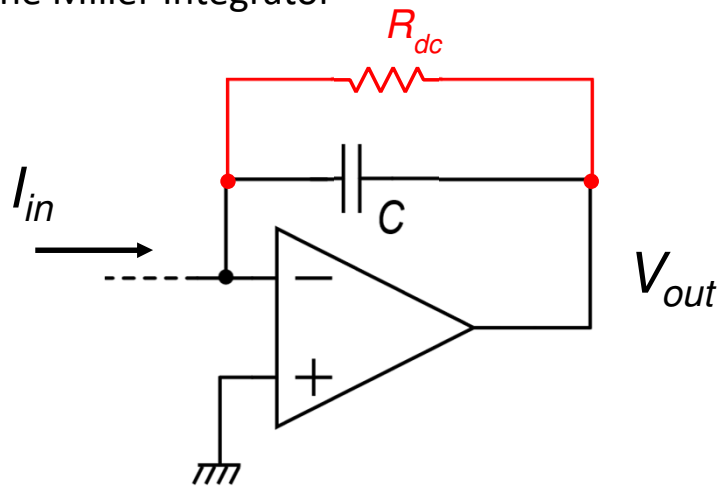


# An interface for capacitive sensors based on the SC charge amplifier

## Charge amplifier

Invented by Walter Kistler in 1950  
It is based on the Miller Integrator



$$V_{out}(t_2) = -\frac{1}{C} \int_{t_1}^{t_2} I_{in} dt + V_{out}(t_1)$$

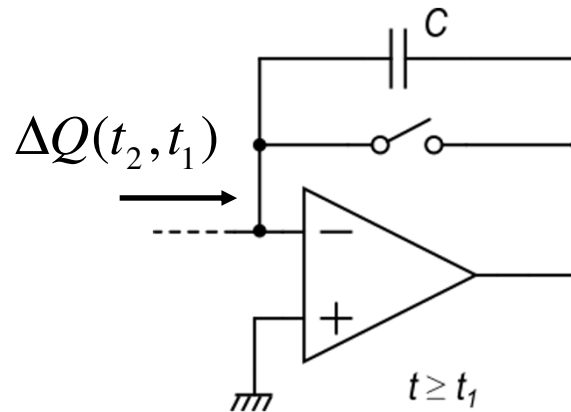
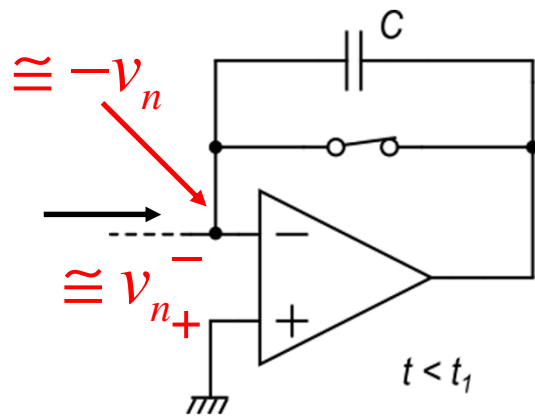
Charge fed to the amplifier from  $t_1$  to  $t_2$   
 $= \Delta Q(t_2, t_1)$

This value is an uncertainty and may also cause saturation of the amplifier

The classical solution consists in placing a resistor of very high resistance ( $R_{dc}$ ) in parallel with the capacitor

Unfortunately, this resistor also discharge the charge accumulated across  $C$ , imposing condition:  $t_2 - t_1 \ll R_{dc} C$

# A Switched Capacitor (SC) charge amplifier



Note:  $v_n(t)$  includes the offset voltage, which can be quite large in CMOS circuits

$$V_{out} = 0$$

$$V_{out}(t_2) = -\frac{\Delta Q(t_2, t_1)}{C}$$

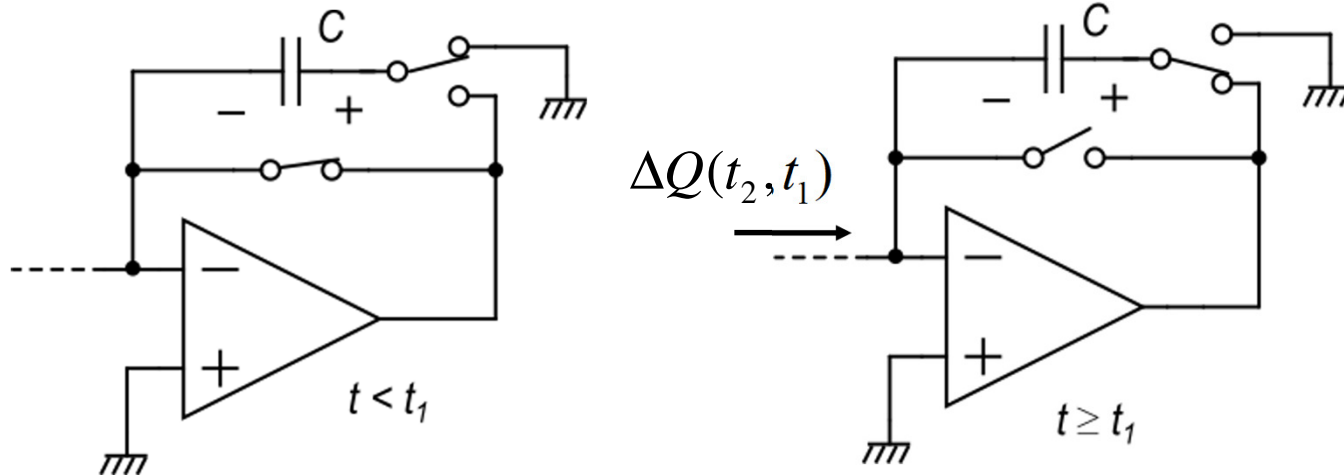
Ideal case

$$V_{out} \cong -v_n(t)$$

$$V_{out}(t_2) = \frac{-\Delta Q(t_2, t_1)}{C} - v_n(t)$$

Real case

## SC charge amplifier with offset cancellation



Two noise samples are subtracted:  
This is the correlated **double** sampling (CDS)

$$V_{out} \cong -v_n(t)$$

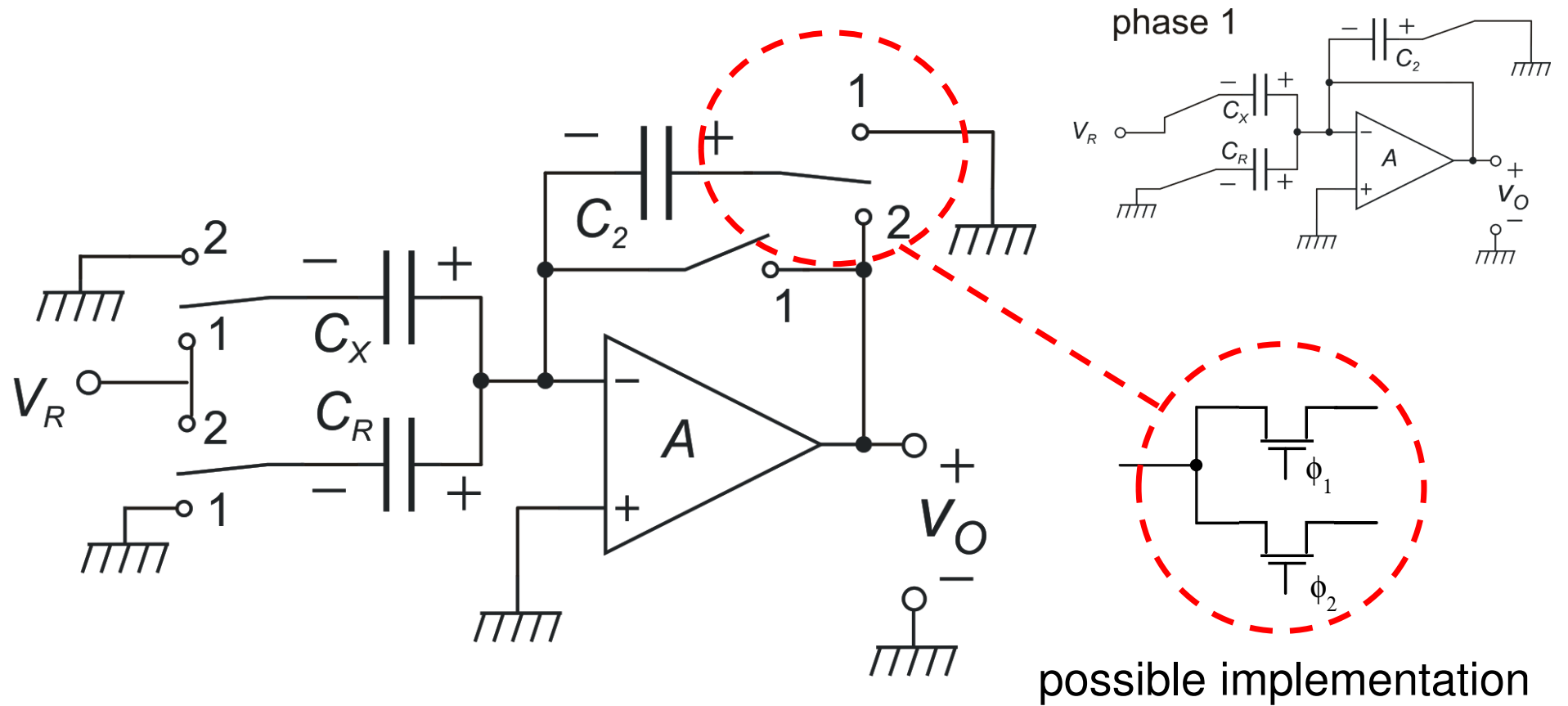
$$V_c \cong 0 - (-v_n(t)) = v_n(t)$$

The feedback capacitor is pre-charged with  $v_n(t)$

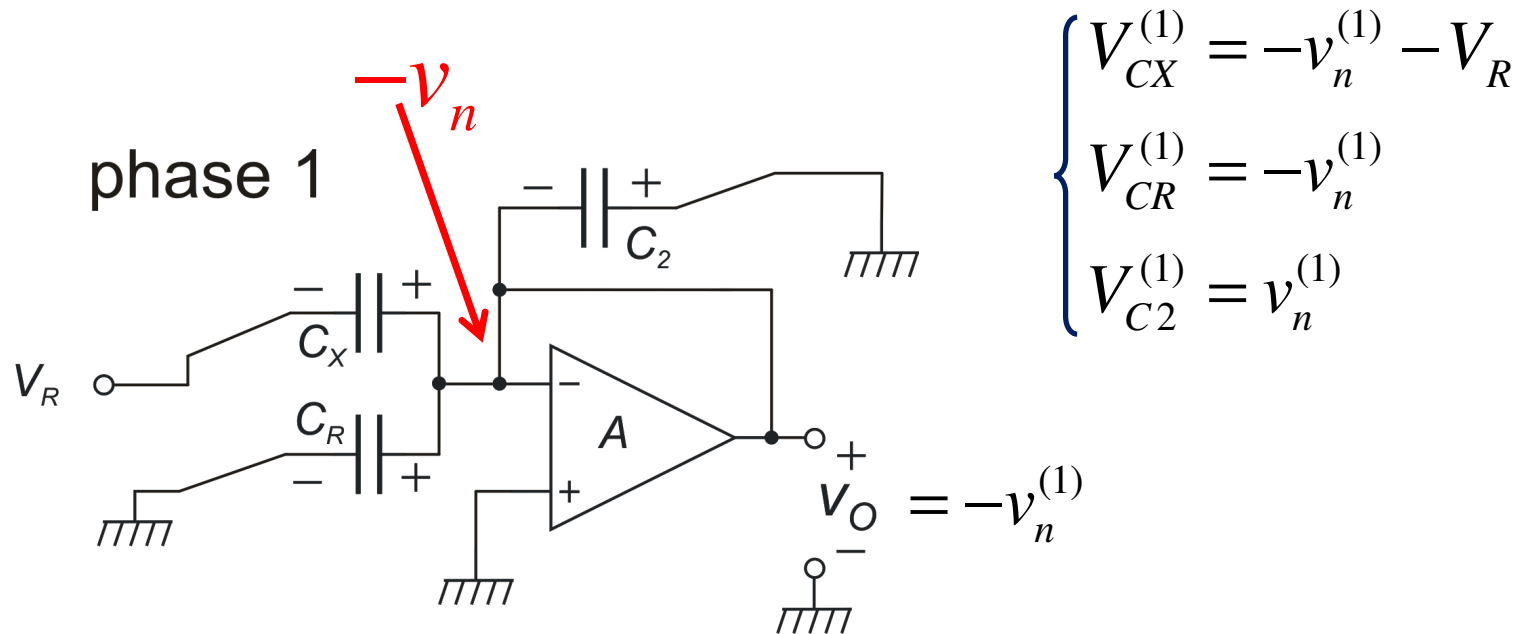
$$V_{out}(t_2) \cong -v_n(t_2) + V_c(t_2) = \underbrace{-v_n(t_2) + v_n(t_1)}_{\text{cancelled}} - \frac{\Delta Q(t_2, t_1)}{C}$$

All  $v_n$  components (offset, low freq. noise) that do not change across the  $t_1, t_2$  interval are cancelled

# SC interface for capacitive sensors



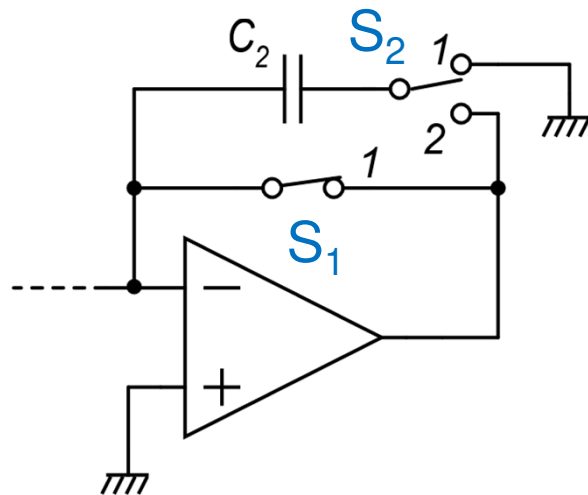
## Phase 1: Capacitor voltages



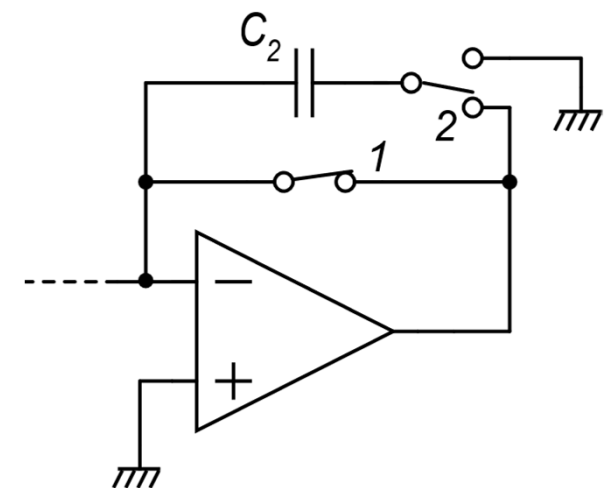
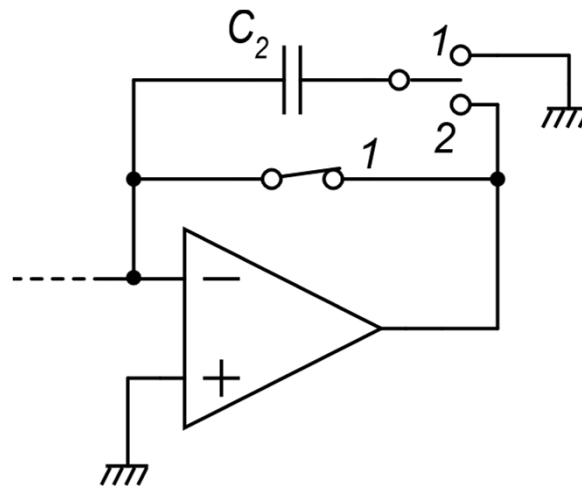
## Transition to phase 2

In SC circuits, timing of the switches is critical, due to the risk to alter the charge stored into the capacitors

If  $S_1$  is still closed in the previous position (phase 1), when  $S_2$  closes to phase 2 position,  $C_2$  is short circuited



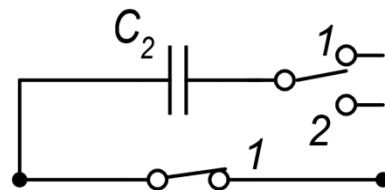
$S_2$  opens and  $C_2$  samples  $v_n^{(1)}$



## The intermediate phase "i"

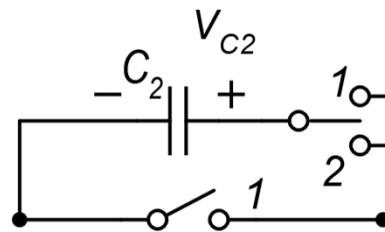
In order to avoid the occurrence of unwanted temporary short-circuits the transition from phase 1 to phase 2 occurs according the following steps:

Switches are closed in position 1



phase 1

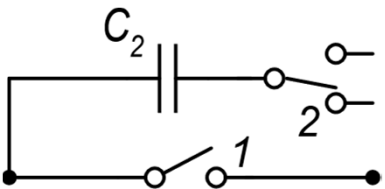
All switches opens (capacitors sample their voltages)



phase "i"

$$V_{C2}^{(i)} = v_n^{(1)} + v_\epsilon$$

Switches close in position 2

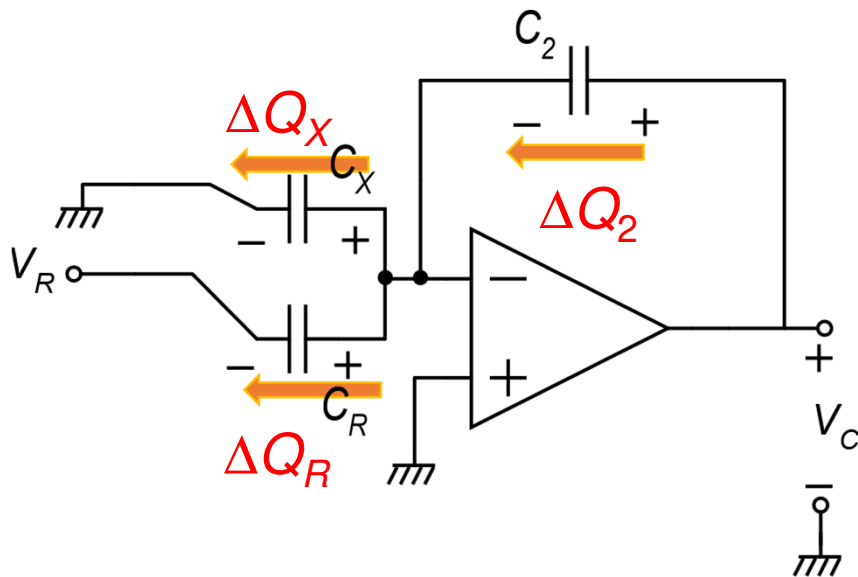
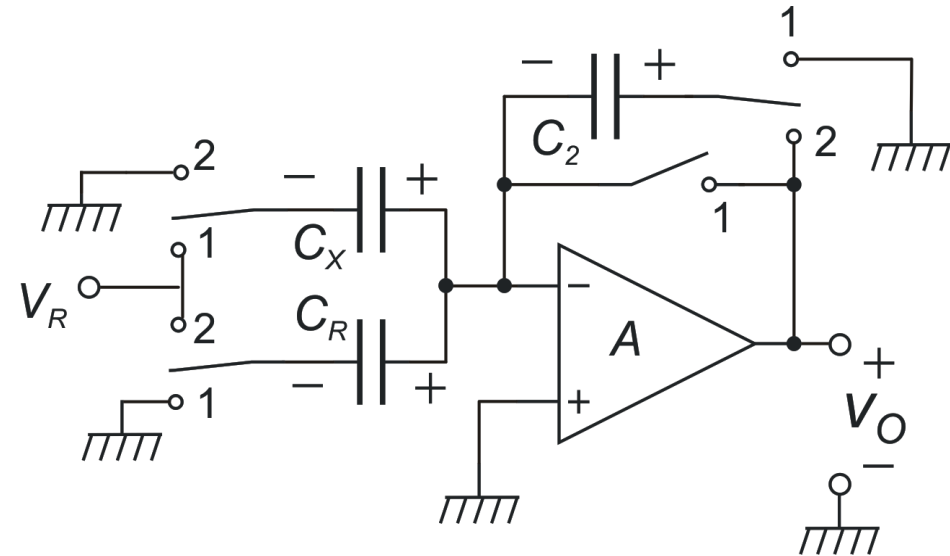


phase 2

kT/C noise

## Phase 1 to phase 2 transition

$$\begin{cases} V_{CX}^{(i)} = -v_n^{(1)} - V_R + v_{\epsilon X} \\ V_{CR}^{(i)} = -v_n^{(1)} + v_{\epsilon R} \\ V_{C2}^{(i)} = v_n^{(1)} + v_{\epsilon 2} \end{cases}$$



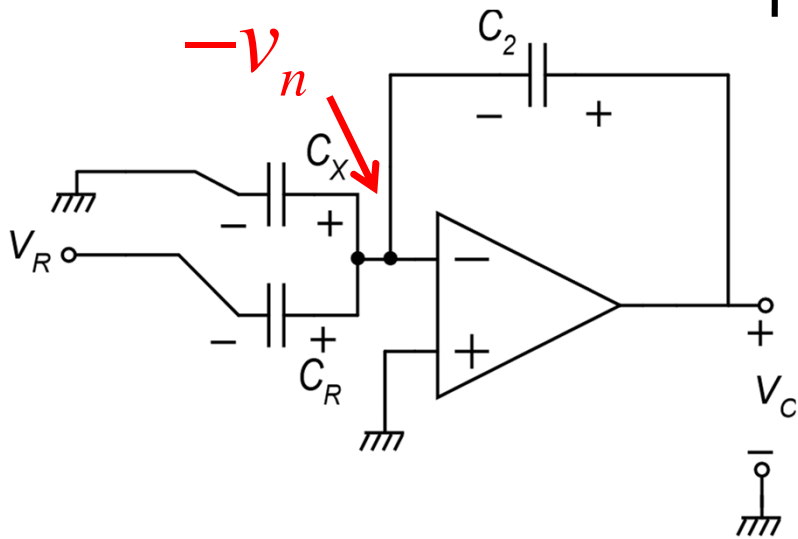
$$\Delta Q_X = C_X (V_{CX}^{(2)} - V_{CX}^{(i)})$$

$$\Delta Q_R = C_R (V_{CR}^{(2)} - V_{CR}^{(i)})$$

$$\Delta Q_2 = \Delta Q_X + \Delta Q_R$$



## Phase 2 voltages



$$V_{CX}^{(2)} = -v_n^{(2)}$$

$$V_{CR}^{(2)} = -v_n^{(2)} - V_R$$

$$V_{C2}^{(2)} = V_{C2}^{(i)} + \frac{\Delta Q_2}{C_2}$$

$$V_o^{(2)} = -v_n^{(2)} + V_{C2}^{(2)} \quad \leftarrow$$

$$\Delta Q_2 = \Delta Q_X + \Delta Q_R = C_X (V_{CX}^{(2)} - V_{CX}^{(i)}) + C_R (V_{CR}^{(2)} - V_{CR}^{(i)})$$

$$V_o^{(2)} = -v_n^{(2)} + V_{C2}^{(i)} + \frac{1}{C_2} \left[ C_X (V_{CX}^{(2)} - V_{CX}^{(i)}) + C_R (V_{CR}^{(2)} - V_{CR}^{(i)}) \right]$$

## Phase 2 output voltage

$$V_o^{(2)} = -v_n^{(2)} + V_{C2}^{(i)} + \frac{1}{C_2} \left[ C_X (V_{CX}^{(2)} - V_{CX}^{(i)}) + C_R (V_{CR}^{(2)} - V_{CR}^{(i)}) \right]$$

$$\left\{ \begin{array}{l} V_{CX}^{(1)} = -v_n^{(1)} - V_R \\ V_{CR}^{(1)} = -v_n^{(1)} \\ V_{C2}^{(1)} = v_n^{(1)} \end{array} \right. \quad \left\{ \begin{array}{l} V_{CX}^{(i)} = -v_n^{(1)} - V_R + v_{\varepsilon X} \\ V_{CR}^{(i)} = -v_n^{(1)} + v_{\varepsilon R} \\ V_{C2}^{(i)} = v_n^{(1)} + v_{\varepsilon 2} \end{array} \right. \quad \begin{array}{l} V_{CX}^{(2)} = -v_n^{(2)} \\ V_{CR}^{(2)} = -v_n^{(2)} - V_R \end{array}$$

$$V_o^{(2)} = -v_n^{(2)} + v_n^{(1)} + v_{\varepsilon 2} + \frac{1}{C_2} \left[ C_X (-v_n^{(2)} + v_n^{(1)} + V_R - v_{\varepsilon X}) + C_R (-v_n^{(2)} - V_R + v_n^{(1)} - v_{\varepsilon R}) \right]$$

$$V_o^{(2)} = \frac{C_X - C_R}{C_2} V_R + (-v_n^{(2)} + v_n^{(1)}) \left( 1 + \frac{C_X}{C_2} + \frac{C_R}{C_2} \right) + v_{\varepsilon 2} - \frac{C_X}{C_2} v_{\varepsilon X} - \frac{C_R}{C_2} v_{\varepsilon R}$$

## Output voltage components

$$\Delta C = C_X - C_R$$

$$V_o^{(2)} = \frac{\Delta C}{C_2} V_R + \underbrace{\left( -v_n^{(2)} + v_n^{(1)} \right) \left( 1 + \frac{C_X}{C_2} + \frac{C_R}{C_2} \right)}_{\text{Amplifier noise (CDS)}} + \underbrace{v_{\epsilon 2} - \frac{C_X}{C_2} v_{\epsilon X} - \frac{C_R}{C_2} v_{\epsilon R}}_{kT/C \text{ noise}}$$

useful signal  
sensitivity:  $\frac{V_R}{C_2}$

Referred to  
the input diff.  
capacitance

$$\Delta C_n^{(2)} = \frac{\left( -v_n^{(2)} + v_n^{(1)} \right) (C_2 + C_X + C_R)}{V_R} + \frac{C_2 v_{\epsilon 2} - C_X v_{\epsilon X} - C_R v_{\epsilon R}}{V_R}$$

Example: DR of the SC interface considering only kT/C noise

$$\Delta C_n = \frac{C_2 v_{\epsilon 2} - C_X v_{\epsilon X} - C_R v_{\epsilon R}}{V_R} \quad DR = \frac{\Delta C_{FS}}{\Delta C_{n-pp}} = \frac{\Delta C_{FS}}{4\Delta C_{rms}} \quad \Delta C_{rms} = \sqrt{\langle \Delta C_n^2 \rangle}$$

$$\langle (\Delta C_n)^2 \rangle = \frac{1}{V_R^2} \left( C_2^2 \langle (v_{\epsilon 2})^2 \rangle + C_X^2 \langle (v_{\epsilon X})^2 \rangle + C_R^2 \langle (v_{\epsilon R})^2 \rangle \right)$$

$$\langle (\Delta C_n)^2 \rangle = \frac{1}{V_R^2} \left( C_2^2 \frac{kT}{C_2} + C_X^2 \frac{kT}{C_X} + C_R^2 \frac{kT}{C_R} \right) = \frac{kT}{V_R^2} (C_2 + C_X + C_R)$$

$$DR = \frac{\Delta C_{FS}}{4 \sqrt{\frac{kT}{V_R^2} (C_2 + C_X + C_R)}}$$

Example: DR of the SC interface considering only kT/C noise

$$DR = \frac{\Delta C_{FS}}{4 \sqrt{\frac{kT}{V_R^2} (C_2 + C_X + C_R)}}$$

$$= \frac{1}{4 \sqrt{\frac{kT}{\Delta C_{FS} V_R^2} \frac{(C_2 + C_X + C_R)}{\Delta C_{FS}}}}$$

$$DR = \frac{1}{\frac{4}{\Delta C_{FS}} \sqrt{\frac{kT}{V_R^2} (C_2 + C_X + C_R)}}$$

$$= \frac{1}{4} \sqrt{\frac{1}{\frac{kT}{\Delta C_{FS} V_R^2} \frac{(C_2 + C_X + C_R)}{\Delta C_{FS}}}}$$

$$DR = \frac{V_R}{4 \sqrt{kT / \Delta C_{FS}}} \sqrt{\frac{\Delta C_{FS}}{(C_2 + C_X + C_R)}}$$

rms value of the kT/C noise associated to  $\Delta C_{FS}$