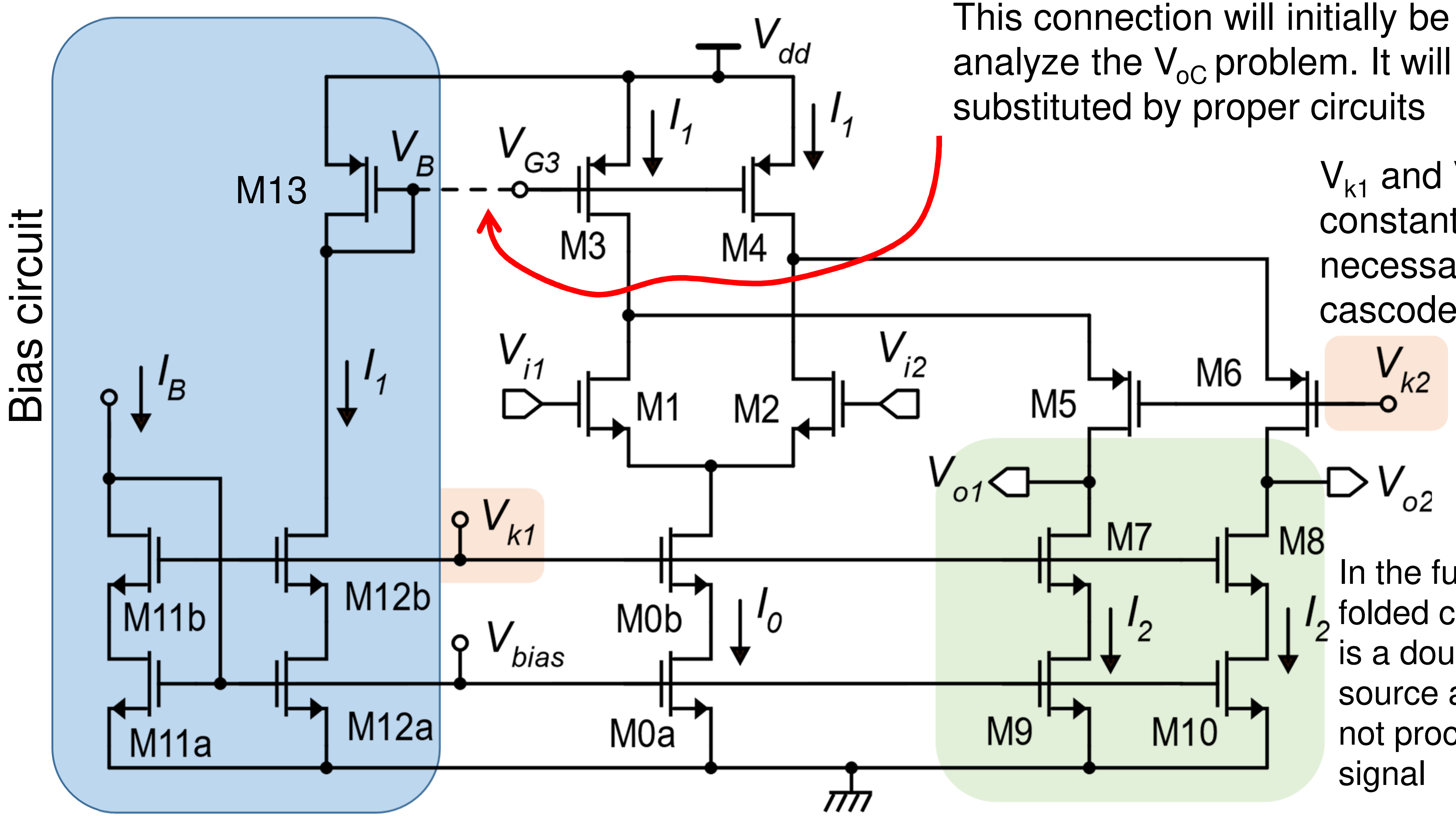
Single stage (OTA) fully differential op-amp



This connection will initially be made to analyze the V_{oC} problem. It will be later substituted by proper circuits

V_{k1} and V_{k2} are constant bias voltages necessary for p and n cascode structures

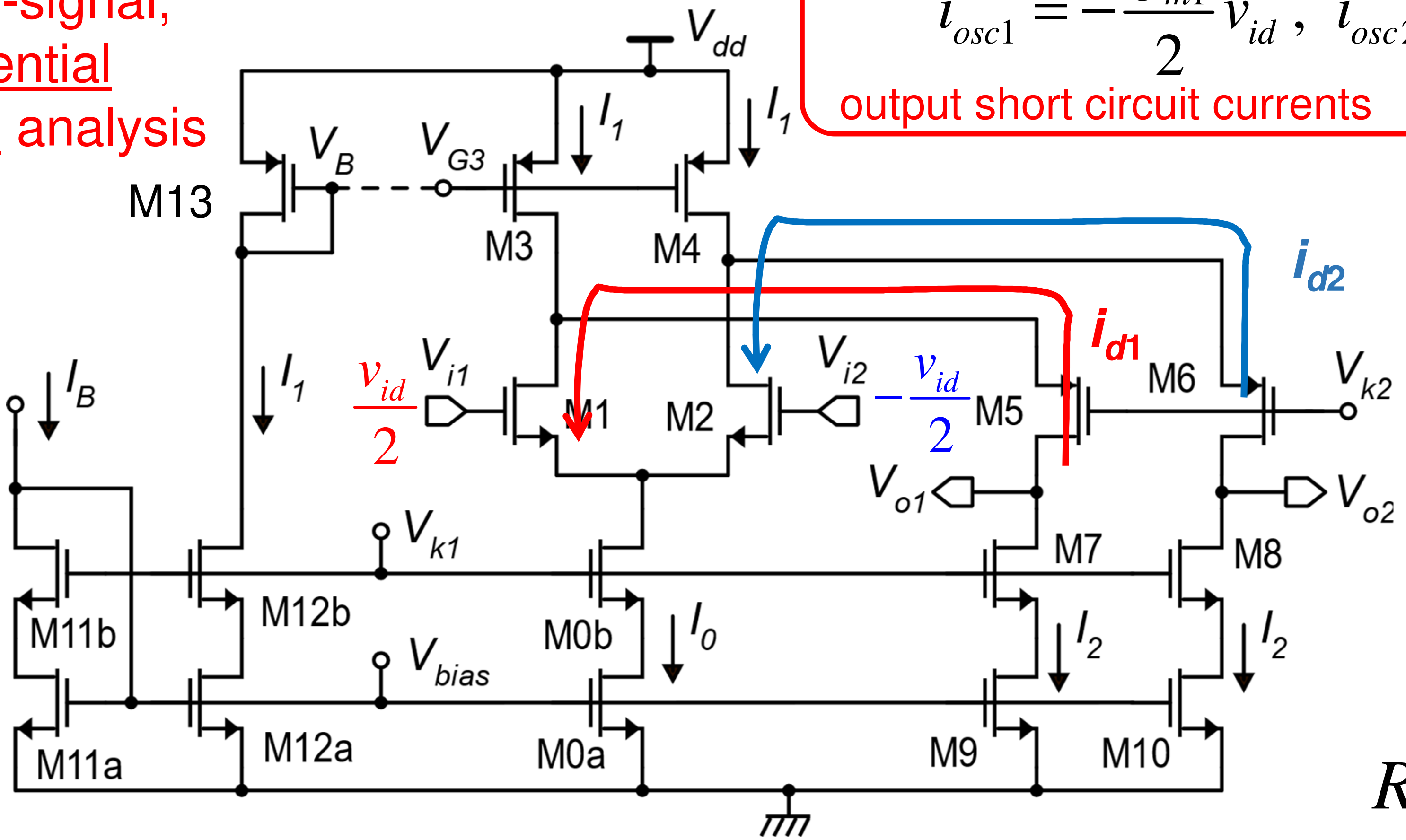
In the fully-diff. folded cascode this is a double current source and does not process the signal

Intuitive idea of the operating principle

Small-signal, <u>differential</u> <u>mode</u> analysis

$$i_{osc1} = -\frac{g_{m1}}{2} v_{id}\ ,\ \ i_{osc2} = \frac{g_{m1}}{2} v_{id}$$

output short circuit currents



$$v_{o1} = -\frac{g_{m1}}{2} R_{out} v_{id}$$

$$v_{o2} = \frac{g_{m1}}{2} R_{out} v_{id}$$

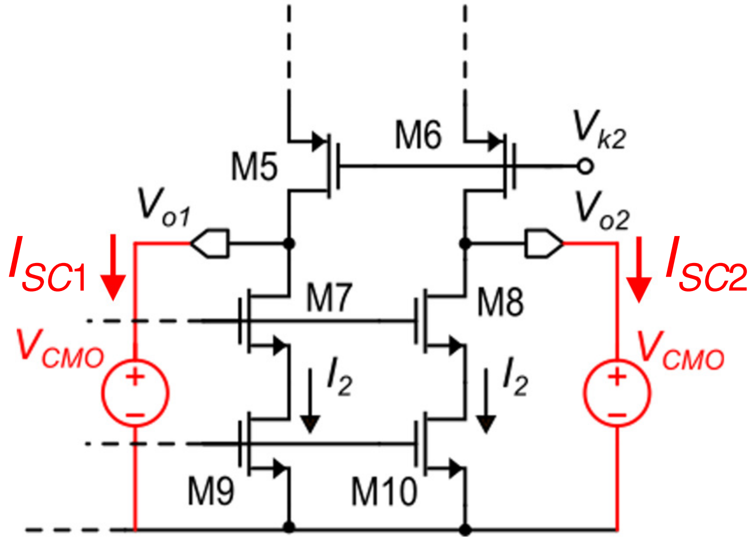
$$v_{od} = v_{o2} - v_{o1} =$$

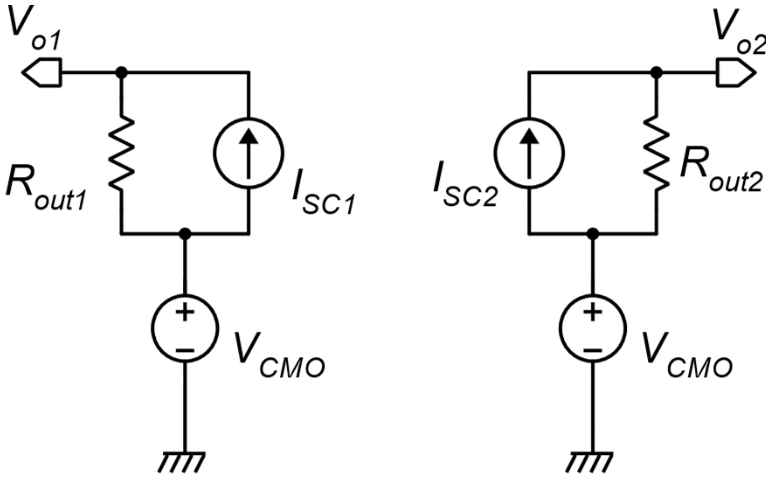
$$= g_{m1} R_{out} v_{id}$$

$$A_{dd} = g_{m1} R_{out}$$

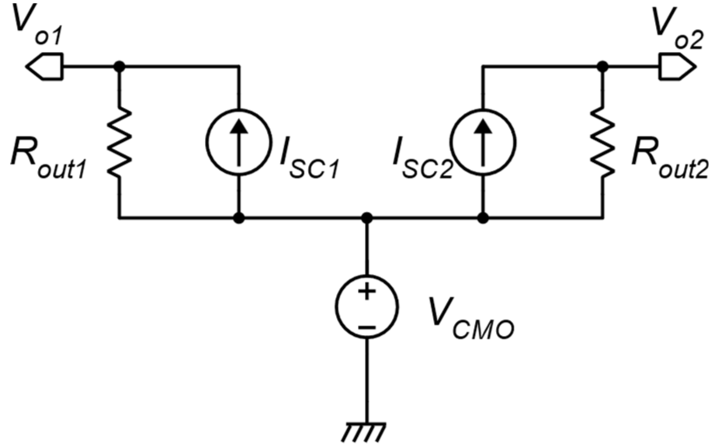
$$R_{out} \approx \frac{r_d \left(g_m r_d \right)}{2}$$

Including common mode / differential and quiescent / small signal components



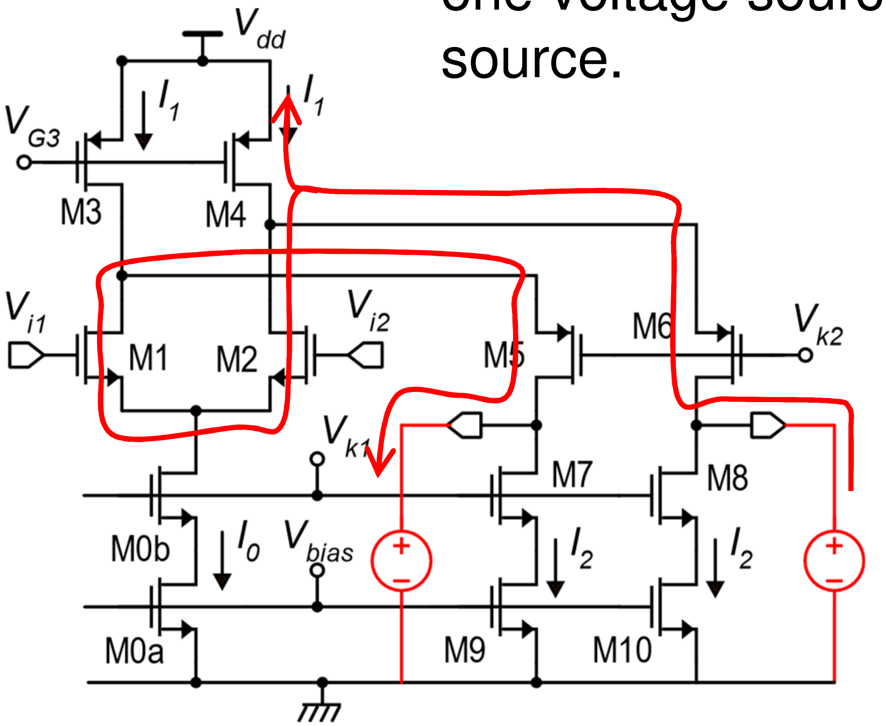


We probe the output ports with voltage sources set to the desired output common mode voltage (V_{CMO})

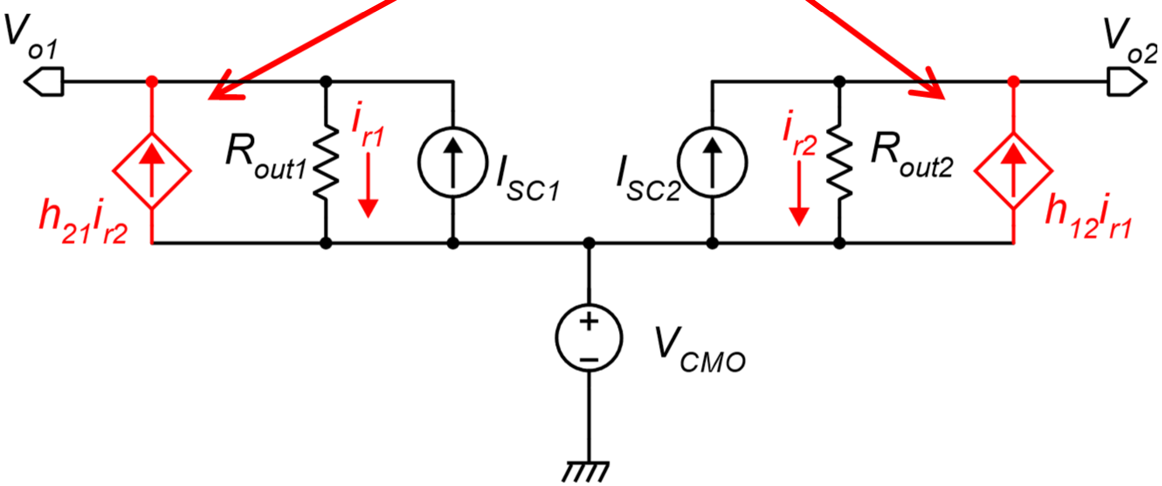


A more accurate model of the output resistances.

When calculating the output resistance, we note that part of the current injected by one voltage source flows into the other source.



It is possible to model this interaction by means of two current-controlled current sources (cccs)



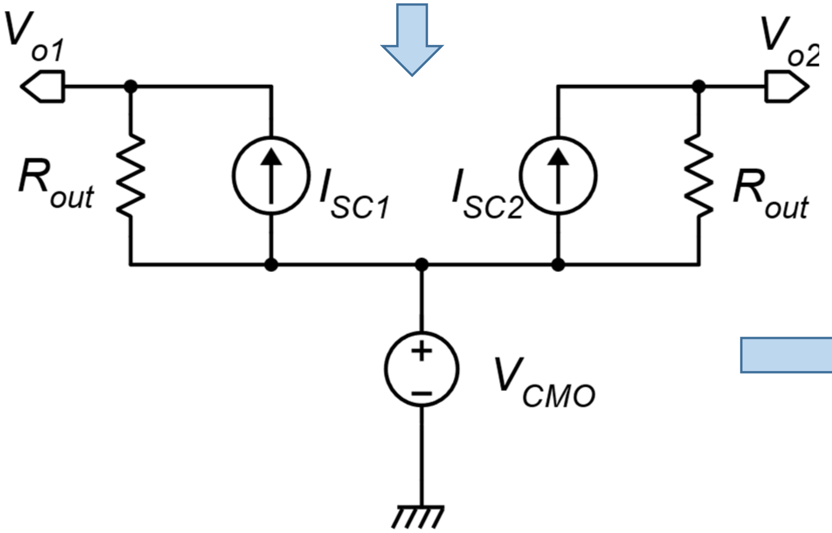
Simplified model

- For the sake of simplicity, in the following study we will use a reduced model where **we neglect the effect of the cross-talk between** the two output ports. It can be shown that it simply turns out into a slightly smaller gain.

- It is possible to show that a **moderate asymmetry** between the two ports does not produce significant effects once an output common mode stabilization circuit is included.

$$h_{12} = h_{21} = 0$$

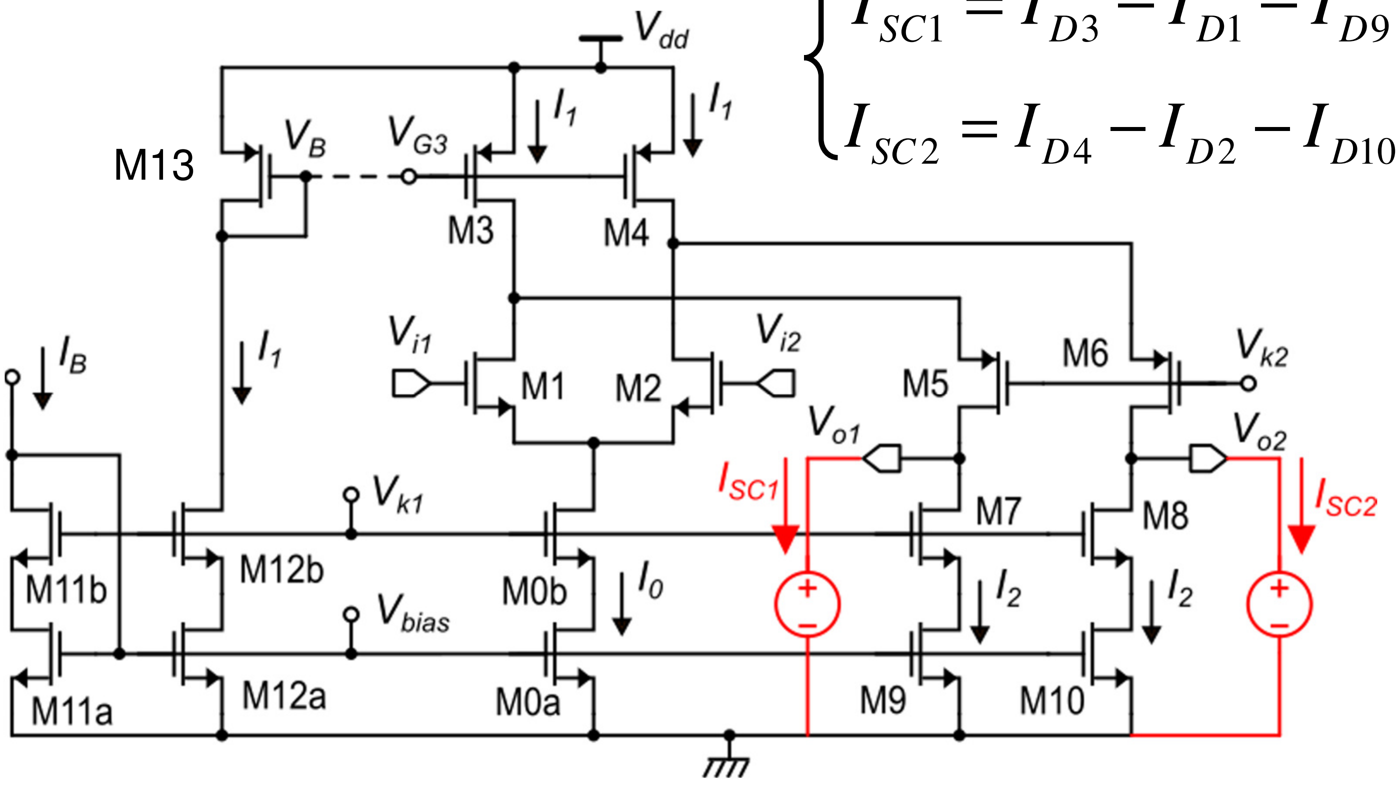
$$R_{out1} = R_{out2} = R_{out}$$



We need to calculate the output short circuit currents

$$\begin{cases} V_{o1} = V_{CMO} + R_{out} \underline{I_{SC1}} \\ V_{o2} = V_{CMO} + R_{out} \underline{I_{SC2}} \end{cases}$$

Output short circuit currents



$$\begin{cases} I_{SC1} = I_{D3} - I_{D1} - I_{D9} \\ I_{SC2} = I_{D4} - I_{D2} - I_{D10} \end{cases}$$

Nominally:

$$I_{D3} = I_{D4} = I_1$$

$$I_{D9} = I_{D10} = I_2$$

$$I_{D1} = \frac{I_0}{2} + g_{m1}\frac{v_{id}}{2}$$

$$I_{D2} = \frac{I_0}{2} - g_{m1}\frac{v_{id}}{2}$$

$$\text{with}: \begin{cases} I_0 = k_0 I_B \\ I_1 = k_1 I_B \\ I_2 = k_2 I_B \end{cases}$$

Output short circuit currents: real case with matching errors

$$\begin{cases} I_{D3} = I_1 + \Delta I_{D3} \\ I_{D4} = I_1 + \Delta I_{D4} \end{cases}$$

$$\begin{cases} I_{D9} = I_2 + \Delta I_{D9} \\ I_{D10} = I_2 + \Delta I_{D10} \end{cases}$$

$$\begin{cases} I_{D1} = \dfrac{I_0}{2} + g_{m1}\dfrac{v_{id}}{2} + \dfrac{\Delta I_{D0-1}}{2} \\ I_{D2} = \dfrac{I_0}{2} - g_{m1}\dfrac{v_{id}}{2} + \dfrac{\Delta I_{D0-2}}{2} \end{cases}$$

Errors in the gain of current mirrors due to device mismatch

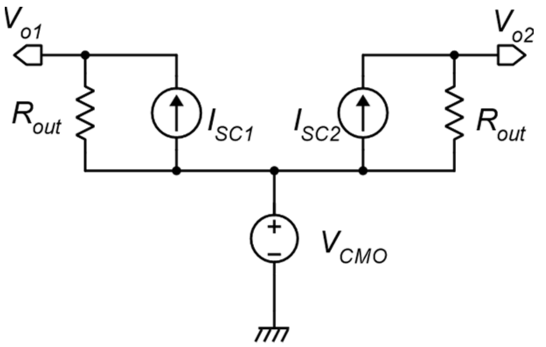
$$\begin{cases} I_0 = k_0 I_B \\ I_1 = k_1 I_B \\ I_2 = k_2 I_B \end{cases}$$

$$\begin{cases} I_{SC1} = I_{D3} - I_{D1} - I_{D9} \\ I_{SC2} = I_{D4} - I_{D2} - I_{D10} \end{cases}$$

All the matching errors can be combined into only these two terms

$$\begin{cases} I_{SC1} = I_1 - \dfrac{I_0}{2} - I_2 - g_{m1}\dfrac{v_{id}}{2} + \boxed{I_{\varepsilon 1}} \\ I_{SC2} = I_1 - \dfrac{I_0}{2} - I_2 + g_{m1}\dfrac{v_{id}}{2} + \boxed{I_{\varepsilon 2}} \end{cases}$$

Output short circuit currents: real case



$$\begin{cases} I_{SC1} = I_1 - \frac{I_0}{2} - I_2 - g_{m1}\frac{v_{id}}{2} + \boxed{I_{\varepsilon 1}} \\ I_{SC2} = I_1 - \frac{I_0}{2} - I_2 + g_{m1}\frac{v_{id}}{2} + \boxed{I_{\varepsilon 2}} \end{cases} \Leftarrow \boxed{\begin{cases} I_{\varepsilon 1} = I_\varepsilon + \frac{\Delta I_\varepsilon}{2} \\ I_{\varepsilon 2} = I_\varepsilon - \frac{\Delta I_\varepsilon}{2} \end{cases}}$$

$$\begin{cases} V_{o1} = V_{CMO} + R_{out} I_{SC1} \\ V_{o2} = V_{CMO} + R_{out} I_{SC2} \end{cases}$$

$$\begin{cases} V_{o1} = V_{CMO} + R_{out}\left(I_1 - \frac{I_0}{2} - I_2 - g_{m1}\frac{v_{id}}{2} + I_\varepsilon + \frac{\Delta I_\varepsilon}{2} \right) \\ V_{o2} = V_{CMO} + R_{out}\left(I_1 - \frac{I_0}{2} - I_2 + g_{m1}\frac{v_{id}}{2} + I_\varepsilon - \frac{\Delta I_\varepsilon}{2} \right) \end{cases}$$

Differential mode

$$V_{od} = V_{o2} - V_{o1}$$

$$\begin{cases} V_{o1} = V_{CMO} + R_{out}\left(I_1 - \dfrac{I_0}{2} - I_2 - g_{m1}\dfrac{v_{id}}{2} + I_\varepsilon + \dfrac{\Delta I_\varepsilon}{2} \right) & - \\ V_{o2} = V_{CMO} + R_{out}\left(I_1 - \dfrac{I_0}{2} - I_2 + g_{m1}\dfrac{v_{id}}{2} + I_\varepsilon - \dfrac{\Delta I_\varepsilon}{2} \right) & + \end{cases}$$

$$V_{od} = R_{out}\left(g_{m1} v_{id} - \Delta I_\varepsilon \right)$$

Gain: $A_{dd} = g_{m1} R_{out}$

$$V_{od} = g_{m1} R_{out}\left(v_{id} - \frac{\Delta I_\varepsilon}{g_{m1}} \right)$$

Offset: $v_{io} = \dfrac{\Delta I_\varepsilon}{g_{m1}}$

Common mode

$$\begin{cases} V_{o1} = V_{CMO} + R_{out}\left(I_1 - \frac{I_0}{2} - I_2 - g_{m1}\frac{v_{id}}{2} \; + \; I_\varepsilon + \frac{\Delta I_\varepsilon}{2}\right) & \times\frac{1}{2} \\ V_{o2} = V_{CMO} + R_{out}\left(I_1 - \frac{I_0}{2} - I_2 + g_{m1}\frac{v_{id}}{2} \; + \; I_\varepsilon - \frac{\Delta I_\varepsilon}{2}\right) & \times\frac{1}{2} \end{cases}$$

$$V_{oc} = \frac{V_{o1} + V_{o2}}{2} = V_{CMO} + \boxed{R_{out}\left(I_1 - \frac{I_0}{2} - I_2 + I_\varepsilon\right)}$$

This is an unwanted term, because we want only V_{CMO}

$$\begin{cases} I_0 = k_0 I_B \\ I_1 = k_1 I_B \\ I_2 = k_2 I_B \end{cases}$$

By design:

$$k_1 - \frac{k_0}{2} - k_2 = 0$$

$$I_1 - \frac{I_0}{2} - I_2 = k_1 I_B - \frac{k_0}{2} I_B - k_2 I_B = 0$$

$$V_{oc} = V_{CMO} + R_{out} I_\varepsilon$$

Common mode error

$$V_{oc} = V_{CMO} + \boxed{R_{out} I_\varepsilon}$$

Error

I_ε is the sum of current mismatches of several current mirror

$$R_{out} I_\varepsilon = \frac{R_{out}\, g_{m1}}{g_{m1}} I_\varepsilon = A_{dd} \frac{V_{TE1}}{I_{D1}} I_\varepsilon = A_{dd} \cdot 2V_{TE1} \frac{I_\varepsilon}{I_0}$$

$$\frac{I_0}{2}$$

In a current mirror:

$$\frac{\Delta I_{out}}{I_{out}} \approx \frac{1}{100}$$

I_0 is the output current of one of the several mirror that contribute to I_ε

50 mV

just an example:

$$R_{out} I_\varepsilon = A_{dd} \cdot 2V_{TE1} \boxed{\frac{I_\varepsilon}{I_0}} > \frac{1}{100} \qquad > 10 \text{ V}$$

possible gain of a folded cascode : 10^4

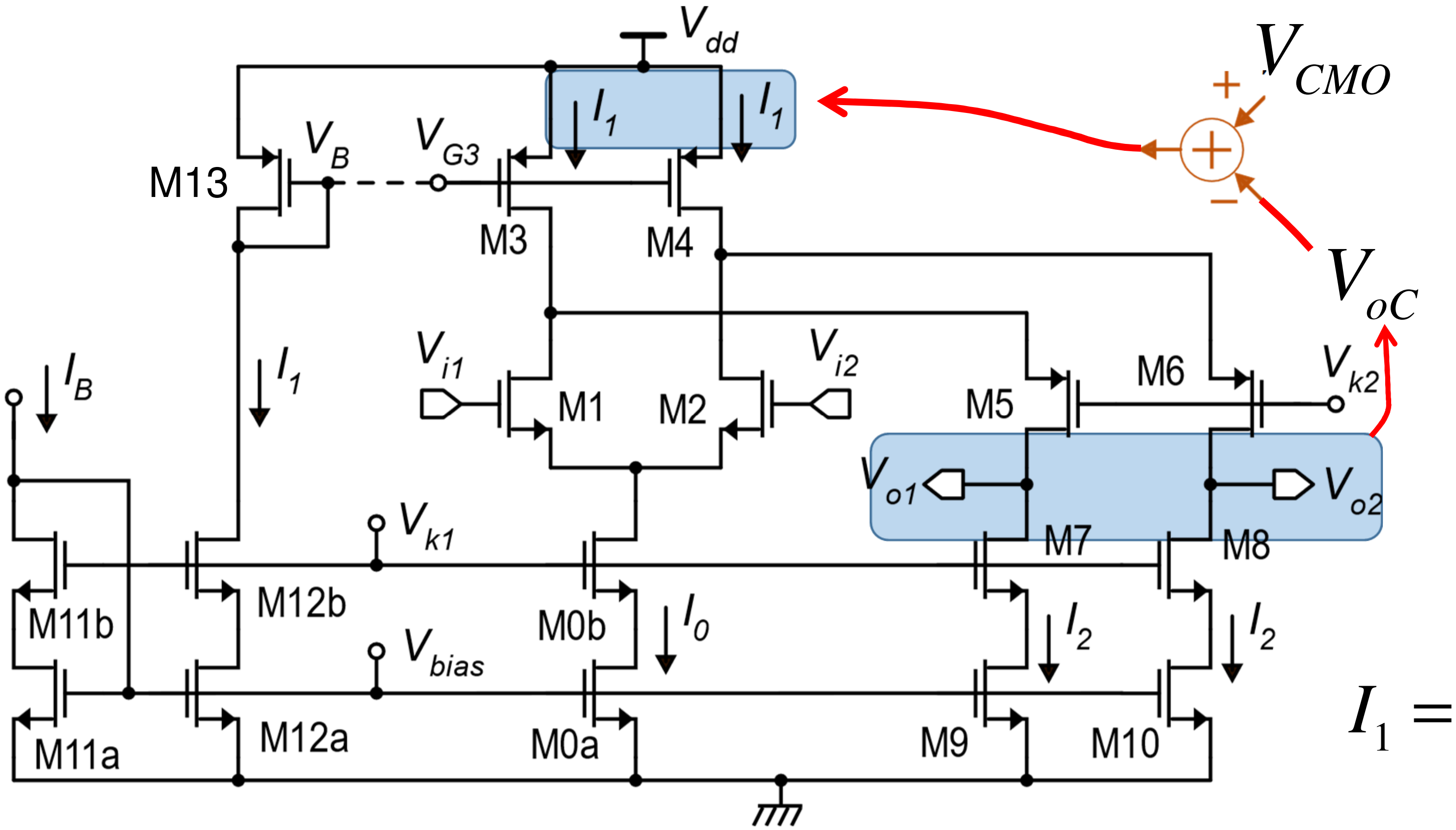
Common Mode Stabilization

$$V_{oc} - V_{CMO} = R_{out} I_{\varepsilon}$$

With the configuration that we have analyzed so far, the error in the common mode is too large for any practical application. It is very likely that the error exceeds the supply voltage, meaning that in quiescent conditions, both the outputs are saturated at either the upper or at the lower bound of the output range

A circuit that stabilizes the output common mode voltage to a value close to V_{CMO} is required.
This circuit is called Common Mode Feed-Back loop, or simply **CMFB**

CMFB: the principle



The CMFB circuit, calculates V_{oc}, compares it with the target value V_{CMO} and adjust one of the bias currents (I_1 , I_0, or I_2) to make $V_{oc} \cong V_{CMO}$

$$I_1 = k_1 I_B - g_m^* \left(V_{oc} - V_{CMO}\right)$$

Generic transconductance

CMFB: the effect

$$\begin{cases} I_1 = k_1 I_B - g_m^* \left(V_{oc} - V_{CMO}\right) \\ I_0 = k_0 I_B \\ I_2 = k_2 I_B \end{cases} \qquad \text{with: } k_1 - \frac{k_0}{2} - k_2 = 0$$

$$V_{oc} = \frac{V_{o1} + V_{o2}}{2} = V_{CMO} + R_{out}\left(I_1 - \frac{I_0}{2} - I_2 + I_\varepsilon\right)$$

$$V_{oc} = V_{CMO} + R_{out}\left(\underline{k_1 I_B} - g_m^* \left(V_{oc} - V_{CMO}\right) - \frac{k_0 I_B}{\underline{2}} - \underline{k_2 I_B} + I_\varepsilon\right)$$

$$V_{oc} = V_{CMO} + R_{out}\left(- g_m^* \left(V_{oc} - V_{CMO}\right) + I_\varepsilon\right)$$

$$V_{oc} = V_{CMO} - g_m^* R_{out} V_{oc} + g_m^* R_{out} V_{CMO} + R_{out} I_\varepsilon$$

CMFB: the effect

$$V_{oc} = V_{CMO} - g_m^* R_{out} V_{oc} + g_m^* R_{out} V_{CMO} + R_{out} I_\varepsilon$$

$$V_{oc}\left(1+g_m^* R_{out}\right) = V_{CMO}\left(1+g_m^* R_{out}\right) + R_{out} I_\varepsilon$$

$$V_{oc} = V_{CMO} + \frac{R_{out} I_\varepsilon}{\left(1+g_m^* R_{out}\right)}$$

If g_m^* is of the same order of g_{m1}, then the product $g_m^* R_{out}$ is of the same order as A_{dd}

⇨ $$g_m^* R_{out} >> 1$$

$$V_{oc} \cong V_{CMO} + I_\varepsilon \frac{1}{g_m^*}$$

Again, this is the ratio of one current mismatch (I_ε) over a full current (I_D^*). We expect this ratio to be << 1

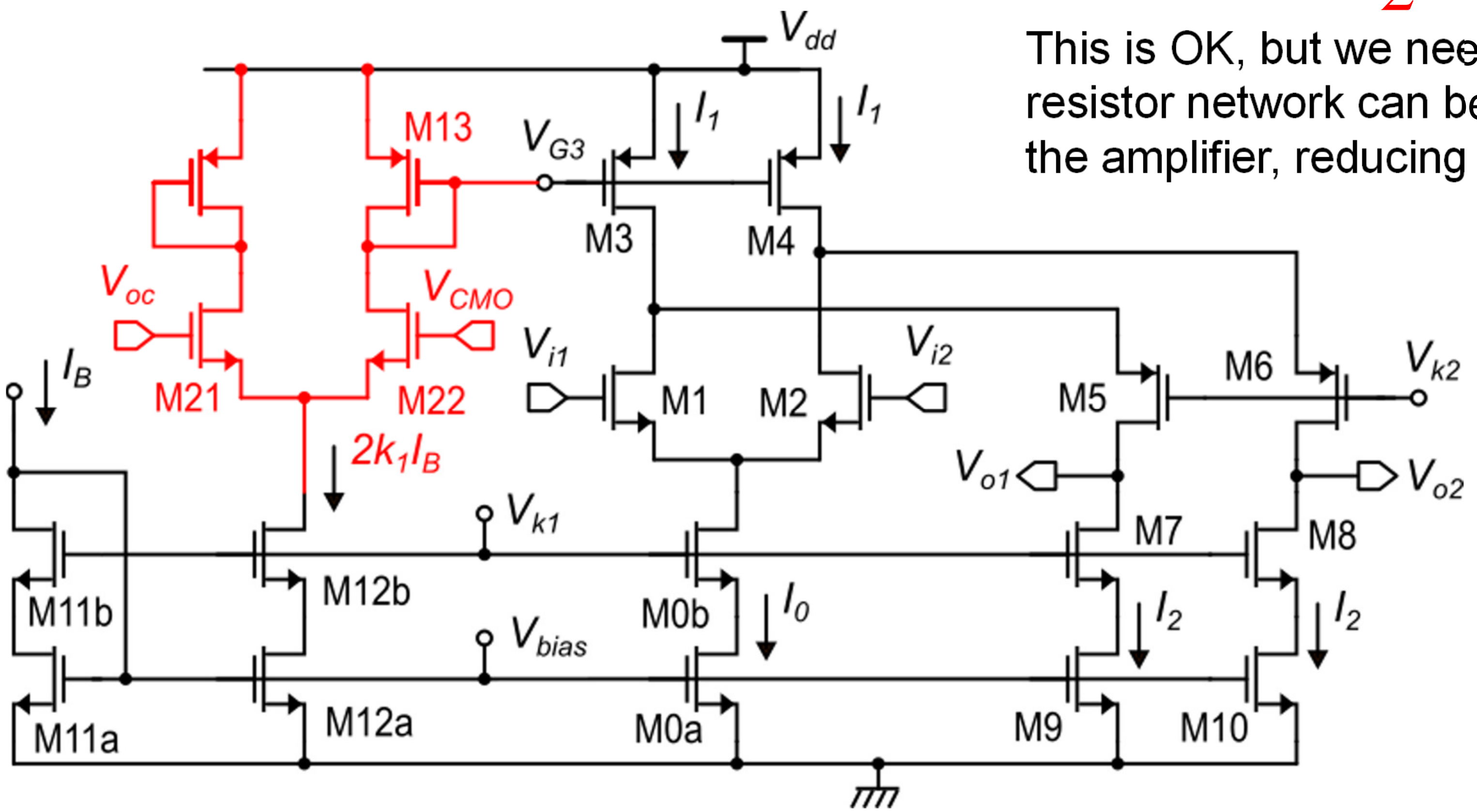
$$g_m^* = \frac{I_D^*}{V_{TE}^*} \qquad V_{oc} - V_{CMO} \cong V_{TE}^* \boxed{\frac{I_\varepsilon}{I_D^*}}$$

With the introduction of the CMFB, the error decreases from several Volt to a few mV

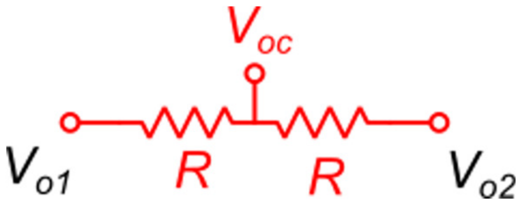
A first idea to obtain the CMFB

$$I_1 = k_1 I_B - g_m^* \left(V_{oc} - V_{CMO}\right)$$

$$I_1 = k_1 I_B - \frac{g_{m21}}{2}\left(V_{oc} - V_{CMO}\right)$$



This is OK, but we need to produce V_{oc}. A resistor network can be used, but this load the amplifier, reducing the gain dramatically.

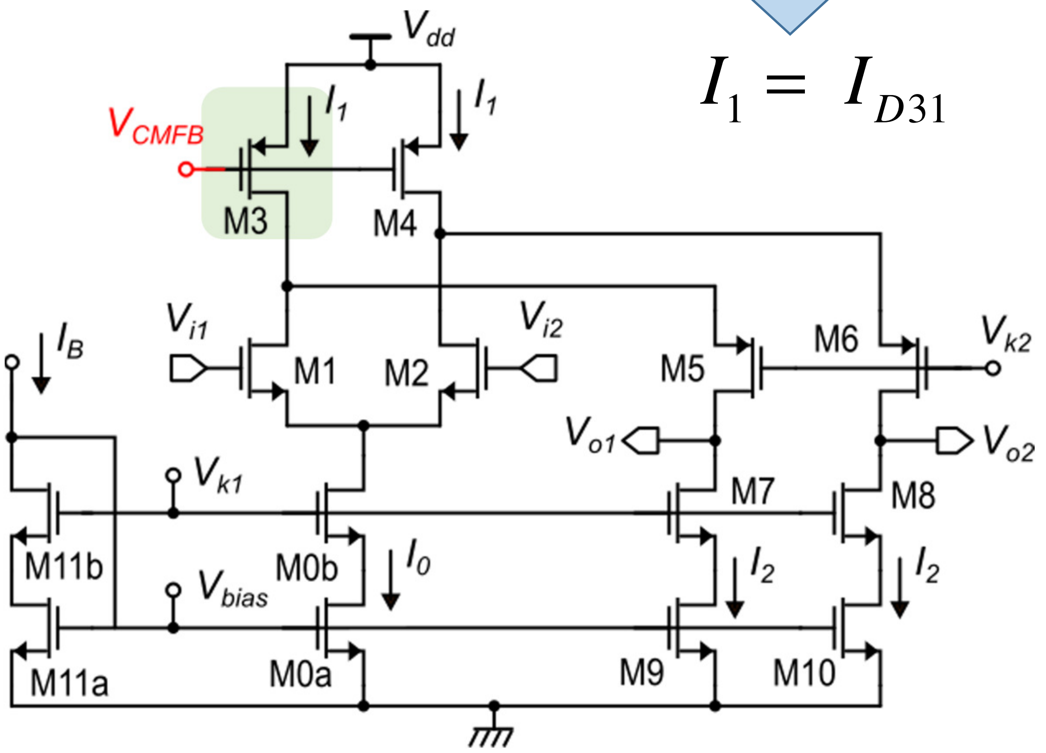


Applicable only when a resistor across the output terminals was already included in the design (e.g. in in-amps)

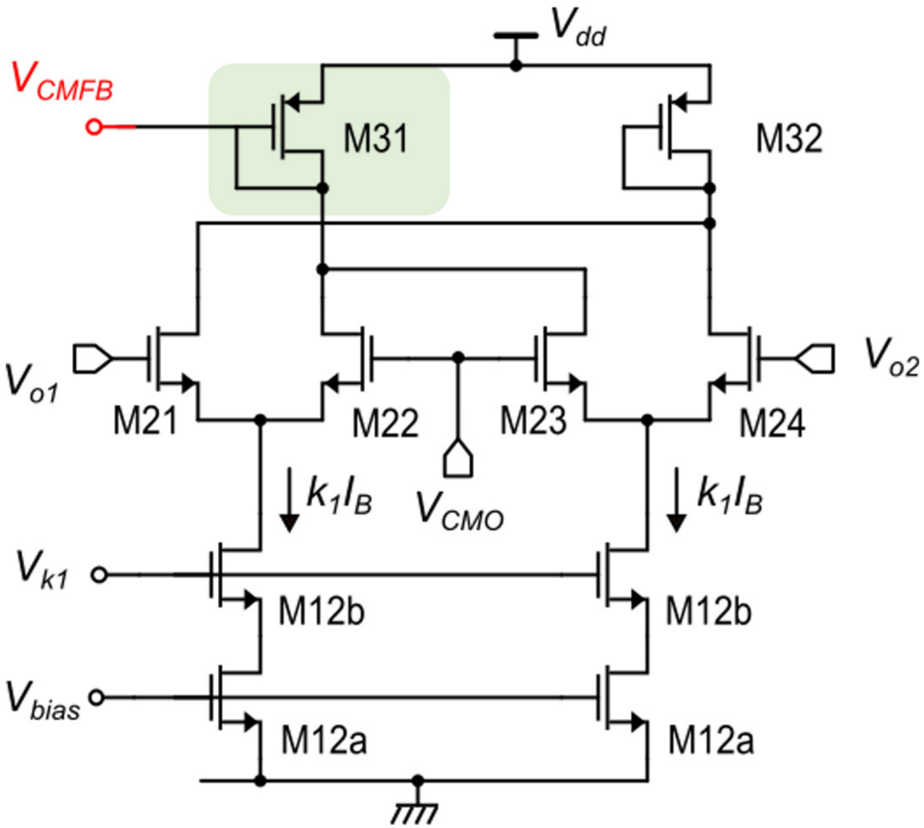
First solution: static CMFB

To simplify the analysis: M31=M3

$$I_1 = I_{D31}$$

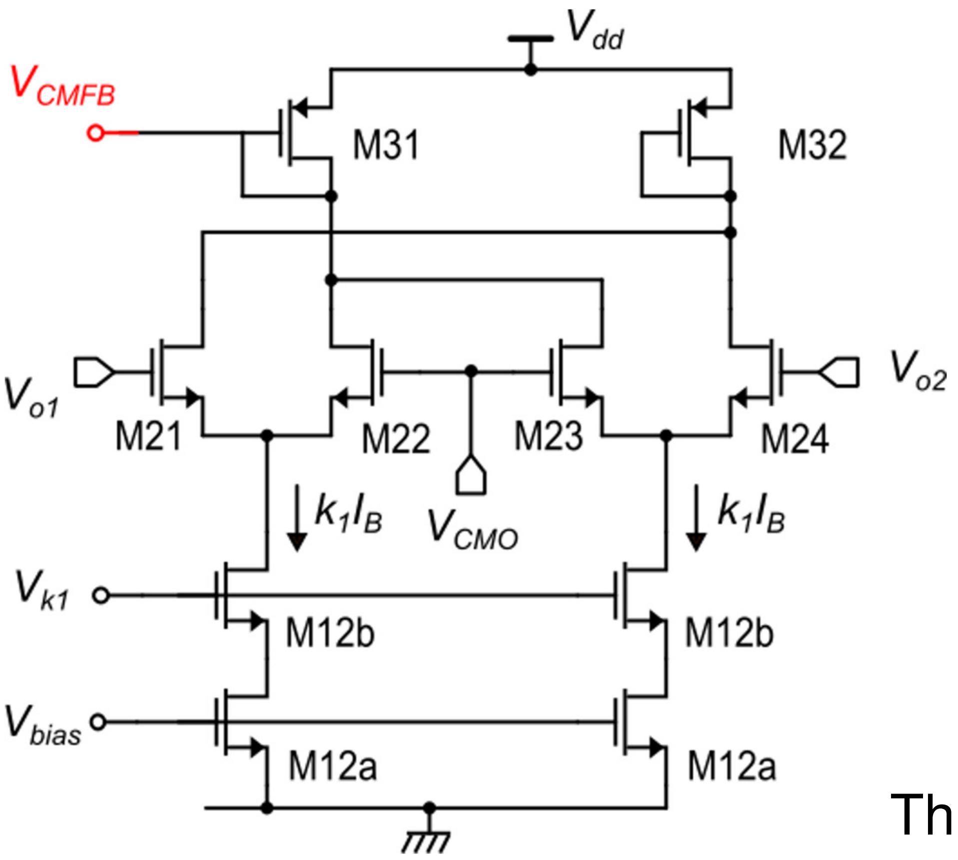


Fully-differential single-stage op-amp



CMFB circuit

Analysis of the static CMFB



$$I_1 = I_{D31} = I_{D22} + I_{D23}$$

$$\begin{cases} I_{D22} = \dfrac{k_1 I_B}{2} - \dfrac{g_{m21}}{2}\left(V_{o1} - V_{CMO}\right) \\ I_{D23} = \dfrac{k_1 I_B}{2} - \dfrac{g_{m23}}{2}\left(V_{o2} - V_{CMO}\right) \end{cases}$$

$$g_{m21} = g_{m22} = g_{m23} = g_{m24}$$

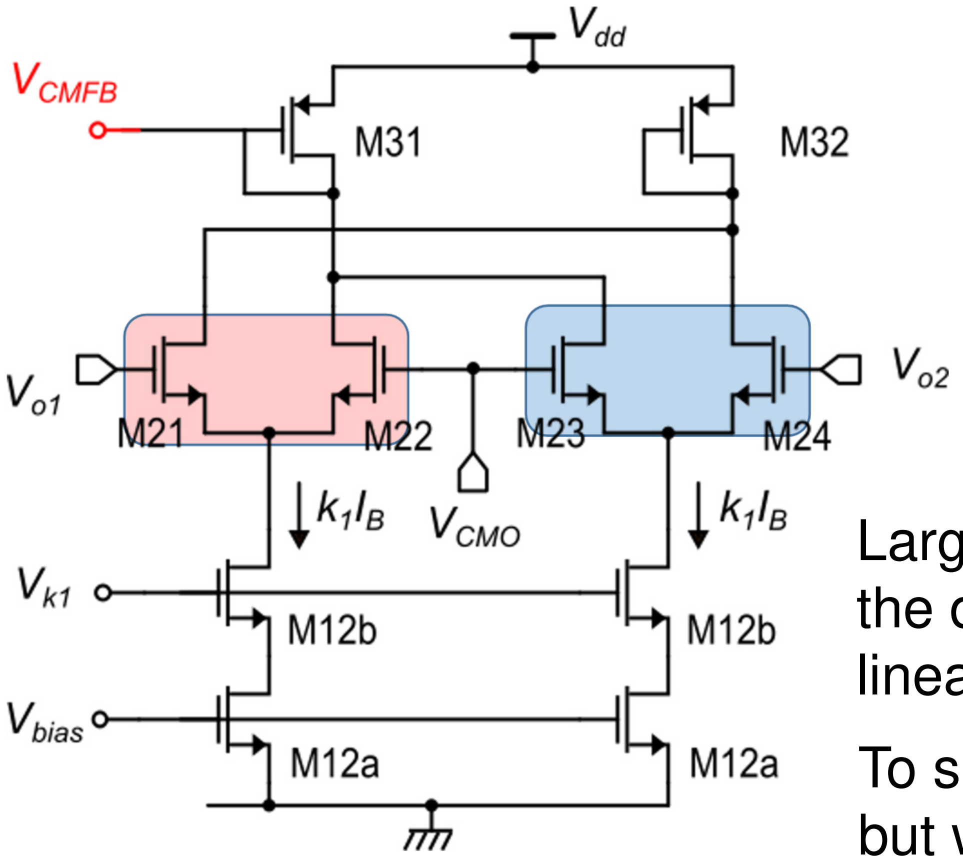
$$I_1 = k_1 I_B - g_{m21}\left(\frac{V_{o1} + V_{o2}}{2} - V_{CMO}\right)$$

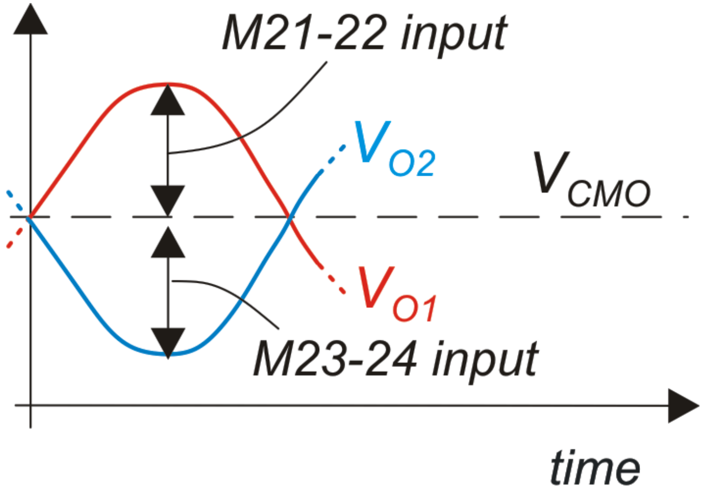
$$I_1 = k_1 I_B - g_{m21}\left(V_{oc} - V_{CMO}\right)$$

This is the required relationship

Note that: $g_m^* = g_{m21}$

Limits of the static CMFB





Large output differential voltages cause the differential pairs to exceed their input linearity range, which is fraction of V_{dmax}

To set large V_{dmax}, we need large $(V_{GS}-V_t)_{21}$, but we need also to satisfy:

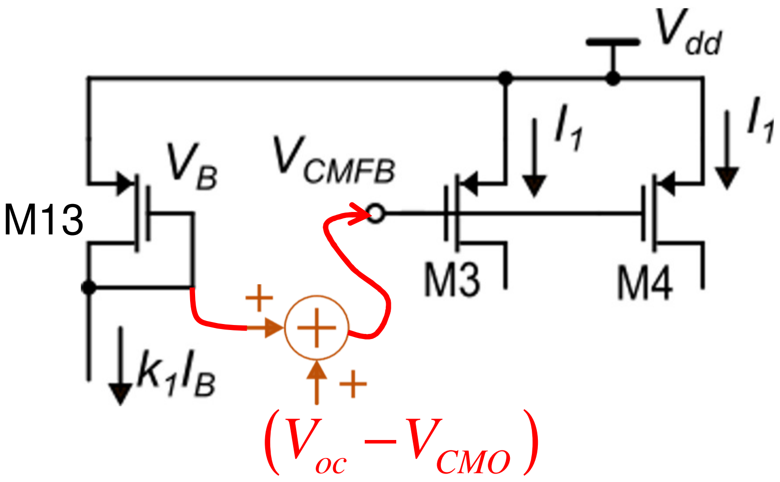
$$V_{CMO} - V_{GS21} > 2V_{DSAT}$$

Dynamic CMFB

With the static CMFB, the maximum output differential voltage is much smaller than the actual capabilities of the folded cascode op-amp, which has potentially a rail-to-rail output range. Furthermore, a static CMFB increases the power consumption of the amplifier.

Dynamic CMFBs are based on passive switched capacitor networks.

Preliminary consideration



$$\text{goal: } I_1 = k_1 I_B - g_m^* (V_{oc} - V_{CMO})$$

$$\text{if } V_{CMFB} = V_B \quad \Rightarrow \quad I_1 = k_1 I_B$$

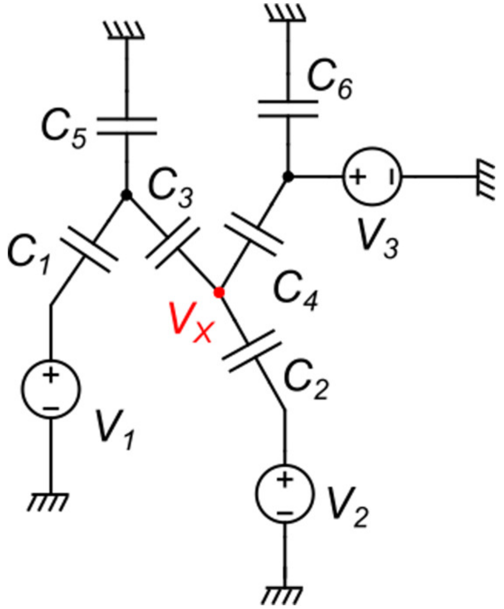
$$\text{if } V_{CMFB} = V_B + \Delta V \quad \Rightarrow \quad I_1 = k_1 I_B - g_{m3} \Delta V$$

$$\boxed{\text{we need: } V_{CMFB} = V_B + (V_{oc} - V_{CMO})}$$

$$I_1 = k_1 I_B - g_{m3} (V_{oc} - V_{CMO})$$

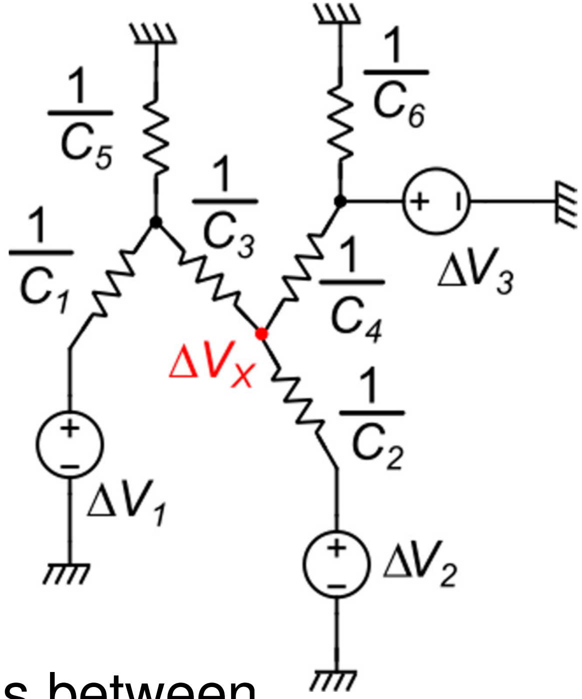
A premise: properties of all-capacitor networks

In a network made up by only capacitors and independent voltage sources, I cannot determine the value of nodal voltages (such as V_X), since it is affected by the initial voltages stored across the capacitors.



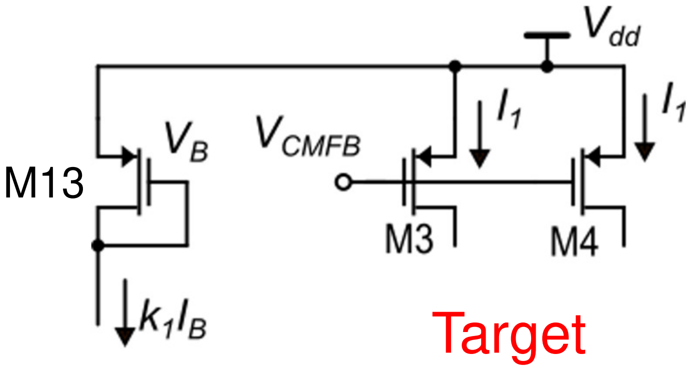
Given two instants t_f and t_i, let us define:

$$\Delta V_k = V_k\left(t_f\right) - V_k\left(t_i\right)$$



I can find the voltage variations between two instants, once the variations of the voltage sources are known. To this aim, an equivalent resistive network can be used

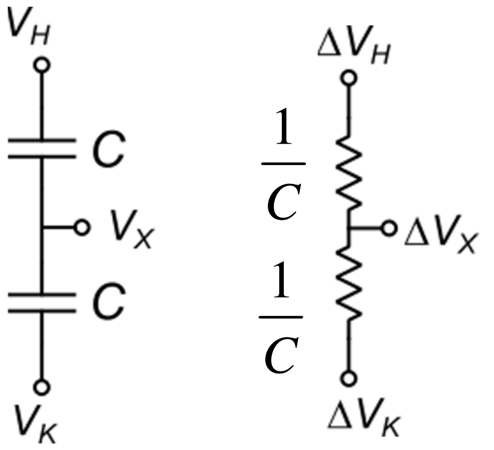
Dynamic CMFB: implementation



Target

$$V_{CMFB} = V_B + (V_{oc} - V_{CMO}) = V_B + \left(\frac{V_{o1} + V_{o2}}{2} - V_{CMO}\right)$$

Let us consider a capacitive voltage divider



$$= \frac{\Delta V_H + \Delta V_K}{2}$$

Considering two phases: (1) = precharge, (2) = calculate $V_X = V_{CMFB}$

$$V_X^{(2)} - V_X^{(1)} = \frac{\left(V_H^{(2)} - V_H^{(1)}\right)}{2} + \frac{\left(V_K^{(2)} - V_K^{(1)}\right)}{2}$$

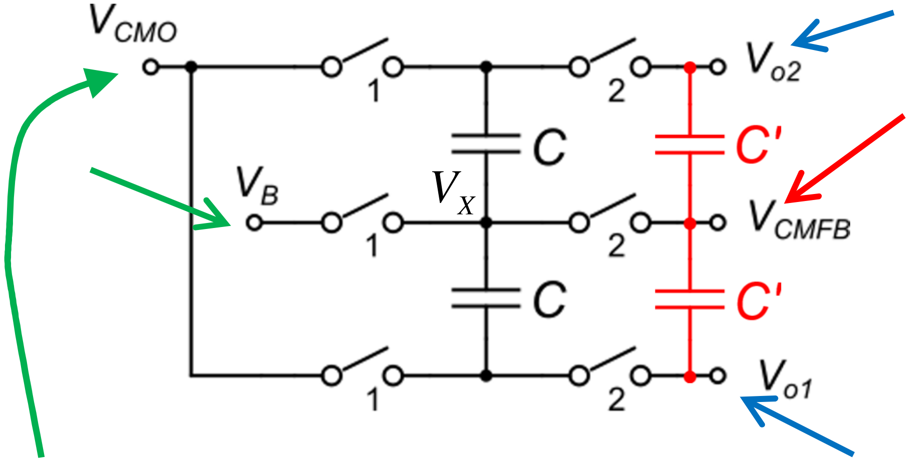
$$V_X^{(2)} = \underset{\underset{V_B}{\uparrow}}{V_X^{(1)}} + \frac{\left(\underset{\underset{V_{o2}}{\uparrow}}{V_H^{(2)}} + \underset{\underset{V_{o1}}{\uparrow}}{V_K^{(2)}}\right)}{2} - \frac{\left(\underset{\underset{V_{CMO}}{\uparrow}}{V_H^{(1)}} + \underset{\underset{V_{CMO}}{\uparrow}}{V_K^{(1)}}\right)}{2}$$

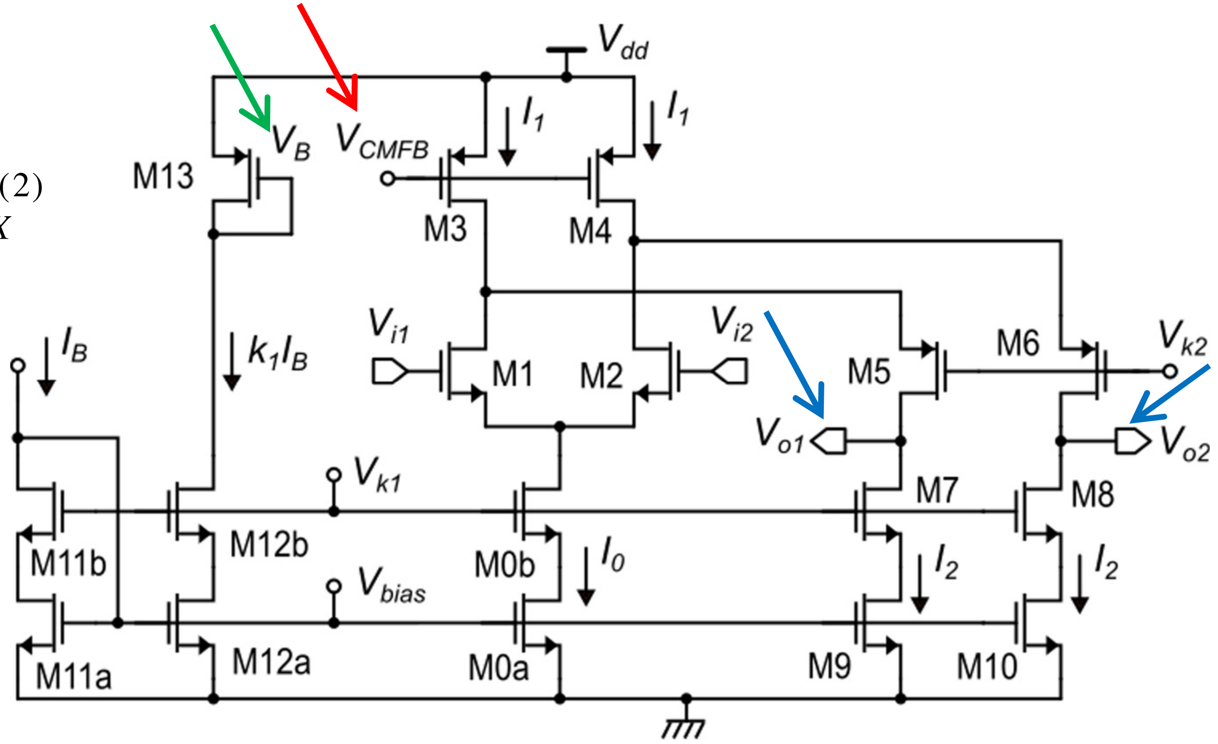
$$V_{CMFB}^{(2)} = V_B^{(1)} + V_{oc}^{(2)} - V_{CMO}^{(1)}$$

Dynamic CMFB: implementation

$$V_H^{(1)} = V_K^{(1)} = V_{CMO} \quad , V_X^{(1)} = V_B$$

$$V_H^{(2)} = V_{o2}, V_K^{(2)} = V_{o1} \quad , V_{CMFB} = V_X^{(2)}$$





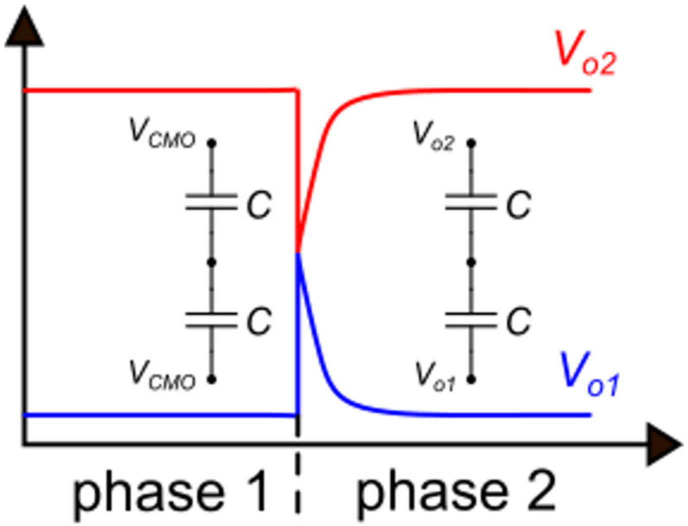
V_{CMO} is an input that has to be connected to the desired value (e.g. $V_{dd}/2$)

Capacitor *C*' are added to allow V_{CMFB} to track the variations of V_{oC} during phase 1, where it is disconnected from the outputs

Dynamic CMFB: final considerations

Advantages:

- It uses a passive networks: high linearity and no adverse effects on the available output range of the amplifier.
- Static consumption is limited to the network that generates V_B, which can be biased with a very small current.



Drawback

- At any transition from phase 1 to phase 2, the output terminals are temporarily shorted together. They have to recover by supplying current into the capacitors. The resulting spikes <u>are not acceptable in a continuous time application</u>. For an SC application, the transient must be finished when the output signal is sampled.