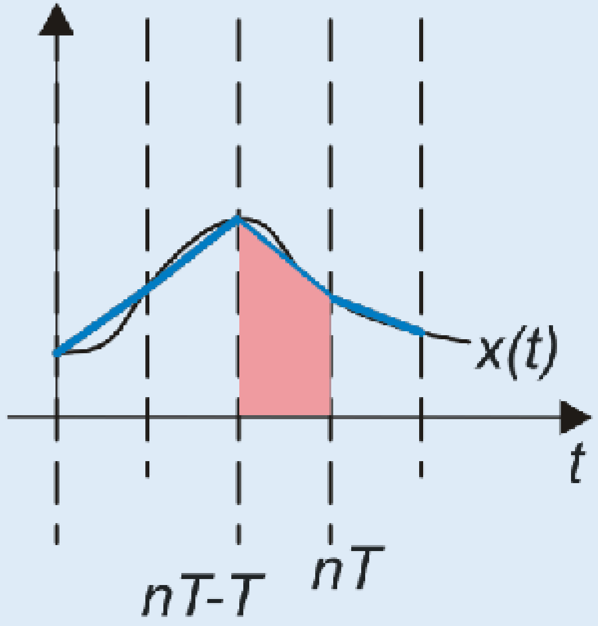
$$\left|H_1\left(e^{j\omega T}\right)\right| = \left|H_2\left(e^{j\omega T}\right)\right|$$

CT: continuous time integrator

INSIGHTS

DT integrator: bilinear approximation method

Trapezoidal integration



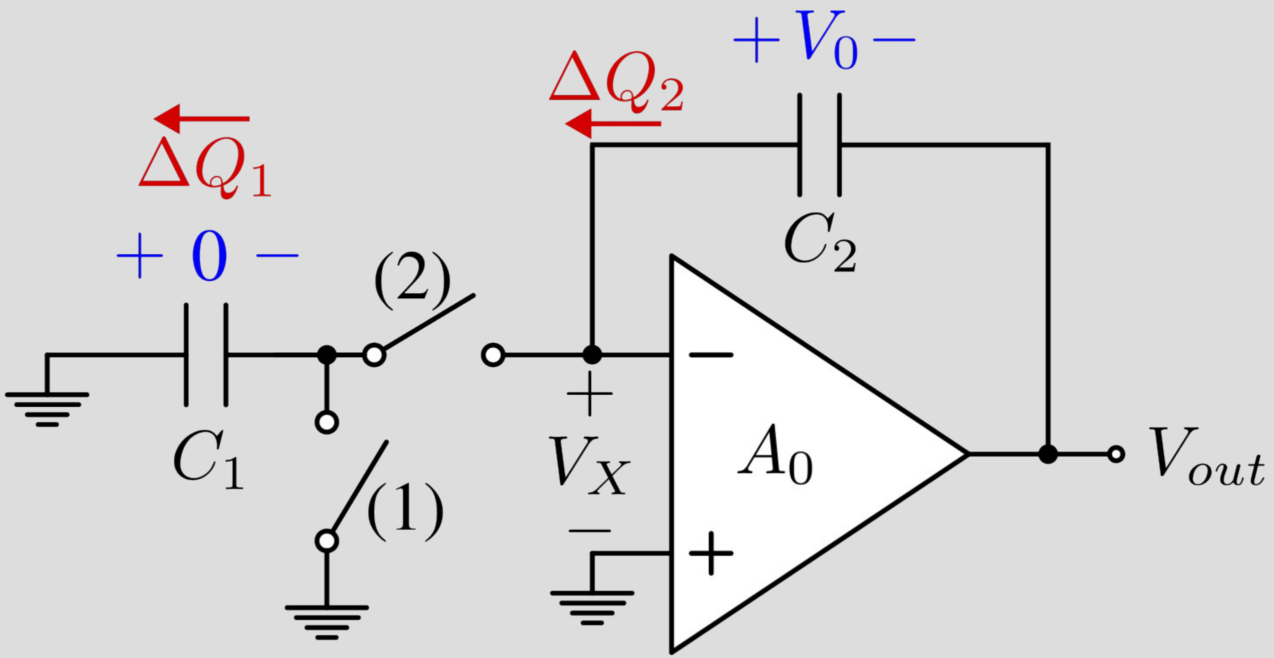
$$y(n) = y(n-1) + T\left[\frac{x(n)+x(n-1)}{2}\right]$$

$$\frac{Y(z)}{X(z)} = \frac{T}{2}\frac{z+1}{z-1} \leftrightarrow \frac{1}{s}$$

$$\frac{2}{T}\frac{z-1}{z+1} = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} \rightarrow s$$

INSIGHTS

DT integrator: Imperfections due to OTA finite gain



The opamp has a finite gain A_0;
C_2 initially charged at V_0, C_1 at 0 V

$$\begin{cases} V_{out} = A_0(V_{ip} - V_{in}) = A_0(-V_X) \\ V_{out} = V_X - V_0 \end{cases} \qquad V_X = \frac{V_0}{1 + A_0}$$

When the switch closes, the charge transfer begins, resulting finally in a new V_0' and V_X':

$$\begin{cases} \Delta Q_1 = C_1 V'_X = \frac{V'_0}{1+A_0} \\ \Delta Q_2 = \Delta Q_1 = C_2(V_0 - V'_0) \end{cases}$$

$$V'_0 = \left(\frac{1}{\frac{C_1/C_2}{1+A_0} + 1} \right) V_0$$

After N cycles:

$$V_0[N] = \left(\frac{1}{\frac{C_1/C_2}{1+A_0} + 1} \right)^N V_0$$

C_2 is discharging!

INSIGHTS

DT integrator: Imperfections due to OTA finite gain

The discharge process can be assimilated to an exponential decay of (sampled every TCK) with characteristic constant τ:

$$V_0[N] = \left(\frac{1}{\frac{C_1/C_2}{1+A_0} + 1} \right)^N V_0 = e^{-NT_{CK}/\tau} V_0$$

$$\tau = \frac{T_{CK}}{\ln\left(\frac{C_1/C_2}{1+A_0} + 1\right)} \approx (1 + A_0)\frac{C_2}{C_1} T_{CK}$$

The finite gain moved the pole from the origin (ideal integrator) to fp = 1/τ: coherent with the analysis of a CT integrator with R and C.