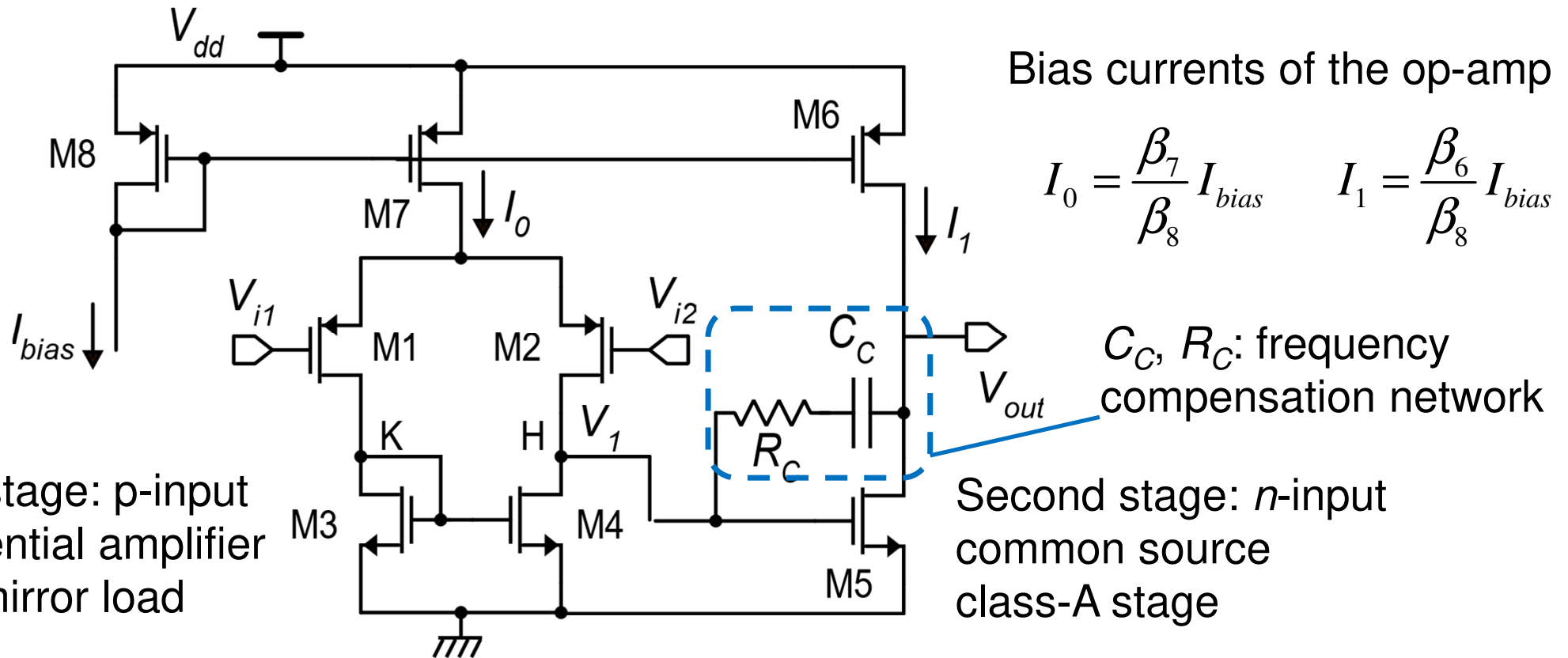


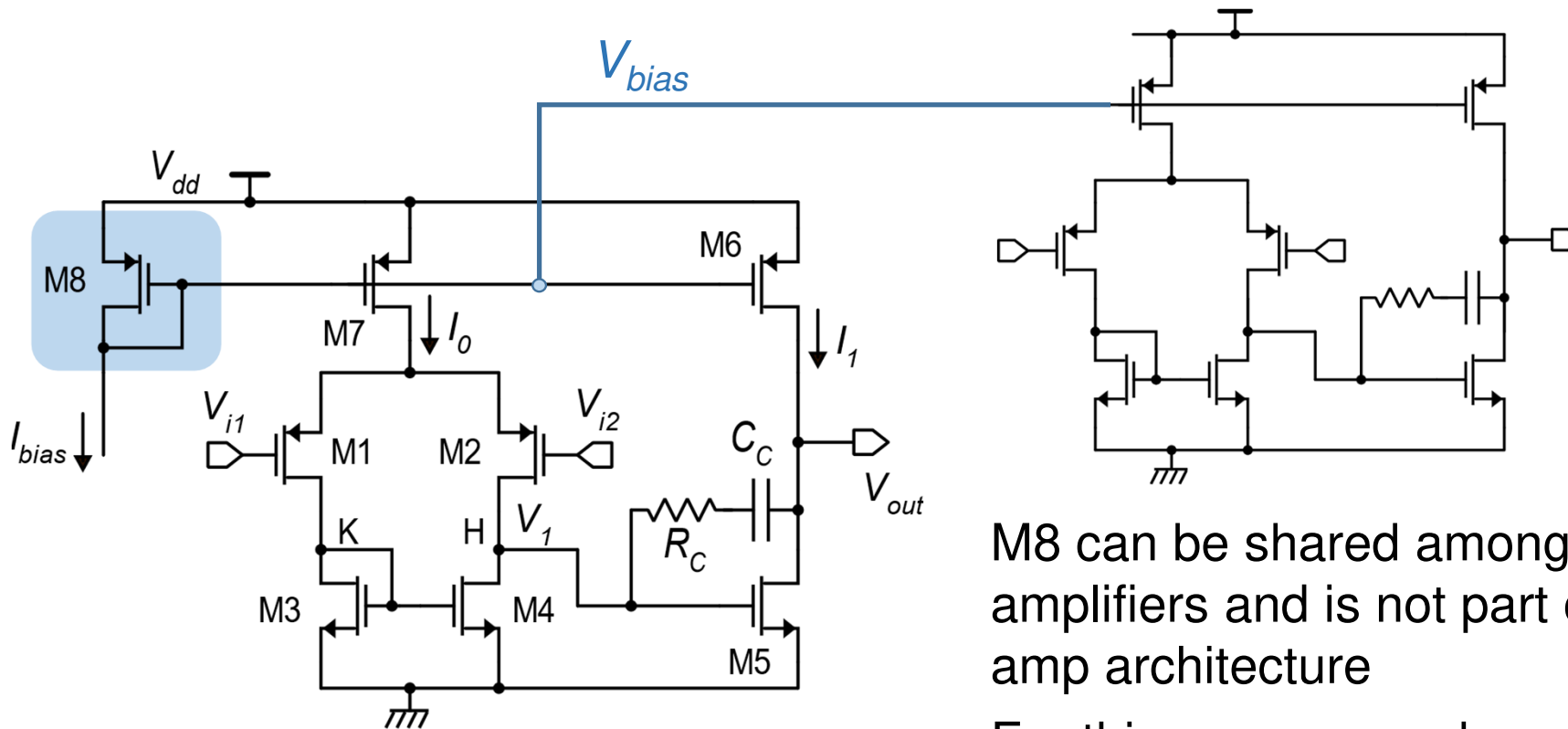
# The simplest CMOS two-stage op-amp



The output of the first stage is "referred to *gnd*"

The input of the second stage is "referred to *gnd*"

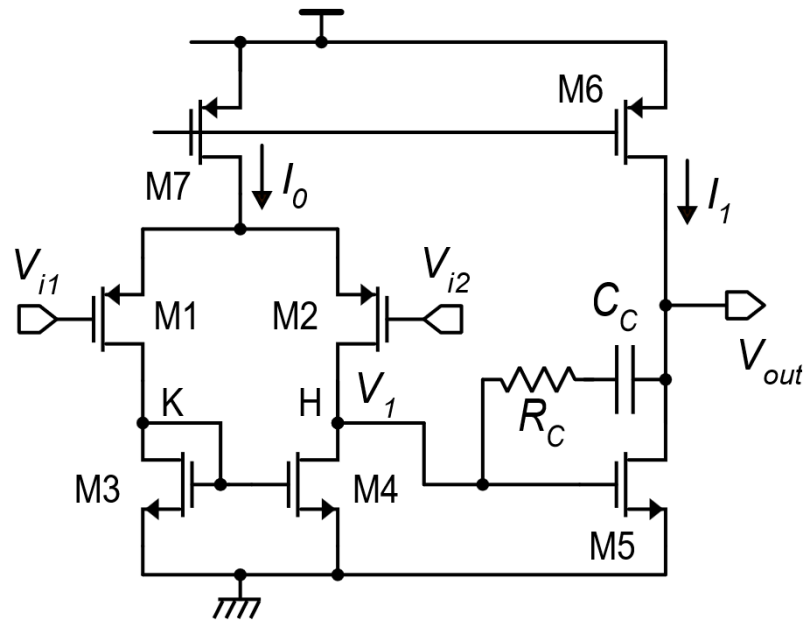
## Use of a single M8 device to bias multiple op-amp



M8 can be shared among different amplifiers and is not part of the op-amp architecture

For this reason, we do not consider M8 in the amplifier topology

## Degrees Of Freedom (DOFs)



Possible DOFs:

$W, L$  of all devices (14 DOFs)

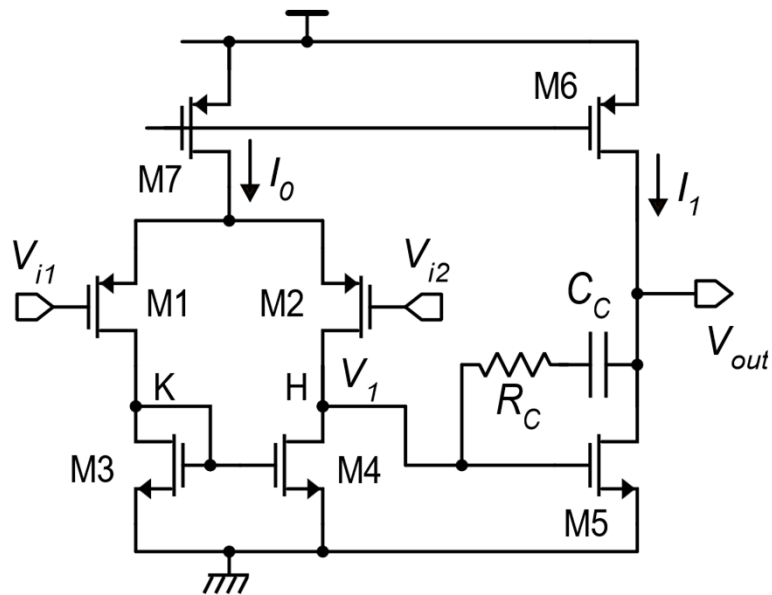
$I_0, I_1$   
 $C_C, R_C$

First estimate: Total number of DOFs: 18

But ...

- Not all DOFs are independent.
- It is necessary to choose a set of independent DOFs

## Constraints



Constraints are relationships among DOFs

Two types of constraints:

- Equality constraints:

For example: 
$$\frac{I_1}{I_0} = \frac{\beta_6}{\beta_7} = \frac{W_6 / L_6}{W_7 / L_7}$$

- Inequality constraints:

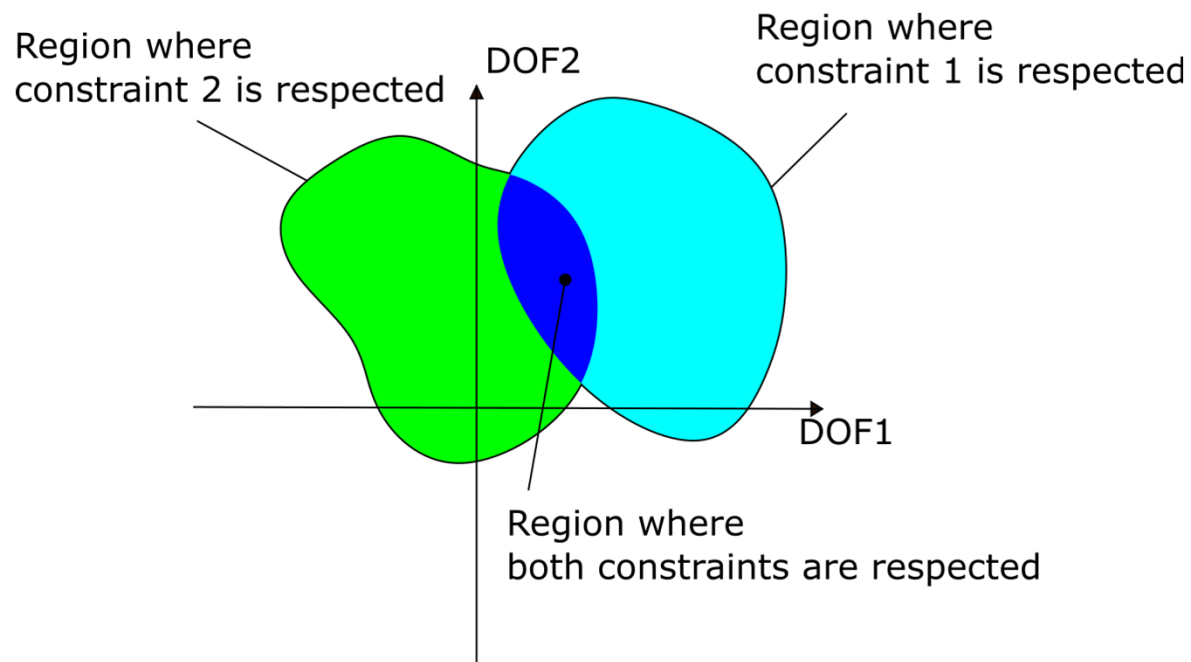
For example: 
$$GBW(DOFs) \geq GBW_{\min}$$

# Constraints

- Every **equality constraint** reduces the dimension of the DOF space. Equality constraints represent exact conditions that has to be fulfilled in order to guarantee correct operation of the circuit. Some equality constraints derive from simple considerations, such as symmetry:  $M1=M2$ ,  $M3=M4$ . With a few exceptions, equality constraints are specific of the topology and does not depend on the specifications
- **Inequality constraints** are derived from the circuit specifications. They do not reduce the dimension of the DOF space but select regions of the DOF space where the specs are met.

# The sizing process: role of multiple inequality constraints

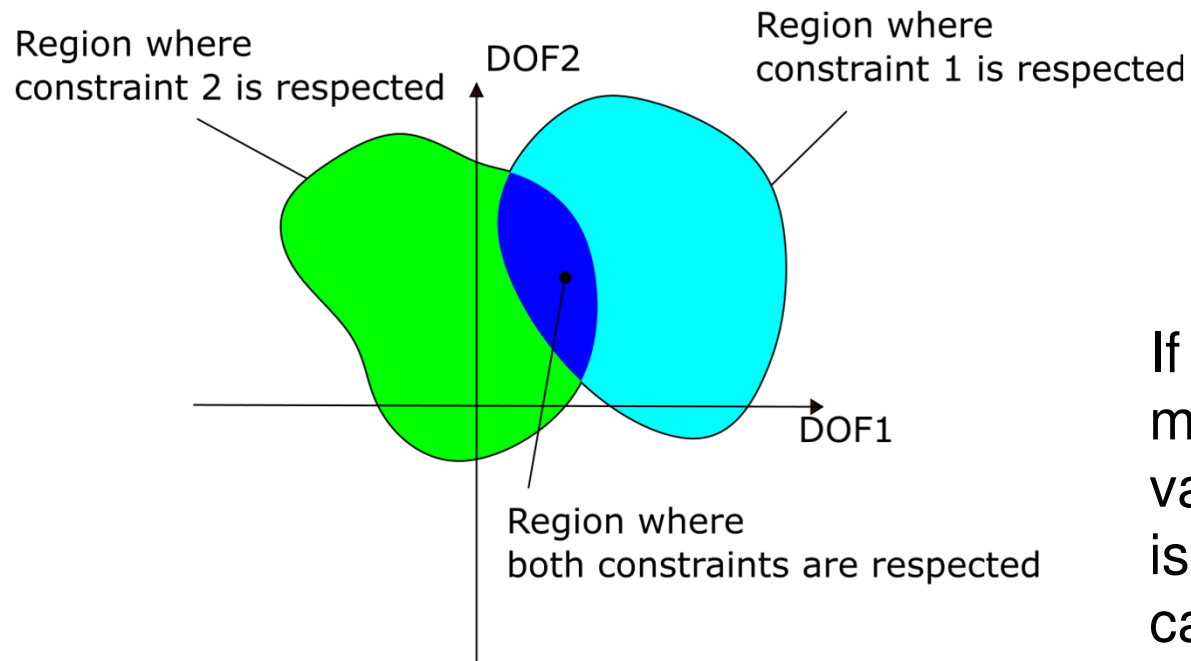
A very simple case with only two independent DOFs and two inequality constraints



Combining the various inequality constraints, we find a domain (the intersection of all regions) where all points satisfy all constraints. All points in the domain are valid solutions.

If such region does not exist (null intersection), the sizing problem is: "unfeasible".

## The sizing process: automatic algorithms



Computer programs that perform automatic sizing, are not compatible with an infinite number of feasible solutions. To find a single solution, an optimization condition is often added.

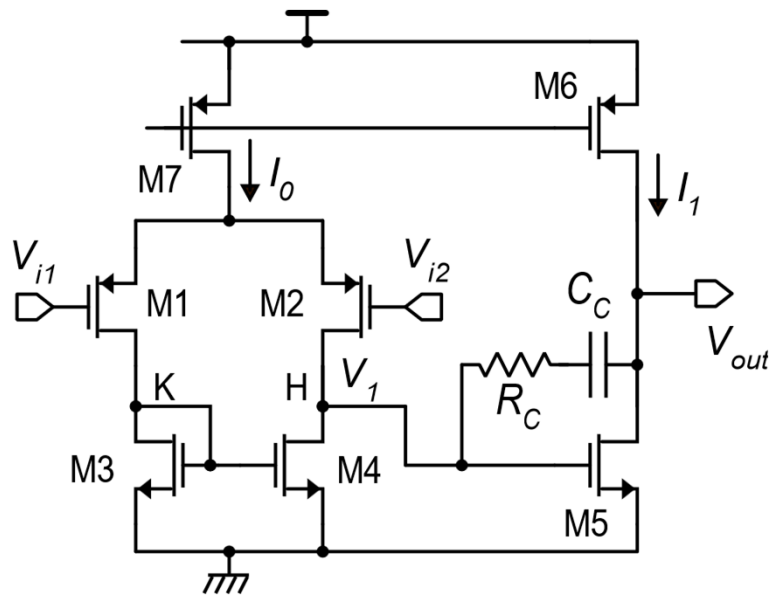
If the design is performed manually, any point (set of DOFs values) in the intersection domain is a good solution. Also in this case, optimization or arbitrary techniques can be used to operate the choice

## Sizing of a new topology: steps

1. Find **equality constraints** to reduce the number of independent DOFs. These constraints will be of two types:
  - (a) Strictly necessary constraints (if not respected the circuit does not work properly)
  - (b) Arbitrary constraints: they are added to further reduce the DOF set and simplify the design. These constraints should be motivated.
2. Choose a set of DOFs that have the following properties:
  - (a) the remaining dependent DOFs can be easily derived from this set;
  - (b) the specifications (**inequality constraints**) can be written easily and in a simple form as a function of the selected DOFs
3. Write the specifications in terms of the selected DOFs and try to find general design rules.



## Equality constraints for the simple 2-stage op-amp



**Symmetry** (necessary to obtain low offset and low CMRR)

N. of equality constraints

$$M1=M2 \quad (W_1=W_2, \quad L_1=L_2) \quad \text{-----} \quad 2$$

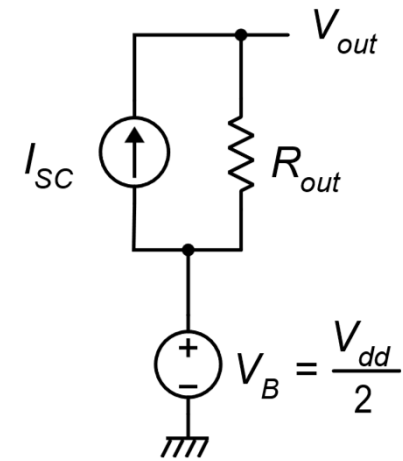
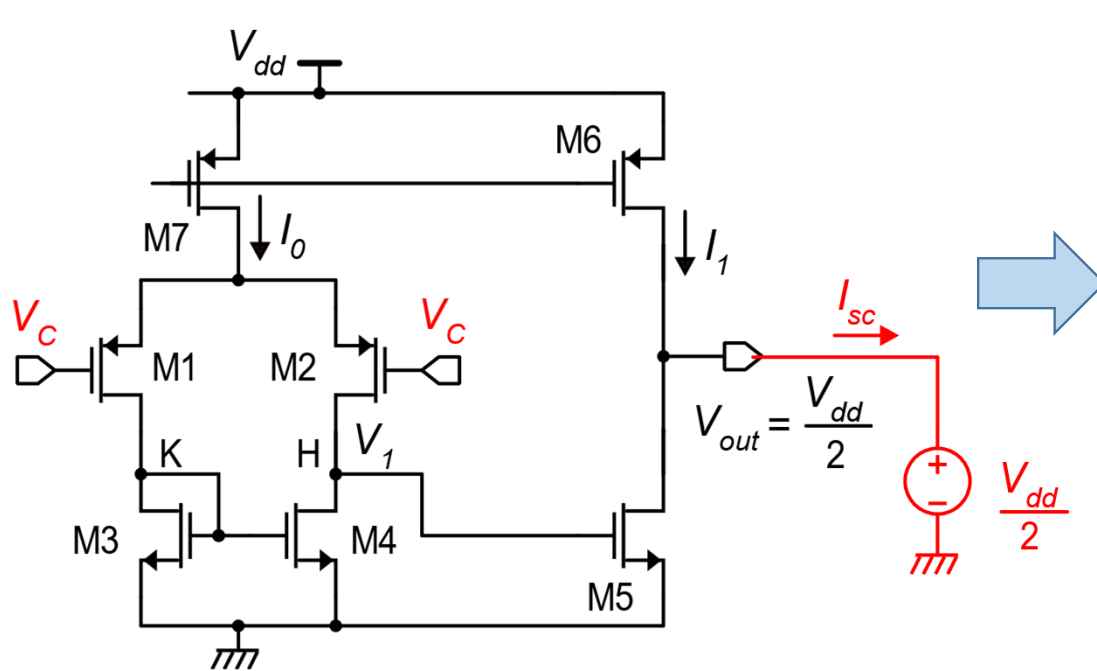
$$M3=M4 \quad (W_3=W_4, \quad L_3=L_4) \quad \text{-----} \quad 2$$

**Current ratios**

$$\frac{I_1}{I_0} = \frac{\beta_6}{\beta_7} = \frac{W_6 / L_6}{W_7 / L_7} \quad \text{-----} \quad 1$$

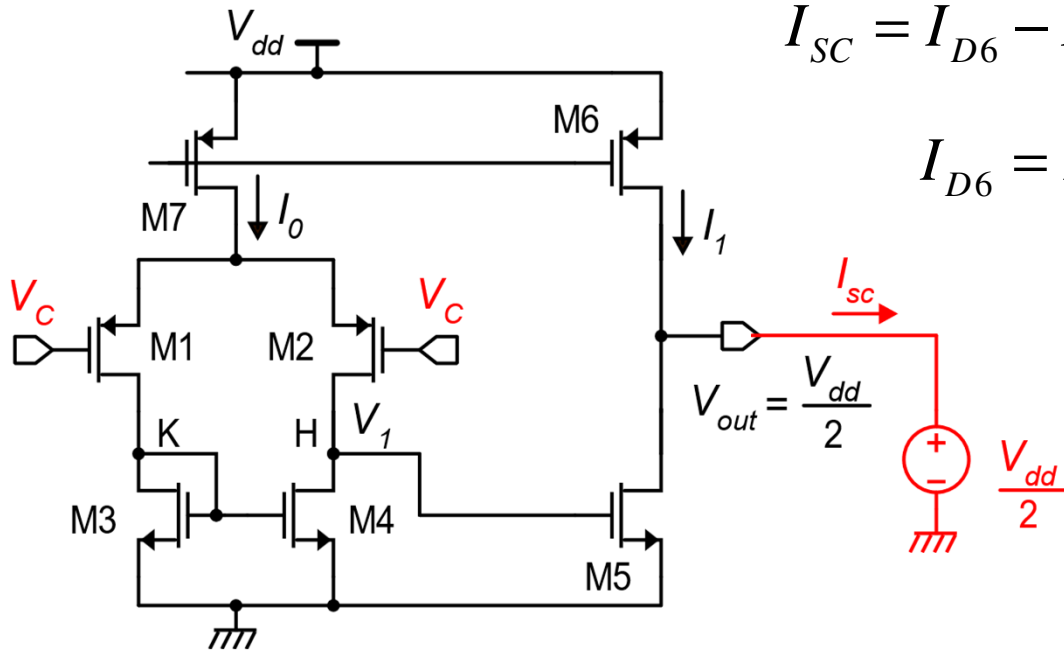
Initial DOF number: 18 , Resulting DOFs after reduction: 18-5=13

## Necessary constraint: null systematic offset



It is not possible to exactly predict  $V_{out}$ , but it will be far from  $V_{dd}/2$  if  $I_{sc}$  is consistently different from zero

# Necessary constraint: null systematic offset



$$I_{SC} = I_{D6} - I_{D5} = 0 \Rightarrow I_{D6} = I_{D5}$$

$$I_{D6} = I_0 \frac{\beta_6}{\beta_7}$$

Since only a common mode voltage is applied to the input:

$$V_H = V_K \Rightarrow V_{GS5} = V_{GS3}$$

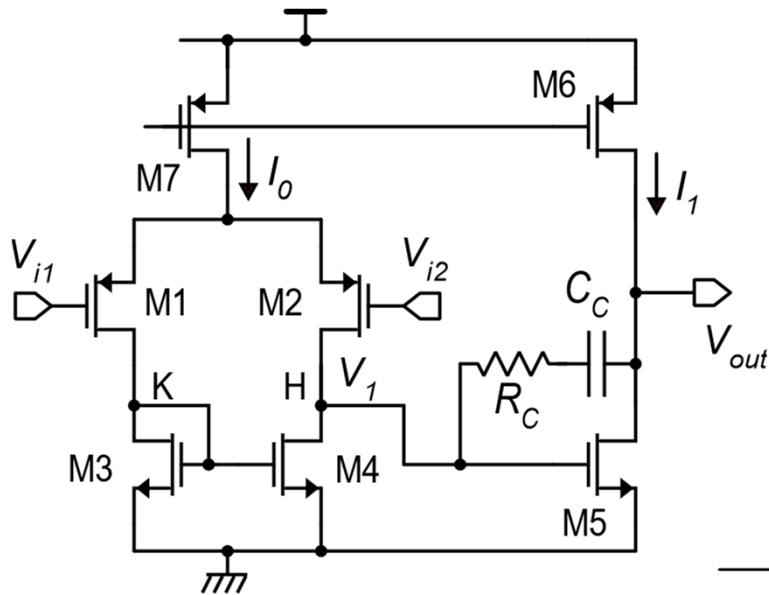
$$I_{D3} = I_{D4} = \frac{I_0}{2}$$

$$I_{D5} = \frac{I_0}{2} \frac{\beta_5}{\beta_3}$$

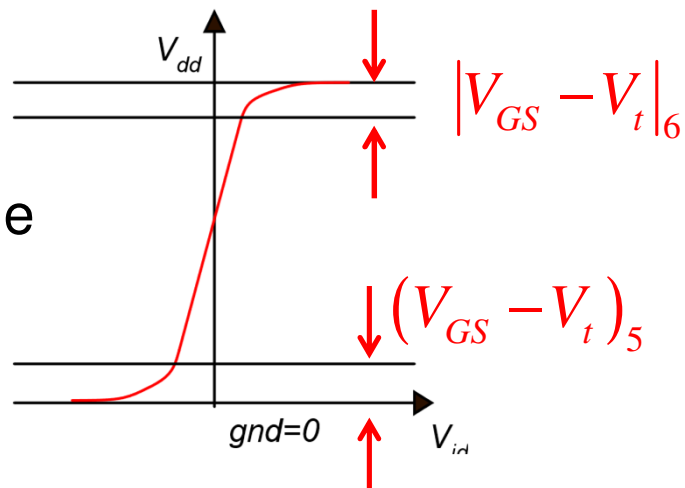
$$I_{D5} = I_{D6} \Rightarrow \frac{I_0}{2} \frac{\beta_5}{\beta_3} = I_0 \frac{\beta_6}{\beta_7}$$

$$\frac{1}{2} \frac{\beta_5}{\beta_3} = \frac{\beta_6}{\beta_7}$$

## More constraints



Let's now consider the output characteristic



$$\frac{1}{2} \frac{\beta_5}{\beta_3} = \frac{\beta_6}{\beta_7} \quad \text{Null (small) systematic offset} \quad \leftarrow$$

Arbitrary constraints

Good matching M5-M3:  $L_5=L_3$   $\leftarrow$

Good matching M6-M7:  $L_6=L_7$   $\leftarrow$

Symmetric output swing  
(same margins to Vdd  
and gnd)

$$(V_{GS} - V_t)_5 = |V_{GS} - V_t|_6 \quad \leftarrow$$

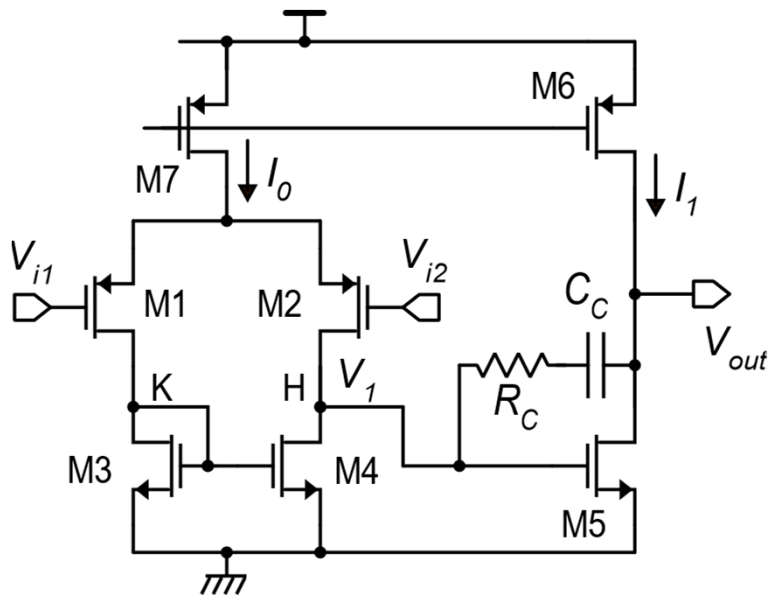
## Residual number of DOFs

$13 - 4 = 9$  Of these residual DOFs we can separate two ones ( $C_C$  and  $R_C$ ) that do not affect the dc performances and the operating point. We will come back to them later. Then we will focus on **7** DOFs (bias current  $I_0$  and device size) that affect the operating point and we will call them "static" DOFs).

We could select 7 DOFs within the original set of 16 DOFs ( $R_C$  and  $C_C$  are not included), and then try to derive the remaining ones using the equations that tie them (equality constraints).

It is more useful to choose a set of DOFs that may not necessarily include the original 18, in a way that the other ones can be easily derived.

## Selection of the 7 DOFs



Rationale: the most important MOSFETs of the circuits are M1 (=M2) and M5, since these are the devices that are at the heart of the two stages, where they perform the V-to-I conversion.

We include all possible DOFs of M1 and M5 into the selected set

$$M1: W_1, L_1, |V_{GS} - V_t|_1 \quad 6 \text{ DOFs}$$

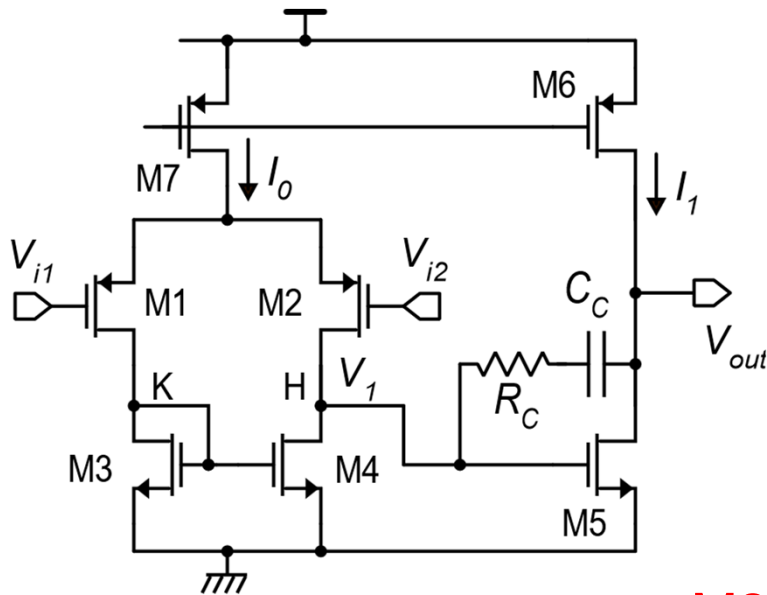
$$M5: W_5, L_5, (V_{GS} - V_t)_5$$

To complete the set, let us include also  $L_6$  into the DOFs

$$\text{Final set of static DOFs: } \{W_1, L_1, |V_{GS} - V_t|_1, W_5, L_5, (V_{GS} - V_t)_5, L_6\}$$

# Derivation of all the op-amp parameters from the 7 DOFs

All conditions will refer to the operating point ( $V_{id}=0$ )



M2 is identical to M1, then M1 DOFs specify also M2 parameters

$$I_0 = 2I_{D1}, \quad I_1 = I_{D5}$$

$$I_{D1} = f_p(W_1, L_1, |V_{GS} - V_t|_1) = \frac{\mu_p C_{OX}}{2} \frac{W_1}{L_1} (V_{GS} - V_t)_1^2$$

$$I_{D5} = f_n[W_5, L_5, (V_{GS} - V_t)_5] = \frac{\mu_n C_{OX}}{2} \frac{W_5}{L_5} (V_{GS} - V_t)_5^2$$

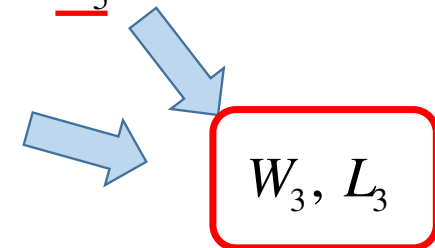
**M3=M4:**

$$L_3 = L_5$$

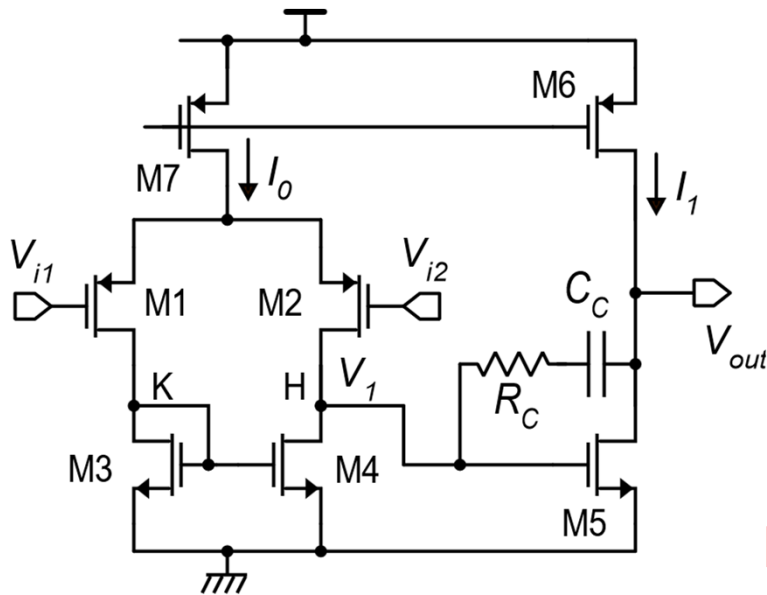
$$I_{D3} = I_{D1}$$

$$(V_{GS} - V_t)_3 = (V_{GS} - V_t)_5$$

$$\frac{\mu_n C_{OX}}{2} \frac{W_3}{L_3} = \frac{I_{D3}}{(V_{GS} - V_t)_3^2} \Rightarrow \frac{W_3}{L_3}$$



# Derivation of all the op-amp parameters from the 7 DOFs



**M6:**  $I_{D6} = I_{D5}$   
 $|V_{GS} - V_t|_6 = (V_{GS} - V_t)_5$

$$\frac{\mu_p C_{OX}}{2} \frac{W_6}{L_6} = \frac{I_{D6}}{(V_{GS} - V_t)_6^2} \Rightarrow \frac{W_6}{L_6} \Rightarrow W_6, L_6$$

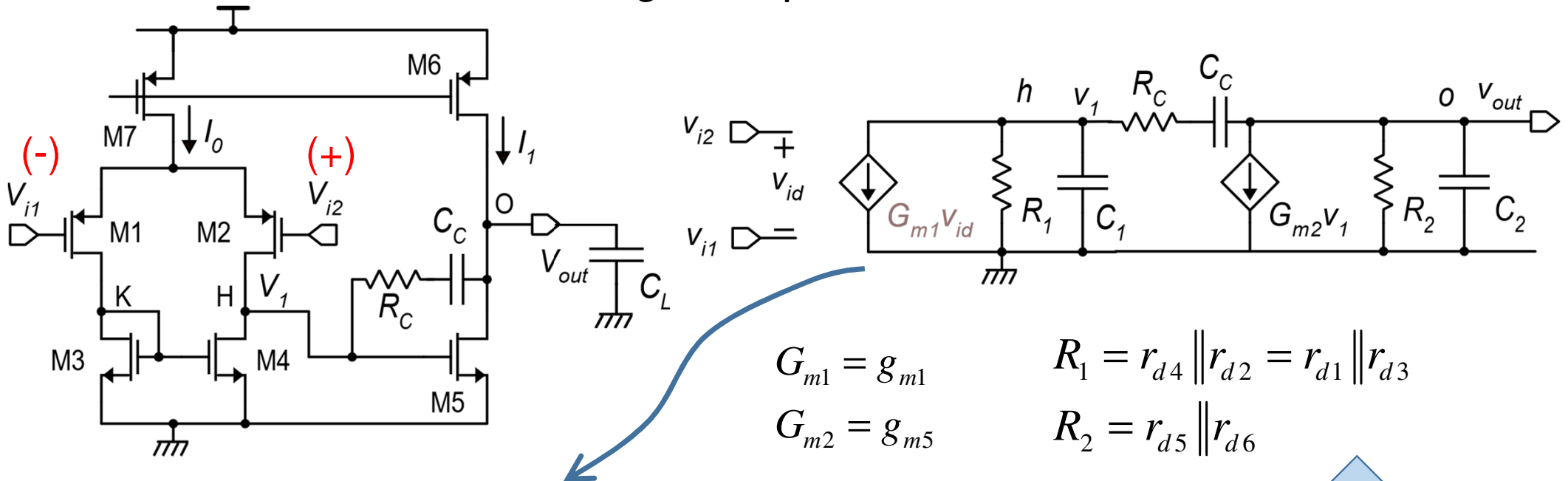
**M7:**  $I_{D7} = 2I_{D1}$   
 $|V_{GS} - V_t|_7 = |V_{GS} - V_t|_6$

$$\frac{W_7}{L_7} \Rightarrow W_7, L_7$$

$L_7 = L_6$



## Small-signal equivalent circuit



Note: the equivalent circuit can be used to represent the behavior of most two-stage topologies, not just the simple amplifier of the figure.

$$G_{m1} = g_{m1} \quad R_1 = r_{d4} \parallel r_{d2} = r_{d1} \parallel r_{d3}$$

$$G_{m2} = g_{m5} \quad R_2 = r_{d5} \parallel r_{d6}$$

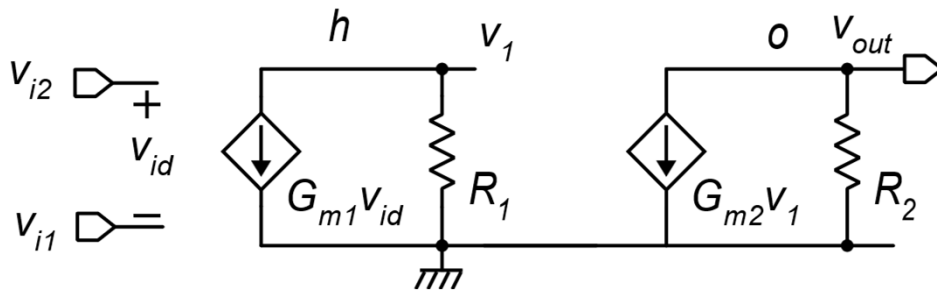
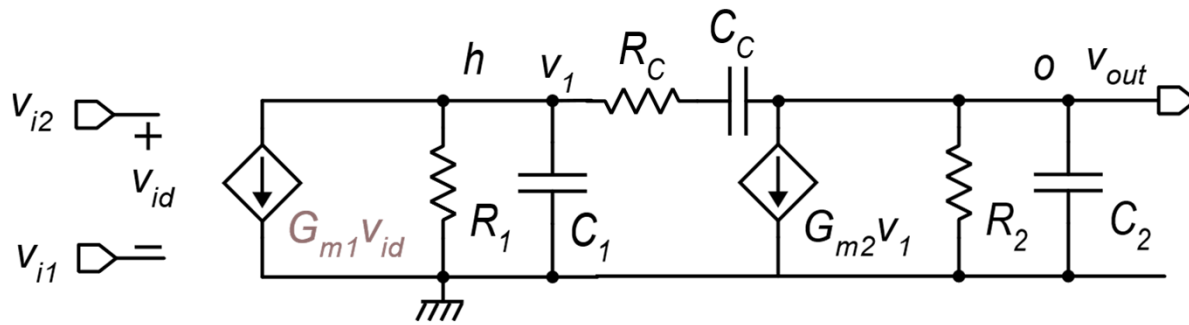
$$C_1 = C_{DB2} + C_{DB4} + C_{GS5}$$

$$C_2 = C_2' + C_L$$

$$C_2' = C_{DB5} + C_{DB6}$$

All these values are functions of the DOFs

## dc gain

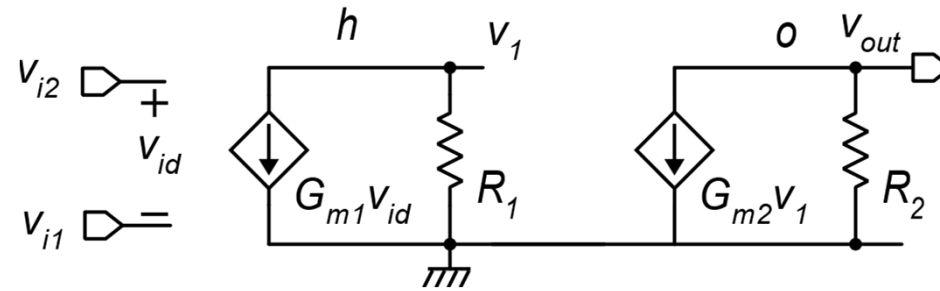
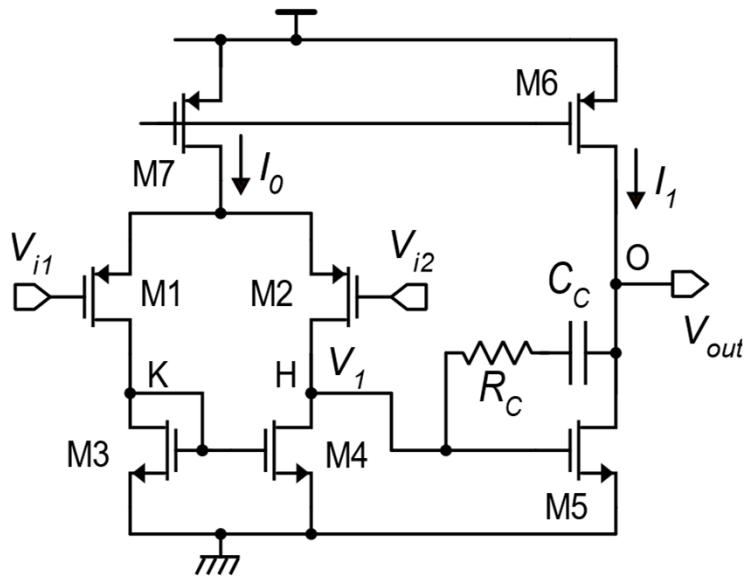


$$v_1 = -G_{m1} v_{id} \cdot R_1$$

$$v_{out} = -G_{m2} v_1 \cdot R_2$$

$$v_{out} = -G_{m2} (-G_{m1} v_{id} \cdot R_1) R_2 = G_{m1} R_1 G_{m2} R_2 v_{id} \Rightarrow A_0 = G_{m1} R_1 G_{m2} R_2$$

## dc gain as a function of the DOFs



$$A_0 = G_{m1} R_1 G_{m2} R_2$$

$$G_{m1} = g_{m1}$$

$$G_{m2} = g_{m5}$$

$$A_0 = g_{m1} \frac{1}{\frac{1}{r_{d1}} + \frac{1}{r_{d3}}} g_{m5} \frac{1}{\frac{1}{r_{d5}} + \frac{1}{r_{d6}}}$$

$$R_1 = r_{d4} \parallel r_{d2} = r_{d1} \parallel r_{d3}$$

$$R_2 = r_{d5} \parallel r_{d6}$$

$$g_m = \frac{I_D}{V_{TE}}$$

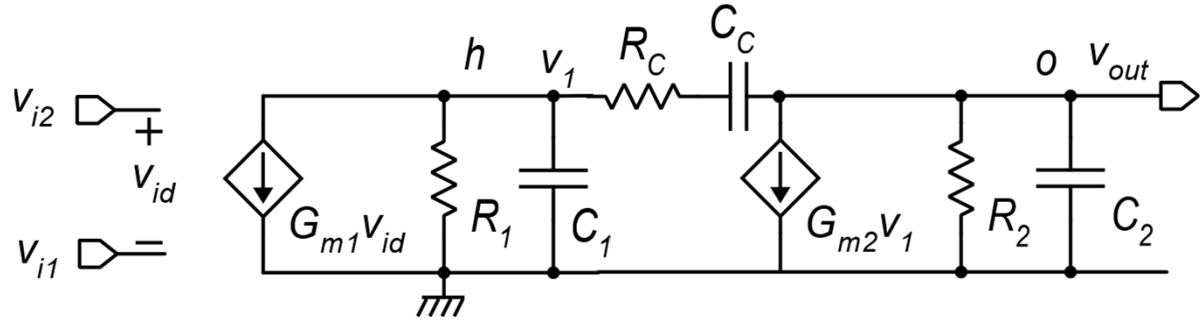
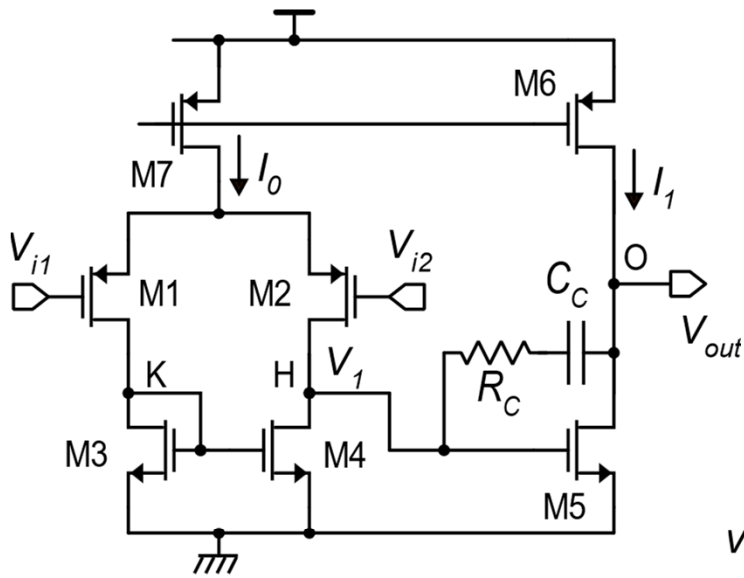
$$I_{D1} = I_{D3}$$

$$I_{D5} = I_{D6}$$

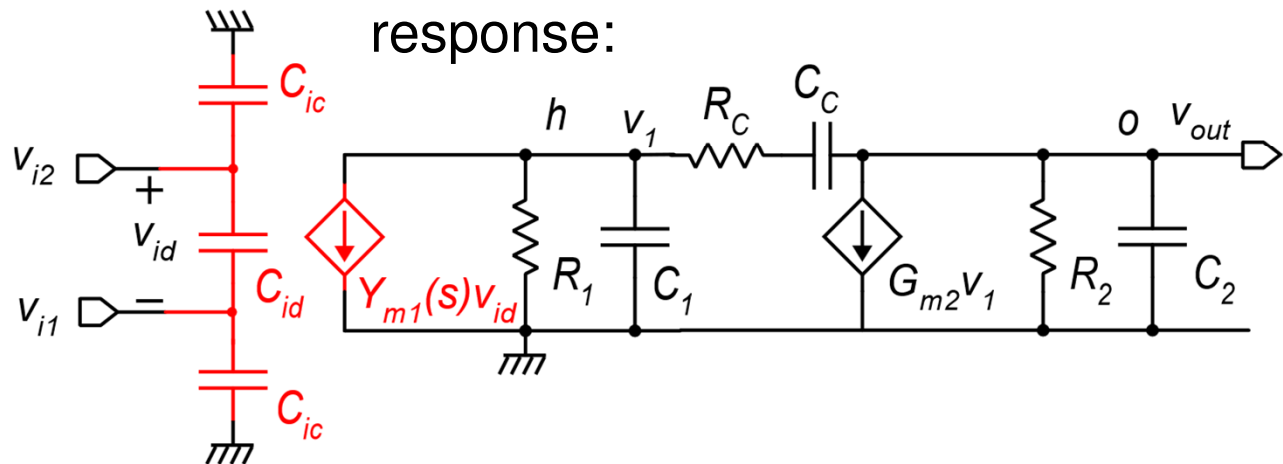
$$\frac{1}{r_d} = g_d = \lambda I_D$$

$$A_0 = \frac{1}{V_{TE1}} \frac{1}{V_{TE5}} \frac{1}{(\lambda_1 + \lambda_3)} \frac{1}{(\lambda_5 + \lambda_6)}$$

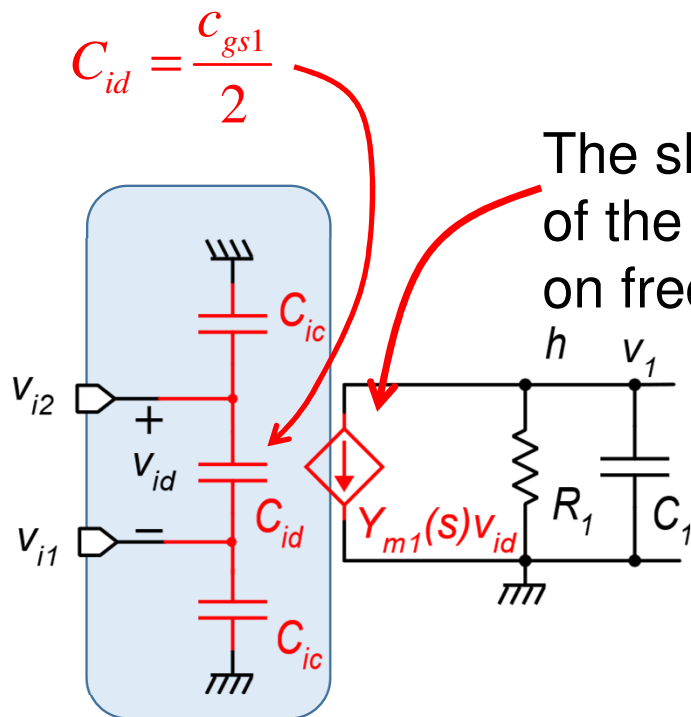
# Frequency response of a two-stage op-amp



This circuit does not include all components that affect the frequency response:

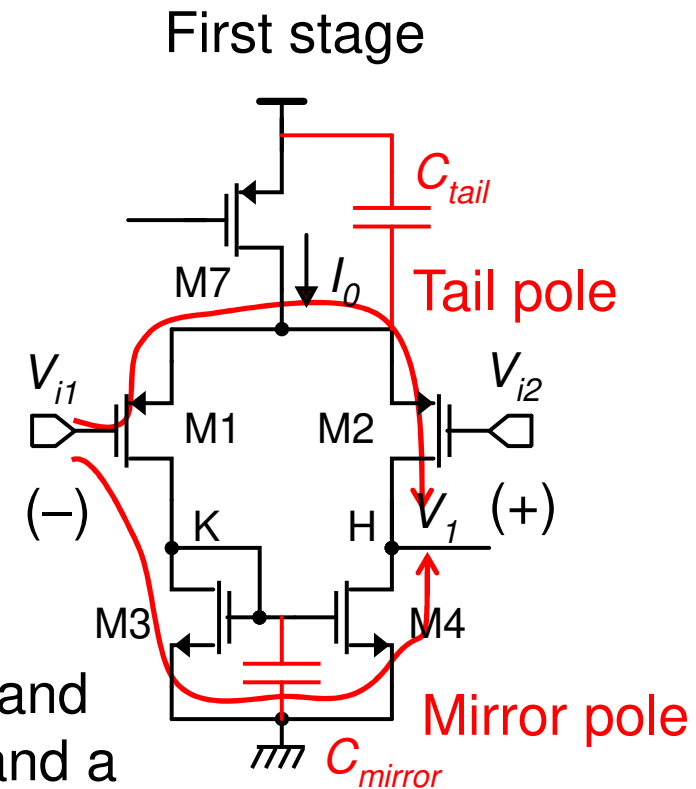


## Frequency response



The short circuit output current of the first stage also depends on frequency

$-\beta V_{out}$   $\rightarrow$

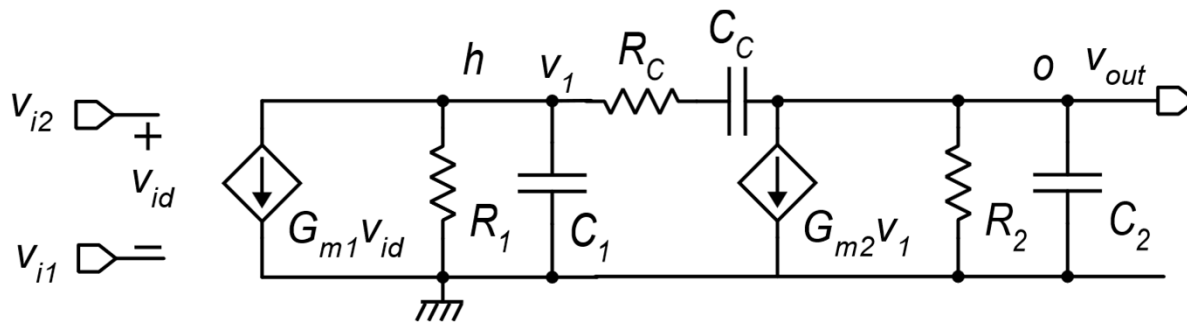


The tail pole and mirror pole (and a zero created by their combination) affect  $Y_m(s)$ . Frequencies are of the order of

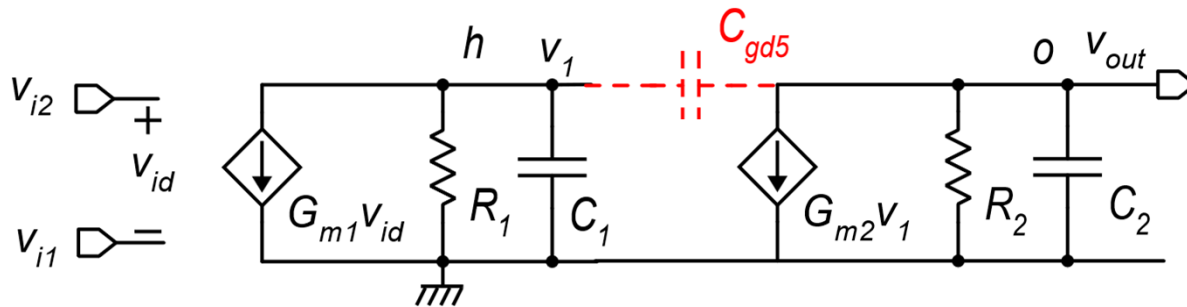
$f \sim \frac{1}{2\pi} \frac{g_m}{C_p}$

These capacitances determine the load presented by the amplifier to the signal source. We will suppose that  $v_{i1}$  and  $v_{i2}$  are produced by ideal voltage sources, thus no loading effect will be considered

## Frequency response, simplified small-signal circuit

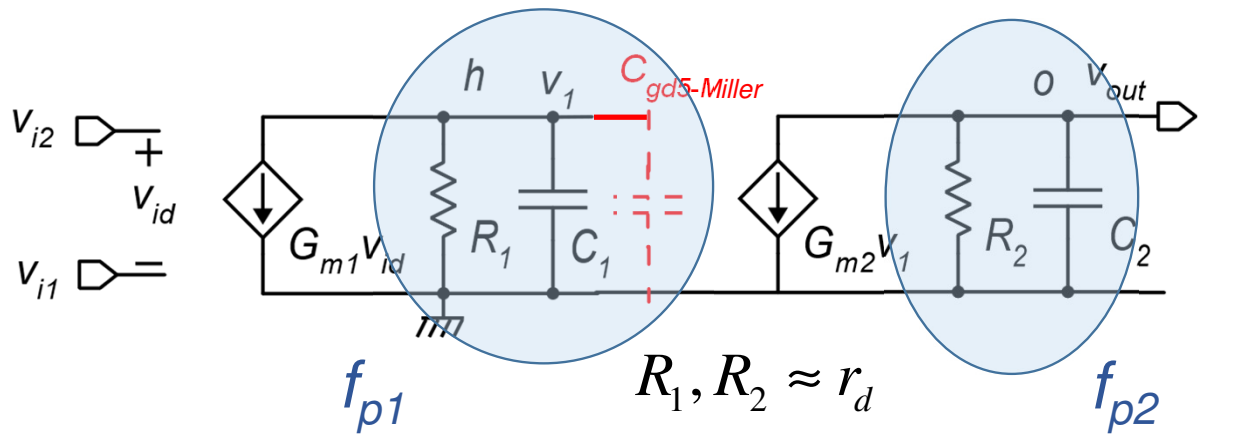


If the compensation group  $C_C$ - $R_C$  is not present:



We still have  $C_{gd5}$  parasitic capacitance, which is not sufficient to produce the compensation effect

## Uncompensated frequency response

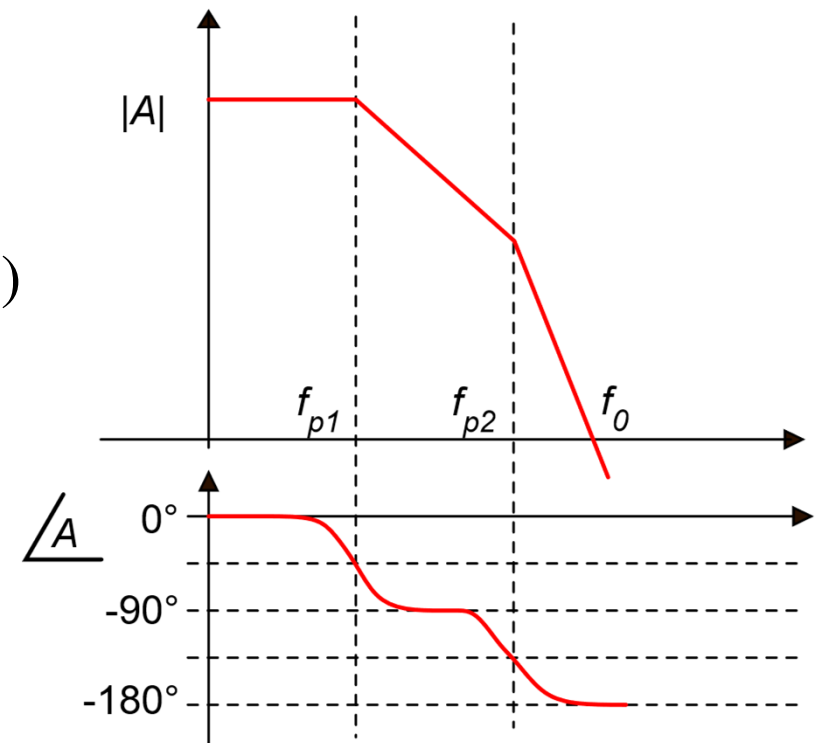


$$C_1, C_2, C_{gd5} \approx C_p \text{ (parasitic cap.)}$$

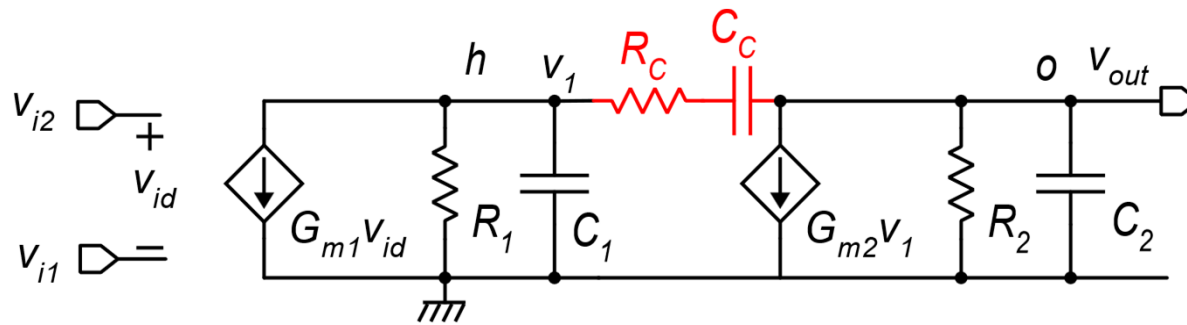
Without compensation, we have two poles at frequencies  $f_{p1}$ ,  $f_{p2}$ , which are of the same order of magnitude and none of them is dominant.

The result is a very small or even negative (= instability) phase margin.

$$f_{p1}, f_{p2} \approx \frac{1}{C_p r_d} = \frac{g_d}{C_p} \ll \frac{g_m}{C_p}$$

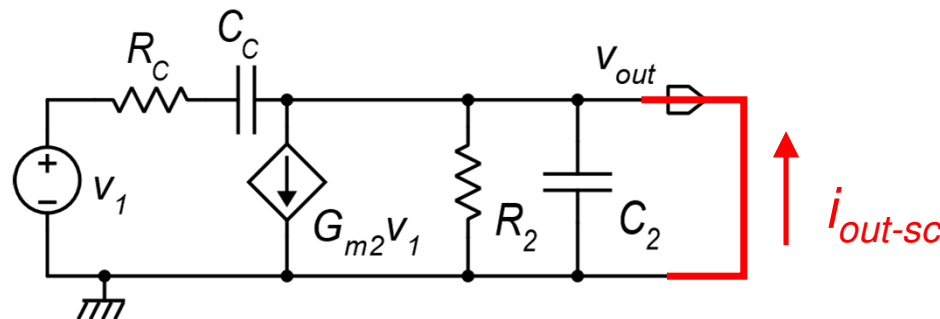


## Miller compensation $\Rightarrow$ Pole splitting



It is convenient to divide the bridge impedance  $R_C$ - $C_C$  into two impedances by means of the Miller theorem

To do this, we need to calculate the Miller factor  $K = v_{out}/v_1$ . We force voltage  $v_1$  and use the Norton equivalent model of the output port.

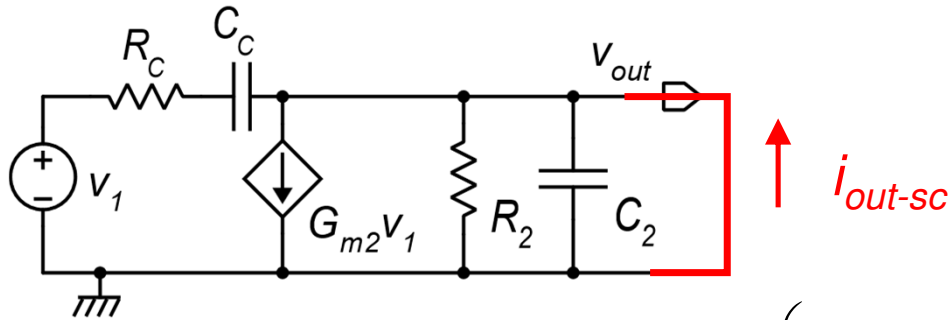


$$i_{out-sc} = G_{m2}v_1 - \frac{v_1}{R_C + \frac{1}{sC_C}} = v_1 \left( G_{m2} - \frac{sC_C}{1 + sC_C R_C} \right)$$

$$i_{out-sc} = v_1 \left( \frac{G_{m2} + sC_C R_C G_{m2} - sC_C}{1 + sC_C R_C} \right)$$

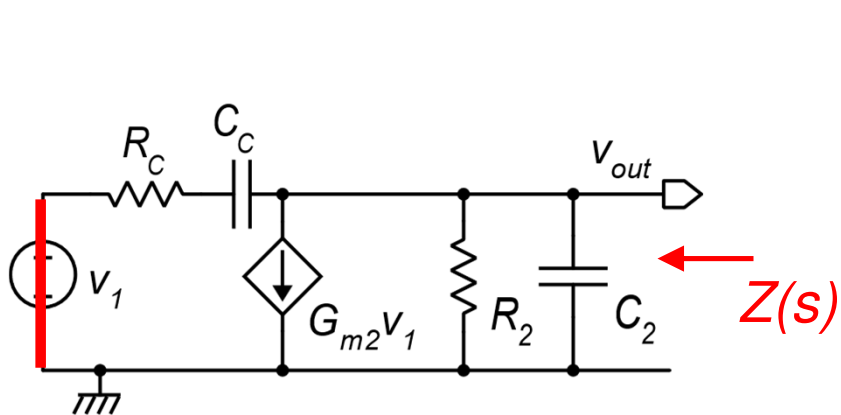


## Miller factor



$$i_{out-sc} = v_1 \left( \frac{G_{m2} + sC_C R_C G_{m2} - sC_C}{1 + sC_C R_C} \right)$$

$$i_{out-sc} = v_1 G_{m2} \left( \frac{1 + sC_C R_C - s \frac{C_C}{G_{m2}}}{1 + sC_C R_C} \right) = v_1 G_{m2} \left( \frac{1 + sC_C \left( R_C - \frac{1}{G_{m2}} \right)}{1 + sC_C R_C} \right)$$

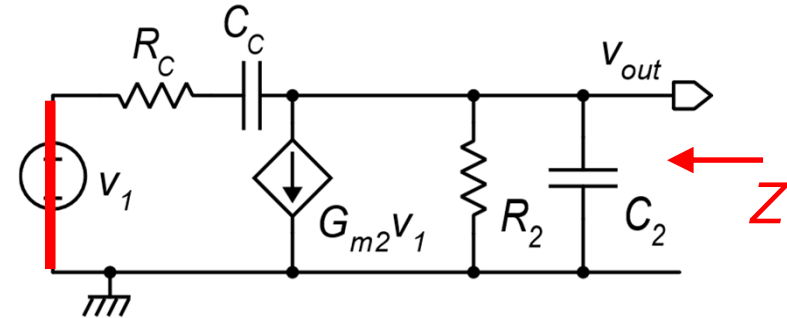


$$v_{out} = -Z(s) i_{out-sc} = -Z(s) v_1 G_{m2} \left( \frac{1 + sC_C \left( R_C - \frac{1}{G_{m2}} \right)}{1 + sC_C R_C} \right)$$

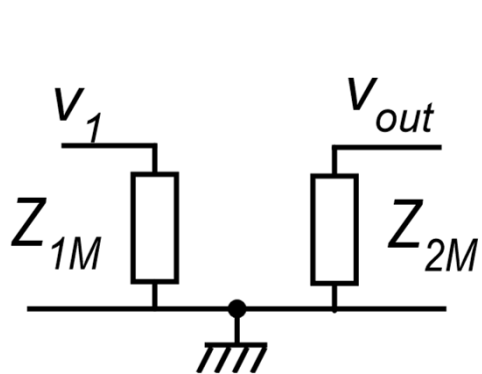
Miller factor:  $K(s) = \frac{v_{out}}{v_1}$

## Transformation of the bridge impedance $Z_C$ by the Miller Theorem

$$K = \frac{v_{out}}{v_1} = -ZG_{m2} \left( \frac{1 + sC_C \left( R_C - \frac{1}{G_{m2}} \right)}{1 + sC_C R_C} \right)$$

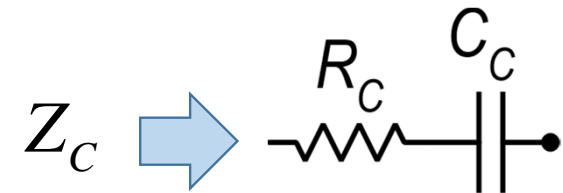


The low frequency limit of the K factor:  $Z(f \rightarrow 0) = R_2$        $K(f \rightarrow 0) = -G_{m2}R_2$



$$Z_{1M} = \frac{Z_C}{1-K} = \frac{1}{j\omega C_C(1-K)}$$

$$Z_{2M} = \frac{-KZ_C}{1-K} = \frac{-K}{j\omega C_C(1-K)}$$



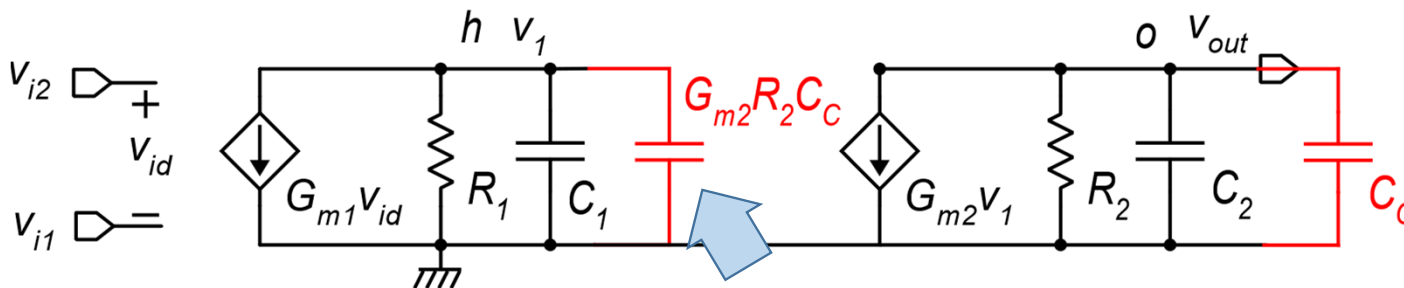
$$Z_C \left( f \ll \frac{1}{2\pi C_C R_C} \right) = \frac{1}{j\omega C_C}$$

Miller transformations: shifting the input pole to very low frequencies

$$Z_{1M} = \frac{Z_C}{1-K} = \frac{1}{j\omega C_C(1-K)} \quad Z_{2M} = \frac{-KZ_C}{1-K} = \frac{-K}{j\omega C_C(1-K)}$$

Using  
 $K = K(0) = -G_{m2}R_2$

$$Z_{1M} = \frac{1}{j\omega C_C(1+G_{m2}R_2)} \cong \frac{1}{j\omega C_C G_{m2}R_2} \quad Z_{2M} \cong \frac{1}{j\omega C_C}$$

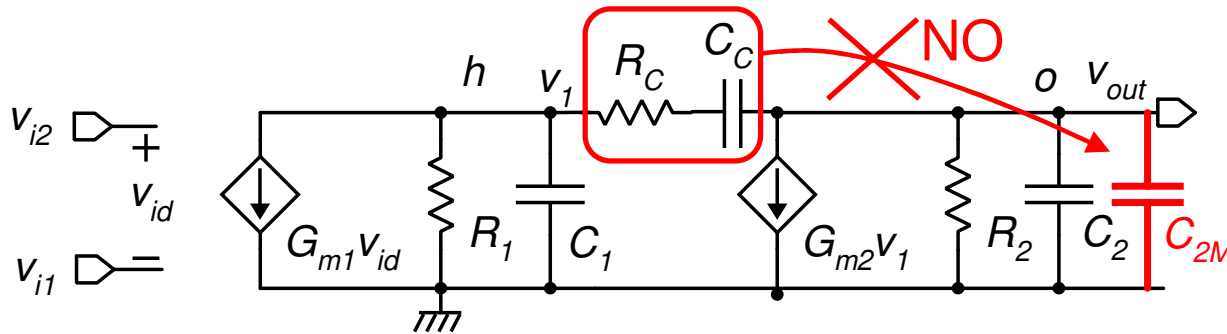


First effect of Miller compensation: a very large capacitor  $G_{m2}R_2C_C$  is brought back to the input mesh, shifting the input pole back to low frequencies:

$$\omega_p \cong \frac{1}{R_1 G_{m2} R_2 C_C}$$

This sets the dominant pole:

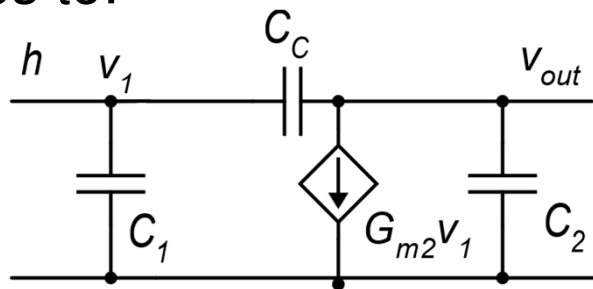
Second effect of Pole Splitting:  
shifting the output pole to high frequencies



We cannot use the Miller theorem again, because the resulting pole would fall at frequencies where  $K$  is very different from  $\mathbf{K(0)}$ .

At frequencies such that:  $\frac{1}{\omega C_2} \ll R_2$ ,  $\frac{1}{\omega C_1} \ll R_1$  and still:  $\frac{1}{\omega C_C} \gg R_C$

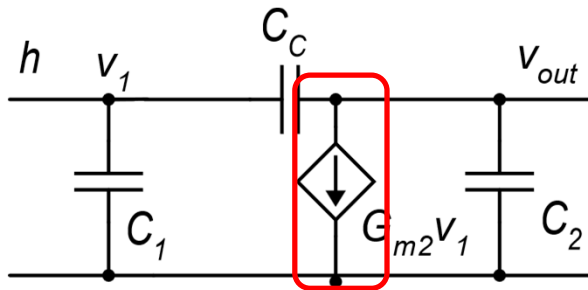
The equivalent circuit reduces to:



Current source  $G_{m2}v_1$  is controlled by the voltage across it:

$$v_1 = v_{out} \frac{C_C}{C_1 + C_C}$$

## Second effect of Miller Compensation: shifting the output pole to high frequencies



Then, it is equivalent to a resistance:  $R_V = \frac{v_{out}}{i}$

$$v_1 = v_{out} \frac{C_c}{C_1 + C_c} \quad i = G_{m2} v_1 = v_{out} G_{m2} \frac{C_c}{C_1 + C_c}$$

$$R_V = \frac{v_{out}}{i} = \frac{1}{G_{m2}} \frac{C_1 + C_c}{C_c}$$

This resistance "sees" a capacitance:  $C_V = C_2 + \frac{C_1 C_c}{C_1 + C_c}$

This sets a pole at:  $\omega_2 = \frac{1}{R_V C_V}$

$$\omega_2 = \frac{1}{\frac{1}{G_{m2}} \frac{C_1 + C_c}{C_c} \left( C_2 + \frac{C_1 C_c}{C_1 + C_c} \right)}$$

**Note:**  $R_V$  is actually the op-amp output resistance at medium and high frequencies. It is of the order of  $1/G_{m2}$  and is much smaller than the value in dc (order of  $r_d$ ).

## Second effect of Miller Compensation: shifting the output pole to high frequencies

$$\omega_2 = \frac{1}{R_V C_V} = \frac{1}{\frac{1}{G_{m2}} \frac{C_1 + C_C}{C_C} \left( C_2 + \frac{C_1 C_C}{C_1 + C_C} \right)} = \frac{G_{m2}}{\frac{C_1 C_2 + C_C C_2 + C_1 C_C}{C_C}}$$

$$\omega_2 = \frac{G_{m2}}{\left( \frac{C_1 C_2}{C_C} + C_2 + C_1 \right)} = \frac{G_{m2}}{(C_1 + C_2) \left( 1 + \frac{1}{C_C} \cdot \frac{C_1 C_2}{C_1 + C_2} \right)}$$

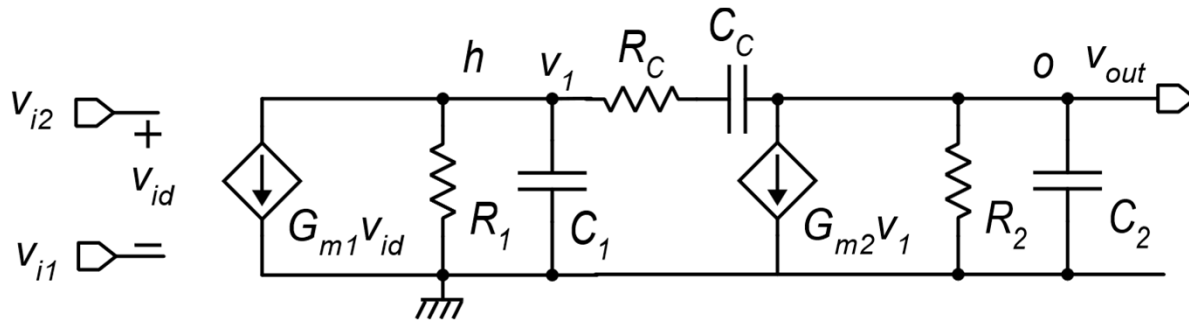
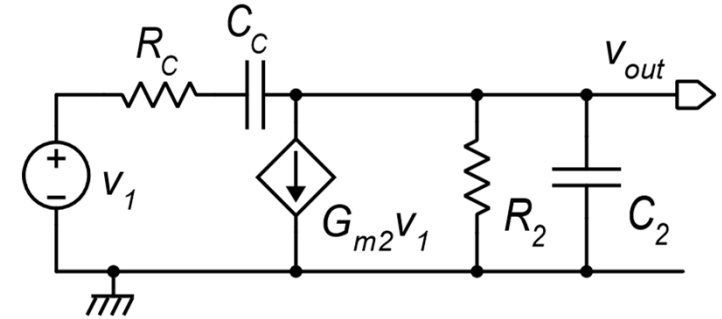
$$C_S = \frac{C_1 C_2}{C_1 + C_2}$$

Series of  $C_1$   
and  $C_2$

$$\omega_2 = \frac{G_{m2}}{(C_1 + C_2) \left( 1 + \frac{C_S}{C_C} \right)}$$


## Miller Factor and overall transfer function

$$\text{Miller factor: } K(s) = \frac{v_{out}}{v_1} = -ZG_{m2} \left( \frac{1 + sC_C \left( R_C - \frac{1}{G_{m2}} \right)}{1 + sC_C R_C} \right)$$



$$\text{Factorizing A: } A = \frac{v_{out}}{v_{id}} = \frac{v_1}{v_{id}} \frac{v_{out}}{v_1} = \frac{v_1}{v_{id}} K(s)$$

The zero introduced by  $R_C$ - $C_C$

$$K(s) = \frac{v_{out}}{v_1} = -ZG_{m2} \left( \frac{1 + sC_C \left( R_C - \frac{1}{G_{m2}} \right)}{1 + sC_C R_C} \right)$$


$$A = \frac{v_{out}}{v_{id}} = \frac{v_1}{v_{id}} \frac{v_{out}}{v_1} = \frac{v_1}{v_{id}} K(s)$$

There is a zero in K(s):

$$s_z = \frac{-1}{C_C \left( R_C - \frac{1}{G_{m2}} \right)}$$

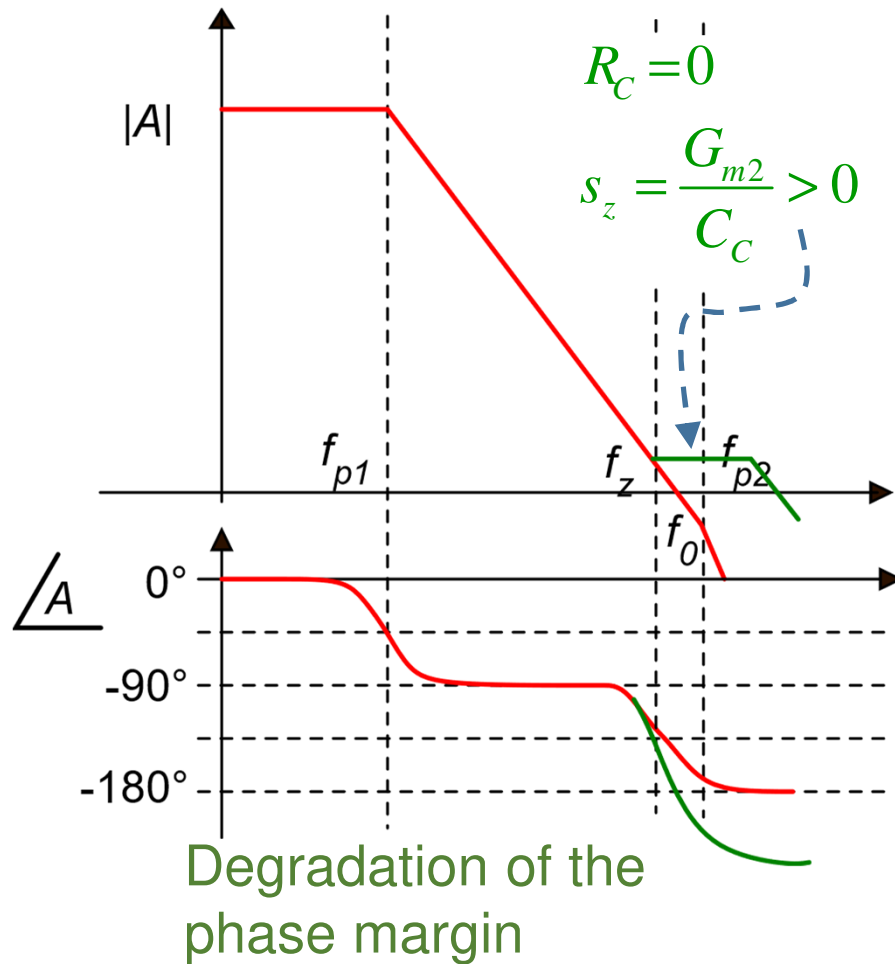
If  $R_C=0$   
(no  $R_C$ )

$$s_z = \frac{-1}{C_C \left( \frac{-1}{G_{m2}} \right)} = \frac{G_{m2}}{C_C} > 0$$

This zero occurs also in the overall transfer-function of the amplifier (A). It cannot be cancelled by an equal zero, since an unstable pole ( $>0$ ) would be required



# Effects of the zero in the transfer function



Thanks to  $R_C$ :

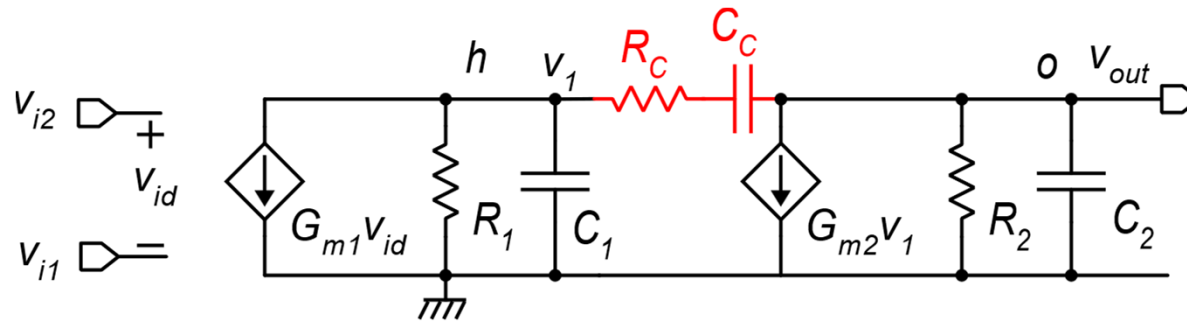
$$s_z = \frac{-1}{C_C \left( R_C - \frac{1}{G_{m2}} \right)}$$

$$R_C = \frac{1}{G_{m2}} \quad s_z \rightarrow \infty$$

With this choice for  $R_C$ , we can eliminate the zero and cancel its bad effect on the phase delay.

Other possible choices are possible: for  $R_C > 1/G_{m2}$  it is possible to change the positive zero into a negative one and use it to compensate  $f_{p2}$ .

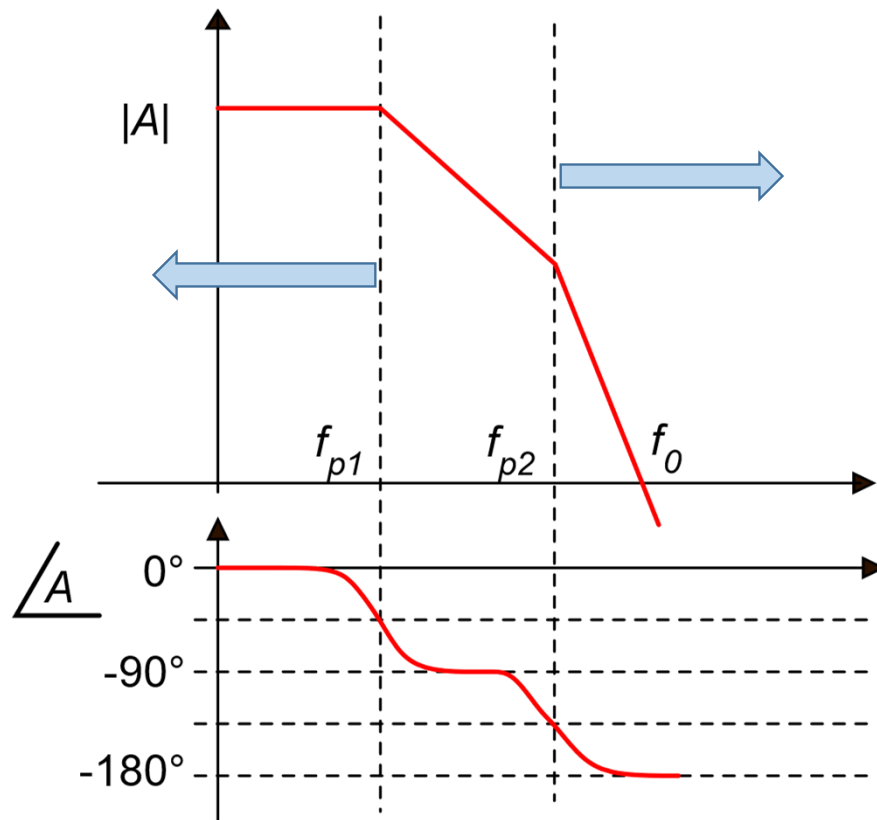
## Summary of pole splitting



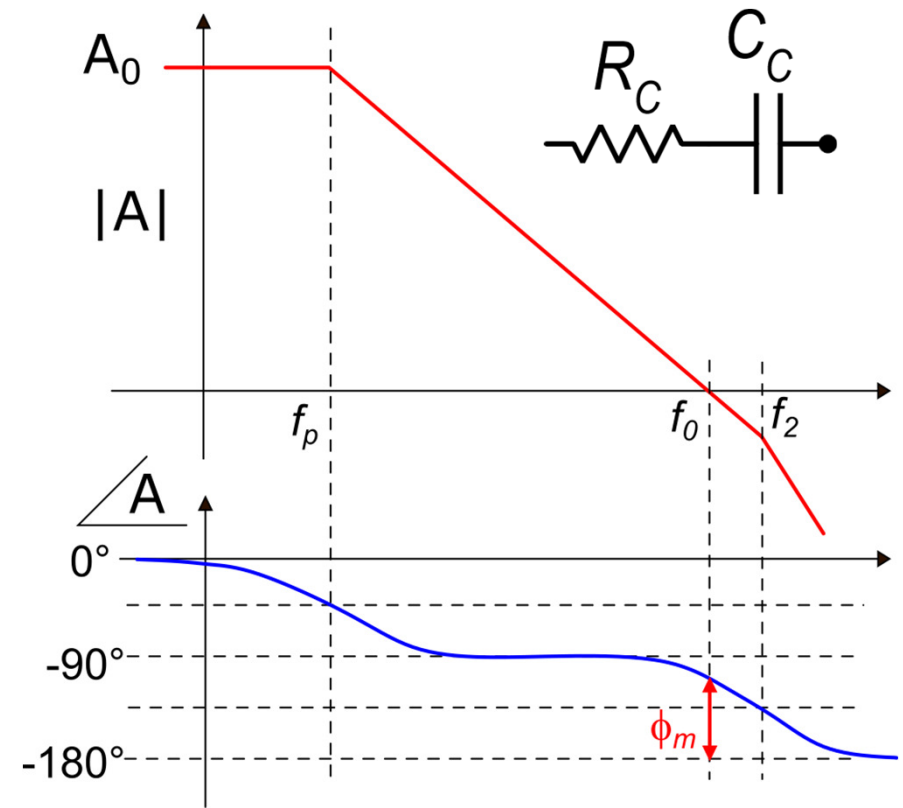
Capacitor  $C_C$  introduces a feedback across the second stage that:

1. Puts an equivalent large capacitor ( $C_C G_{m2} R_2 \gg C_1$ ) across the output resistance of the first stage ( $R_1$ ) shifting the first pole back to very low frequencies
2. Reduces the output resistance ( $R_V$ ) at medium/high frequencies from  $R_2$  to a value close to  $1/G_{m2}$ . This shifts the output pole to much higher frequencies.
3. **Resistor  $R_C$**  is significant only at high frequencies and "shapes" the zero, either cancelling it or turning it into a negative zero

## Pole splitting, graphical view

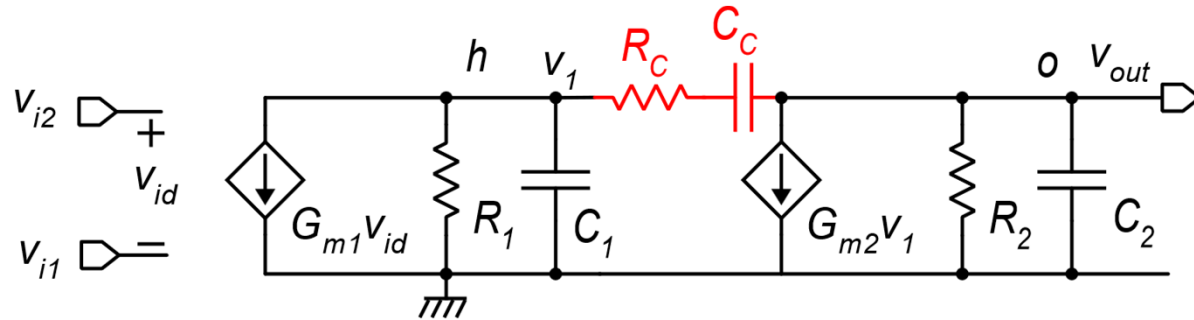


Before compensation



After compensation

## Summary of singularities



$$\omega_p \cong \frac{1}{R_1 G_{m2} R_2 C_C}$$

$$s_z = \frac{-1}{C_C \left( R_C - \frac{1}{G_{m2}} \right)}$$

$$\omega_2 \cong \frac{G_{m2}}{(C_1 + C_2) \left( 1 + \frac{C_S}{C_C} \right)} = \frac{G_{m2}}{(C_1 + C_2) \left( 1 + \frac{C_S}{C_C} \right)^{-1}} \quad C_S = \frac{C_1 C_2}{C_1 + C_2}$$

$$\omega_3 \cong \frac{1}{R_C \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_C} \right)^{-1}} \gg \omega_2$$

This third pole ( $s_3 = -\omega_3$ ), can be guessed considering that at very high frequencies the whole network reduces to the three capacitors and resistor  $R_C$ .