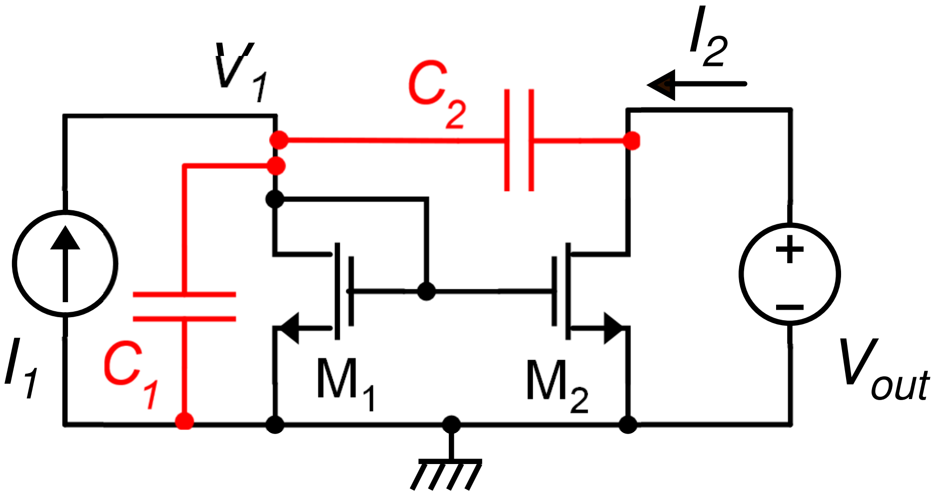
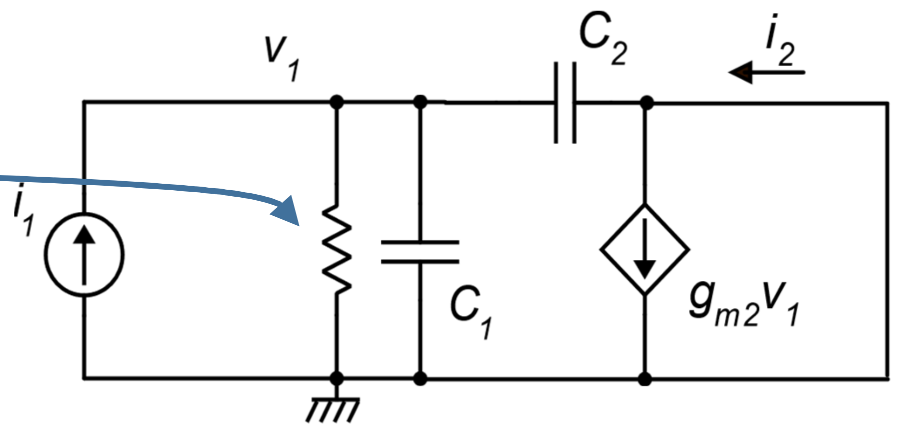
Frequency response of current mirrors



$$C_1 = C_{GS1} + C_{GS2} + C_{DB1}$$

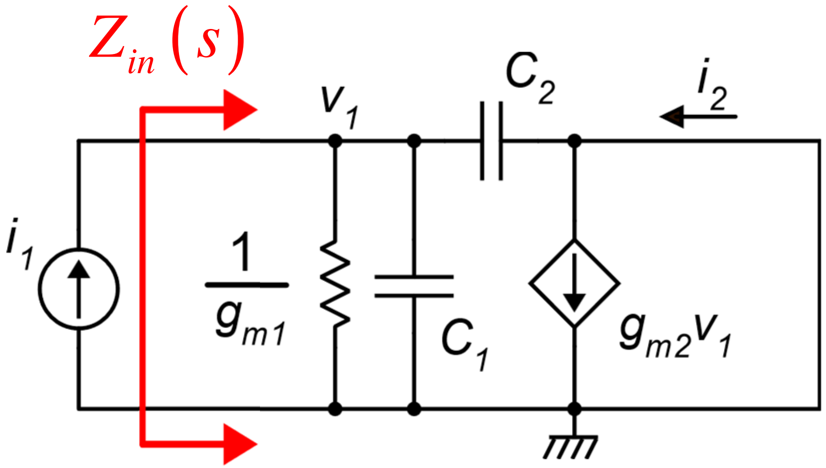
$$C_2 = C_{GD2}$$

small signal equivalent circuit



$$\frac{1}{g_{m1}} // r_{d1} \cong \frac{1}{g_{m1}}$$

Small signal equivalent circuit



$$i_2 = g_{m2}v_1 - sC_2v_1 = v_1\left(g_{m2} - sC_2\right)$$

$$v_1 = Z_{in}\left(s\right)i_1$$

$$Z_{in}\left(s\right) = \frac{1}{g_{m1} + s\left(C_1 + C_2\right)}$$

$$i_2 = i_1\frac{g_{m2} - sC_2}{g_{m1} + s\left(C_1 + C_2\right)}$$

$$A_I\left(s\right) = \frac{i_2}{i_1} = \frac{g_{m2}}{g_{m1}}\frac{1 - s\dfrac{C_2}{g_{m2}}}{1 + s\dfrac{\left(C_1 + C_2\right)}{g_{m1}}}$$

$$A_I = A_I(0)\left(\frac{1 - \dfrac{s}{\omega_z}}{1 + \dfrac{s}{\omega_p}}\right)$$

$$\omega_p = \frac{g_{m1}}{C_1 + C_2}$$

$$\omega_z = \frac{g_{m2}}{C_2}$$

$$A_I\left(0\right) = \frac{g_{m2}}{g_{m1}}$$

Frequency response of the current gain A_I

$$A_I = A_I(0)\left(\frac{1-\dfrac{s}{\omega_z}}{1+\dfrac{s}{\omega_p}}\right)$$

$$\omega_p = \frac{g_{m1}}{C_1 + C_2}$$

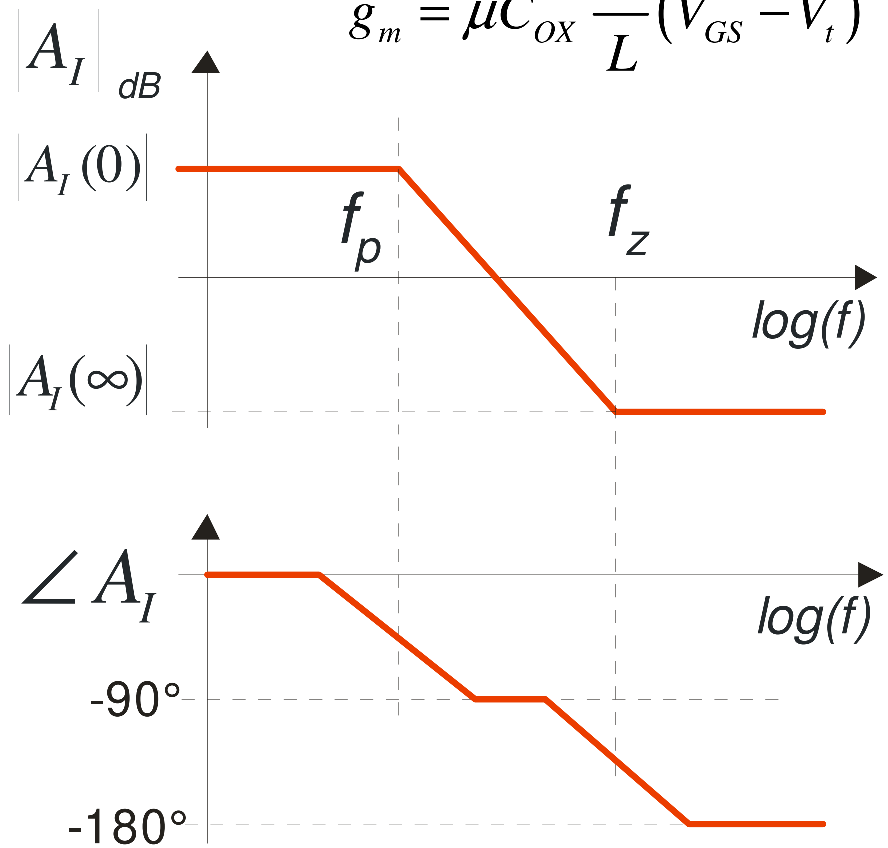
$$\omega_z = \frac{g_{m2}}{C_2}$$

$$g_m = \mu C_{OX}\frac{W}{L}\left(V_{GS} - V_t\right)$$

$$A_I(0) = \frac{g_{m2}}{g_{m1}} = \frac{W_2/L_2}{W_1/L_1} = k_M$$

$$A_I(\infty) = -A_I(0)\frac{\omega_P}{\omega_Z} = -\frac{g_{m2}}{g_{m1}}\frac{g_{m1}}{(C_1+C_2)}\frac{C_2}{g_{m2}}$$

$$A_I(\infty) = -\frac{C_2}{C_1+C_2} < 1$$



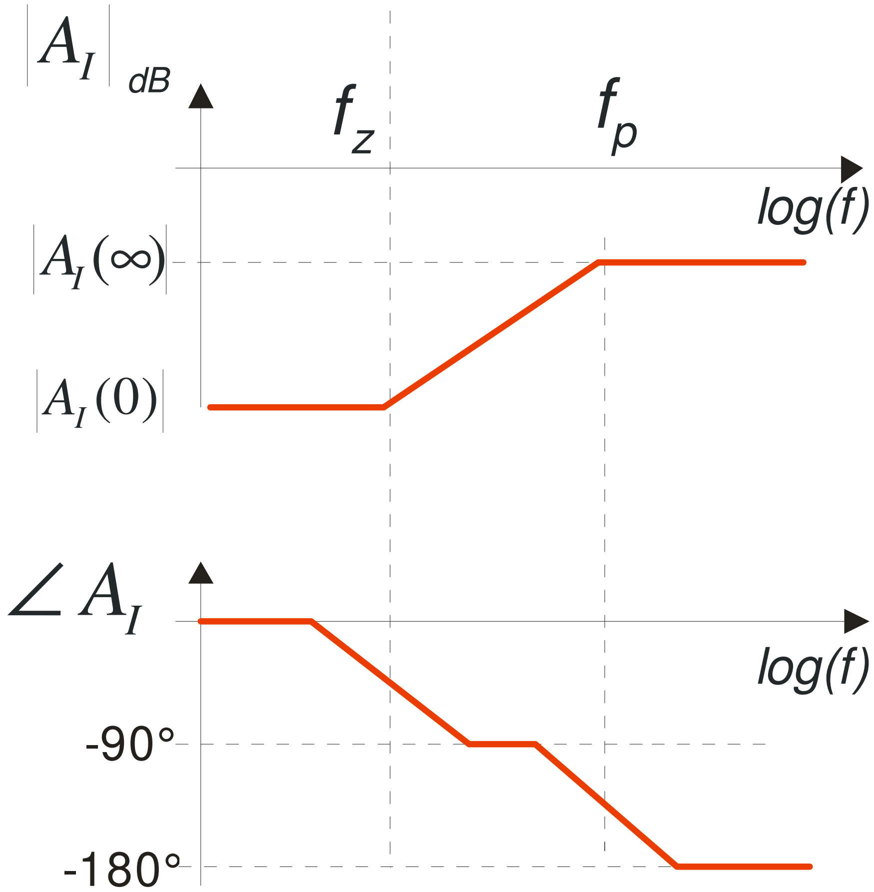
Frequency response of current mirrors with A_I<<1

$$A_I(0) \cong \frac{g_{m2}}{g_{m1}} \cong \frac{W_2/L_2}{W_1/L_1} = k_M$$

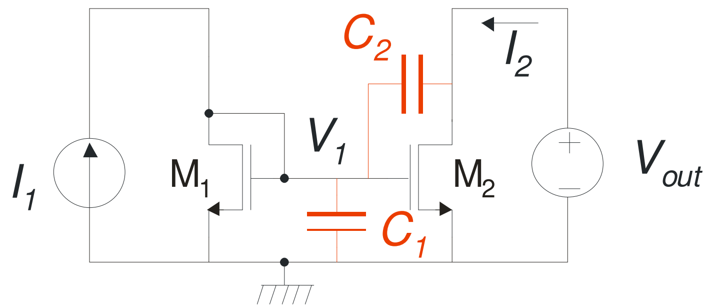
If $A_I(0) << 1$

we can expect that:

$$A_I(0) << |A_I(\infty)| = \frac{C_2}{C_1 + C_2}$$



Upper band limit (f_H) of a current mirror



Hypothesis: $f_p < f_z$ then: $f_H = f_p$

$$f_p = \frac{\omega_p}{2\pi} \cong \frac{g_{m1}}{2\pi C_1} \cong \frac{g_{m1}}{2\pi\left(C_{gs1} + C_{gs2}\right)} = \frac{g_{m1}}{2\pi C_{gs1}} \frac{1}{\left(1 + \frac{C_{gs2}}{C_{gs1}}\right)}$$

neglecting C_{DB1} in C_1

f_{T1}

$$f_p = \frac{f_{T1}}{\left(1 + \frac{C_{gs2}}{C_{gs1}}\right)}$$

$$C_{gs2} \cong \frac{2}{3} C_{OX} W_2 L_2$$

$$C_{gs1} \cong \frac{2}{3} C_{OX} W_1 L_1$$

$$f_p \cong \frac{f_{T1}}{\left(1 + \frac{W_2 L_2}{W_1 L_1}\right)}$$

Upper band limit (f_H) of a current mirror

$$f_H = f_p = \frac{f_{T1}}{\left(1 + \frac{W_2 L_2}{W_1 L_1}\right)}$$

In strong inversion

$$f_T = \frac{3}{4\pi}\mu\frac{1}{L_1^2}(V_{GS} - V_t)$$

Fast current mirrors:

- Short channel length
- Large overdrive voltage (V_{GS}-V_t)

Precision current mirror:

$$L_1 = L_2 \Rightarrow \frac{W_2 L_2}{W_1 L_1} = \frac{W_2}{W_1} = \frac{W_2 / L_2}{W_1 / L_1} = k_M$$

$$f_p \cong \frac{f_{T1}}{(1 + k_M)}$$

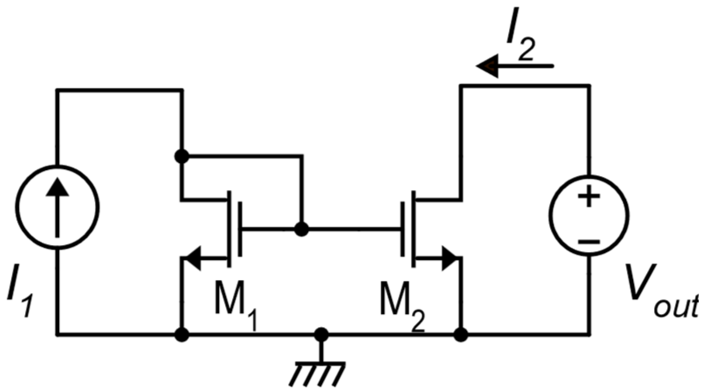
Summary

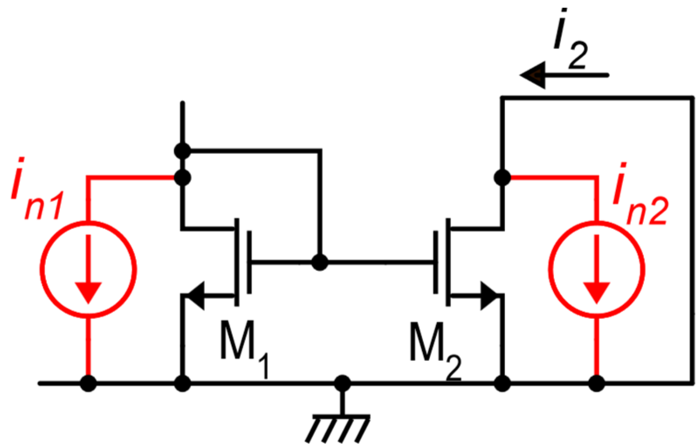
1. The frequency response of a current mirror is marked by a **pole** (angular frequency ω_p) and a **zero** (angular frequency ω_z).
2. The zero (s_z) is positive, then it gives a phase contribution similar to that of the pole. In total, the pole and zero give an asymptotic phase variation of 180°.
3. Generally, the pole falls at lower frequencies than the zero, thus the upper band limit is given by the pole.
4. The frequency of the zero is smaller than the pole frequency only for mirrors designed to have a current gain << 1.
5. In all other cases, which include unity gain current mirrors, the upper band limit is given by:

$$f_H = f_p = \frac{f_{T1}}{\left(1 + \frac{W_2 L_2}{W_1 L_1}\right)}$$

Noise in current mirrors

Simple MOSFET current mirror





$$i_{n-out} = i_2 = i_{n2} - A_I i_{n1}$$

$$S_{Iout}(f) = S_{In2}(f) + \left|A_I(f)\right|^2 S_{In1}(f)$$

For $f << f_H \quad A_I \cong A_I(0)$

$$S_{Iout}(f) = S_{In2}(f) + \left|A_I(0)\right|^2 S_{In1}(f)$$

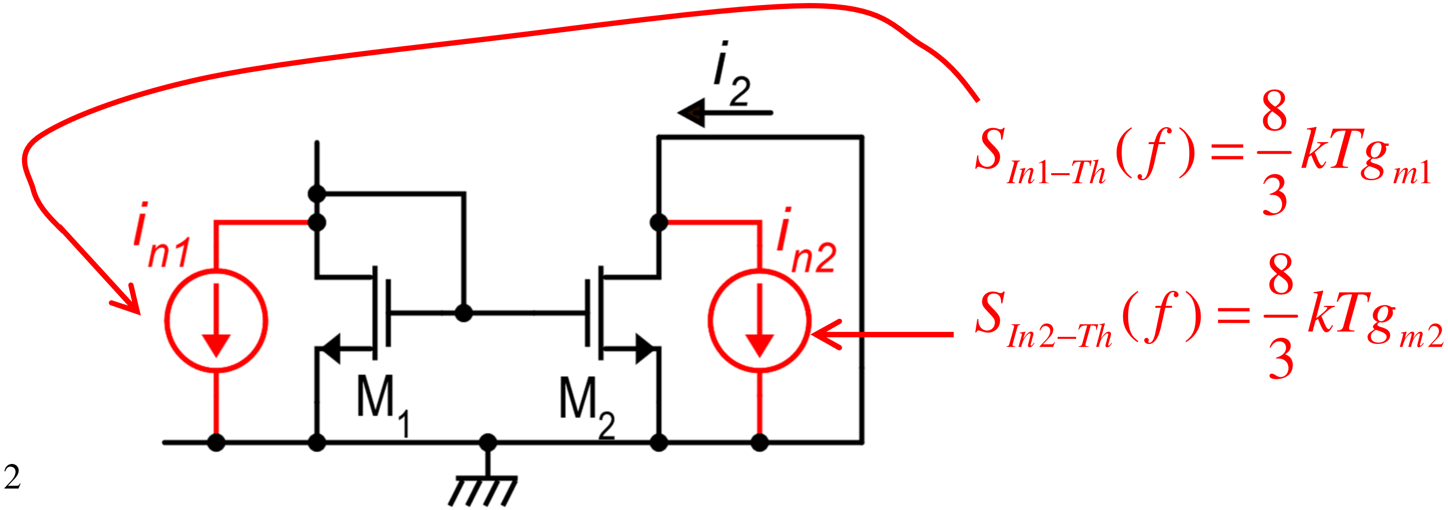
$$S_{Iout}(f) = S_{In2}(f) + \left(\frac{g_{m2}}{g_{m1}}\right)^2 S_{In1}(f)$$

Thermal noise

Let us assume:

$$S_{In-Th}(f)=\frac{8}{3}kTg_m$$

$$\frac{g_{m2}}{g_{m1}}=A_I(0)=k_M$$



$$S_{Iout}(f)=S_{In2}(f)+\left(\frac{g_{m2}}{g_{m1}}\right)^2 S_{In1}(f)$$

$$S_{Iout-th}(f)=\frac{8}{3}kTg_{m2}+\frac{8}{3}kTg_{m1}\frac{g_{m2}^2}{g_{m1}^2}$$

k_M

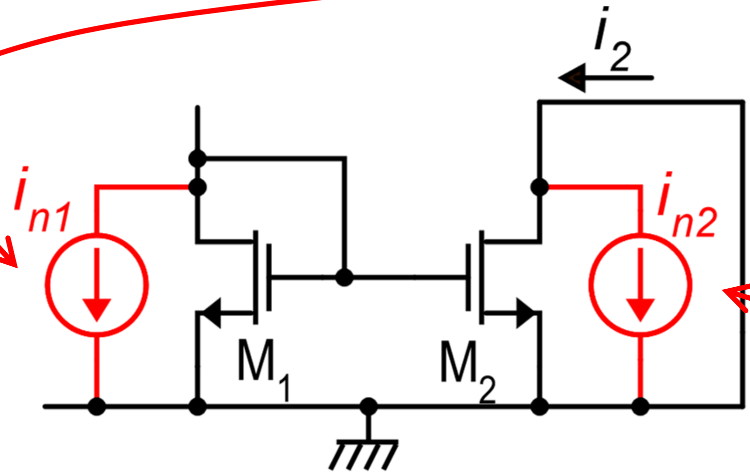
$$S_{Iout-th}(f)=\frac{8}{3}kTg_{m2}+\frac{8}{3}kTg_{m2}\frac{g_{m2}}{g_{m1}}$$

$$S_{Iout-th}(f)=\frac{8}{3}kTg_{m2}(1+k_M)$$

Flicker Noise

Let us assume:

$$S_{In-F}(f) = g_m^2 \frac{N_f}{WL}\frac{1}{f}$$



$$S_{In1-F}(f) = g_{m1}^2 \frac{N_f}{W_1 L_1}\frac{1}{f}$$

$$S_{In1-F}(f) = g_{m2}^2 \frac{N_f}{W_2 L_2}\frac{1}{f}$$

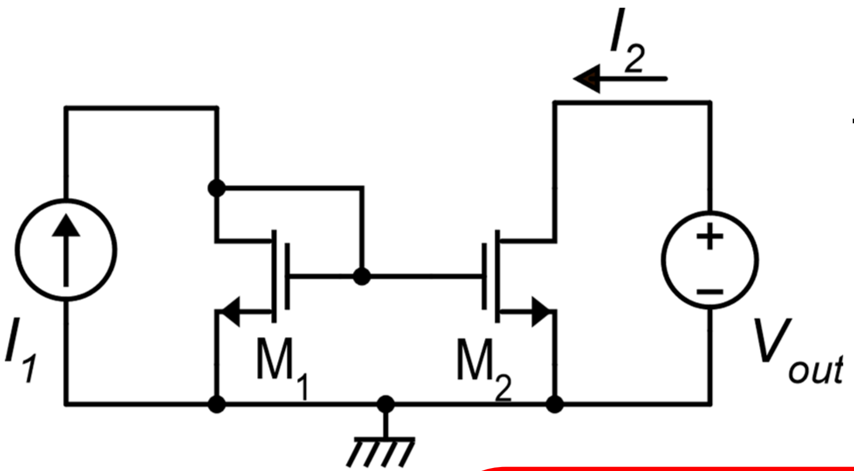
$$S_{Iout}(f) = S_{In2}(f) + \left(\frac{g_{m2}}{g_{m1}}\right)^2 S_{In1}(f)$$

$$S_{Iout-F}(f) = g_{m2}^2 \frac{N_f}{W_2 L_2}\frac{1}{f} + \frac{g_{m2}^2}{g_{m1}^2}\left(g_{m1}^2 \frac{N_f}{W_1 L_1}\frac{1}{f}\right)$$

$$S_{Iout-F}(f) = g_{m2}^2 \frac{N_f}{W_2 L_2}\frac{1}{f} + g_{m2}^2 \frac{N_f}{W_1 L_1}\frac{1}{f}$$

$$S_{Iout-F}(f) = g_{m2}^2 \frac{N_f}{W_2 L_2}\left(1 + \frac{W_2 L_2}{W_1 L_1}\right)\frac{1}{f}$$

Parameters that affect the output noise



Thermal noise: $$S_{Iout-th}(f) = \frac{8}{3} kT g_{m2} (1 + k_M)$$

Flicker noise: $$S_{In-F}(f) = g_{m2}^2 \frac{N_f}{W_2 L_2} \left(1 + \frac{W_2 L_2}{W_1 L_1}\right) \frac{1}{f}$$

Using: $$g_m = \frac{I_D}{V_{TE}}$$

$$\begin{cases} S_{Iout-th}(f) = \frac{8}{3} kT \frac{I_{D2}}{V_{TE2}} (1 + k_M) \\ S_{In-F}(f) = \left(\frac{I_{D2}}{V_{TE2}}\right)^2 \frac{N_f}{W_2 L_2} \left(1 + \frac{W_2 L_2}{W_1 L_1}\right) \frac{1}{f} \end{cases}$$

- The higher the current I_{D2}, the higher the output current PSD
- High values of V_{TE} reduce noise

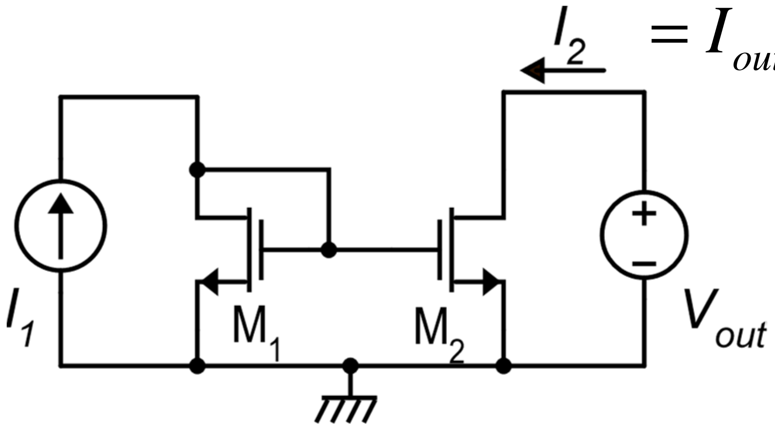
Dynamic range of a current mirror

$$I_1 = I_{1Q} + i_1(t)$$

dc bias → I_{1Q}

signal → $i_1(t)$

$$I_2 = I_{out} = I_{out-Q} + i_{out}(t)$$



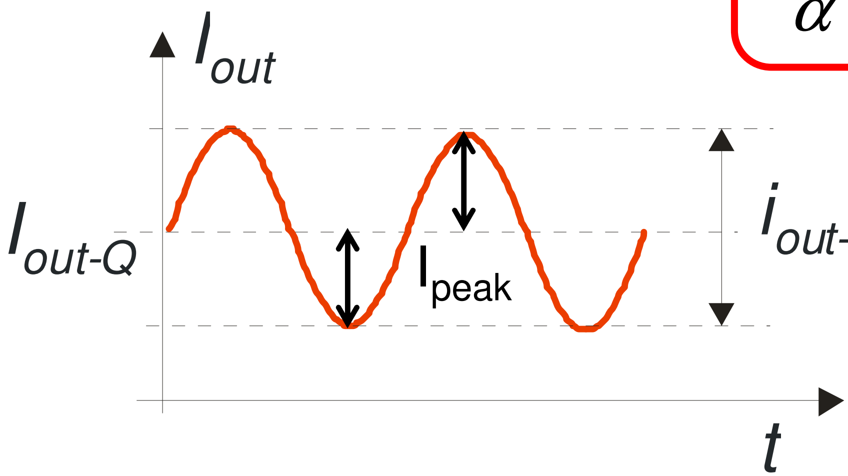
$$DR = \frac{i_{out-FS}}{i_{out-n}}$$

$$i_{out-FS} = \alpha I_{out-Q}$$

$$\alpha < 2$$

For simplicity, let us consider a sinusoidal stimulus

α=2 only if we allow the minimum value of I_{out} to be zero -> distortion



$$i_{out\text{-}FS} = 2I_{peak} < 2I_{out-Q}$$

DR of a current mirror

$$DR = \frac{i_{out-FS}}{i_{out-n}}$$

$$i_{out-FS} = \alpha I_{out-Q} \qquad (\alpha < 2)$$

$$i_{np-p} = 4 i_{n-rms} = 4\sqrt{\int_{f_L}^{f_H} S_{Iout}(f)\,df}$$

$$(DR)^2 = \frac{\alpha^2 I_{out-Q}^2}{16\int_{f_L}^{f_H} S_{Iout}(f)\,df}$$

$$\frac{\alpha^2 I_{out-Q}^2}{16 S_{IBB} B_S} \qquad \text{Thermal}$$

$$\frac{\alpha^2 I_{out-Q}^2}{16 k_F \ln\left(\frac{f_H}{f_L}\right)} \qquad \text{Flicker}$$

DR of a current mirror

$$S_{Iout-th}(f) = \boxed{\frac{8}{3}kT\frac{I_{D2}}{V_{TE2}}(1+k_M)}$$

S_{IBB}

$$S_{In-F}(f) = \boxed{\left(\frac{I_{D2}}{V_{TE2}}\right)^2 \frac{N_f}{W_2L_2}\left(1+\frac{W_2L_2}{W_1L_1}\right)} \cdot \frac{1}{f}$$

$I_{D2} = I_{out-Q}$

k_f

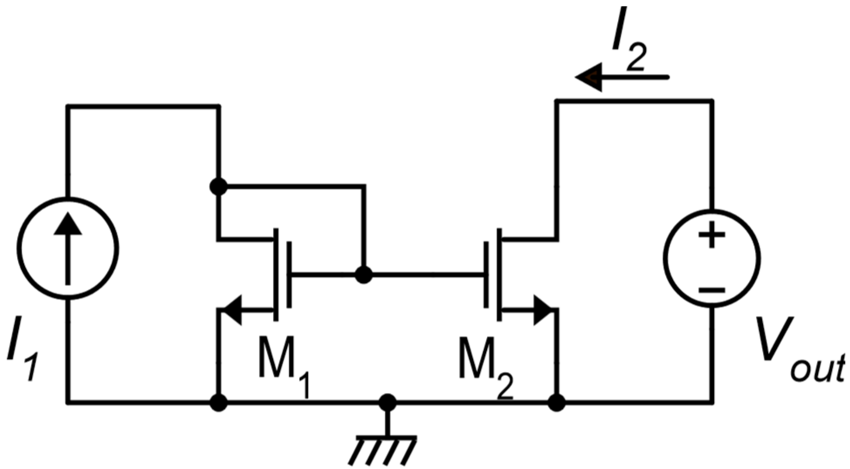
Thermal:

$$(DR)^2 = \frac{\alpha^2 I_{out-Q}^2}{16 \cdot \frac{8}{3}kT\frac{I_{out-Q}}{V_{TE2}}(1+k_M)B_S} = \frac{3}{128}\frac{\alpha^2 V_{TE2} I_{out-Q}}{kT \cdot B_S(1+k_M)}$$

Flicker:

$$(DR)^2 = \frac{\alpha^2 I_{out-Q}^2}{16\left(\frac{I_{out-Q}}{V_{TE2}}\right)^2 \frac{N_f}{W_2L_2}\left(1+\frac{W_2L_2}{W_1L_1}\right)\ln\left(\frac{f_H}{f_L}\right)} = \frac{\alpha^2 V_{TE2}^2 W_2 L_2}{16 N_f\left(1+\frac{W_2L_2}{W_1L_1}\right)\ln(f_H / f_L)}$$

Examples



$$I_{out-Q} = 1\ \mu\text{A}$$

$$V_{TE2} = 100\ \text{mV}$$

$$k_M = 1,\ \alpha = 1.5$$

$$B_S = 1\text{kHz}$$

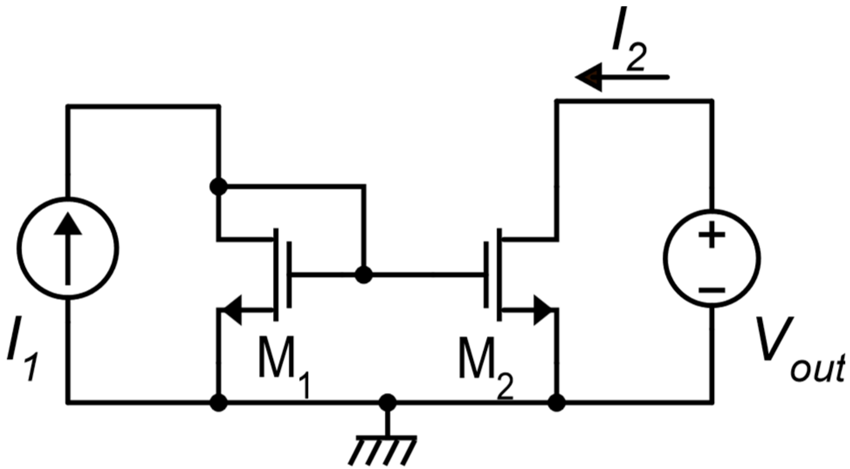
Thermal

$$DR = \sqrt{\frac{3}{128}\frac{\alpha^2 V_{TE2} I_{out-Q}}{kT \cdot B_S\left(1+k_M\right)}}$$

$$DR \cong 25.7\times 10^3$$

$$(88\ \text{dB},\ \approx 14\ \text{bit})$$

Examples



Flicker

$$DR = \sqrt{\frac{\alpha^2 V_{TE2}^2 W_2 L_2}{16 N_f \left(1+\frac{W_2 L_2}{W_1 L_1}\right)\ln(f_H / f_L)}}$$

$$I_{out-Q} = 1\ \mu\text{A}$$

$$V_{TE2} = 100\ \text{mV}$$

$$k_M = 1,\ \alpha = 1.5$$

$$B_S = 1\text{kHz}$$

$$WL = 1\ \mu\text{m}^2$$

$$N_f = N_{fn} = 6\times 10^{-10}\ V^2 \mu\text{m}^2$$

$$f_L = 0.01\ \text{Hz}$$

$$DR \cong 319$$

$$(50\ \text{dB},\ \approx 8\ \text{bit})$$

Flicker noise dominates
This is the total DR