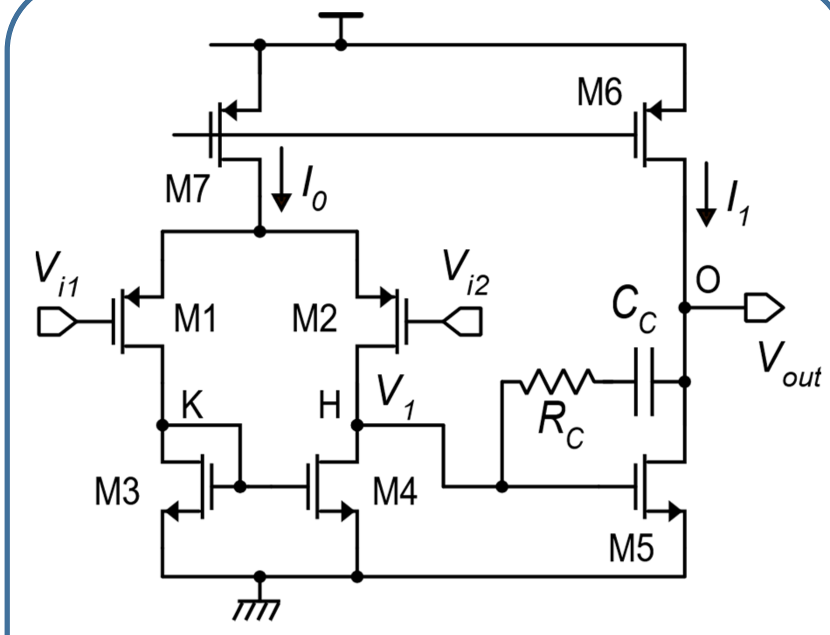
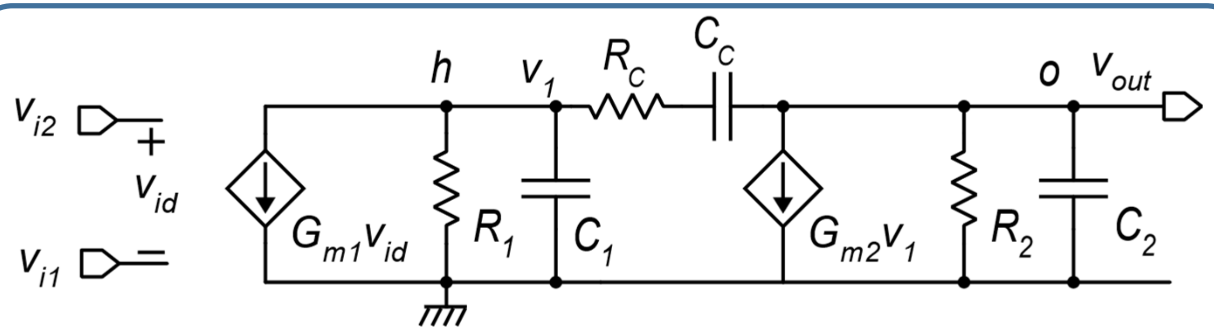
# Two-stage op-amp: design for *GBW* and Phase Margin




Topology




Small signal equivalent circuit

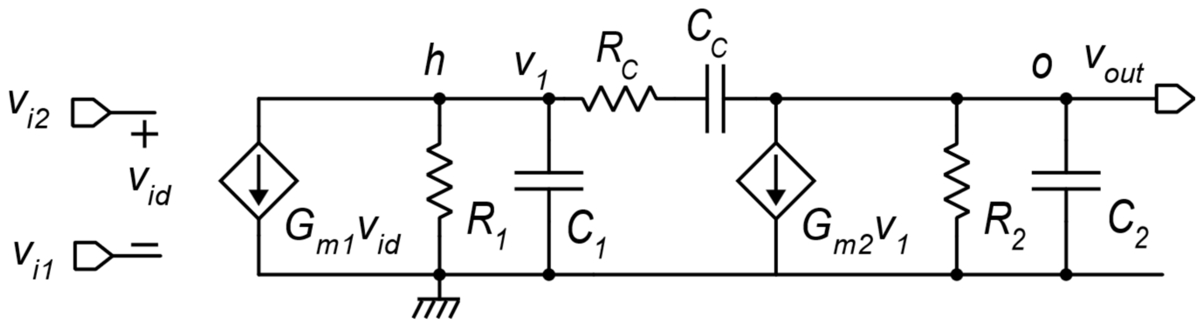
## Singularities

$$\omega_p \cong \frac{1}{R_1 G_{m2} R_2 C_C}$$

$$\omega_2 \cong \frac{G_{m2}}{(C_1 + C_2)}\left(1 + \frac{C_S}{C_C}\right)^{-1} \qquad C_S = \frac{C_1 C_2}{C_1 + C_2}$$

$$\omega_3 \cong \frac{1}{R_C\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_C}\right)^{-1}} \qquad s_z = \frac{-1}{C_C\left(R_C - \frac{1}{G_{m2}}\right)}$$

# Preliminary assumptions




$$s_z = \frac{-1}{C_C\left(R_C - \frac{1}{G_{m2}}\right)}$$

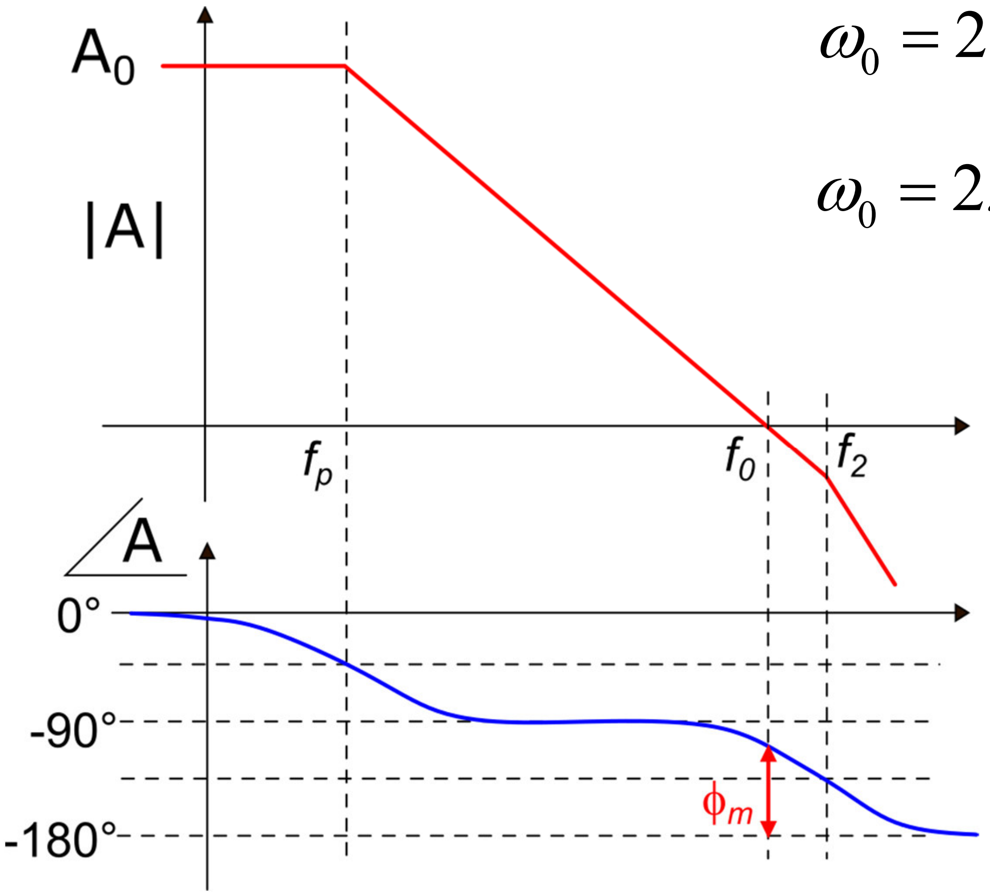
In this design procedure, we decide to cancel the zero:

$$R_C = \frac{1}{G_{m2}}$$

We consider that the *GBW* and phase margin can be estimated by using a two-pole approximation of the frequency response, with a dominant pole ($\omega_p$):

$$\omega_p << \omega_2 \qquad \omega_2 << \omega_3,\ \omega_{tail},\ \omega_{mirror}$$

# GBW and unity-gain angular frequency ($\omega_0$)




$$\omega_0 = 2\pi f_0 \qquad f_0 \cong GBW = A_0 f_p$$

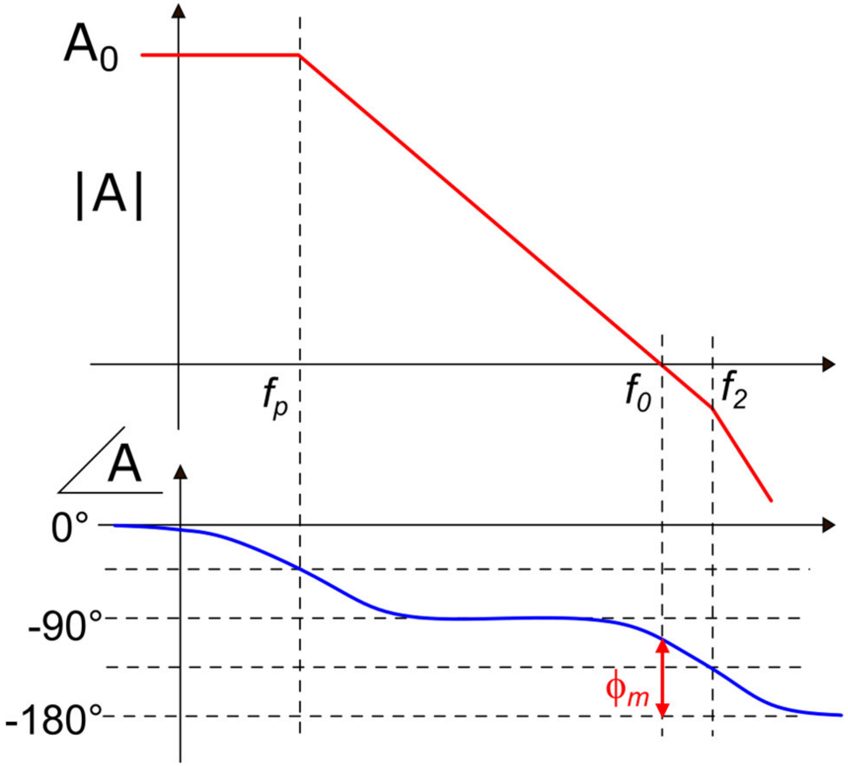
$$\omega_0 = 2\pi A_0 f_p \rightarrow \omega_0 = A_0 \omega_p$$

$$\omega_p \cong \frac{1}{R_1 G_{m2} R_2 C_C}$$

$$A_0 \cong G_{m1} R_1 G_{m2} R_2$$

$$\omega_0 \cong \frac{G_{m1} R_1 G_{m2} R_2}{R_1 G_{m2} R_2 C_C} = \frac{G_{m1}}{C_C}$$

# GBW and stability specifications



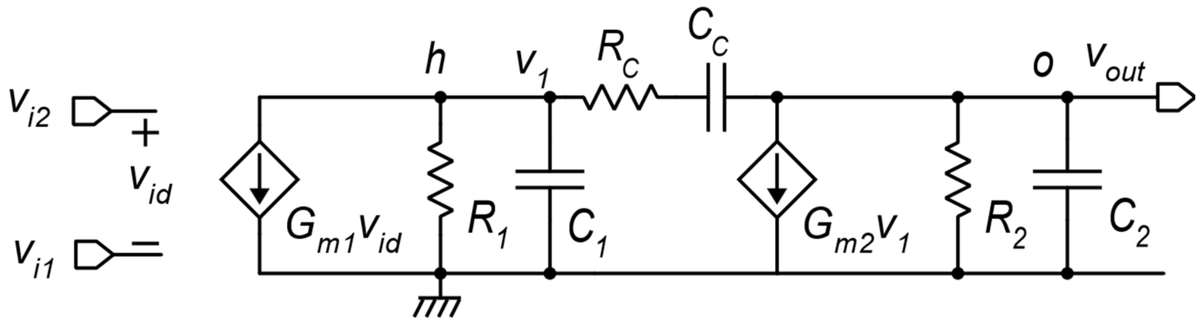

$$\omega_0 \cong \frac{G_{m1}}{C_C} \qquad GBW = \frac{\omega_0}{2\pi}$$

$$\varphi_m \cong 70° \Rightarrow \sigma = \frac{f_2}{f_0} = 3$$

Then, we impose: $\omega_2 = \sigma\omega_0$

$$\omega_2 \cong \frac{G_{m2}}{(C_1 + C_2)}\left(1 + \frac{C_S}{C_C}\right)^{-1}$$

# An approximate approach: Hypothesis 1




Hypothesis 1: $C_1$ is much smaller than $C_2$ and $C_C$:

$$C_2 >> C_1$$

$$C_C >> C_1$$

$$C_S < C_1 << C_C$$

Motivation:
$C_2$ includes the load capacitance, $C_L$
$C_C$ can be made arbitrarily large to satisfy the hypothesis

$$\omega_2 = \frac{G_{m2}}{(C_1 + C_2)}\left(1 + \frac{C_S}{C_C}\right)^{-1} \cong \frac{G_{m2}}{(C_1 + C_2)}$$

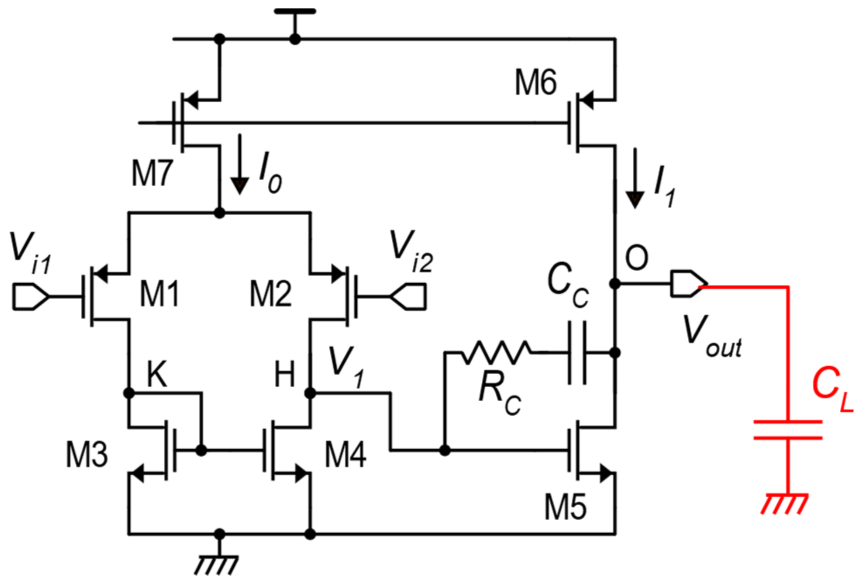
$$\omega_2 \cong \frac{G_{m2}}{C_2}$$

# An approximate approach: Hypothesis 2

Hypothesis 2: The parasitic component $C_2'$ is much smaller than $C_L$:

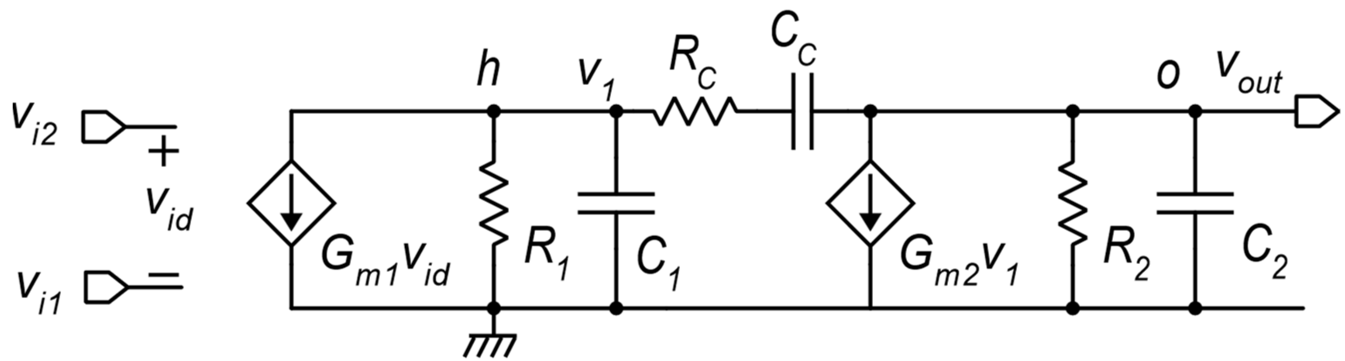
$$C_2 = C'_2 + C_L \cong C_L \qquad \omega_2 \cong \frac{G_{m2}}{C_2} \cong \frac{G_{m2}}{C_L}$$

Now the expression of $\omega_2$ is strongly simplified




Hypotheses 1 and 2 correspond to consider that all parasitic capacitances of the amplifier are negligible with respect to $C_L$ and $C_C$, which are external to the amplifier ($C_L$) or are purposely placed devices ($C_C$).

# The *GBW* specification shapes the second stage ($G_{m2}$)



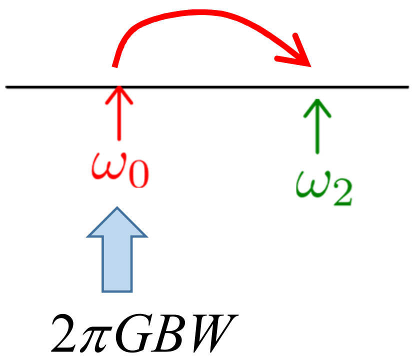

$$\omega_2 \cong \frac{G_{m2}}{C_L}$$

For the stability requirement: $\omega_2 = \sigma\omega_0$

$$\omega_2 = \sigma \cdot 2\pi f_0 \cong \sigma \cdot 2\pi GBW$$

stability specification

$\times\sigma$

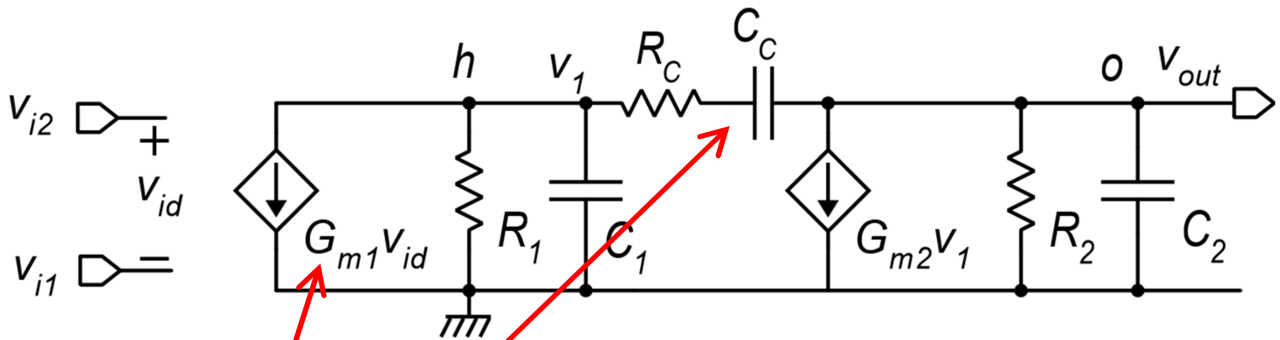



GBW specification

$$\frac{G_{m2}}{C_L} \cong \sigma \cdot 2\pi GBW$$

$$G_{m2} \cong 2\pi\sigma \cdot GBW \cdot C_L$$

# Back to the first stage




$$G_{m2} \cong 2\pi\sigma \cdot GBW \cdot C_L$$

$$R_C = \frac{1}{G_{m2}}$$

$$\omega_0 \cong \frac{G_{m1}}{C_C}$$

We have two degrees of freedom, $G_{m1}$ and $C_C$: a smaller $G_{m1}$ would allow for a smaller $C_C$ value, saving area. However, $C_C$ cannot get too small, otherwise hypothesis 1 risks to fail.

To guide the choice, it is convenient to relate $C_C$ to $C_L$ and to the $G_{m1}/G_{m2}$ ratio, through the stability condition

$$\omega_2 = \sigma\omega_0$$

stability condition

$$\frac{G_{m2}}{C_L} = \sigma \frac{G_{m1}}{C_C}$$

$$\frac{G_{m1}}{G_{m2}} = \frac{1}{\sigma}\frac{C_C}{C_L}$$

# $C_C$ and $C_L$, the "rule of thumb" and its limits

$$\frac{G_{m1}}{G_{m2}} = \frac{1}{\sigma}\frac{C_C}{C_L}$$

(a) $C_2 \cong C_L >> C_1$

(b) $C_C >> C_1$

} Hypothesis 1

If we make $C_C$=$C_L$, validity of condition (a) makes also condition (b) automatically true. Making $C_C$ even larger than $C_L$ does not add validity to hypothesis 1 and requires more area.

Rule of thumb: $C_C = C_L$ $\quad G_{m1} = \frac{1}{\sigma} G_{m2}$ $\quad$ With σ=3 ($\varphi_m \cong 70°$)

We have determined both $G_{m1}$ and $G_{m2}$

$$G_{m1} = \frac{1}{3} G_{m2}$$

## Limits of the rule of thumb

Rule of thumb: $C_C = C_L$

Often, it is convenient to apply a different choice

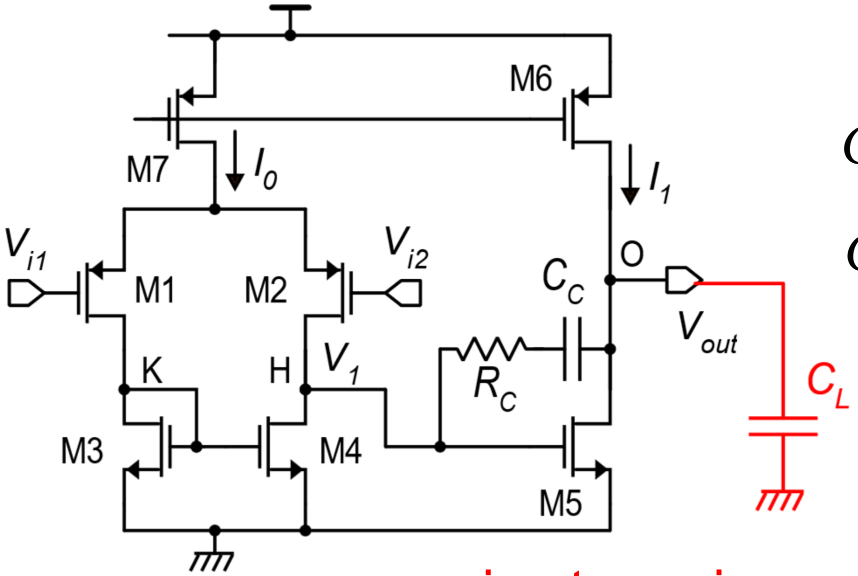
**Case of large $C_L$**: If the maximum load capacitance is particularly large (> tens pF), using the rule of thumb can result in too large a compensation capacitance, and then, in non-acceptable chip area occupation. In those cases, the $C_C/C_L$ ratio can be made smaller than one ($C_C<C_L$) in order to make $C_C$ easily integrable.

For example: with $C_L$=100 pF,
I can choose: $C_C$=10 pF

$$\frac{G_{m1}}{G_{m2}} = \frac{1}{\sigma}\frac{C_C}{C_L} = \frac{1}{10\sigma} \cong \frac{1}{30}$$

This does not risk to break the hypothesis $C_C>>C_1$, because in the example, $C_L$ is so large that even choosing $C_C$ ten times smaller results in a capacitance value that is >> of typical parasitic capacitances. However, it is always necessary to check if the hypothesis still holds.

# Application of the design procedure to our simple two-stage op-amp




$$G_{m2} = g_{m5} \cong 2\pi\sigma \cdot GBW \cdot C_L$$

$$G_{m1} = g_{m1} = G_{m2}\frac{1}{\sigma}\frac{C_C}{C_L} = g_{m5}\frac{1}{\sigma}\frac{C_C}{C_L}$$

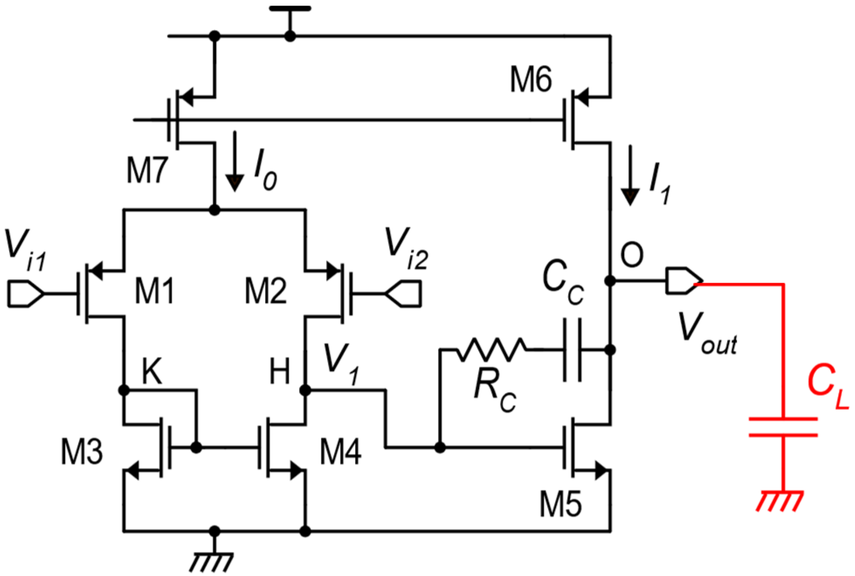
in strong inversion

$$g_{m1} = \mu_p C_{OX}\frac{W_1}{L_1}\left|V_{GS} - V_t\right|_1 \Rightarrow \frac{W_1}{L_1} = \frac{g_{m1}}{\mu_p C_{OX}\left|V_{GS} - V_t\right|_1}$$

$$g_{m5} = \mu_n C_{OX}\frac{W_5}{L_5}\left(V_{GS} - V_t\right)_5 \Rightarrow \frac{W_5}{L_5} = \frac{g_{m5}}{\mu_n C_{OX}\left(V_{GS} - V_t\right)_5}$$

Once the M1 and M5 overdrive voltages have been chosen, the *GBW* specification determines the aspect ratios (*W*/*L*) of both MOSFETs

# *GBW* and supply current




$$I_{supply} = 2I_{D1} + I_{D5}$$

$$g_m = \frac{I_D}{V_{TE}} \Rightarrow I_D = g_m V_{TE}$$

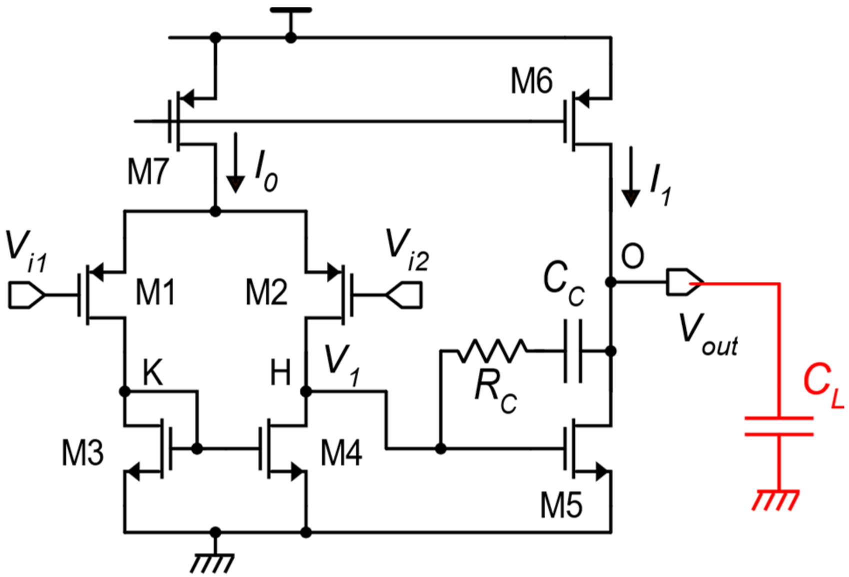
$$I_{supply} = 2g_{m1}V_{TE1} + g_{m5}V_{TE5}$$

$$I_{supply} = g_{m5}\left(V_{TE5} + 2\frac{g_{m1}}{g_{m5}}V_{TE1}\right)$$

$$g_{m5} \cong 2\pi\sigma \cdot GBW \cdot C_L$$

$$I_{supply} = 2\pi\sigma \cdot GBW \cdot C_L \cdot \left(V_{TE5} + 2\frac{g_{m1}}{g_{m5}}V_{TE1}\right)$$

# *GBW*, $C_L$ and supply current

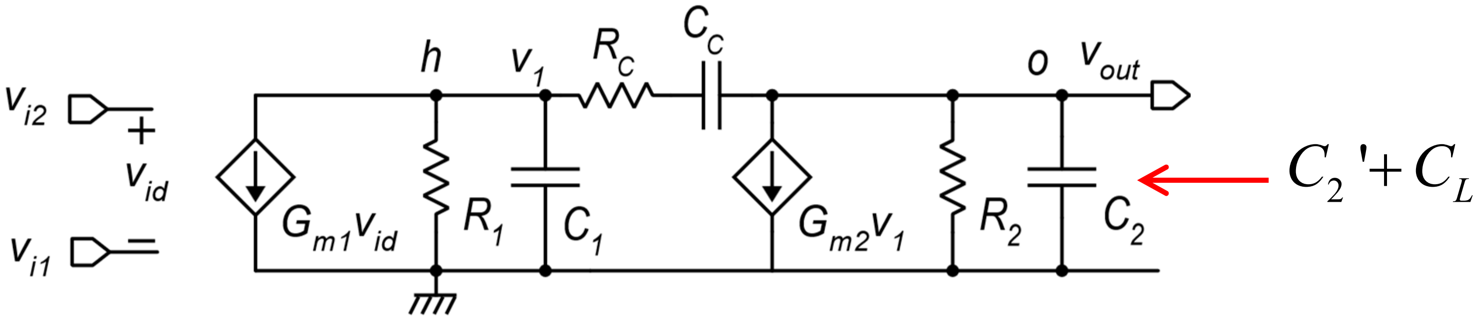



$$I_{supply} = 2\pi\sigma \cdot GBW \cdot C_L \cdot \left( V_{TE5} + 2\frac{g_{m1}}{g_{m5}} V_{TE1} \right)$$

$$\frac{g_{m1}}{g_{m5}} = \frac{G_{m1}}{G_{m2}} = \frac{1}{\sigma}\frac{C_C}{C_L}$$

- The higher the *GBW and load capacitance* $C_L$, the higher the supply current and then the power consumption
- For the same *GBW* and $C_L$ specifications, lower supply currents can be obtained with the lowest $V_{TE5}$ and $V_{TE1}$.

# Robustness against $C_L$ variations




We have designed the op-amp for the maximum $C_L$. It must be stable also for smaller values

If we **reduce** $C_L$, then $C_2$ reduces and hypothesis 1 may be no more valid. Then we have to use the complete expression for $\omega_2$:

$$\omega_2 = \frac{G_{m2}}{(C_1 + C_2)} \cdot \frac{1}{1 + \frac{C_S}{C_C}}$$

$\omega_2$ increases!

Both denominators reduces

Effects of $C_2$ reduction:

$$C_S = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$$ gets smaller

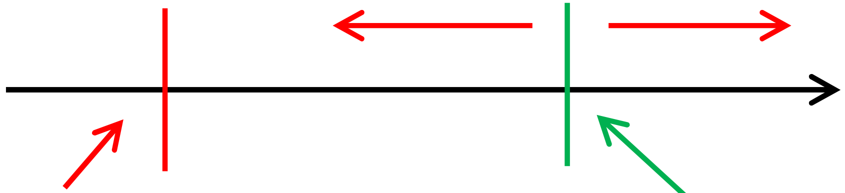
$$\frac{C_S}{C_C}$$ gets smaller

# Robustness against $C_L$ variations

$$\varphi_m \cong \arctan\left(\frac{\omega_2}{\omega_0}\right)$$

$C_L$ increases: smaller phase margin

$C_L$ decreases: greater phase margin



$$\omega_0 \cong \frac{G_{m1}}{C_C}$$ Is nearly unaffected by $C_L$

$$\omega_2 = \frac{G_{m2}}{(C_1 + C_2)} \cdot \frac{1}{1 + \frac{C_S}{C_C}}$$

In a two-stage amplifier:

- Reducing (or even removing) the load capacitance improves stability
- Increasing the load capacitance reduces stability and eventually causes instability

## Limits of the simplified design procedure

Given a *GBW* specification, the procedure can be summarized in the following way

1. Find the required $G_{m2}$ value from the equation: $$\omega_2 \cong \sigma \cdot 2\pi GBW = \frac{G_{m2}}{C_L}$$

2. Choose a proper $C_C/C_L$ ratio, depending on the value of $C_L$

3. Find the required $G_{m1}$: $$G_{m1} = \frac{1}{\sigma}\frac{C_C}{C_L}G_{m2}$$

It seems that using the required current to set $G_{m2}$ and $G_{m1}$ to the correct value, we can reach an arbitrarily high GBW, independently of the process being used. This is clearly not reasonable.

# Limits of the simplified design procedure

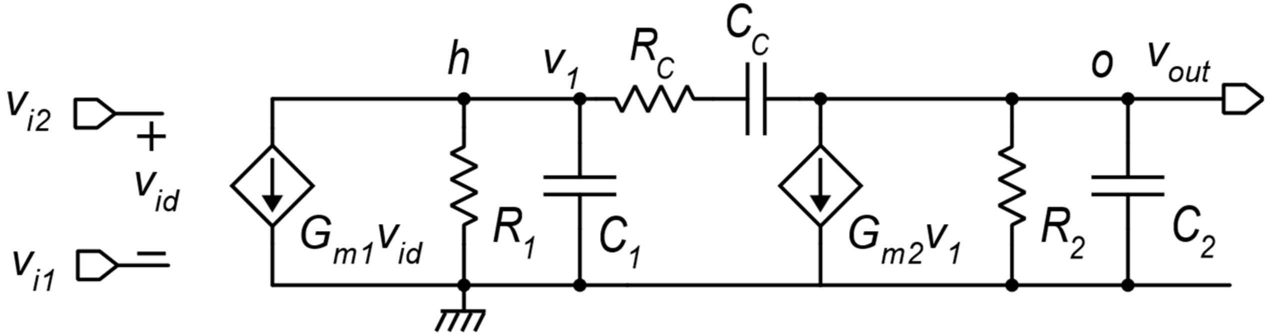
The problem stands in the hypotheses the procedure is based on.

The hypotheses derive from the fact that $C_L$ is generally prevalent over the parasitic capacitances:

Hyp. 1: $C_2 >> C_1$ ⇐ $C_L >> C_1, C_2'$ ⇒ $C_2 = C'_2 + C_L \cong C_L$ Hyp. 2

$C_C >> C_1$ ⇐ can be satisfied choosing a large enough $C_C$.

Trying to get larger and larger $G_{m2}$ and $G_{m1}$ increases also the size of the MOSFETs in the first and second stage, causing violation of the hypotheses



# Limits of the simplified design procedure

$$\omega_2 = \sigma \cdot 2\pi f_0 \cong \sigma \cdot 2\pi GBW$$

$$GBW = \frac{1}{2\pi\sigma}\omega_2$$

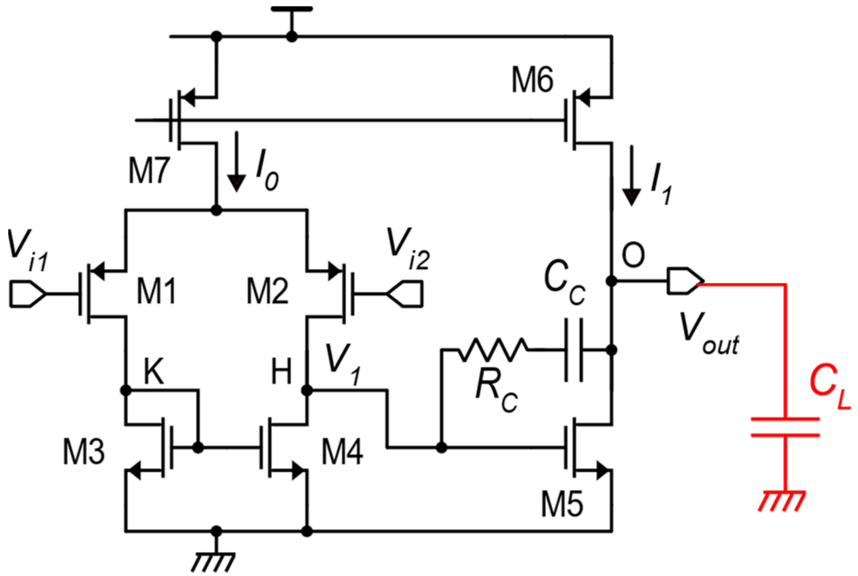
$$\omega_2 = \frac{G_{m2}}{\left(C_1 + C_2\right)} \cdot \frac{1}{1 + \frac{C_S}{C_C}}$$

I can still set $C_C$ as large as to make $C_S/C_C$ <<1

$$\omega_2 \cong \frac{G_{m2}}{\left(C_1 + C_2\right)}$$

$$GBW = \frac{1}{2\pi\sigma}\frac{G_{m2}}{\left(C_1 + C_2\right)} = \frac{1}{2\pi\sigma}\frac{G_{m2}}{\left(C_1 + C_2' + C_L\right)}$$

# Limits of the simplified design procedure




$$GBW = \frac{1}{2\pi\sigma}\frac{G_{m2}}{\left(C_1 + C_2' + C_L\right)}$$

$$C_1 = C_{GS5} + C_{DB2} + C_{DB4} \cong C_{GS5}$$

$$C_2' = C_{DB5} + C_{DB6} \ll C_{GS5}$$

$$GBW \cong \frac{1}{2\pi\sigma}\frac{g_{m5}}{\left(C_{GS5} + C_L\right)}$$

$$GBW \cong \frac{1}{2\pi\sigma}\frac{g_{m5}}{C_{GS5}}\frac{1}{\left(1 + \frac{C_L}{C_{GS5}}\right)}$$

$$\frac{1}{2\pi}\frac{g_{m5}}{C_{GS5}} \cong f_{T5}$$

$$GBW \cong \frac{f_{T5}}{\sigma}\frac{1}{\left(1 + \frac{C_L}{C_{GS5}}\right)}$$

# Maximum achievable *GBW*

$$C_{gs5} = \frac{g_{m5}}{2\pi f_{T5}}$$

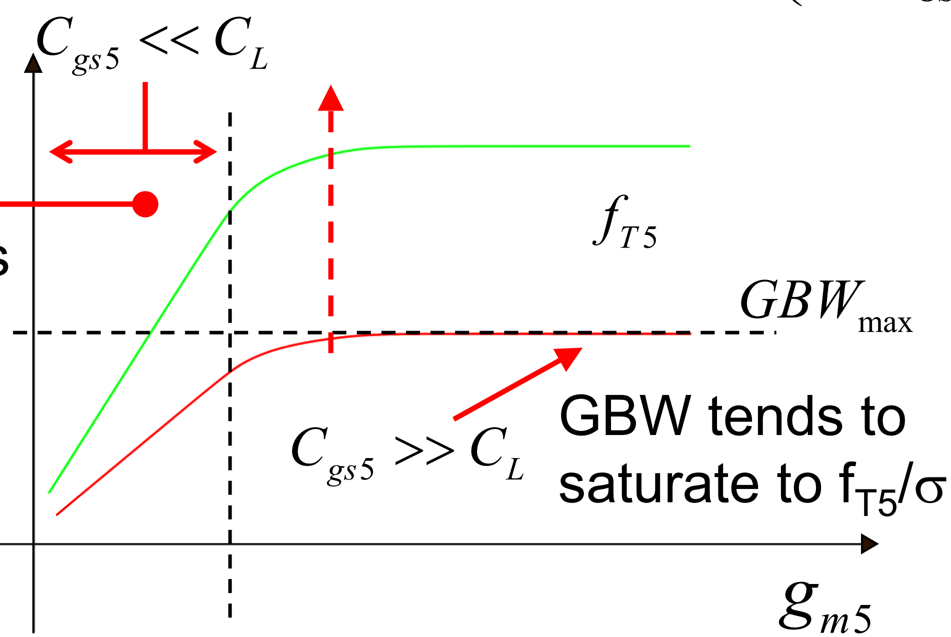
Increasing $g_{m5}$ for a given $f_{T5}$, increases $C_{gs5}$ proportionally

$$GBW \cong \frac{f_{T5}}{\sigma} \frac{1}{\left(1 + \frac{C_L}{C_{GS5}}\right)} \begin{cases} \frac{1}{2\pi\sigma} \frac{g_{m5}}{C_L} & C_{gs5} << C_L \\ \frac{f_{T5}}{\sigma} & C_{gs5} >> C_L \end{cases}$$

GBW

In this region the hypotheses are valid

$$GBW = \frac{1}{2\pi\sigma} \frac{g_{m5}}{C_L}$$



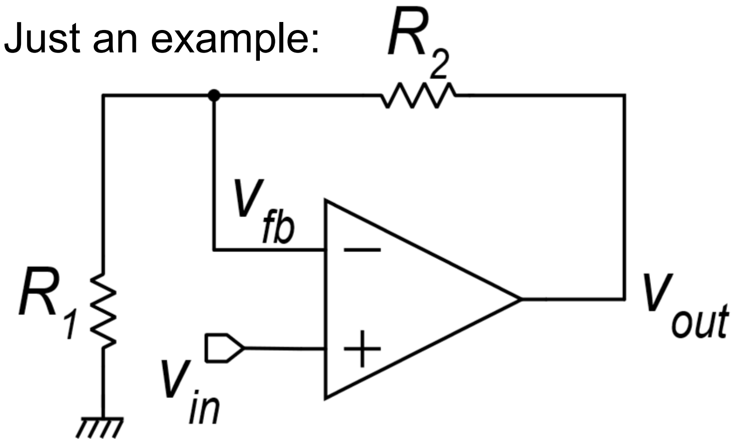

In strong inversion

$L_5$ $(V_{GS}-V_t)_5$

$$f_{T5} \cong \frac{3}{4\pi} \mu_n \frac{1}{L_5^2} \left(V_{GS} - V_t\right)_5$$

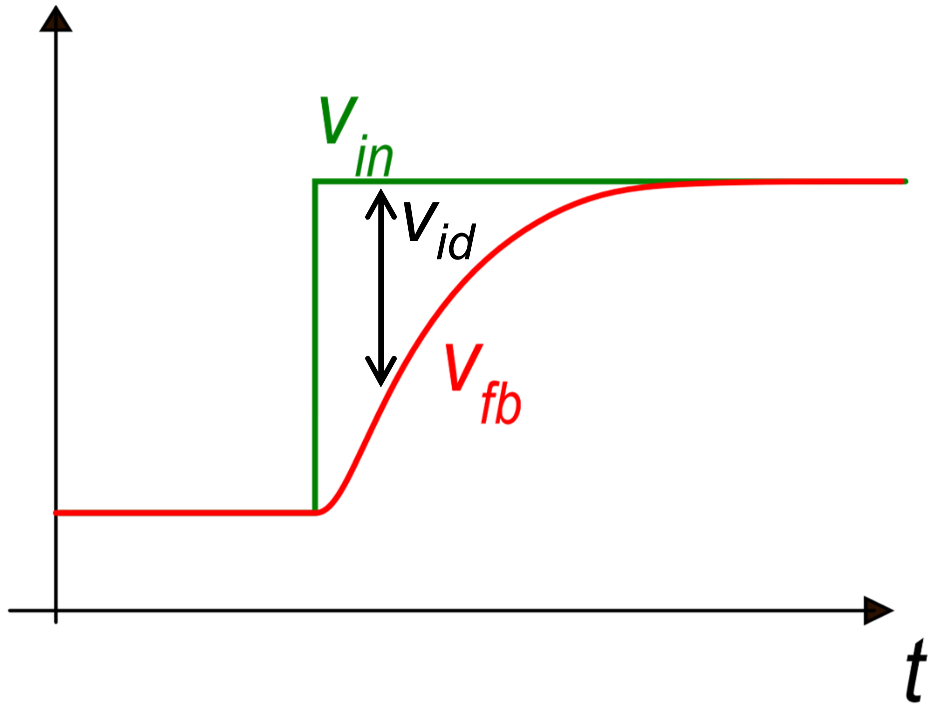
# The slew rate problem

Just an example:

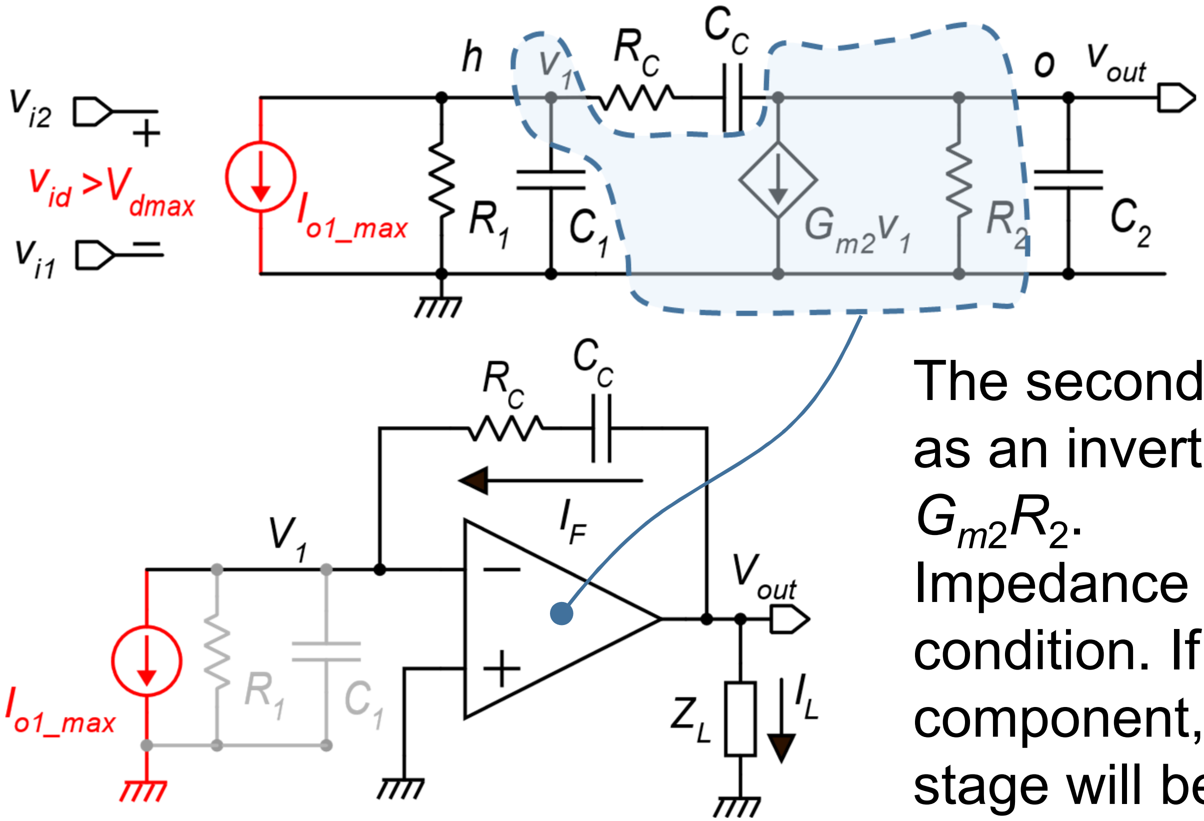



Just after a step variation of the input voltage, the feedback voltage, derived by the output voltage, cannot change instantaneously and the input differential voltage of the op-amp can be driven out of the linearity region of the first stage

If the input step is large enough, the input stage saturates



# Slew rate in Miller-compensated two-stage op-amps

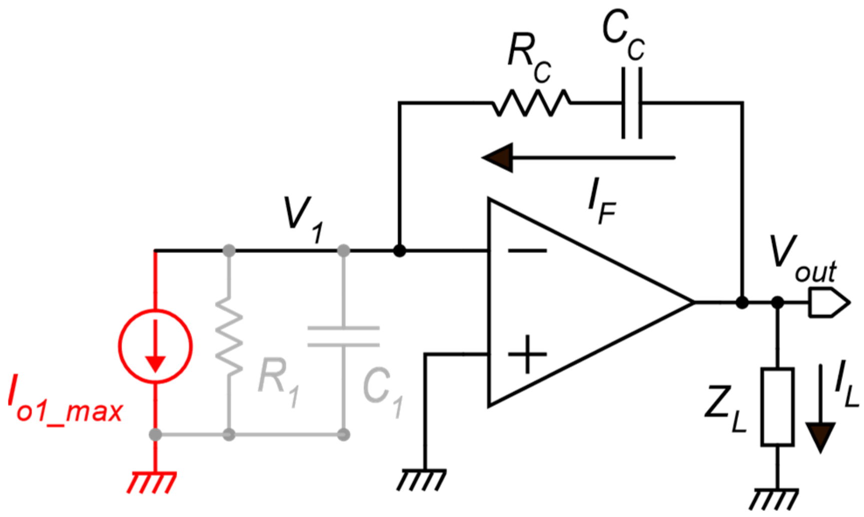



The second stage can be represented as an inverting amplifier with gain $G_{m2}R_2$.

Impedance $Z_L$ represents the load condition. If $Z_L$ has a resistive component, the gain of the second stage will be smaller than $G_{m2}R_2$.

$Z_L$ includes the load capacitance

# Slew rate in Miller-compensated two-stage op-amps




$$I_F \cong I_{o1-\max}$$

$$v_{out} = v_1 + I_{o1-\max} R_C + \frac{1}{C_C}\int I_{o1-\max}\,dt$$

since $V_1 \cong constant$ $\quad \dfrac{dv_{out}}{dt} \cong \dfrac{I_{o1-\max}}{C_C}$

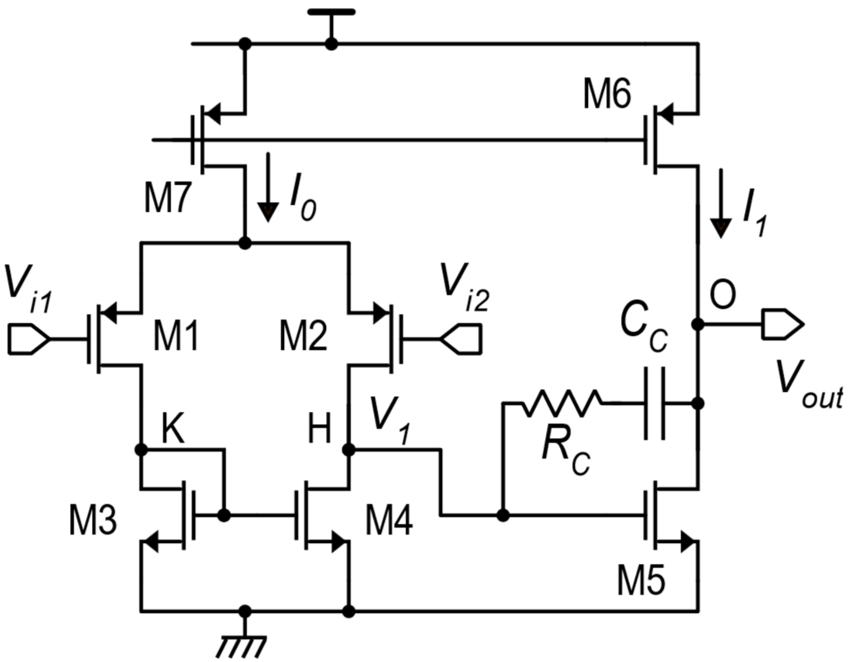
($I_{o1\text{-}max}$ can be either negative or positive)

$$s_R = \max\left|\frac{dv_{out}}{dt}\right| = \frac{|I_{01-\max}|}{C_C}$$

If the gain of the second stage is high enough, we can consider virtual gnd at input $v_1$.

Note that current $I_F$ flows also through the output terminal of the amplifier. Then, the analysis shown above is applicable if the second stage (output stage) is capable of producing the total current $I_F$+$I_L$

# Slew rate of the simple class-A, two stage op-amp.




$$s_R = \frac{|I_{01-\max}|}{C_C}$$

From simple inspection of the first stage:

$$|I_{01-\max}| = I_0$$

$$s_R = \frac{I_0}{C_C} = \frac{2I_{D1}}{C_C} \qquad I_{D1} = V_{TE1} g_{m1}$$

$$s_R = 2V_{TE1}\frac{g_{m1}}{C_C} = 2V_{TE1}\omega_0$$

$$s_R = GBW \cdot 4\pi V_{TE1}$$

For a given *GBW*, the higher $V_{TE1}$ , the higher the slew rate